# Analysis of muon decay with lepton-number-nonconserving interactions

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We analyze muon decay using the most general local, derivative-free, Lorentz-invariant, lepton-number-nonconserving interactions. We show that, neglecting neutrino masses, there is a one-to-one correspondence between the coupling constants in the lepton-number-conserving case and combinations of coupling constants in the lepton-number-violating case; i.e., it is not possible, even in principle, to test lepton-number conservation in muon decay if the final neutrinos are massless and are not observed. Using these relations, we are able to use previous analyses of (lepton-number-conserving) muon and inverse muon decay to put stringent limits on certain combinations of parameters. If fine-tuned cancellations are not allowed, these limits constrain the individual coupling constants. We discuss to what extent it is directly tested that the (left-handed) neutrino emitted in muon decay is the same as that which is produced in $\pi_{\mu 2}$ decay.

Muon decay is one of the few processes in which the $V-A$ nature of the weak interactions can be tested in precision experiments. For this reason, muon decay has been analyzed in terms of the most general local, derivative-free, Lorentz-invariant, lepton-number-conserving effective four-fermion Lagrangian.[1] (By lepton-number conservation or nonconservation we refer both to the individual lepton flavors $L_e$, $L_\mu$, and $L_\tau$, and to the total lepton number $L=L_e+L_\mu+L_\tau$.) Using this, the muon decay process in which the emitted (massless) neutrinos are not observed can be described in terms of the total rate $\tau_\mu$ and nine real parameters.[2]

However the imposition of lepton-number conservation on this Lagrangian is somewhat *ad hoc*. In the standard model with massless neutrinos lepton number is automatically conserved. However, if one goes outside the standard model and introduces scalar, pseudoscalar, and tensor interactions, there is no reason to assume that lepton number should be conserved. Even within the context of gauge theories (beyond the standard model), lepton number can be violated in a number of ways, including Dirac or Majorana masses for neutrinos[3] and spontaneous lepton-number violation in supersymmetric models.[4] (Dirac masses can violate lepton-flavor conservation but conserve the total lepton number; Majorana masses violate both.)

We have been led to consider the question of lepton-number violation in muon decay from a somewhat different angle. In examining the constraints the charged-current experiments put on mixings of the ordinary fermions with exotic fermions,[5] we were led to the possibility of lepton-number violation via nonorthogonal neutrinos.[6] Consider the case in which one makes a modest expansion of the two-family standard model by adding a pair of SU(2)-singlet Weyl neutrinos $N_L^0$ and $N_R^0$, and in which one allows all mass terms which conserve the total lepton number. Because of the mismatch between the number of left- and right-handed states, the diagonalization of the mass matrix yields two massless neutrinos $\nu_1$ and $\nu_2$ and one heavy Dirac neutrino $N$. (In this example, the massless neutrinos are Weyl, but there are also cases in which one ends up with Dirac or Majorana neutrinos.) Since both weak eigenstates $\nu_e^0$ and $\nu_\mu^0$ mix with the singlet neutrinos, the electron and muon will couple to states which are linear combinations of the two light and the one heavy states. However, in low-energy weak interactions, the neutrinos which are produced are just the light pieces of $\nu_e^0$ or $\nu_\mu^0$. These effective light states, that is, those states obtained by projecting out the light pieces of $\nu_e^0$ and $\nu_\mu^0$, will not, in general, be orthogonal, even if the $\nu_i$ are massless. Therefore, there is a nonzero amplitude for both the electron and muon to couple to the same light-mass eigenstate. In this case, there is the possibility that lepton-number violation can occur in muon decay.

We have therefore recalculated the muon decay parameters in terms of the most general lepton-number-nonconserving couplings. We will first review the results of the lepton-number-conserving (LNC) calculation and the limits on the coupling constants from both muon decay and inverse muon decay. We then present the calculation of muon decay with lepton-number violation, and discuss differences between it and the LNC calculation. We will show that the form of the muon decay distribution in the lepton-number-nonconserving case is identical to that of the LNC case, and that there is a one-to-one correspondence between the parameters of the LNC case and sets of indistinguishable parameters in the lepton-number-nonconserving case. This implies that it is not possible, even in principle, to test lepton-number conservation in muon decay (assuming massless, unobserved final neutrinos, and that higher-order effects can be neglected). We will then see that, except for the possibility of fine-tuned cancellations, the previous analyses of muon decay can be used to put constraints on the lepton-number-violating coupling constants. Finally, we discuss the constraints that inverse muon decay puts on the coupling constants in the lepton-number-nonconserving case.

The most general lepton-number-conserving (LNC) four-fermion interaction can be written in the helicity-projection form[1] as

$$\mathcal{H}=\frac{4G_0}{\sqrt{2}}(g^{++}\bar{e}_L\nu_{eR}\bar{\nu}_{\mu L}\mu_R+g^{-+}\bar{e}_R\nu_{eL}\bar{\nu}_{\mu L}\mu_R+g^{+-}\bar{e}_L\nu_{eR}\bar{\nu}_{\mu R}\mu_L+g^{--}\bar{e}_R\nu_{eL}\bar{\nu}_{\mu R}\mu_L$$
$$+g^{LL}\bar{e}_L\gamma^\lambda\nu_{eL}\bar{\nu}_{\mu L}\gamma_\lambda\mu_L+g^{RL}\bar{e}_R\gamma^\lambda\nu_{eR}\bar{\nu}_{\mu L}\gamma_\lambda\mu_L+g^{LR}\bar{e}_L\gamma^\lambda\nu_{eL}\bar{\nu}_{\mu R}\gamma_\lambda\mu_R$$
$$+g^{RR}\bar{e}_R\gamma^\lambda\nu_{eR}\bar{\nu}_{\mu R}\gamma_\lambda\mu_R+g^{T+}\bar{e}_L t^{\alpha\beta}\nu_{eR}\bar{\nu}_{\mu L}t_{\alpha\beta}\mu_R+g^{T-}\bar{e}_R t^{\alpha\beta}\nu_{eL}\bar{\nu}_{\mu R}t_{\alpha\beta}\mu_L)\ , \quad (1)$$

where the subscript $L$ [$R$] denotes multiplication by $\frac{1}{2}(1-\gamma_5)$ [$\frac{1}{2}(1+\gamma_5)$], and $t_{\alpha\beta}=(i/2\sqrt{2})[\gamma_\alpha,\gamma_\beta]$. The standard-model limit is obtained by taking $g^{LL}=1$, and all other $g$'s equal to zero. One can see that there will be interference between certain terms in Eq. (1). For example, since the $g^{LR}$, $g^{T-}$, and $g^{--}$ terms all create a right-handed muon neutrino and a right-handed electron antineutrino, they will interfere. Similar statements apply to ($g^{RL}$,$g^{T+}$,$g^{++}$), ($g^{LL}$,$g^{-+}$), and ($g^{RR}$,$g^{+-}$). It has been shown[2] that if the neutrino masses are neglected, then all electron observables can be expressed in terms of the following ten real constants:

$$\left.\begin{matrix} a \\ a' \end{matrix}\right\}=16(|g^{RL}|^2\pm|g^{LR}|^2) +(|g^{--}+6g^{T-}|^2\pm|g^{++}+6g^{T+}|^2)\ , \quad (2)$$

$$\left.\begin{matrix} b \\ b' \end{matrix}\right\}=4(|g^{RR}|^2\pm|g^{LL}|^2)+(g|^{-+}|^2\pm|g^{+-}|^2)\ , \quad (3)$$

$$\left.\begin{matrix} c \\ c' \end{matrix}\right\}=\tfrac{1}{2}(|g^{--}-2g^{T-}|^2\pm|g^{++}-2g^{T+}|^2)\ , \quad (4)$$

$$\left.\begin{matrix} \alpha \\ \alpha' \end{matrix}\right\}=8\left\{\begin{matrix} \mathrm{Re} \\ \mathrm{Im} \end{matrix}\right\}[g^{LR}(g^{--}+6g^{T-})^* +g^{RL*}(g^{++}+6g^{T+})]\ , \quad (5)$$

$$\left.\begin{matrix} \beta \\ \beta' \end{matrix}\right\}=-4\left\{\begin{matrix} \mathrm{Re} \\ \mathrm{Im} \end{matrix}\right\}(g^{LL}g^{-+*}+g^{RR*}g^{+-})\ . \quad (6)$$

Of these ten quantities, only the transverse-polarization parameters $\alpha$, $\alpha'$, $\beta$, and $\beta'$ are measurable directly. The other six are measured indirectly through the measurements of the following observables: $A$, which is related to the muon lifetime; the spectrum parameters $\delta$ and $\rho$; the end-point asymmetry $\xi\delta/\rho$; the longitudinal electron polarization $\xi'$; and the angular dependence of the longitudinal electron polarization $\xi''$. These can be written as

$$A=a+4b+6c\ , \quad (7)$$

$$\rho=\frac{1}{A}(3b+6c)\ , \quad (8)$$

$$\delta=\frac{3b'-6c'}{3a'+4b'-14c'}\ , \quad (9)$$

$$\frac{\xi\delta}{\rho}=\frac{-3b'+6c'}{3b+6c}\ , \quad (10)$$

$$\xi'=-\frac{1}{A}(a'+4b'+6c')\ , \quad (11)$$

$$\xi''=\frac{1}{A}(3a+4b-14c)\ . \quad (12)$$

One point is worth noting. If one considers only $V$ and $A$ interactions, as is the case relevant to the mixing of ordinary fermions with exotic fermions, for example, examination of Eqs. (5) and (6) shows that the transverse-polarization pieces $\alpha,\alpha'\beta,\beta'$, which have been measured rather precisely, are predicted to vanish. However, as we will see, because of Fierz transformations, $(V,A)$ terms are related to $(S,P)$ terms. Therefore the meaning of "considering only $V$ and $A$ interactions" must be more clearly defined. We will address this point in more detail when we consider lepton-number-nonconserving interactions.

Fetscher, Gerber, and Johnson[7] have obtained limits on all the LNC coupling constants by considering the chiralities of the decaying muon and the produced electron. After absorbing the overall strength of the interaction into the Fermi constant (which is determined by the total rate), one can define renormalized couplings for which the common factor $A$ [Eq. (7)] is 16:

$$A\equiv4(|g^{++}|^2+|g^{+-}|^2+|g^{-+}|^2+|g^{--}|^2) +16(|g^{LL}|^2+|g^{LR}|^2+|g^{RL}|^2+|g^{RR}|^2) +48(|g^{T+}|^2+|g^{T-}|^2)=16\ . \quad (13)$$

The probability of producing an $\epsilon$-handed electron in the decay of a $\mu$-handed muon ($\epsilon,\mu=L,R$) is $Q_{\epsilon\mu}$, where

$$Q_{RR}=2(b+b')/A=\tfrac{1}{4}|g^{-+}|^2+|g^{RR}|^2\ , \quad (14)$$

$$Q_{LR}=[(a-a')+6(c-c')]/2A =\tfrac{1}{4}|g^{++}|^2+|g^{LR}|^2+3|g^{T+}|^2\ , \quad (15)$$

$$Q_{RL}=[(a+a')+6(c+c')]/2A =\tfrac{1}{4}|g^{--}|^2+|g^{RL}|^2+3|g^{T-}|^2\ , \quad (16)$$

$$Q_{LL}=2(b-b')/A=\tfrac{1}{4}|g^{+-}|^2+|g^{LL}|^2\ . \quad (17)$$

Since all the $Q_{\epsilon\mu}$ are positive definite and $\sum_{\epsilon\mu}Q_{\epsilon\mu}=1$, the authors of Ref. 7 were able to put limits on all the parameters using the current experimental data for muon decay,[8] along with inverse muon decay.[9] These measurements give strong upper limits on the $Q_{RR}$, $Q_{LR}$, and $Q_{RL}$, which in turn give upper limits on the squares of the coupling constants in (14)–(16). For $Q_{LL}$ there is a good lower limit, and $|g^{+-}|^2$ is separated from $|g^{LL}|^2$ in the following way. The helicity of the $\nu_\mu$ in pion decay has been measured to be $-1$ to excellent precision.[10] Thus the $g^{+-}$ term does not contribute to inverse muon decay $\nu_\mu e^- \to \mu^- \nu_e$ (the $g^{+-}$ interaction requires an incoming $\nu_\mu$ of helicity $+1$), and the measurement of inverse muon decay therefore puts a lower limit on $|g^{LL}|^2$ alone. $Q_{LL} \le 1$ gives the corresponding upper limit on $|g^{+-}|^2$. The limits on the coupling constants are listed in Table I, which is taken from Ref. 7.

We now consider the possibility of lepton-number-nonconserving interactions. We allow both lepton-flavor violation and total-lepton-number violation ($\Delta L = 2$, Majorana mass terms). It is convenient, when dealing with Majorana neutrinos, to denote all left-handed states $n_L$ and all right-handed states $n_R^c$. The two are not independent, but are essentially *CP* conjugates, being related by

$$n_R^c = C \bar{n}_L^T \,, \tag{18}$$

where $C$ is the charge-conjugation matrix. In the usual case, $n_L$ and $n_R^c$ refer to left-handed neutrinos and right-handed antineutrinos, respectively. When considering Majorana neutrinos, however, the superscript $c$ simply means that the two are *CP* conjugates. It does *not* mean that the $n_L$ are leptons and the $n_R^c$ are antileptons; there is no distinction for Majorana neutrinos. For the special case in which there is a distinction (i.e., Dirac masses only), $n_L$ refers to both left-handed leptons and antileptons. [For example, a single Dirac neutrino has (in conventional notation) the four components $\nu_L \leftrightarrow \nu_R^c$ (doublets) and $N_R \leftrightarrow N_L^c$ (singlets), where $\nu_L$ and $N_R$ are leptons and $\nu_R^c, N_L^c$ are antileptons. In our notation we would denote $(\nu_L, N_L^c) \to (n_{1L}, n_{2L})$ and $(\nu_R^c, N_R) \to (n_{1R}^c, n_{2R}^c)$.] This can lead to some confusion when considering the lepton-number-conserving limit, and we will discuss this in more detail below.

The lepton-number-nonconserving Hamiltonian is then written as

$$\begin{aligned}\mathcal{H} = \frac{4G_0}{\sqrt{2}} \sum_{i,j} (&g_{ij}^{++} \bar{e}_L n_{iR}^c \bar{n}_{jL} \mu_R + g_{ij}^{-+} \bar{e}_R n_{iL} \bar{n}_{jL} \mu_R + g_{ij}^{+-} \bar{e}_L n_{iR}^c \bar{n}_{jR}^c \mu_L \\ &+ g_{ij}^{--} \bar{e}_R n_{iL} \bar{n}_{jR}^c \mu_L + g_{ij}^{LL} \bar{e}_L \gamma^\lambda n_{iL} \bar{n}_{jL} \gamma_\lambda \mu_L + g_{ij}^{RL} \bar{e}_R \gamma^\lambda n_{iR}^c \bar{n}_{jL} \gamma_\lambda \mu_L \\ &+ g_{ij}^{LR} \bar{e}_L \gamma^\lambda n_{iL} \bar{n}_{jR}^c \gamma_\lambda \mu_R + g_{ij}^{RR} \bar{e}_R \gamma^\lambda n_{iR}^c \bar{n}_{jR}^c \gamma_\lambda \mu_R + g_{ij}^{T+} \bar{e}_L t^{\alpha\beta} n_{iR}^c \bar{n}_{jL} t_{\alpha\beta} \mu_R + g_{ij}^{T-} \bar{e}_R t^{\alpha\beta} n_{iL} \bar{n}_{jR}^c t_{\alpha\beta} \mu_L) \,,\end{aligned} \tag{19}$$

where $i,j$ run over all the light neutrino species. Just as in the LNC case, certain terms (e.g., $g_{ij}^{LR}$, $g_{ij}^{T-}$, and $g_{ij}^{--}$) interfere with one another. In addition, there are new types of interference terms due to the Majorana nature of the neutrinos. For example, the $g_{ij}^{RL}$ term produces two left-handed states, so that, for a given $i,j$, a $g_{ij}^{RL}$ term will interfere with its corresponding $g_{ji}^{RL}$ term. This same sort of "internal interference" occurs also for the $g_{ij}^{++}$, $g_{ij}^{T+}$, $g_{ij}^{LR}$, $g_{ij}^{--}$, and $g_{ij}^{T-}$ terms. Also, since the $g_{ji}^{+-}$, $g_{ij}^{-+}$, $g_{ij}^{LL}$, and $g_{ji}^{RR}$ terms all produce one left-handed $n_{iL}$ and one right-handed $n_{jR}^c$, they will all interfere (in the LNC case, the only interference terms were $g^{LL}$-$g^{-+}$ and $g^{RR}$-$g^{+-}$). Therefore, at first sight one expects the expression for the decay rate to be very much more complicated than in the LNC case. However, we will see that this is not the case.

TABLE I. 90%-C.L. limits on the coupling constants in lepton-number-conserving muon decay, from Fetscher, Gerber, and Johnson (Ref. 7). The data from both muon decay and inverse muon decay are used to obtain the limits.

| | |
|---|---|
| Muon decay | |
| $Q_{RR} < 2.0 \times 10^{-3}$ | $\lvert g^{-+}\rvert < 9.1 \times 10^{-2}$ |
| | $\lvert g^{RR}\rvert < 4.5 \times 10^{-2}$ |
| $Q_{LR} < 3.9 \times 10^{-3}$ | $\lvert g^{++}\rvert < 0.137$ |
| | $\lvert g^{LR}\rvert < 6.2 \times 10^{-2}$ |
| | $\lvert g^{T+}\rvert < 4.0 \times 10^{-2}$ |
| $Q_{RL} < 4.5 \times 10^{-2}$ | $\lvert g^{--}\rvert < 0.448$ |
| | $\lvert g^{RL}\rvert < 0.114$ |
| | $\lvert g^{T-}\rvert < 0.112$ |
| $Q_{LL} > 0.949$ | |
| Inverse muon decay | |
| | $\lvert g^{+-}\rvert < 0.961$ |
| | $\lvert g^{LL}\rvert > 0.877$ |

The lepton-number-conserving limit is complicated somewhat by the notation for the neutrinos. However, it can be seen by writing out the components of $n_L$ and $n_R^c$ explicitly for the case in which all neutrinos are Dirac: $n_L = (\nu_{eL}, \nu_{eL}^c, \nu_{\mu L}, \nu_{\mu L}^c, \ldots)$, $n_R^c = (\nu_{eR}^c, \nu_{eR}, \nu_{\mu R}^c, \nu_{\mu R}, \ldots)$. By comparing (1) and (19), it is seen that the lepton-number-conserving limit is recovered by taking $g_{23}^{++}$, $g_{23}^{RL}$, $g_{23}^{T+}$, $g_{14}^{--}$, $g_{14}^{LR}$, $g_{14}^{T-}$, $g_{13}^{LL}$, $g_{24}^{+-}$, $g_{24}^{RR}$, and $g_{13}^{-+}$ nonzero, with all other coupling constants set equal to zero.

We have found by explicit calculation that the full expression for muon decay in the lepton-number-nonconserving case (for unobserved massless final neutrinos[11]) is identical in form to the LNC case—i.e., there are no new observables, even though there are enough appropriate Lorentz invariants to construct them. In particular, all electron observables can again be expressed in terms of $a$, $a'$, $b$, $b'$, $c$, $c'$, $\alpha$, $\alpha'$, $\beta$, and $\beta'$, which have physical meanings identical to the LNC case. The parameters $a$, $a'$, $b$, $b'$, $c$, and $c'$ have similar forms to the LNC case:

$$\left.\begin{matrix} a \\ a' \end{matrix}\right\} = 16(|\tilde{g}^{RL}|^2 \pm |\tilde{g}^{LR}|^2) + (|\tilde{g}_1^{T---}|^2 \pm |\tilde{g}_1^{T+++}|^2) , \tag{20}$$

$$\left.\begin{matrix} b \\ b' \end{matrix}\right\} = 4(|\tilde{g}^{RR-+}|^2 \pm |\tilde{g}^{LL+-}|^2) , \tag{21}$$

$$\left.\begin{matrix} c \\ c' \end{matrix}\right\} = \tfrac{1}{2}(|\tilde{g}_2^{T---}|^2 \pm |\tilde{g}_2^{T+++}|^2) , \tag{22}$$

where

$$\begin{aligned} |\tilde{g}^{LL+-}|^2 &\equiv \sum_{i,j} |g_{ij}^{LL} + \tfrac{1}{2} g_{ji}^{+-}|^2 , \\ |\tilde{g}^{RR-+}|^2 &\equiv \sum_{i,j} |g_{ij}^{RR} + \tfrac{1}{2} g_{ji}^{-+}|^2 , \end{aligned} \tag{23}$$

and

$$\begin{aligned} |\tilde{g}^{LR}|^2 &\equiv \tfrac{1}{2} \sum_{i,j} |g_{ij}^{LR} + g_{ji}^{LR}|^2 , \\ |\tilde{g}^{RL}|^2 &\equiv \tfrac{1}{2} \sum_{i,j} |g_{ij}^{RL} + g_{ji}^{RL}|^2 , \\ |\tilde{g}_1^{T---}|^2 &\equiv \tfrac{1}{2} \sum_{i,j} |g_{ij}^{--} + g_{ji}^{--} + 6g_{ij}^{T-} + 6g_{ji}^{T-}|^2 , \\ |\tilde{g}_1^{T+++}|^2 &\equiv \tfrac{1}{2} \sum_{i,j} |g_{ij}^{++} + g_{ji}^{++} + 6g_{ij}^{T+} + 6g_{ji}^{T+}|^2 , \\ |\tilde{g}_2^{T---}|^2 &\equiv \tfrac{1}{2} \sum_{i,j} |g_{ij}^{--} - g_{ji}^{--} - 2g_{ij}^{T-} + 2g_{ji}^{T-}|^2 , \\ |\tilde{g}_2^{T+++}|^2 &\equiv \tfrac{1}{2} \sum_{i,j} |g_{ij}^{++} - g_{ji}^{++} - 2g_{ij}^{T+} + 2g_{ji}^{T+}|^2 . \end{aligned} \tag{24}$$

In Eq. (24), the origin of the factor $\frac{1}{2}$ in front of the sums is different for $i \neq j$ and for $i=j$. For $i \neq j$, the factor comes about because of double counting in the unrestricted sum; for $i=j$, it is due to the presence of identical fermions in the final state. (The factor of 2 in the amplitude for $i=j$ is because there are two distinct but numerically equal diagrams.)

The transverse-polarization pieces also have a similar form to the LNC case:

$$\left.\begin{matrix} \alpha \\ \alpha' \end{matrix}\right\} = 4 \sum_{i,j} \left\{\begin{matrix} \mathrm{Re} \\ \mathrm{Im} \end{matrix}\right\} [(g_{ij}^{LR} + g_{ji}^{LR})(g_{ij}^{--} + g_{ji}^{--} + 6g_{ij}^{T-} + 6g_{ji}^{T-})^* + (g_{ij}^{RL} + g_{ji}^{RL})^*(g_{ij}^{++} + g_{ji}^{++} + 6g_{ij}^{T+} + 6g_{ji}^{T+})] , \tag{25}$$

$$\left.\begin{matrix} \beta \\ \beta' \end{matrix}\right\} = -8 \sum_{i,j} \left\{\begin{matrix} \mathrm{Re} \\ \mathrm{Im} \end{matrix}\right\} [(g_{ij}^{LL} + \tfrac{1}{2} g_{ji}^{+-})(g_{ji}^{RR} + \tfrac{1}{2} g_{ij}^{-+})^*] . \tag{26}$$

By comparison of Eqs. (2)–(6) and (20)–(22), (25), and (26), it is clear that there are great similarities between the lepton-number-violating and the lepton-number-conserving cases. We now demonstrate explicitly that there is a 1-1 correspondence (a homomorphism) between the parameters in the LNC case and sets of parameters in the lepton-number-nonconserving case.

First of all, $\alpha$ and $\alpha'$ in the LNC case are given by

$$\left.\begin{matrix} \alpha \\ \alpha' \end{matrix}\right\} = 8|g^{LR}(g^{--} + 6g^{T-})^* + g^{RL*}(g^{++} + 6g^{T+})| \left\{\begin{matrix} \cos\theta_\alpha \\ \sin\theta_\alpha \end{matrix}\right\} , \tag{27}$$

where $\theta_\alpha$ is the phase of

$$[g^{LR}(g^{--} + 6g^{T-})^* + g^{RL*}(g^{++} + 6g^{T+})] .$$

Using the triangle inequality, this can be written

$$\left.\begin{matrix} \alpha \\ \alpha' \end{matrix}\right\} = 8(|g^{LR}||g^{--} + 6g^{T-}| + |g^{RL}||g^{++} + 6g^{T+}|) \left\{\begin{matrix} \cos\theta_\alpha \\ \sin\theta_\alpha \end{matrix}\right\} \cos\phi_\alpha , \tag{28}$$

where $\cos\theta_\alpha$ is positive, and is given by

$$\cos\phi_\alpha \equiv \frac{|g^{LR}(g^{--} + 6g^{T-})^* + g^{RL*}(g^{++} + 6g^{T+})|}{[|g^{LR}||g^{--} + 6g^{T-}| + |g^{RL}||g^{++} + 6g^{T+}|]} . \tag{29}$$

Similarly, $\beta$ and $\beta'$ in the LNC case can be written

$$\left.\begin{matrix} \beta \\ \beta' \end{matrix}\right\} = -4(|g^{LL}||g^{-+}| + |g^{RR}||g^{+-}|) \left\{\begin{matrix} \cos\theta_\beta \\ \sin\theta_\beta \end{matrix}\right\} \cos\phi_\beta . \tag{30}$$

For the lepton-number-nonconserving case, we must also use the Schwarz inequality, which reads

$$\left| \sum_i x_i^* y_i \right|^2 \leq \sum_i |x_i|^2 \sum_i |y_i|^2 , \tag{31}$$

for the complex numbers $x_i$ and $y_i$. Using the Schwarz and triangle inequalities, $\alpha$ and $\alpha'$ [Eq. (25)] can be written

$$\left.\begin{matrix} \alpha \\ \alpha' \end{matrix}\right\} = 8(|\tilde{g}^{LR}||\tilde{g}_1^{T---}| + |\tilde{g}^{RL}||\tilde{g}_1^{T+++}|) \times \left\{\begin{matrix} \cos\tilde{\theta}_\alpha \\ \sin\tilde{\theta}_\alpha \end{matrix}\right\} \cos\tilde{\phi}_\alpha , \tag{32}$$

where $|\tilde{g}^{LR}|$, $|\tilde{g}_1^{T---}|$, $|\tilde{g}^{RL}|$, and $|\tilde{g}_1^{T+++}|$ are defined in Eq. (24). For $\beta$ and $\beta'$ in the lepton-number-violating case, the situation is a bit more complicated. We must divide up the $LL+-$ and $RR-+$ coupling constants into two pieces:

$$\begin{aligned} |\tilde{g}^{LL+-}|^2 &= |\tilde{g}_1^{LL+-}|^2 + |\tilde{g}_2^{LL+-}|^2 , \\ |\tilde{g}^{RR-+}|^2 &= |\tilde{g}_1^{RR-+}|^2 + |\tilde{g}_2^{RR-+}|^2 , \end{aligned} \tag{33}$$

where

$$\begin{aligned} |\tilde{g}_1^{LL+-}|^2 &\equiv \sum_{i\le j} |g_{ij}^{LL} + \tfrac{1}{2} g_{ji}^{+-}|^2 , \\ |\tilde{g}_2^{LL+-}|^2 &\equiv \sum_{i>j} |g_{ij}^{LL} + \tfrac{1}{2} g_{ji}^{+-}|^2 , \\ |\tilde{g}_1^{RR-+}|^2 &\equiv \sum_{i<j} |g_{ij}^{RR} + \tfrac{1}{2} g_{ji}^{-+}|^2 , \\ |\tilde{g}_2^{RR-+}|^2 &\equiv \sum_{i\ge j} |g_{ij}^{RR} + \tfrac{1}{2} g_{ji}^{-+}|^2 . \end{aligned} \tag{34}$$

Using these and the Schwarz and triangle inequalities, Eq. (26) can be rewritten as

$$\left.\begin{matrix}\beta \\ \beta'\end{matrix}\right\} = -8(|\tilde{g}_1^{LL+-}||\tilde{g}_2^{RR-+}| + |\tilde{g}_2^{LL+-}||\tilde{g}_1^{RR-+}|) \times \left\{\begin{matrix}\cos\tilde{\theta}_\beta \\ \sin\tilde{\theta}_\beta\end{matrix}\right\} \cos\phi_\beta . \tag{35}$$

The homomorphism relating the LNC coupling constants with sets of lepton-number-nonconserving parameters is now evident. It is

$$\begin{aligned} & |g^{LR}|^2 \leftrightarrow |\tilde{g}^{LR}|^2, \quad |g^{RL}|^2 \leftrightarrow |\tilde{g}^{RL}|^2 , \\ & |g^{++} + 6g^{T+}|^2 \leftrightarrow |\tilde{g}_1^{T+++}|^2, \\ & |g^{--} + 6g^{T-}|^2 \leftrightarrow |\tilde{g}_1^{T---}|^2 , \\ & |g^{++} - 2g^{T+}|^2 \leftrightarrow |\tilde{g}_2^{T+++}|^2 , \\ & |g^{--} - 2g^{T-}|^2 \leftrightarrow |\tilde{g}_2^{T---}|^2 , \\ & |g^{LL}|^2 \leftrightarrow |\tilde{g}_1^{LL+-}|^2, \quad |g^{RR}|^2 \leftrightarrow |\tilde{g}_1^{RR-+}|^2 , \\ & \tfrac{1}{4}|g^{+-}|^2 \leftrightarrow |\tilde{g}_2^{LL+-}|^2, \quad \tfrac{1}{4}|g^{-+}|^2 \leftrightarrow |\tilde{g}_2^{RR-+}|^2 . \end{aligned} \tag{36}$$

In addition, the various angles for the transverse-polarization pieces must be mapped into one another:

$$\begin{aligned} & \theta_\alpha \leftrightarrow \tilde{\theta}_\alpha, \quad \theta_\beta \leftrightarrow \tilde{\theta}_\beta , \\ & \phi_\alpha \leftrightarrow \tilde{\phi}_\alpha, \quad \phi_\beta \leftrightarrow \tilde{\phi}_\beta . \end{aligned} \tag{37}$$

An interpretation of the homomorphism is that there are classes of couplings which yield indistinguishable results for the electron observables in muon decay, and each class contains one set of LNC parameters [the mapping in (36) and (37) is just the identity in this case]. It is therefore not possible, even in principle, to distinguish between the two cases using only muon decay.

Actually, with the benefit of hindsight, it is possible to see this correspondence at the Hamiltonian level. The only complication in going from the LNC case to the lepton-number-nonconserving case is the interference between the $g_{ji}$ and $g_{ij}$ pieces. This complication can be removed through the use of Fierz transformations.

Consider first the $RL$ pieces. A $g_{ji}^{RL}$ term can be written using Fierz transformations (see the Appendix) as

$$g_{ji}^{RL} \bar{e}_R \gamma^\lambda n_{jR}^c \bar{n}_{iL} \gamma_\lambda \mu_L = g_{ji}^{RL} \bar{e}_R \gamma^\lambda n_{iR}^c \bar{n}_{jL} \gamma_\lambda \mu_L , \tag{38}$$

which has the same form as a $g_{ij}^{RL}$ terms. Therefore, in the Hamiltonian, the $RL$ pieces can be written

$$\sum_{i<j} (g_{ij}^{RL} + g_{ji}^{RL}) \bar{e}_R \gamma^\lambda n_{iR}^c \bar{n}_{jL} \gamma_\lambda \mu_L + \sum_i g_{ii}^{RL} \bar{e}_R \gamma^\lambda n_{iR}^c \bar{n}_{iL} \gamma_\lambda \mu_L . \tag{39}$$

For $i \ne j$, there is no longer any "internal interference," and the calculation of the contribution of this term proceeds just as in the LNC case. This gives a coefficient

$$\sum_{i<j} |g_{ij}^{RL} + g_{ji}^{RL}|^2 . \tag{40}$$

For $i=j$, there are two distinct diagrams which contribute to the amplitude. However, because of Fierz transformations, these are equal. In addition, there is a factor $\frac{1}{2}$ due to the presence of identical particles in the final state. Therefore the coefficient of this term is

$$\tfrac{1}{2} \sum_i |2g_{ii}^{RL}|^2 . \tag{41}$$

Combining (40) and (41) and allowing an unrestricted sum gives the coefficient

$$\tfrac{1}{2} \sum_{i,j} |g_{ij}^{RL} + g_{ji}^{RL}|^2 , \tag{42}$$

which is to be mapped onto $|g^{RL}|^2$ in the LNC case. A similar analysis follows for the $LR$ pieces.

For the $--$ and $T-$ pieces, the Fierz transformations are more complicated, but the calculation proceeds in a similar way. We first reexamine the LNC case. The terms

$$g^{--} \bar{e}_R \nu_{eL} \bar{\nu}_{\mu R} \mu_L + g^{T-} \bar{e}_R t^{\alpha\beta} \nu_{eL} \bar{\nu}_{\mu R} t_{\alpha\beta} \mu_L \tag{43}$$

can be written as

$$(g^{--} + 6g^{T-})(\tfrac{1}{4} \bar{e}_R \nu_{eL} \bar{\nu}_{\mu R} \mu_L + \tfrac{1}{8} \bar{e}_R t^{\alpha\beta} \nu_{eL} \bar{\nu}_{\mu R} t_{\alpha\beta} \mu_L) + (g^{--} - 2g^{T-})(\tfrac{3}{4} \bar{e}_R \nu_{eL} \bar{\nu}_{\mu R} \mu_L - \tfrac{1}{8} \bar{e}_R t^{\alpha\beta} \nu_{eL} \bar{\nu}_{\mu R} t_{\alpha\beta} \mu_L) . \tag{44}$$

It has been shown by explicit calculation in the LNC case that the above two terms do not interfere. For the lepton-number-nonconserving case, the $g_{ji}^{--}$ and $g_{ji}^{T-}$ pieces have the following Fierz transformations (see the Appendix):

$$\begin{aligned} g_{ji}^{--} \bar{e}_R n_{jL} \bar{n}_{iR}^c \mu_L &= g_{ji}^{--} (-\tfrac{1}{2} \bar{e}_R n_{iL} \bar{n}_{jR}^c \mu_L + \tfrac{1}{4} \bar{e}_R t^{\alpha\beta} n_{iL} \bar{n}_{jR}^c t_{\alpha\beta} \mu_L) , \\ g_{ji}^{T-} \bar{e}_R t^{\alpha\beta} n_{jL} \bar{n}_{iR}^c t_{\alpha\beta} \mu_L &= g_{ji}^{T-} (3 \bar{e}_R n_{iL} \bar{n}_{jR}^c \mu_L + \tfrac{1}{2} \bar{e}_R t^{\alpha\beta} n_{iL} \bar{n}_{jR}^c t_{\alpha\beta} \mu_L) . \end{aligned} \tag{45}$$

Therefore, the $--$ and $T-$ pieces in the Hamiltonian

$$g_{ij}^{--}\bar{e}_R n_{iL}\bar{n}_{jR}^c\mu_L+g_{ij}^{T-}\bar{e}_R t^{\alpha\beta}n_{iL}\bar{n}_{jR}^c t_{\alpha\beta}\mu_L+g_{ji}^{--}\bar{e}_R n_{jL}\bar{n}_{iR}^c\mu_L+g_{ji}^{T-}\bar{e}_R t^{\alpha\beta}n_{jL}\bar{n}_{iR}^c t_{\alpha\beta}\mu_L \ , \tag{46}$$

can be written

$$(g_{ij}^{--}+g_{ji}^{--}+6g_{ij}^{T-}+6g_{ji}^{T-})S_{ij}^{T---} +(g_{ij}^{--}-g_{ji}^{--}-2g_{ij}^{T-}+2g_{ji}^{T-})A_{ij}^{T---} \ , \tag{47}$$

where we have used Eq. (45), and

$$\begin{aligned} S_{ij}^{T---}&=\tfrac{1}{4}\bar{e}_R n_{iL}\bar{n}_{jR}^c\mu_L+\tfrac{1}{8}\bar{e}_R t^{\alpha\beta}n_{iL}\bar{n}_{jR}^c t_{\alpha\beta}\mu_L \ , \\ A_{ij}^{T---}&=\tfrac{3}{4}\bar{e}_R n_{iL}\bar{n}_{jR}^c\mu_L-\tfrac{1}{8}\bar{e}_R t^{\alpha\beta}n_{iL}\bar{n}_{jR}^c t_{\alpha\beta}\mu_L \ . \end{aligned} \tag{48}$$

$S_{ij}^{T---}$ is symmetric in $i,j$ under a Fierz transformation, and $A_{ij}^{T---}$ is antisymmetric. Just as in the LNC case, these two terms do not interfere. The analysis now follows that of the $RL$ case. For $i<j$, there is no internal interference; for $i=j$, there are two diagrams, which are equal by the symmetry of $S_{ij}^{T---}$ (which is the only piece which contributes). There is also a factor $\frac{1}{2}$ due to identical particles in the final state. Therefore, allowing an unrestricted sum, the coefficient

$$\tfrac{1}{2}\sum_{i,j}|g_{ij}^{--}\ g_{ji}^{--}+6g_{ij}^{T-}+6g_{ji}^{T-}|^2 \tag{49}$$

is mapped onto $|g^{--}+6g^{T-}|^2$ in the LNC case;

$$\tfrac{1}{2}\sum_{i,j}|g_{ij}^{--}-g_{ji}^{--}-2g_{ij}^{T-}+2g_{ji}^{T-}|^2 \tag{50}$$

is mapped onto $|g^{--}-2g^{T-}|^2$. The analysis is identical for the $++$ and $T+$ pieces.

It will be useful in the following to rewrite Eq (47) as

$$\tilde{g}_{ij}^{--}\bar{e}_R n_{iL}\bar{n}_{jR}^c\mu_L+\tilde{g}_{ij}^{T-}\bar{e}_R t^{\alpha\beta}n_{iL}\bar{n}_{jR}^c t_{\alpha\beta}\mu_L \ , \tag{51}$$

where

$$\begin{aligned} \tilde{g}_{ij}^{--}&=g_{ij}^{--}-\tfrac{1}{2}g_{ji}^{--}+3g_{ji}^{T-} \ , \\ \tilde{g}_{ij}^{T-}&=\tfrac{1}{4}g_{ji}^{--}+g_{ij}^{T-}+\tfrac{1}{2}g_{ji}^{T-} \ , \end{aligned} \tag{52}$$

with similar definitions for $\tilde{g}_{ij}^{++}$ and $\tilde{g}_{ij}^{T+}$. [The form of Eq. (51) and (52) follow directly from the Fierz transformations in Eq. (45).] Using these variables, it is clear that an equivalent statement of the homomorphism in Eq. (36) is

$$\begin{aligned} &|g^{++}|^2\leftrightarrow|\tilde{g}^{++}|^2, \quad |g^{--}|^2\leftrightarrow|\tilde{g}^{--}|^2 \ , \\ &|g^{T+}|^2\leftrightarrow|\tilde{g}^{T+}|^2, \quad |g^{T-}|^2\leftrightarrow|\tilde{g}^{T-}|^2 \ , \end{aligned} \tag{53}$$

where

$$\begin{aligned} &|\tilde{g}^{++}|^2=\tfrac{1}{2}\sum_{i,j}|\tilde{g}_{ij}^{++}|^2, \quad |\tilde{g}^{--}|^2=\tfrac{1}{2}\sum_{i,j}|\tilde{g}_{ij}^{--}|^2 \ , \\ &|\tilde{g}^{T+}|^2=\tfrac{1}{2}\sum_{i,j}|\tilde{g}_{ij}^{T+}|^2, \quad |\tilde{g}^{T-}|^2=\tfrac{1}{2}\sum_{i,j}|\tilde{g}_{ij}^{T-}|^2 \ . \end{aligned} \tag{54}$$

For the $LL$ and $+-$ pieces, there is no internal interference. However, because of the Fierz transformations, one can write

$$g_{ji}^{+-}\bar{e}_L n_{jR}^c\bar{n}_{iR}^c\mu_L=\tfrac{1}{2}g_{ji}^{+-}\bar{e}_L\gamma^\lambda n_{iL}\bar{n}_{jL}\gamma_\lambda\mu_L \ , \tag{55}$$

which has the same form as the $g_{ij}^{LL}$ term. Similarly, the $g_{ji}^{-+}$ pieces can be transformed into the same form as the $g_{ij}^{RR}$ terms. From this it is clear that only the combinations

$$\sum_{i,j}|g_{ij}^{LL}+\tfrac{1}{2}g_{ji}^{+-}|^2, \quad \sum_{i,j}|g_{ij}^{RR}+\tfrac{1}{2}g_{ji}^{-+}|^2 \tag{56}$$

will appear in the expressions in the lepton-number-nonconserving case. To complete the homomorphism, it is necessary to separate these combinations as in Eq. (34), and to relate them to $|g^{LL}|^2$, $|g^{+-}|^2$, $|g^{RR}|^2$, and $|g^{-+}|^2$ in the LNC case. As was already derived, the correspondence follows directly. It is therefore possible to see the homomorphism at the Hamiltonian level, although this was done *a posteriori*.

One point should be noted. An examination of Eq. (26) appears to indicate that, unlike the LNC case, $\beta$ and $\beta'$ are nonzero when one considers only $V$ and $A$ interactions. However, Eq. (55) shows that, because of Fierz transformations, the lepton-number-nonconserving $V$ and $A$ interactions can be related to $S$ and $P$ interactions; i.e., the interactions are not well defined as $(V,A)$ in the lepton-number-nonconserving case. Even in the LNC case there is some ambiguity, as mentioned earlier. A lepton-number-conserving $(S,P)$ interaction can be written (using the Fierz transformations) as a $(V,A)$ interaction in which lepton-number conservation is *not manifest*. Therefore the correct statement concerning the transverse-polarization pieces in the LNC case is "when one considers only $V$ and $A$ interactions *with manifest lepton-number conservation*, the transverse-polarization pieces vanish."

Because of the homomorphism, it is possible to use the analysis of Ref. 7 to put limits on certain combinations of parameters in the lepton-number-violating case. As in the lepton-number-conserving case, we normalize the coupling constants:

$$\begin{aligned} A\equiv 16(&|\tilde{g}_1^{RR\,-+}|^2+|\tilde{g}_2^{RR\,-+}|^2+|\tilde{g}_1^{LL\,+-}|^2 \\ &+|\tilde{g}_2^{LL\,+-}|^2+|\tilde{g}^{RL}|^2+|\tilde{g}^{LR}|^2) \\ &+4(|\tilde{g}^{--}|^2+|\tilde{g}^{++}|^2)+48(|\tilde{g}^{T-}|^2+|\tilde{g}^{T+}|^2) \\ =16 \ , \end{aligned} \tag{57}$$

where we have used the variables introduced in Eqs. (52) and (54). The $Q$'s then take the form

$$Q_{RR}=|\tilde{g}_1^{RR\,-+}|^2+|\tilde{g}_2^{RR\,-+}|^2 \ , \tag{58}$$

$$Q_{LR}=|\tilde{g}^{LR}|^2+\tfrac{1}{4}|\tilde{g}^{++}|^2+3|\tilde{g}^{T+}|^2 \ , \tag{59}$$

$$Q_{RL}=|\tilde{g}^{RL}|^2+\tfrac{1}{4}|\tilde{g}^{--}|^2+3|\tilde{g}^{T-}|^2 \ , \tag{60}$$

$$Q_{LL}=|\tilde{g}_1^{LL\,+-}|^2+|\tilde{g}_2^{LL\,+-}|^2 \ . \tag{61}$$

The constraints

$$a \geq 0,\quad b \geq 0,\quad c \geq 0\ ,$$
$$a^2 \geq a'^2 + \alpha'^2 + \alpha^2,\quad b^2 \geq b'^2 + \beta'^2 + \beta^2\ ,$$
$$c^2 \geq c'^2\ , \tag{62}$$

which were used to derive the results in Table I, hold in the lepton-number-nonconserving case as well as in the LNC case.

Using Table I and the homomorphism given in Eqs. (36) and (53), it is seen that the quantities $|\tilde{g}_1^{RR-+}|^2$, $|\tilde{g}_2^{RR-+}|^2$, $|\tilde{g}^{LR}|^2$, $|\tilde{g}^{RL}|^2$, $|\tilde{g}^{++}|^2$, $|\tilde{g}^{T+}|^2$, $|\tilde{g}^{--}|^2$, and $|\tilde{g}^{T-}|^2$ are quite stringently constrained. This in turn can put quite good limits on the coupling constants themselves, except that one must allow for the possibility of fine-tuned cancellations between them.

From Table I and Eqs. (33), (34), and (61) we have

$$\sum_{i,j} |\tilde{g}_{ij}^{LL+-}|^2 \geq 0.949 \quad (90\%\ \text{C.L.})\ , \tag{63}$$

where

$$|\tilde{g}_{ij}^{LL+-}|^2 \equiv |g_{ij}^{LL} + \tfrac{1}{2} g_{ji}^{+-}|^2\ . \tag{64}$$

We can go one step further if we incorporate the experimental value $S = 0.98 \pm 0.12$ (Ref. 9), where $S$ is the total rate for inverse muon decay (normalized to the lepton-number-conserving $V-A$ case). As in the LNC case one can use the experimental fact that the incident neutrino from pion decay has helicity $-1$. Then, neglecting the small $LR$, $RL$, and $RR$ terms, one obtains

$$S = \sum_i \left| \sum_j c_j^* \tilde{g}_{ij}^{LL+-} \right|^2 = 0.98 \pm 0.12\ , \tag{65}$$

where $\nu_{\pi_{\mu\nu}} \equiv \sum_j c_j n_{jL}$ is the neutrino state produced in the decay $\pi^+ \to \mu^+ \nu_{\pi_{\mu\nu}}$. Without loss of generality we can choose a basis for the light neutrinos such that $\nu_{\pi_{\mu\nu}} \equiv n_{3L}$. Combining Eq. (65) with the fact that $Q_{LL} \leq 1$, one therefore obtains[7]

$$\sum_i |\tilde{g}_i^{LL+-}|^2 > 0.77\ ,$$
$$\sum_{i,j\neq 3} |\tilde{g}_{ij}^{LL+-}|^2 < 0.23\ . \tag{66}$$

That is, the left-handed neutrino emitted in muon decay is, to first approximation, the same as the neutrino produced in $\pi_{\mu 2}$ decay, but there is considerable room for admixtures of other states. Of course, one cannot disentangle the contributions of different $i$'s, since the final neutrino is not observed, or separate the $g_{i3}^{LL}$ and $g_{3i}^{+-}$.

In conclusion, we have calculated the parameters in muon decay using lepton-number-nonconserving interactions. We have shown that there is a one-to-one correspondence between the coupling constants in the lepton-number-conserving case, and combinations of parameters in the lepton-number-violating case. It is therefore impossible, even in principle, to test lepton number in muon decay if the final neutrinos are massless and unobserved (and if higher-order corrections can be ignored). Using this correspondence, good limits on certain combinations of the coupling constants are obtained for the cases in which a right-handed muon and/or electron is involved. If we do not allow fine-tuned cancellations to take place, the individual coupling constants are quite stringently constrained. However, we cannot discount the logical possibility that such cancellations might occur, evading the individual constraints. Combining data for muon decay with inverse muon decay, one can constrain the couplings involving left-handed muons and electrons. It is found, to first approximation, that the left-handed neutrinos emitted in muon decay is the same as that which is produced in $\pi_{\mu 2}$ decay, but there is considerable room for deviation.

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## APPENDIX

The Fierz identities useful for deriving the results in this paper are

$$\bar{\psi}_{1L}\gamma^\lambda \psi_{2L} \bar{\psi}_{3L}\gamma_\lambda \psi_{4L} = \bar{\psi}_{1L}\gamma^\lambda \psi_{4L} \bar{\psi}_{3L}\gamma_\lambda \psi_{2L}\ , \tag{A1}$$

$$\bar{\psi}_{1R}\psi_{2L}\bar{\psi}_{3L}\psi_{4R} = -\tfrac{1}{2}\bar{\psi}_{1R}\gamma^\lambda \psi_{4R}\bar{\psi}_{3L}\gamma_\lambda \psi_{2L}\ , \tag{A2}$$

$$\bar{\psi}_{1R}\psi_{2L}\bar{\psi}_{3R}\psi_{4L} = -\tfrac{1}{2}\bar{\psi}_{1R}\psi_{4L}\bar{\psi}_{3R}\psi_{2L} - \tfrac{1}{4}\bar{\psi}_{1R}t^{\alpha\beta}\psi_{4L}\bar{\psi}_{3R}t_{\alpha\beta}\psi_{2L}\ , \tag{A3}$$

where $\psi_{iL}$ and $\psi_{jR}$ are anticommuting left- and right-handed chiral fields, and $t^{\alpha\beta} = (i/2\sqrt{2})[\gamma^\alpha, \gamma^\beta]$. There is a relation analogous to (A1) with $\psi_{iL} \to \psi_{iR}$, $i = 1, \ldots, 4$. A related identity is

$$\bar{\psi}_{1R}t^{\alpha\beta}\psi_{2L}\bar{\psi}_{3L}t_{\alpha\beta}\psi_{4R} = 0\ . \tag{A4}$$

Defining the charge-conjugate fields $\psi_L^c \equiv C\bar{\psi}_R^T$, $\psi_R^c \equiv C\bar{\psi}_L^T$, where $C$ is the charge-conjugation matrix $(C\gamma_\mu C^{-1} = -\gamma_\mu^T)$, one has

$$\bar{\psi}_{1L}\gamma^\lambda \psi_{2L} = -\bar{\psi}_{2R}^c \gamma^\lambda \psi_{1R}^c\ , \tag{A5}$$

$$\bar{\psi}_{1L}\psi_{2R} = +\bar{\psi}_{2L}^c \psi_{1R}^c\ , \tag{A6}$$

$$\bar{\psi}_{1L}t^{\alpha\beta}\psi_{2R} = -\bar{\psi}_{2L}^c t^{\alpha\beta}\psi_{1R}^c\ . \tag{A7}$$

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