$K_L \to \mu e$ in $SU(2)_L \times U(1)$ and $SU(2)_L \times SU(2)_R \times U(1)$ models with large neutrino masses

Paul Langacker* and S. Uma Sankar*
Deutsches Electronen-Synchrotron DESY, D-2000 Hamburg 52, Federal Republic of Germany

K. Schilcher
Institut für Physik, Johannes Gutenberg Universität, D-6500 Mainz, Federal Republic of Germany
(Received 27 June 1988)

The expectations for $B(K_L \to \mu e)$ generated by massive neutrinos are studied in detail for the $SU(2)_L \times U(1)$ and $SU(2)_L \times SU(2)_R \times U(1)$ models. The effects of Dirac and Majorana masses for the ordinary as well as for fourth-family and $SU(2)_L$-singlet (right-handed) neutrinos are considered, taking into account all existing laboratory and cosmological constraints on the neutrino masses and mixings, as well as plausible (i.e., not requiring extreme fine-tuning) expectations for mixing between light and heavy neutrinos. We consider general $SU(2)_L \times SU(2)_R \times U(1)$ models, in which the right-handed quark mixing matrix U^R is arbitrary, as well as the special case of manifest or pseudomanifest left-right symmetry ($U^R = U^L$ or U^{L*}), and incorporate existing limits on the mass and mixing of the W_R and on U^R. It is found that all plausible versions of the $SU(2)_L \times U(1)$ model lead to immeasurably small branching ratios. Branching ratios as large as 2×10^{-13} are allowed in the $SU(2)_L \times SU(2)_R \times U(1)$ model, but only for tiny corners of parameter space. For most parameter values (including manifest and pseudomanifest left-right symmetry) one has $B(K_L \to \mu e) \leq 10^{-15}$.

I. INTRODUCTION

Lepton-flavor violation, absolutely forbidden in the standard model, occurs almost inevitably in possible extensions invented to overcome some of the shortcomings of that model. The decay $K_L \to \mu e$ is, in this respect, one of the most interesting reactions because it involves both quark- and lepton-flavor changes. The experimental limit has recently been improved significantly to $B(K_L \to \mu e) < 6.7 \times 10^{-9}$ (Ref. 1). [This limit should be compared to the observed branching ratio $B(K_L \to \mu \bar{\mu}) = (9.1 \pm 1.9) \times 10^{-9}$ (Ref. 2).] The experimental bound on $K_L \to \mu e$ can be expected to be improved to the level of 10^{-10} to 2×10^{-13} (Ref. 3) when current experiments at BNL and KEK are completed. Experiments at the projected kaon factories would probably allow one to increase the sensitivity by an additional 2 orders of magnitude.

On the theoretical side one can distinguish two mechanisms by which the decay $K_L \to \mu e$ can occur in extensions of the standard model.

(1) New lepton-flavor-violating interactions: $K_L \to \mu e$ has been studied in models based on technicolor,[4] supersymmetry,[5] compositeness,[6] horizontal symmetry,[7] extended scalar sectors,[8] exotic leptoquarks,[9] and superstrings.[10]

(2) Nonvanishing and nondegenerate neutrino masses within the standard model or minimal extensions of it, such as the $SU(2)_L \times SU(2)_R \times U(1)$ model.

In Ref. 11 the effect of Dirac neutrino masses in such models was studied. Since there are stringent upper limits on the masses of the three observed neutrinos, the $K_L \to \mu e$ rate turns out to be unmeasurably small unless a fourth family with a very heavy neutrino with significant mixings with the ν_e and ν_μ is postulated. Such a model is highly unnatural and can only be obtained by an extreme fine-tuning of the Yukawa couplings. On the other hand, if large Majorana masses are introduced for the $SU(2)_L$-singlet (right-handed) neutrinos then the very small masses of the ordinary $SU(2)_L$-doublet (left-handed) neutrinos arise naturally via the seesaw mechanisms.[12] We therefore chose to investigate the decay $K_L \to \mu e$ in such a general scheme. Heavy singlet neutrinos in the $SU(2)_L \times U(1)$ model only interact via small light-heavy neutrino mixings. We therefore also considered the extension to the $SU(2)_L \times SU(2)_R \times U(1)$ gauge group, in which the heavy neutrinos have full-strength interactions. We consider general $SU(2)_L \times SU(2)_R \times U(1)$ models and do not restrict ourselves to the manifest or the pseudomanifest left-right (L-R) symmetry [i.e., the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrices of the left- and right-handed fermions are allowed to be different].

Of course one cannot consider limits on $K_L \to \mu e$ in isolation. From studying the K_L-K_S mass difference and bottom-quark decay[13–15] one obtains restrictions on the mass M_R of the right-handed gauge boson and on the right-handed quark CKM mixing matrix. From $\mu \to e\gamma$, limits on the right-handed leptonic mixing matrix are obtained. In addition, there exist many experimental constraints on neutrino masses and mixings. We find that in the case of the $SU(2)_L \times U(1)$ model, even the introduction of heavy Majorana mass terms still leads to an unobservable $K_L \to \mu e$ rate. In the $SU(2)_L \times SU(2)_R \times U(1)$ model the W_L-W_R exchange diagram can lead to a possibly observable branching ratio (up to 2×10^{-13}) without violating the existing constraints provided by other reasons. However, this happens only in small corners of parameter space which do not correspond to manifest or pseudomanifest left-right symmetry. For most other values of the parameters the branching ratio associated with gauge interactions is unobservably small.

The paper is arranged as follows. In Sec. II we introduce the formalism and existing constraints on neutrino masses and lepton flavor mixing and specify the charged-current interactions. In Sec. III we calculate the decay rate $\Gamma(K_L \to \mu e)$ in the $SU(2)_L \times U(1)$ model. In Sec. IV the same calculation is done for the general $SU(2)_L \times SU(2)_R \times U(1)$ model. An analysis of the predicted branching ratios is presented, which incorporates all of the relevant experimental constraints. The effects of flavor-changing neutral-Higgs-boson exchange are also briefly discussed. We conclude with a summary of our results in Sec. V.

II. FORMALISM

The decay $K_L \to \mu e$ is absolutely forbidden in the standard model with massless neutrinos because the individual lepton flavor numbers L_e, L_μ, and L_τ are conserved. However, the decay can proceed if the standard model is extended to include Dirac or Majorana neutrino masses (which generally lead to generational mixing in the lepton sector) and/or new interactions.

The existing limits on neutrino masses are extremely stringent.[16] The direct laboratory limits are

$$m_{\nu_e} < 18 \text{ eV} \quad (\text{Ref. } 17) \, ,$$
$$m_{\nu_\mu} < 250 \text{ keV} \quad (\text{Ref. } 18) \, , \tag{1}$$
$$m_{\nu_\tau} < 35 \text{ MeV} \quad (\text{Ref. } 19) \, ,$$

assuming that the generational mixing effects are small enough so that the weak and mass eigenstates are almost identical. In addition, if the ν_e is Majorana, then from limits on neutrinoless double-beta decay[16,20] one has[21]

$$m_{\nu_e} < 1 \text{ eV} \, . \tag{2}$$

Finally, the limits on the energy density of the present Universe imply[16]

$$\sum_i m_{\nu_i} < 40 \text{ eV} \, , \tag{3}$$

where the sum runs over the light, stable neutrinos. A variety of astrophysical constraints on the unstable neutrinos decaying via normal weak processes (e.g., $\nu_2 \to \nu_1 \gamma, \nu_1 e^+ e^-$), combined with laboratory limits imply[16]

$$m_{\nu_\mu} < 40 \text{ eV}, \quad m_{\nu_\tau} < 40 \text{ eV} \, . \tag{4}$$

The only way to evade the limits in Eqs. (3) and (4) is to invent new physics to allow fast invisible decays or annihilations for the heavy neutrinos.[22]

There are also stringent limits on ν_e-ν_μ-ν_τ mixing from neutrino oscillation experiments. For masses in the range given in Eq. (4), the limits on the mixing angles are[16]

$$\Theta_{e\mu} < 0.029, \quad \Theta_{e\tau} < 0.2, \quad \Theta_{\mu\tau} < 0.032 \, . \tag{5}$$

We will see in Sec. III that the $K_L \to \mu e$ branching ratio in the $SU(2)_L \times U(1)$ model due to the exchange of virtual ν_e, ν_μ, and ν_τ in the box diagrams is immeasurably small ($< 10^{-24}$), because of the constraints in Eqs. (1)–(5). Hence, we are led to consider models in which there are additional very heavy neutrinos. However, heavy $SU(2)_L$-singlet neutrinos added to the standard model have no gauge interactions except for very small light-heavy neutrino mixing effects. Similarly, the contribution of a possible fourth-family neutrino would be strongly suppressed by small intergeneration mixing angles. Therefore, in Sec. IV we will consider the possibility of new gauge interactions [associated with the group $SU(2)_L \times SU(2)_R \times U(1)$] with full strength couplings to heavy Majorana neutrinos. In this case it is barely possible to achieve a measurable branching ratio for $K_L \to \mu e$, but only under the most optimistic assumptions for the values of the various parameters.

To illustrate the possible types of neutrino masses, consider first a single-family model. The ordinary left-handed electron neutrino ν_{eL} is related by CP to the right-handed antineutrino ν_{eR}^C, where

$$\nu_{eR}^C = C \bar{\nu}_{eL}^T \, , \tag{6}$$

and C is the charge-conjugate matrix defined by $C \gamma_\mu C^{-1} = -\gamma_\mu^T$. ν_{eL} and ν_{eR}^C transform as doublets under $SU(2)_L$; they are related to e_L^- and e_R^+, respectively.

To generate a conventional Dirac mass for the neutrino, one must add to the theory a new $SU(2)_L$-singlet Weyl neutrino N_R and its CP conjugate $N_L^C = C \bar{N}_R^T$. A Dirac mass term is then of the form

$$-L_{\text{Dirac}} = m_D \bar{\nu}_L N_R + \text{H.c.}, \tag{7}$$

where m_D is generated by the ordinary Higgs doublet. Similarly, one can have Majorana ($\Delta L = \pm 2$) mass terms

$$-L_M = \frac{m_T}{2} \bar{\nu}_L \nu_R^C + \frac{m_S}{2} \bar{N}_L^C N_R + \text{H.c.} \tag{8}$$

for ν_L and N_R, where m_T and m_S can be generated by new Higgs triplets[23] and by Higgs singlets (or bare masses), respectively.[24]

In general Dirac and Majorana masses can be present simultaneously. One then has a 2×2 mass matrix

$$-L_M = \tfrac{1}{2} (\bar{\nu}_L \;\; \bar{N}_L^C) \begin{bmatrix} m_T & m_D \\ m_D^T & m_S \end{bmatrix} \begin{bmatrix} \nu_R^C \\ N_R \end{bmatrix} + \text{H.c.}$$
$$= \tfrac{1}{2} (\bar{\nu}_L \;\; \bar{N}_L^C) M \begin{bmatrix} \nu_R^C \\ N_R \end{bmatrix} + \text{H.c.}, \tag{9}$$

where $m_D^T = m_D$ for the single-family case. M can be diagonalized by a biunitary transformation

$$\mathcal{U}_L^\dagger M \mathcal{U}_R = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \, ,$$

where $\mathcal{U}_L = \mathcal{U}_R^* K^\dagger$ (K is a diagonal matrix of phases[16]). m_1 and m_2 are the physical masses of the two Majorana mass eigenstates ν_{iL} and ν_{iR}^C, where

$$\begin{bmatrix} \nu_L \\ N_L^C \end{bmatrix} = \mathcal{U}_L \begin{bmatrix} \nu_{1L} \\ \nu_{2L} \end{bmatrix} \, , \tag{10}$$

and similarly for ν_{iR}^C.

The interesting physical case for our purposes is that in which ν_{1L} is mainly the $SU(2)_L$ doublet ν_L and ν_{2L} is mainly the singlet N_L^C, and $m_1 \ll m_2$. Essentially the only way to achieve this is the seesaw model,[12] in which $m_S \gg m_D, m_T$ (Ref. 25). Then one has

$$\begin{aligned} \nu_L &= \nu_{1L}\cos\theta + \nu_{2L}\sin\theta \ , \\ N_L^C &= -\nu_{1L}\sin\theta + \nu_{2L}\cos\theta \ , \end{aligned} \tag{11}$$

where the masses are

$$m_1 = \left| m_T - \frac{m_D^2}{m_S} \right| , \quad m_2 \sim m_S \ , \tag{12}$$

and

$$\theta \simeq \frac{m_D}{m_S} \ll 1 \ . \tag{13}$$

Usually one assumes $m_T = 0$ (no Higgs triplets), and that m_D is comparable to a quark or charged-lepton mass. Then one has a natural explanation of why $m_1 \sim m_D^2/m_S$ $(\ll m_D)$ is much smaller than other fermion masses. Also, in this case

$$\theta \simeq \left[\frac{m_1}{m_2} \right]^{1/2} \simeq \frac{m_D}{m_S} \ll 1 \ . \tag{14}$$

There are a variety of limits on the mixing of ν_e and ν_μ with heavy neutrinos from universality, the lepton spectrum in β, π, and K decays, searches for the heavy neutrino decay products (e.g., $\nu_2 \to \nu_1 e^+ e^-$) in beam dumps, neutrino scattering, and e^+e^- annihilation.[16,26] These limits on θ^2 are comparable to or smaller than the seesaw predictions in Eq. (14) for m_{ν_e} or m_{ν_μ} in the 10-eV range and for m_2 as large as 20 GeV. They are much smaller than the seesaw predictions for $m_{\nu_\mu} \sim 250$ keV (and m_2 up to 20 GeV). In the following we will therefore assume $m_2 \geq 20$ GeV. As an example, for $m_1 \sim m_{\nu_\mu} \sim 250$ keV and a heavy-neutrino mass of 100 GeV, we expect a light-heavy mixing angle $\theta^2 \sim m_1/m_2 \sim 2.5 \times 10^{-6}$.

We note in passing that the mixing between the ν_e, ν_μ, or ν_τ and a very heavy fourth-family neutrino also will be most likely described by a seesaw-type mechanism (although in this case all the terms in the mass matrix are generated by Higgs doublets, and there is no natural explanation for the hierarchy of masses). In particular, one expects relations analogous to Eqs. (13) or (14), i.e., $\theta \sim m_1/m_2$ or $\sqrt{m_1/m_2}$ for $m_T \sim m_D$ or $m_T = 0$, respectively.

The seesaw model is easily extended to three (or more) generations. The expression for the mass Lagrangian in Eq. (9) still holds, provided ν_L, N_L^C are interpreted as three-component vectors [i.e., $\nu_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T$, etc.], and similarly for ν_R^C and N_R. Also, m_T, m_D, and m_S are 3×3 matrices, with $m_T^T = m_T$ and $m_S^T = m_S$. If the eigenvalues of m_S are all large compared to the components of m_T and m_D there will be three light and three heavy Majorana neutrinos, corresponding to the $SU(2)_L$ doublets and singlets, respectively, up to small mixings of order $m_D m_S^{-1}$. One has

$$\begin{bmatrix} \nu_L \\ N_L^C \end{bmatrix} = \mathcal{U}_L \begin{bmatrix} \nu_{lL} \\ \nu_{hL} \end{bmatrix} , \quad \begin{bmatrix} \nu_R^C \\ N_R \end{bmatrix} = \mathcal{U}_R \begin{bmatrix} \nu_{lR}^C \\ \nu_{hR}^C \end{bmatrix} , \tag{15}$$

where ν_l and ν_h represent the light- and heavy-mass eigenstates, respectively. $\mathcal{U}_L$ and $\mathcal{U}_R$ are unitary matrices which diagonalize M:

$$\mathcal{U}_L^\dagger M \mathcal{U}_R = \begin{bmatrix} m_l & 0 \\ 0 & m_h \end{bmatrix} , \tag{16}$$

where m_l and m_h are diagonal 3×3 matrices of the light and heavy eigenvalues, respectively. Because M is symmetric, $\mathcal{U}_L$ and $\mathcal{U}_R$ are related by $\mathcal{U}_L^\dagger = K \mathcal{U}_R^T$, where K is a diagonal phase matrix.

$\mathcal{U}_L$ may be written in block form:

$$\mathcal{U}_L = \begin{bmatrix} V^{Ll} & V^{Lh} \\ V^{Rl*} K_1^\dagger & V^{Rh*} K_2^\dagger \end{bmatrix} , \tag{17}$$

where V^{Ll}, V^{Lh}, V^{Rl}, and V^{Rh} are 3×3 matrices and $K = \mathrm{diag}(K_1, K_2)$. The form in Eq. (17) has been chosen for later convenience. $\mathcal{U}_R$ can also be written in a similar block form.

From the unitarity of $\mathcal{U}_L$ one has

$$\begin{aligned} &V^{Ll} V^{Ll^\dagger} + V^{Lh} V^{Lh^\dagger} = I \ , \\ &V^{Rl} V^{Rl^\dagger} + V^{Rh} V^{Rh^\dagger} = I \ , \\ &V^{Ll} K_1 V^{Rl^T} + V^{Lh} K_2 V^{Rh^T} = 0 \ . \end{aligned} \tag{18}$$

Substituting the block forms of $\mathcal{U}_L$ and $\mathcal{U}_R$ in Eq. (16) one obtains, to leading order in m_S^{-1} (Ref. 16),

$$\begin{aligned} m_l &= V^{Ll^\dagger} (m_T - m_D m_S^{-1} m_D^T) V^{Ll*} K_1^* \ , \\ m_h &= K_2 V^{Rh^T} m_S V^{Rh} \ , \end{aligned} \tag{19}$$

and

$$\begin{aligned} V^{Lh} &= (m_D m_S^{-1}) V^{Rh*} K_2^* \ , \\ V^{Rl} &= -(m_D m_S^{-1})^T V^{Ll*} K_1^* \ . \end{aligned} \tag{20}$$

From Eq. (19) we see that the mass matrix of the light neutrinos is $(m_T - m_D m_S^{-1} m_D^T)$, which is diagonalized by V^{Ll}, while the heavy-neutrino mass matrix is m_S, which is diagonalized by V^{Rh}. Again, if $m_T = 0$ one has a natural explanation of why the light neutrinos are much lighter than quark and lepton masses (which are of order m_D). From Eq. (20) we see that the mixing between the light and heavy sectors, which is given by the matrices V^{Lh} and V^{Rl}, is of order $M_D m_S^{-1}$, as in the case of a single generation. This also implies that V^{Rl} and V^{Lh} are not unitary. From the unitarity relations of $\mathcal{U}_L$ we note that V^{Ll} and V^{Rh} are approximately unitary and the deviations from the unitarity are of order $(m_D m_S^{-1})^2$.

The leptonic weak charged-current interaction can be written as

$$-L_L^{CC} = \frac{g}{\sqrt{2}} (J_L^\mu W_\mu^- + J_L^{\mu^\dagger} W_\mu^+) \ . \tag{21}$$

In Eq. (21),

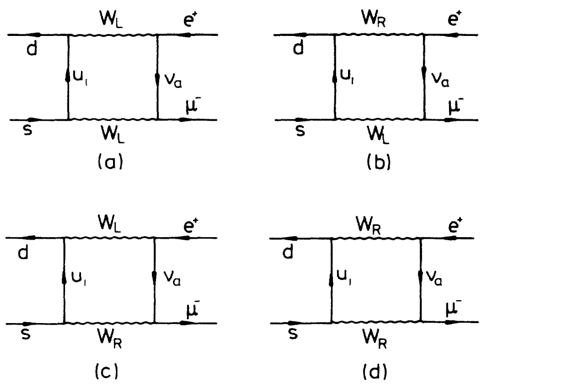

FIG. 1. (a) Box diagram for $K\rightarrow\mu e$ in the $SU(2)_L\times U(1)$ model. (b)–(d) Additional diagrams in the $SU(2)_L\times SU(2)_R\times U(1)$ model ignoring W_L-W_R mixing. u_i, $i=1,2,3$, represent the u, c, and t quarks, while ν_a represents the ath neutrino, which can be either light or heavy.

$$J_L^\mu=\bar{l}_L\gamma^\mu\nu_L \ , \tag{22}$$

where $l_L=(e_L^-,\mu_L^-,\tau_L^-)^T$ and $\nu_L=(\nu_{eL},\nu_{\mu L},\nu_{\tau L})^T$, and we have chosen the l_L weak-eigenstate basis to coincide with the mass eigenstate basis. In terms of the neutrino mass eigenstates, J_L^μ can be rewritten as

$$J_L^\mu=\bar{l}_L\gamma^\mu(V^{Ll}\nu_{lL}+V^{Lh}\nu_{hL}) \ . \tag{23}$$

V^{Ll} is the leptonic analogue of the CKM quark mixing matrix, while V^{Lh} represents the small admixture of heavy neutrinos into the ordinary left-handed current.

In considering $SU(2)_L\times SU(2)_R\times U(1)$ models in Sec. IV, we will be concerned with the right-handed leptonic currents

$$J_R^\mu=\bar{l}_R\gamma^\mu N_R \ , \tag{24}$$

where (in an appropriate basis) $l_R=(e_R^-,\mu_R^-,\tau_R^-)^T$. Rewriting N_R in terms of the mass eigenstates, one has

$$J_R^\mu=\bar{l}_R\gamma^\mu(V^{Rl}\nu_{lR}^C+V^{Rh}\nu_{hR}^C) \ , \tag{25}$$

where the V^{Rl} term represents the small admixture of light neutrinos. The right-handed hadronic current will be discussed in Sec. IV.

III. $K_L\rightarrow\mu e$ IN THE $SU(2)_L\times U(1)$ MODEL

The major contribution to the decay $K_L\rightarrow\mu e$ in the $SU(2)_L\times U(1)$ model comes from the W-W box diagram shown in Fig. 1(a). Neglecting the external momenta, the amplitude is found to be

$$A_{SM}(\bar{K}^0\rightarrow\mu^-e^+)=\left[\frac{g}{\sqrt{2}}\right]^4\langle 0|\bar{d}\gamma_\alpha\gamma_L s|\bar{K}^0\rangle\bar{u}_\mu\gamma^\alpha\gamma_L v_e\frac{1}{(4\pi)^2}\frac{1}{M_W^2}\times\sum_i U_{id}^{L*}U_{is}^L\left[\sum_a V_{\mu a}^{Ll}V_{ea}^{Ll*}I_1(x_i,x_{la})+(l\rightarrow h)\right] , \tag{26}$$

where $x_i=m_i^2/M_W^2$ and $x_a=m_a^2/M_W^2$. The index i refers to the intermediate charge $\frac{2}{3}$ quarks and the indices la and ha refer to the light and heavy intermediate neutrino states, respectively. U^L is the usual CKM matrix and the V's are mixing matrices for the leptons defined in the previous section. The function I_1 is defined by

$$I_1(x_i,x_a)=-i(4\pi)^2M_W^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-m_i^2)(k^2-m_a^2)(k^2-M_W^2)^2}=\left[\frac{x_i^2\ln x_i}{(x_a-x_i)(1-x_i)^2}+\frac{x_a^2\ln x_a}{(x_i-x_a)(1-x_a)^2}-\frac{1}{(1-x_i)(1-x_a)}\right] . \tag{27}$$

It is convenient to define the reduced amplitude $\tilde{A}$ by

$$A=\left[\frac{g_L}{\sqrt{2}}\right]^4\langle 0|\bar{d}\gamma_\alpha\gamma_L s|\bar{K}^0\rangle\bar{u}_\mu\gamma^\alpha\gamma_L v_e\frac{1}{(4\pi)^2}\frac{1}{M_W^2}\tilde{A} \ . \tag{28}$$

Thus, in the $SU(2)_L\times U(1)$ model, one has

$$\tilde{A}_{SM}=\sum_i U_{id}^{L*}U_{is}^L\left[\sum_a V_{\mu a}^{Ll}V_{ea}^{Ll*}I_1(x_i,x_{la})+(l\rightarrow h)\right] . \tag{29}$$

Using isospin invariance to relate the amplitude for $\bar{K}^0\rightarrow\mu^-e^+$ to that of $K^-\rightarrow\mu^-\bar{\nu}_\mu$, and neglecting the electron mass relative to the muon mass in the phase-space factors, one obtains

$$B(K_L\rightarrow\mu e)=4.1\times 10^{-4}|\mathbf{A}|^2 \ . \tag{30}$$

For $B(K_L\rightarrow\mu e)$ to be of order 10^{-12}, $\tilde{A}$ must be of order 10^{-4}.

Let us estimate $\tilde{A}_{SM}$ from Eq. (29). Using the unitarity of U^L [i.e., the Glashow-Iliopoulos-Maiani (GIM) mechanism] one can replace I_1 in Eq. (29) by

$$I_1'(x_i,x_a)=x_i\left[\frac{x_i\ln x_i}{(x_a-x_i)(1-x_i)^2}+\frac{x_a\ln x_a}{(x_i-x_a)(1-x_a)^2}-\frac{1}{(1-x_i)(1-x_a)}\right]. \quad (31)$$

V^{Ll} is unitary to order $(m_D m_S^{-1})^2$. From the unitary part of V^{Ll}, a double GIM mechanism is operative and one can replace I_1' by

$$I_1''(x_i,x_a)=x_i x_a\left[\frac{\ln x_i}{(x_a-x_i)(1-x_i)^2}+\frac{\ln x_a}{(x_i-x_a)(1-x_a)^2}-\frac{1}{(1-x_i)(1-x_a)}\right]. \quad (32)$$

Writing I_1 in Eq. (29) in the form of I_1' (GIM reduced form) and I_1'' (double GIM reduced form) guarantees that the amplitude is not proportional to the small differences of large numbers. In this way, the amplitude in Eq. (29) becomes

$$\begin{aligned}\tilde{A}_{SM}&=\sum_i U_{id}^{L*}U_{is}^{L}\sum_a V_{\mu a}^{Ll}V_{ea}^{Ll*}I_i''(x_i,x_{la})\\&\quad+O(m_D m_S^{-1})^2\sum_i U_{id}^{L*}U_{is}^{L}[I_1'(x_i,x_{ha})-I_1'(x_i,x_{la})]\\&=\tilde{A}_{SM}(1)+\tilde{A}_{SM}(2)\,. \end{aligned} \quad (33)$$

In estimating $\tilde{A}_{SM}$ we consider only the intermediate up and the charm quarks for simplicity (the top contribution is at most of the same order[27]). First we consider $\tilde{A}_{SM}(1)$ (double GIM reduced term) in several cases. We take $m_u=5.6$ MeV, $m_c=1.35$ GeV, and $M_W=80.9$ GeV.

(1) *Cosmological bounds respected.* For $m_{\nu_\mu}\le 40$ eV and the limits on the mixing between generations from Eq. (5), we have a branching ratio of $B(K_L\to\mu e)\le 1.2\times10^{-43}$.

(2) *Cosmological bounds not respected.* For $m_{\nu_\mu}\le 250$ keV and $|V_{e\mu}^{Ll}|\le 0.07$ (approximate for heavy ν_μ) from Ref. 16, the branching ratio is $B(K_L\to\mu e)\le 9.7\times10^{-26}$.

(3) *Large mixing with the third generation.* For $m_{\nu_\tau}\le 35$ MeV and $|V_{e\tau}^{Ll}V_{\mu\tau}^{Ll}|\le 6\times10^{-6}$ from Ref. 16, we have $B(K_L\to\mu e)\le 1.4\times10^{-27}$.

(4) *Possible fourth generation.* A neutrino belonging to a possible fourth generation can be heavier than 20 GeV. Its mixing with the first and second generations are given by expressions similar to the seesaw formula given in Eq. (13). Therefore, $V_{e4}\sim m_e/m_{\nu_4}\le 3\times10^{-5}$ and $V_{\mu4}\sim m_\mu/m_{\nu_4}\le 5\times10^{-3}$. The branching ratio in this case is $B(K_L\to\mu e)\le 7.5\times10^{-25}$ (Ref. 28).

(5) *Heavy $SU(2)_L$ singlet (right-handed) neutrino.* The estimates for the light-heavy mixing and for $B(K_L\to\mu e)$ are identical to case 4. The results of these cases are summarized in Table I.

Considering $\tilde{A}_{SM}(2)$ [the singlet GIM reduced term in Eq. (33)] we note that it is not directly proportional to the light-neutrino masses. Therefore, one might hope that it might be significant even though it is proportional to the small number $(m_D m_S^{-1})^2$. As a matter of fact, we find that

$$\tilde{A}_{SM}(2)\sim\frac{m_D^2}{m_S^2}x_c\sim\frac{m_D^4}{m_S^2M_W^2}\sim\frac{m_l^2}{M_W^2}\sim x_{la}\,, \quad (34)$$

i.e., $\tilde{A}_{SM}(2)$ is of the same order of magnitude as $\tilde{A}_{SM}(1)$. In the above equation we assumed $m_D\sim m_c$. Any deviations from this will not make a significant difference. Qualitatively, the discussion given for $\tilde{A}_{SM}(1)$ still holds and the branching ratio in the $SU(2)_L\times U(1)$ model could at most be of order 10^{-24}.

The standard model, extended by both Dirac and Majorana neutrino masses, leads invariably to an uninterestingly small decay rate for $K_L\to\mu e$. The reason for this is that the ordinary left-handed neutrinos are very light, while the heavy neutrinos couple to the electron and the muon only through small light-heavy mixings. This situation is altered in the $SU(2)_L\times SU(2)_R\times U(1)$ model,

TABLE I. Expectations for $\tilde{A}$ and the $K_L\to\mu e$ branching ratio in various cases for $SU(2)_L\times U(1)$ and $SU(2)_L\times SU(2)_R\times U(1)$ models. For the W_R-W_R case we have assumed $g_R=g_L$.

Standard model	Case number	m_ν	$V_{a\mu}V_{ae}^*$	$\tilde{A}_{SM}(1)$	$B(K_L\to\mu e)$
	1	40 eV	0.03	1.7×10^{-20}	1.2×10^{-43}
	2	250 keV	0.07	1.5×10^{-11}	9.7×10^{-26}
	3	35 MeV	6×10^{-6}	1.9×10^{-12}	1.4×10^{-27}
	4,5	20 GeV	1.4×10^{-7}	4.3×10^{-11}	7.5×10^{-25}
L-R model	Diagram	*L-R* symmetry	β_g	$\tilde{A}$	$B(K_L\to\mu e)$
	W_L-W_R	Yes	3.4×10^{-3}	1.1×10^{-6}	5.2×10^{-16}
	W_L-W_R	No	0.03	2.2×10^{-5}	2.0×10^{-13}
	W_R-W_R	Yes	3.4×10^{-3}	3.2×10^{-9}	4.1×10^{-21}
	W_R-W_R	No	0.025	2.7×10^{-6}	3.0×10^{-15}

where the right-handed fermions have gauge interactions also. Therefore, we may hope to obtain a large branching ratio in this model.

IV. THE $SU(2)_L \times SU(2)_R \times U(1)$ MODEL

In $SU(2)_L \times SU(2)_R \times U(1)$ (L-R) models there are two charged bosons W_L and W_R coupling to left- and right-handed fermions, respectively. W_L is essentially the same as the W of the standard model. In general, W_L and W_R are not mass eigenstates, but their mixing is constrained to be very small. Therefore, we shall neglect gauge-boson mixing for most of our calculations, leaving a discussion of this effect to the end.

The two charged-current interactions result in two distinct mixing matrices for the left- and right-handed fermions. The mixing matrix for the left-handed quarks U^L is the usual CKM matrix. The values of the elements of U^L are affected very little by the presence of the right-handed currents, provided that W_R is massive enough to satisfy existing constraints. The mixing matrix for the right-handed quarks, U^R, is a new unitary matrix which can be parametrized by three angles and six phases. Its elements are unconstrained by experiment and are limited only by unitarity. The mixing in the leptonic sector is discussed in Sec. II.

The mass M_R of W_R is much larger than the mass M_L of W_L. It is tightly constrained by the K_L-K_S mass difference. For the case of manifest L-R symmetry ($U^R=U^L$) or pseudomanifest L-R symmetry ($U^R=U^{L*}$), there is a very stringent lower limit of $M_R>1.4$ TeV (Ref. 13). For the case of no L-R symmetry (arbitrary U^R) the combination of the K_L-K_S mass difference and the bottom-quark decay imply the bound $M_R>300$ GeV (Refs. 14 and 15). However, M_R can be as low as 300 GeV only for some special forms of U^R. Therefore, this limit comes into play only in some corners of the parameter space of the matrix elements U^R_{ij}. For most values of the elements of U^R, much stronger bounds (e.g., $M_R \geq 1.4$ TeV) apply. Note that the coupling constants g_L and g_R of the two SU(2) groups need not be the same for the case of no L-R symmetry.

In this model there are four box diagrams, which are shown in Fig. 1. The calculation of Fig. 1(a) was discussed in Sec. III.

The reduced amplitude for the box diagrams containing one W_L and one W_R is

$$\tilde{A}_{LR}=\beta_g\eta\left[\sum_i \frac{m_i}{M_L}U^{L*}_{id}U^R_{is}\sum_a \frac{m_{ha}}{M_L}V^{Rh}_{\mu a}V^{Lh*}_{ea}J_0(x_i,x_{ha},\beta)\right.$$
$$\left.+(L\leftrightarrow R)+(h\to l)\right], \tag{35}$$

where $\beta=M_L^2/M_R^2, \beta_g=(g_R^2/g_L^2)(M_L^2/M_R^2)$ (Ref. 29) and η is an enhancement factor arising from the different operator entering the kaon to vacuum matrix element:

$$\eta=\frac{\langle 0|\bar{d}\gamma^\rho\gamma^\alpha\gamma_R s|\bar{K}^0\rangle\bar{u}_\mu\gamma_\rho\gamma_\alpha\gamma_L v_e}{\langle 0|\bar{d}\gamma^\alpha\gamma_L s|\bar{K}^0\rangle\bar{u}_\mu\gamma_\alpha\gamma_L v_e}$$
$$\simeq 4\frac{M_K^2}{(m_s+m_d)m_\mu}\simeq 50\,, \tag{36}$$

where the PCAC (partial conservation of axial-vector current) estimate has been used.

The function J_0 is given by

$$J_0(x_i,x_a,\beta)=\frac{x_i\ln x_i}{(x_a-x_i)(1-x_i)(1-\beta x_i)}$$
$$+\frac{x_a\ln x_a}{(x_i-x_a)(1-x_a)(1-\beta x_a)}$$
$$+\frac{\beta\ln\beta}{(1-\beta)(1-\beta x_i)(1-\beta x_a)}\,. \tag{37}$$

Because of the explicit mass factors m_i and m_{ha} appearing in Eq. (35) there is no GIM cancellation at all in $\tilde{A}_{LR}$. Therefore, $\tilde{A}_{LR}$ can make an important contribution to $K_L\to\mu e$, even though it is suppressed by a factor of β_g.

The product of the leptonic mixing matrix elements for both the light and heavy intermediate neutrinos is of the same order, i.e.,

$$V^{Rl}_{\mu a}V^{Ll*}_{ea}=O\left[\frac{m_D}{m_S}\right]\sim V^{Rh}_{\mu a}V^{Lh*}_{ea}\,.$$

We can therefore neglect the light-neutrino terms in Eq. (35) because of the explicit mass dependence. For the same reason the up-quark contribution can be neglected also, but the charm and the top contributions should both be considered. In our estimates below we take $m_c=1.35$ GeV and $m_t=55$ GeV for the quarks and $m_S\sim m_{ha}\sim 100$ GeV for the neutrinos. The Dirac mass m_D should be similar to a charged-lepton mass or a quark mass. We take $m_D\sim m_\mu\sim 100$ MeV, so the light-heavy mixing m_D/m_S is $\sim 10^{-3}$. The results are insensitive to m_t and scale roughly like $1/m_S$ over their allowed ranges.[30] For U^L we take $U^L_{td}=0.02$ and $U^L_{ts}=0.043$.

There are stringent constraints on β_g and the elements of U^R from the K_L-K_S mass difference Δm_K and from the b-quark semileptonic branching ratio. Assuming no fine-tuned cancellations between different contributions to Δm_K, it is found that the weakest limits on β_g (and correspondingly the largest possible values for $\tilde{A}_{LR}$) occur for several special forms for U^R, which are listed in Table II. For each of these forms $\tilde{A}_{LR}$ is dominated by one term in Eq. (35). (We have checked that perturbations consistent with Δm_K around these values of U^R do not have any significant effect.) The upper limits on $\tilde{A}_{LR}$ for each of these cases is given in Table II. It is seen that the largest allowed values for $\tilde{A}_{LR}$ (barring extreme fine-tunings) is

TABLE II. Specific forms for U^R which allow the largest values for β_g and $\tilde{A}_{LR}$, along with the upper limits on β_g and $\tilde{A}_{LR}$ and the dominant contribution for each case. The alternate limits in parentheses for cases (b) and (c) are obtained if one imposes additional constraints from neutrinoless double-β decay (Ref. 31).

Form	U^R	β_g	Dominant exchange	$\tilde{A}_{LR}$
a	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	0.01	$U_{cd}^{R*} U_{cs}^L$	6.9×10^{-6}
b	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	0.07 (0.01)	$U_{cs}^R U_{cd}^{L*}$	1.1×10^{-5} (1.5×10^{-6})
c	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	0.03 (0.01)	$U_{ts}^R U_{td}^{L*}$	1.1×10^{-5} (3.5×10^{-6})
d	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	0.03	$U_{td}^{R*} U_{ts}^L$	2.2×10^{-5}

$$\tilde{A}_{LR} \le 2.2\times 10^{-5} , \tag{38}$$

corresponding to

$$B(K_L \to \mu e) \le 2.0\times 10^{-13} . \tag{39}$$

There are some additional constraints on U^R from neutrinoless double-β decay[31] in the likely case that the heavy neutrino is Majorana (as it is in the seesaw model). These imply limits on β_g (for two of the forms of U^R) that are a factor 5–10 more stringent than those from Δm_K (typical values are given in Table II), although there is considerable uncertainty from the nuclear matrix elements. However, these new limits do not affect the overall limit in Eq. (39) on the branching ratio, because they do not modify form d.

The branching ratio in Eq. (39) is to be compared to the case of manifest or pseudomanifest L-R symmetry, for which $\beta_g \le 3.4\times 10^{-3}$. Then

$$\tilde{A}_{LR} \le 1.1\times 10^{-6} , \tag{40}$$

giving a branching ratio $\le 5.2\times 10^{-16}$.

The reduced amplitude of the box diagram with two W_R's is

$$\tilde{A}_{RR} = \beta_g^2 \sum_i U_{id}^{R*} U_{is}^R \sum_a V_{\mu a}^{Rh} V_{ea}^{Rh*} x_i x_{ha} \left[\frac{\ln \beta x_i}{(x_{ha}-x_i)(1-\beta x_i)^2} + \frac{\ln \beta x_{ha}}{(x_i - x_{ha})(1-\beta x_{ha})^2} - \frac{\beta}{(1-\beta x_i)(1-\beta x_{ha})} \right] . \tag{41}$$

In the above expression we have made use of the unitarity of U^R and the approximate unitarity [to order $(m_D/m_S)^2$] of V^{Rh}. The order-$(m_D/m_S)^2$ corrections coming from the light-neutrino terms and from the nonunitary part of V^{Rh} can be neglected compared to the right-hand side (RHS) of Eq. (41). This is to be contrasted with the situation of $\tilde{A}_{LL}$, where the $(m_D/m_S)^2$ corrections are of the same order as the double GIM reduced terms because the latter are proportional to the light-neutrino mass.

Because of the factor x_i in Eq. (41) only the top-quark contribution is significant for both the cases of L-R symmetry and no symmetry. The relevant constraints for the quantity $\tilde{A}_{RR}$ are

$$\beta_g \, | U_{td}^R U_{ts}^R | \le 3.5\times 10^{-4} . \tag{42}$$

from the K_L-K_S mass difference[15] and

$$\frac{g_L^2}{g_R^2} \beta_g^2 \, | V_{\mu a}^{Rh*} V_{ea}^{Rh} x_{ha} | \le 4.7\times 10^{-4} , \tag{43}$$

from the upper limit $B(\mu \to e\gamma) < 4.9\times 10^{-11}$ (Refs. 32 and 33). The largest value for $\tilde{A}_{RR}$ consistent with the above constraints is obtained for $| V_{\mu a}^{Rh} V_{ea}^{Rh*} | = 0.5$ (the unitarity limit), $\beta_g = 0.025 g_R/g_L$, and $| U_{td}^{R*} U_{ts}^R |$ given in Eq. (42). One obtains

$$\tilde{A}_{RR} \le 2.7\times 10^{-6} \frac{g_R}{g_L} , \tag{44}$$

and

$$B(K_L \to \mu e) \le 3.0\times 10^{-15} \frac{g_R^2}{g_L^2} . \tag{45}$$

Of course, for reasonable models one expects $g_R/g_L \sim 1$.

For the special case of manifest or pseudomanifest L-R symmetry we obtain

$$\tilde{A}_{RR} \leq 3.2\times 10^{-9} , \quad (46)$$

implying a branching ratio of 4.1×10^{-21}, much smaller than that due to $\tilde{A}_{LR}$. The results for the $SU(2)_L \times SU(2)_R \times U(1)$ model are summarized in Table I.

We now briefly discuss the effect of the gauge-boson mixing on the various amplitudes. The mass eigenstates of the gauge bosons are

$$W_1 = \cos\zeta W_L - \sin\zeta W_R , \quad W_2 = \sin\zeta W_L + \cos\zeta W_R . \quad (47)$$

The angle ζ is very tightly constrained by the universality of the lepton and quark couplings to the gauge bosons and by nonleptonic K decays:

$$|\tan\zeta| < 0.005 \quad (48)$$

for $U^L = U^R$ (Ref. 34) and

$$|\tan\zeta| < 0.015 \quad (49)$$

for arbitrary U^R (Ref. 15).

For the amplitudes $\tilde{A}_{LR}$ and $\tilde{A}_{RR}$ one can safely neglect the corrections due to mixing because the leading (no mixing) term is not proportional to light-neutrino masses. The leading term of $\tilde{A}_{LL}$ is proportional to m_{la}^2 and we must check if the corrections due to gauge-boson mixing make a significant contribution to the overall amplitude. The first-order correction due to gauge-boson mixing (proportional to $\tan\zeta$) is also proportional to the external momenta (i.e., proportional to m_K). So we reconsider the box diagram with two light gauge bosons, with nonzero external momenta. Without mixing, the double GIM mechanism is still operative and nothing is gained by having nonzero external momenta. With mixing, the $\tan\zeta$ correction consists of two terms, one for which quark GIM is operative and the other for which leptonic GIM is operative. Comparing the term with the quark GIM to $\tilde{A}_{LR}$ (which is the largest of all the amplitudes in the L-R model) we find the ratio to be

$$\sim \frac{\tan\zeta}{\beta_g} \frac{m_i m_K}{m_{ha}^2} \quad (50)$$

which is $< 10^{-3}$ for the values of parameters assumed. In the above expression we have set the external momentum to be equal to the kaon mass. The ratio of the term with the leptonic GIM to $\tilde{A}_{LR}$ is

$$\sim \frac{\tan\zeta}{\beta_g} \frac{m_K}{m_D} \left| \frac{m_{la}^2}{m_i^2} + \frac{m_D^2}{m_S^2} \right| , \quad (51)$$

which is very small. The second-order corrections, proportional to $\tan^2\zeta$, are of the form

$$\frac{\tan^2\zeta}{\beta_g} \tilde{A}_{LR} . \quad (52)$$

For arbitrary U^R as well as for $U^L = U^R$, we have $\tan^2\zeta \ll \beta_g$. So the second-order corrections can also be neglected.

L-R models always contain flavor-changing neutral Higgs fields (denote them by ϕ). In the simplest model the Dirac masses of the fermions are generated by a single $(\frac{1}{2},\frac{1}{2})$ Higgs multiplet. Then the coupling of the flavor-changing neutral Higgs field to the d-s quarks is of the form[35]

$$\sum_i (\bar{d} U_{id}^{L*} m_i U_{is}^R \gamma_R s\phi + \bar{d} U_{id}^{R*} m_i U_{is}^L \gamma_L s\phi^\dagger) . \quad (53)$$

Similarly the coupling of ϕ to the charged leptons is proportional to the neutrino masses and the corresponding leptonic mixing matrices. Demanding that the contribution of the tree diagram with an intermediate ϕ to the K_L-K_S mass difference be small gives us a lower limit on the mass of the ϕ. For example, for the case of (pseudo)manifest L-R symmetry and an extra constraint[35] on the Higgs potential which ensures

$$M_{\mathrm{Re}\phi} = M_{\mathrm{Im}\phi} , \quad (54)$$

one has

$$M_\phi \geq 5.7 \text{ TeV} . \quad (55)$$

Calculating the tree diagram for $K_L \to \mu e$ and using the above limit gives us the value

$$\tilde{A}_\phi \leq 9.1\times 10^{-6} \quad (56)$$

leading to a branching ratio of 3.4×10^{-14}.

If one chooses certain special forms for U^R (such as those in Table II) which violate (pseudo)manifest L-R symmetry and still assume the condition in Eq. (54) then the interaction in Eq. (53) no longer contributes to Δm_K at the tree level. It then appears that ϕ could be light (e.g., ~ 100 GeV) and that one can have a large tree-level contribution to $K_L \to \mu e$. This is misleading, however, because models which lead to $|U_{ij}^R| \neq |U_{ij}^L|$ will almost certainly violate the condition in Eq. (54) and/or the exact form of the couplings in Eq. (53). It appears very unlikely that flavor-changing Higgs-boson effects consistent with Δm_K could lead to an observable $K_L \to \mu e$ rate. We have not attempted a detailed investigation of this point, which is outside the scope of this paper.

In all our estimates we have considered the amplitudes of box diagrams containing only the gauge bosons. Since we have used the Feynman-gauge propagators, we must also consider the box diagrams containing Goldstone bosons. We have calculated the Goldstone boson contributions to LL, LR, and RR amplitudes and found that they are negligible for the LL and RR cases. In the LR case, the diagram in which G_L-W_R are exchanged [Fig. 1(c), with G_L replacing the W_L] can make a contribution to $\tilde{A}_{LR}$ that is comparable to that of W_L-W_R box diagram.

A few years ago it was pointed out[36] that the set of box diagrams for the K_L-K_S mass difference in the L-R models is not gauge invariant. Subsequently, additional diagrams containing flavor-changing neutral Higgs field were found, which together with the box diagrams formed a gauge-invariant set.[35] The effect of these additional diagrams, compared to the box diagrams, is found to be small ($<5\%$). For the case of $K_L \to \mu e$, the effect of the diagrams restoring the gauge invariance is found to be

$$\frac{\widetilde{A}_{AD}}{\widetilde{A}_{LR}} \sim \frac{\ln \frac{M_R^2}{M_\phi^2}}{4J_0(x_i, x_{ha}, \beta)} , \tag{57}$$

which is at most of order 1. Therefore, including the additional diagrams that make the box diagrams gauge invariant still gives the same order of magnitude result for $B(K_L \to \mu e)$.

V. SUMMARY AND CONCLUSIONS

We have studied the decay $K_L \to \mu e$ in $SU(2)_L \times U(1)$ and $SU(2)_L \times SU(2)_R \times U(1)$ models with Dirac and Majorana masses for the neutrinos. We investigated the possibility of whether the branching ratio for this decay mode could be as large as 10^{-12}, so that it could be observed in the current round of experiments. We found that even with the largest allowed neutrino masses and mixing angles in the leptonic sector, one can obtain a branching ratio of only about 10^{-24} in the $SU(2)_L \times U(1)$ model. The situation is more promising in the $SU(2)_L \times SU(2)_R \times U(1)$ model because of the large Majorana masses for the right-handed neutrinos. But the branching ratio in this model is still very small because of the large mass of the right-handed charged gauge boson W_R. For the case of L-R symmetry, manifest or pseudomanifest, the bound $M_{W_R} > 1.4$ TeV leads to a $B(K_L \to \mu e) \leq 10^{-15}$. For some special values of the elements of U^R, the right-handed CKM matrix, the lower bound on M_{W_R} can be relaxed down to 300 GeV, and one obtains $B(K_L \to \mu e) \leq 2 \times 10^{-13}$. But we must emphasize that this is so only in some small corners of the parameter space of U_{ij}^R. Another possibility in $SU(2)_L \times SU(2)_R \times U(1)$ models is $K_L \to \mu e$ mediated by flavor-changing neutral-Higgs-boson couplings. As discussed briefly in Sec. IV, it appears unlikely that Higgs-boson parameters consistent with the K_L-K_S mass difference could generate an observable effect, but this point has not been investigated in detail for models without L-R symmetry. If the current or future experiments succeed in finding $B(K_L \to \mu e)$ at a level of 10^{-12} or more, it would require either a completely different mechanism for the decay or a revision of our understanding of massive neutrinos.

ACKNOWLEDGMENTS

This work was supported by DESY, by the U.S. Department of Energy under Grant No. DE-AC02-76-ERO-3071 (P.L. and S.U.), by Alexander von Humboldt-Stiftung (P.L.), and by Bundesministerium für Forschung und Technologie (K.S. and S.U.).

*Permanent address: Department of Physics, University of Pennsylvania, Philadelphia, PA 19104.

[1]H. B. Greenlee *et al.*, Phys. Rev. Lett. **60**, 893 (1988).
[2]M. J. Shochet *et al.*, Phys. Rev. D **10**, 1965 (1979).
[3]J. Markey, talk presented at the 9th Workshop on Grand Unification, Aix-les-Baines, France, 1988 (unpublished).
[4]E. Eichten, I. Hinchliffe, K. D. Lane, and C. Quigg, Phys. Rev. D **34**, 1547 (1986).
[5]B. A. Campbell, Phys. Rev. D **28**, 209 (1983).
[6]J. C. Pati and H. Stremnitzer, Phys. Lett. B **172**, 441 (1986); I. I. Bigi, G. Köpp, and P. M. Zerwas, Phys. Lett. **166B**, 238 (1986).
[7]Wei-Shu Hou and A. Soni, Phys. Rev. Lett. **54**, 2038 (1985).
[8]O. Shanker, Nucl. Phys. **B206**, 253 (1982).
[9]L. J. Hall and L. J. Randall, Nucl. Phys. **B274**, 157 (1986).
[10]B. A. Campbell, J. Ellis, K. Enqvist, M. K. Gaillard, and D. V. Nanopoulos, Int. J. Mod. Phys. **A2**, 831 (1987).
[11]B. W. Lee and R. E. Shrock, Phys. Rev. D **16**, 1444 (1977); A. Barroso, G. C. Branco, and M. C. Bento, Phys. Lett. **134B**, 123 (1986).
[12]M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, Prog. Theor. Phys. **B135**, 66 (1978).
[13]G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **48**, 848 (1982); G. Ecker and W. Grimus, Nucl. Phys. **B258**, 328 (1985).
[14]F. I. Olness and M. E. Ebel, Phys. Rev. D **30**, 1034 (1984).
[15]P. Langacker and S. Uma Sankar (in preparation).
[16]For recent reviews, see P. Langacker, in *Neutrino Physics*, edited by H. V. Klapdor (Springer, Berlin, in press). R. Eichler, in *Lepton and Photon Interactions*, proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, West Germany, 1987, edited by W. Bartel and R. Rückl [Nucl. Phys. B, Proc. Suppl. **3**, 389 (1987)]; S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987); J. D. Vergados, Phys. Rep. **133**, 1 (1986); F. Boehm and P. Vogel, *Physics of Massive Neutrinos* (Cambridge University Press, Cambridge, England, 1987).
[17]M. Fritschi *et al.*, Phys. Lett. B **173**, 485 (1986); The ITEP group [S. Boris *et al.*, Phys. Rev. Lett. **58**, 2019 (1987)] claims evidence for a nonzero ν_e mass in the range 17–40 eV.
[18]R. Abela *et al.*, Phys. Lett. **146B**, 431 (1984).
[19]H. Albrecht *et al.*, Phys. Lett. B **202**, 149 (1988).
[20]D. Caldwell, talk presented at the 9th Workshop on Grand Unification (Ref. 3).
[21]The constraint in Eq. (2) can in principle be relaxed if the ν_e is a mixture of Majorana mass eigenstates, but this is very unlikely in practice. See P. Langacker, B. Sathiapalan, and G. Steigman, Nucl. Phys. **B266**, 669 (1986).
[22]For a recent review see G. Gelmini, in *Neutrino Physics* (Ref. 16).
[23]G. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981); H. Georgi *et al.*, Nucl. Phys. **B193**, 297 (1983).
[24]There is no difference between Dirac and Majorana masses for the $K_L \to \mu e$ amplitude itself since the total lepton number is conserved.
[25]One can also have a light eigenvalue if $m_T \sim m_D \sim m_S$. This case is not physically realistic because to achieve a large value for m_2 (e.g., 20 GeV), one would require m_T, m_D, and m_S to

all be much larger than m_e, m_u, and m_d. Also the mixing angle would be large (45° for $m_T = m_D = m_S$), in violation of weak universality.

[26]Most of these limits depend only weakly (or not at all) on whether the heavy neutrino is an $SU(2)_L$ singlet or doublet.

[27]For a light neutrino $\tilde{A}_{SM}(t)$ is negligible. For a heavy neutrino the ratio of the intermediate-top-quark contribution to that of the charm quark is $\tilde{A}_{SM}(t)/\tilde{A}_{SM}(c) \sim \sin^4\theta_C (m_t/m_c)^2 \sim (m_t/20m_c)^2 \sim 1$.

[28]Barroso *et al.* (Ref. 11), in their calculation of the effect of a fourth generation with a heavy Dirac neutrino, have taken $V^{Ll}_{\mu a} V^{Ll*}_{ea} \sim 0.05$. This value is allowed by the experimental limits, but theoretically this requires an incredibly fine-tuned cancellation between large (order of several hundred MeV) contributions to the light ν_e and ν_μ mass eigenvalues.

[29]The limits from the K_L-K_S mass difference and the b-quark decay constrain β_g rather than β. We will assume that β satisfies the same limits as β_g. The effect of this assumption is very small because $\tilde{A}_{LR}$ is dependent only weakly on β, whereas it is directly proportional to β_g.

[30]$m_t \simeq 180$ GeV is the largest value allowed by neutral-current data: U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987); G. Costa *et al.*, Nucl. Phys. **B297**, 244 (1988). $m_S \geq 20$ GeV from experimental constraints on light-energy mixing (Ref. 16), and $m_S \leq M_R$ is required by vacuum stability: R. N. Mohapatra, Phys. Rev. D **34**, 909 (1986).

[31]Mohapatra (Ref. 30).

[32]R. D. Bolton *et al.*, Phys. Rev. Lett. **56**, 2461 (1986).

[33]T. P. Cheng and L. F. Li, Phys. Rev. Lett. **45**, 1908 (1980).

[34]J. F. Donoghue and B. R. Holstein, Phys. Lett. **113B**, 382 (1982); L. Wolfenstein, Phys. Rev. D **29**, 2130 (1984).

[35]W.-S. Hou, and A. Soni, Phys. Rev. D **32**, 163 (1985); J. Basecq, L. F. Li, and P. B. Pal, *ibid.* **32**, 175 (1985).

[36]D. Chang, J. Basecq, L. F. Li, and P. B. Pal, Phys. Rev. D **30**, 1601 (1984).