A TOPOLOGICAL MODEL FOR BARYON PRODUCTION IN JETS

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We present a conceptual model for baryon production in jets, inspired by the Skyrme picture of baryons as topological defects in a chiral quark–antiquark condensate. High energy collisions produce "hot" partons which split perturbatively into showers of "cool" partons which hadronize non-perturbatively. We visualize each of these as corresponding to a connected domain with a common orientation of the chiral condensate. Topological defects, namely baryons, are formed when there are mismatches in the orientations of adjacent field domains, rather as cosmic strings or monopoles are formed in the early Universe. Our model gives a good qualitative description of various salient features of baryon production in jets, which previously could be described only with a large number of free parameters. In particular, we give a qualitative explanation of the high baryon production rate in Υ decays compared to the e^+e^- continuum. When combined with a perturbative QCD parton shower Monte Carlo it could become a basis for a fully-fledged fragmentation model.

1. Introduction

Although the production of hadrons in high energy hard collisions is in principle calculable, in practice complete calculations are not yet possible. Part of the jet fragmentation process can be computed using perturbative QCD, as a shower initiated by a "hot" parton which is far off mass-shell and which branches into "cooler" partons of successively lower virtuality, until they are within ≈ 1 GeV of mass-shell. At this point, the "cool" partons are believed to hadronize in an as yet non-computable manner. There are two main theoretical approaches to this final hadronization step: string models [1] in which mesons are formed as quark–antiquark pairs popping out of the colour field, and cluster models [2] in which colour-singlet combinations of cool partons are assumed to decay isotropically into hadrons. Baryon production in string models is handled by postulating arbitrary probabilities for various diquark–antidiquark pairs to pop out of the colour field, whereas in models based on colour singlet clusters, these are assigned arbitrary probabilities for decays into final states containing baryons. The string model for baryon production is based on the naive non-relativistic quark model (NQM) for baryons whereas cluster models avoid even this meagre theoretical input.

The baryon production measured in jets shows several interesting properties indicating that the production mechanism for baryons may be considerably different from that for mesons [#1]. The baryon multiplicity was found to be surprisingly high – around 0.6–0.7 protons per event at $\sqrt{s}\sim 30$ GeV [4] whilst the early fragmentation models had anticipated no baryon production at all. This result forced string model builders [1] to introduce a new object, the diquark, which helps to parametrize baryon properties. However, the probability to produce diquarks is chosen freely without any theoretical constraint. The cluster models expected baryon production from the thermodynamical behaviour of cluster decay products [2], however, these models predict no differences as

[#1] See ref. [3] for a review.

a function of jet event type, in clear contradiction to the data. The ARGUS measurement [5] of baryon production on the Υ-resonance (final states dominated by three-gluon decay) and in the jet events of the surrounding continuum (mainly $\bar{q}q$ final states) found a drastic difference of more than a factor 2 [#2]. Moreover, both the string and cluster models have difficulties in reproducing the momentum distribution of baryons within jets whereas they are quite successful in the meson sector.

In our opinion the difficulties of the standard phenomenology of baryon production are due to the fact that it treats baryons and mesons identically: diquarks are introduced like quarks, and thermodynamically protons and pions differ only by their masses. In this paper we propose a new kind of fragmentation model in which baryons and mesons are treated differently from the beginning. We base our new approach on inhomogeneities in the chiral quark–antiquark condensate field U [7]. The fluctuations of the field U around the spontaneously broken vacuum expectation values can be interpreted as pseudoscalar mesons whereas, as was first noticed by Skyrme [8], the topological defects of this field can be interpreted as baryons.

We guess that when each colour-singlet cluster of cool partons prepared by perturbative QCD hadronizes, it breaks chiral symmetry spontaneously and non-perturbatively by choosing at random one of the possible orientations of $\langle 0|U|0\rangle$ in the internal symmetry space. Each cluster forms a domain with a uniform value of $\langle 0|U|0\rangle$ much like a domain of spin orientations in a ferromagnet, or of Higgs VEV $\langle 0|H|0\rangle$ forming in the early Universe as it cools [9]. In the latter case one has at least one domain per horizon volume, since causality does not permit the orientations of $\langle 0|H|0\rangle$ to be correlated beyond the horizon. According to the standard picture of string or monopole formation in the early Universe, these arise as topological defects at junctions between domains with different orientations of $\langle 0|H|0\rangle$. We visualize baryon production in a similar way, as the formation of topological defects at junctions between clusters with different orientations of $\langle 0|U|0\rangle$. String defects correspond to non-trivial elements of $\Pi_1(M)$, monopoles to non-trivial elements of $\Pi_2(M)$ and baryons to non-trivial elements of $\Pi_3(M)$ where M is the vacuum manifold. The different orders of the homotopy groups translate into different probabilities for the formation of the corresponding defect at any individual junction of domains. In an infinite medium, one expects a probability per domain of $\frac{1}{4}$ for string formation, $\frac{1}{8}$ per domain for monopole formation, and by extension $\frac{1}{16}$ per domain for baryon formation. Thus the asymptotic baryon multiplicity is proportional to the cluster multiplicity with a known coefficient. However, if the excitation of the chiral field has only a finite extent in space, as in jets, the coefficient $\frac{1}{16}$ may be strongly modified due to the connectivity of the excited domain and to the influence of the surrounding vacuum. We discuss this below and show that the probability of finding a defect is strongly dependent on the form of the excited region (e.g., cigar-like shapes give much lower probabilities than ball-like ones). It is also easy to see that this picture leads to strong baryon–antibaryon correlation and baryon–baryon anticorrelation in agreement with observation. Some other qualitative features of baryon production in high energy collisions can also be reproduced in this topological model.

2. Chiral symmetry, the Skyrme model and fragmentation

Before describing in more detail our model for baryon production, we first recall some key features of chiral symmetry [7] and the Skyrme model [8,10] [#3]. Non-perturbative phenomena in QCD are believed to be responsible for two phenomena of importance to us here: confinement and spontaneous chiral symmetry breaking. The need for the former is obvious: the latter is motivated by pion and kaon dynamics, which indicate that the u, d and s quarks are very light ($\ll m_B/3$) and that chiral quark–antiquark condensates form: $\langle 0|\bar{u}u, \bar{d}d, \bar{s}s|0\rangle \neq 0$. According to spontaneously broken chiral symmetry, pions and kaons are Goldstone bosons corresponding to fluctuations in the quark–antiquark condensate, analogons to spin waves in a ferromagnet. It is generally thought that, in hot hadronic matter, confinement and sponta-

[#2] For an attempt to reproduce this result within a cluster approach see ref. [6].

[#3] See ref. [11] for the three-flavour extension of the model.

neous chiral symmetry breaking occur at nearby temperatures, which are even likely to be identical [12]. This suggests that chiral symmetry should be an essential ingredient in any model of hadronization in hard collisions. According to chiral symmetry, baryons are to be regarded as coherent "lumps" or solitons corresponding to topological defects in the quark–antiquark condensate field. This may be written as an SU(3) matrix U,

$$U(\boldsymbol{x})=\exp\left(\frac{2i}{f}\sum_{i=1}^{8}\lambda_i\phi_i(\boldsymbol{x})\right), \tag{1}$$

where ϕ_i represents the octet of pseudoscalar mesons $\boldsymbol{\pi}$, K_a, η_8 and f is the pseudoscalar meson decay constant. Baryons are topologically non-trivial configurations

$$U(\boldsymbol{x})=V\cdot U_0\cdot V^{\dagger}, \tag{2}$$

where $U_0=\exp[2i\boldsymbol{\tau}\cdot\boldsymbol{x}F(|\boldsymbol{x}|)]$ takes values in an SU(2) subgroup of SU(3), and V is an SU(3) rotation matrix. Baryons exist because there are configurations $U(\boldsymbol{x})$ which cannot be deformed continuously into a space–time-independent constant matrix U: in mathematical language they appear because of the non-triviality of the third homotopy groups: $\Pi_3(\mathrm{SU}(2)\text{ or }\mathrm{SU}(3))=\mathrm{Z}$. This Skyrme model would be exact in the chiral limit of massless quarks and an infinite number of colours N_c [10]. The former should be a good approximation for the u and d quarks which are believed to weigh only a few MeV, but should be less good for the s quark which weighs ≈ 100 MeV. The $1/N_c$ expansion explains the OZI rule and the narrow widths of mesons [13], but until recently there was no clear evidence of its applicability to baryons.

It has been known for years that the static properties of baryons and their spectroscopy are fitted well by the NQM [#4]. Ratios of static quantities and meson–nucleon scattering phase shifts have also been reproduced using the Skyrme model [10,15], but there used to be no compelling reason to prefer it over the NQM. However, recently EMC data [16] have made it possible to disentangle the contributions (Δu, Δd, Δs) to the proton helicity from u, d and s quarks. Remarkably, (Δu+Δd+Δs) ≈ 0 [17], indicating that (within errors of $\approx 25\%$) none of the proton spin is carried by quarks, in contradiction to the NQM.

[#4] See ref. [14] for a review.

However, this was to be expected in the Skyrme model, according to which (Δu+Δd+Δs)=0 because the baryon is an SU(3) soliton and all the proton spin is due to its slow rotation V in eq. (2) [18]. This phenomenological success inspires us to take the Skyrme model more seriously and invoke it for a model of baryon production in hard collisions.

We adopt the standard QCD picture of the initial perturbative stages of development of the final state in a hard collision, according to which the initial "hot" partons radiate gluons and $\bar{q}q$ pairs, populating the final state with "cool" partons close to mass-shell. To leading order in $\alpha_s=12\pi/(33-2N_q)\ln(Q^2/\Lambda^2)$, where N_q is the number of light quark flavours, the multiplicity of "cool" partons with virtuality Q_0 is [19]

$$n\left(\frac{Q}{Q_0}\right)=\frac{8}{9}\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right)^a \times\frac{\exp\{[(2C_A/\pi b)\ln(Q^2/\Lambda^2)]^{1/2}\}}{\exp\{[(2C_A/\pi b)\ln(Q_0^2/\Lambda^2)]^{1/2}\}}, \tag{3}$$

where

$$a=-\tfrac{1}{4}[1+(2N_f/3\pi b)(1-C_F/C_A)],$$
$$b=(11C_A-2N_f)/12\pi. \tag{4}$$

The leading perturbative corrections to the multiplicity $n(Q/Q_0)$ (3) have been calculated, as has the distribution of "cool" partons in phase space [19], but we will not use them there. Assuming following the analysis of ref. [20] that $Q_0=2\Lambda$ and $\Lambda=0.2$ GeV, eq. (3) predicts 6.7 (11.4) gluons at $\sqrt{s}=10$ (30) GeV.

We follow ref. [20] in assuming that these "cool" partons are in one-to-one correspondence with colour-singlet clusters that hadronize non-perturbatively. For the reasons discussed above, we assume that the confinement associated with hadronization is simultaneous with spontaneous chiral symmetry breaking. If each cluster hadronizes independently, as is conventionally assumed, this means that each cluster chooses independently and at random a local vacuum expectation value for the chiral $\bar{q}q$ condensate field U, just like the formation of a domain in a ferromagnet. Superimposed on the condensate value will be the excitations corresponding to the pseudoscalar mesons, $\phi_i\equiv\boldsymbol{\pi}$, K_a, η_8, just like quantized spin waves. So far, our ignorance of non-perturbative dynamics prevents us from calculating the populations of these excitation levels, so we do not know the average number of pseudoscalar mesons per cluster. However, topological arguments can be used to calculate the average number of baryons per cluster, as we now argue.

3. Description of the fragmentation model and results

Our problem of baryon production is analogous to the cosmological problems of string and monopole formation in the early Universe, to which a solution was proposed by Kibble in 1976 [9]. He pointed out that as the Universe cooled through the temperature of some phase transition where a symmetry was spontaneously broken by the formation of a Higgs VEV, domains of similar orientations could be formed. The sizes of these domains were restricted by causality to be no larger than the horizon size: $d<2ct$ where t is the age of the Universe. Since the directions of the Higgs VEVs in adjacent domains would be uncorrelated, they might mismatch at the boundaries, leading to the formation of topological defects such as strings or monopoles. Kibble [9] proposed the following strategy for estimating the probability that a defect would be formed at any given point, illustrated by the string and monopole cases.

One considers a triangulation of the physical medium (a plane to discuss strings, fig. 1a, 3-space to discuss monopoles, fig. 1b) with each vertex representing the surrounding domain. To answer whether a defect forms at the junction of the domains in the middle of any given triangle (tetrahedron in the case of monopoles) one considers the random and independent directions of the Higgs VEVs at the vertices (on a circle S^1 in the case of strings (fig. 1a), on the surface S^2 of a sphere in the case of monopoles). Each pair of vertices is then connected by the shortest path (in S^1, fig. 2, or S^2), and these are in turn connected by the minimal surfaces in the S^2 case. If the resulting path in S^1 or surface in S^2 has non-zero winding number as in fig. 2 one believes that a defect (string or monopole) is formed in the center. As can be seen in fig. 2, a string is formed if the direction of the Higgs VEV at the vertex 3 is between the points opposite to the vertices 1 and 2, because then the shortest paths

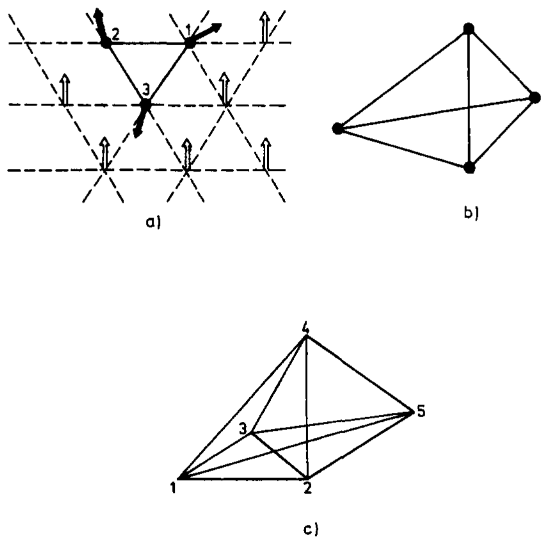

Fig. 1. (a) A decomposition of the plane into triangles (2-simplices), whose vertices are assigned values in U(1) indicated by arrows: closed for the "excited" vertices 1, 2, 3, and open for "background" vertices. (b) An elementary tetrahedron (3-simplex) used in the analysis of monopole formation. (c) An elementary 4-simplex used in the analysis of baryon formation.

between pairs of the vertices 1, 2 and 3 taken cyclically goes around the circle. The average angle between vertices 1 and 2 is $\pi/2$, and the possible range of the azimuthal angle is 2π, so the probability of forming a defect is $\frac{1}{4}$. For monopole formation the Higgs VEV at the fourth vertex has to be in the anti-

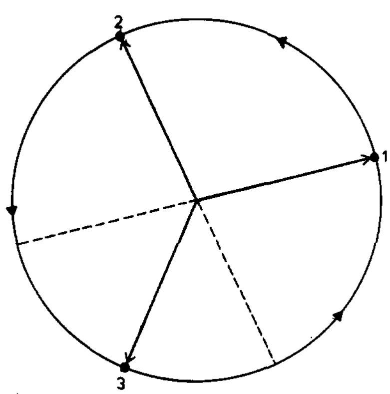

Fig. 2. Illustration of how the shortest paths between the pairs of excited vertices (1,2), (2,3), (3,1) of fig. 1a yield a non-zero winding number configuration (string).

podes of the minimal surface area defined by the directions of the vertices 1, 2 and 3. The average area spanned by three random points on a sphere is $\frac{1}{4}\cdot\frac{1}{2}=\frac{1}{8}$ of the surface area of the sphere, and so the monopole formation probability is also $\frac{1}{8}$.

We generalize this argument to discuss the production in high energy collisions of baryons, which correspond to non-trivial components of $\Pi_3(SU(2))$. Now we must triangulate a sphere in four-dimensional space, S^3, which requires five vertices forming a 4-simplex with tetrahedra as boundaries. Each of the vertices is assigned a random and independent direction in the group SU(2). In the following we describe the procedure [21] which we adopted to determine the winding number first for the case of an isolated simplex and then for more complicated configurations.

To compute the winding number on isolated 4-simplex we first label the vertices on the simplex with $i=1, 2, 3, 4, 5$ as in fig. 1c. This 4-simplex has five boundary tetrahedra: $[2, 3, 4, 5]_+$, $[1, 3, 4, 5]_-$, $[1, 2, 4, 5]_+$, $[1, 2, 3, 5]_-$, $[1, 2, 3, 4]_+$ where the signs denote their orientations relative to the volume of the 4-simplex. To each vertex i we assign a value $U(i)$ of the U field in the SU(2) space, which we represent by 4-vectors, U_μ, $\mu\equiv x, y, z, t$. We also define an arbitrary direction $U(0)$ not associated with any particular vertex of the 4-simplex. For each boundary tetrahedra we first determine the net orientation of its U-directions defined by

$$O(i,j,k,l)=\mathrm{sign}(\epsilon_{xyzt}U_x(i)\,U_y(j)\,U_z(k)\,U_t(l)). \quad (5)$$

We also compute the net orientations of the tetrahedron under consideration with the U direction at each vertex in turn replaced by the arbitrary direction $U(0)$: $O(i,j,k,0)$, $O(i,j,0,l)$, $O(i,0,k,l)$ and $O(0,j,k,l)$. If all these net orientations are equal, and equal to that $[O(i,j,k,l)]$ of the tetrahedron under consideration, then the tetrahedron makes a contribution $O(i,j,k,l)$ to the topological quantum number over the 4-simplex. The sum of the contributions from all five boundary tetrahedra, weighted by their orientations relative to the volume of the 4-simplex, gives the topological quantum number of the field U defined on the 4-simplex. When the $U(i)$ are chosen at random we find that the average probability of forming a topological defect in the 4-simplex is $\frac{1}{16}$, in agreement with the simple counting rule introduced above.

To study the formation of topological defects in high-energy events, we have to model a U field excitation extended in space. We do this starting from a cubic lattice in four dimensions. The field U is now a function of the four lattice indices i_1, i_2, i_3 and i_4. As an exercise, consider first a $2\times2\times2\times2$ hypercube defined at vertices (i_1, i_1+1), (i_2, i_2+1), (i_3, i_3+1), (i_4, i_4+1). To compute the winding numbers of the field U on this cube, we have first to slice it into 4-simplices using the procedure given in ref. [21], which divides such a cube into 16 4-simplices. We then compute the winding number of U on each 4-simplex, and add together the winding numbers from all 4-simplices, distinguishing between positive and negative contributions since we are interested in the total number of baryons and of antibaryons. This procedure may be iterated if there is more than one hypercube.

By comparison with the early Universe, jet final states are quite inhomogeneous. They correspond to a small region of excited chiral field (typically ≈ 10 fm long and ≈ 2 fm broad) which is surrounded by non-excited space in which the chiral field assumes some constant background value (of the open arrows in the string example in fig. 1a). Since the size of each excited domain is ≈ 1 fm, it is important to investigate finite-size effects which may cause the baryon density to depend on the size and shape of the excited region. Baryons may be generated not only on 4-simplices contained entirely within the excited region, but also on those with one vertex in the background region.

To return to our example of a $2\times2\times2\times2$ hypercube, if we choose random values of U at each of its vertices and do not consider 4-simplices involving vertices in the surrounding background field, we find on average one baryon or antibaryon, as expected from the calculated probability of $\frac{1}{16}$ for a baryon or antibaryon to be produced on each of the 16 active 4-simplices. However, if we include 4-simplices involving vertices in the surrounding background field, we find on average 3.5 baryons or antibaryons. This is simply due to the fact that there are 40 4-simplices with one background vertex, and each of these has the same probability of $\frac{1}{16}$ to produce a baryon or antibaryon. We have computed the average baryon

Table 1
Predicted number of baryons or antibaryons for several shapes of excited region.

Shape	Number of B or $\bar{B}$	Number of active simplices
2×2×2×2 no background	1	16
2×2×2×2 with background	3.5	56
2×2×2×1 with background	0.625	10
3×2×2×1 with background	1.25	20
4×2×2×1 with background	1.875	30
5×2×2×1 with background	2.5	40
6×2×2×1 with background	3.125	50
3×3×3×1 with background	5.0	80

numbers for several shapes of the excited region and found that the number of baryons or antibaryons is always given by

$$N_{\mathrm{B\,or\,\bar{B}}} = \tfrac{1}{16}\cdot(E_5+E_4)\,, \qquad (6)$$

where $E_{5(4)}$ is the number of 4-simplices with 5 (4) excited vertices. Examples of results for different shapes of excited region are shown in table 1.

The fourth space dimension has no physical interpretation but is introduced simply to compactify $\mathbb{R}^3$ into a sphere S^3 and thus be able to define the winding number. Therefore we assume that physical states do not extend in the fourth coordinate direction. This assumption has two important consequences; it leads to exact baryon-number conservation and to a strong phase space correlation between baryon and antibaryon production. We illustrate this mechanism with the example of a 2×2×2×1 excited region surrounded by a background field. In fig. 3 we show a subdivision of this cube into tetrahedra, with the fourth dimension ignored for clarity. Let us concentrate first on the tetrahedron given by the vertices [0, 1, 3, 5], where the indexing scheme is shown in fig. 3. This tetrahedron is given on the lattice by the coordinates

$$(i_1, i_2, i_3, i_4),\quad (i_1+1, i_2, i_3, i_4),$$
$$(i_1+1, i_2+1, i_3, i_4),\quad (i_1+1, i_2, i_3+1, i_4)\,. \qquad (7)$$

In a 4-cube with the minimal subdivision into 4-simplices this tetrahedron can be on the boundary of just one 4-simplex, namely that whose fifth vertex is (i_1+1, i_2, i_3, i_4+1)[#5]. On a lattice, however, the tetrahedron (7) can also form a second 4-simplex with the fifth vertex reflected in the hypersurface i_4=const., i.e. (i_1+1, i_2, i_3, i_4-1). According to our assumption the field U is randomly chosen on vertices (7) and assumes some common background value on the fifth vertex and its reflection. This means that the topological quantum numbers on the two 4-simplices are equal and opposite, because they share the same field directions but have, due to the reflection, opposite orientations. This result extends to any configuration with just one value of i_4. This means that all configurations have baryon number conserved locally, baryon and antibaryon production are tightly correlated. The space distance between the baryon and corresponding antibaryon cannot be bigger than a lattice size and their separation should also be of the same order of magnitude.

An important feature of table 1 is the fact that configurations with similar numbers of excited vertices may give very different numbers of baryons or antibaryons. For example, the jet-like 6×2×2×1 configuration gives 3.125 baryons or antibaryons, whilst the more isotropic 3×3×3×1 configuration gives 5 baryons or antibaryons. Another example is the jet-like configuration of fig. 4, which has eight excited vertices but has $E_5+E_4=4$ and therefore only yields $\frac{1}{4}$ baryon or antibaryon, whereas the 2×2×2×1 configuration in table 1 has $E_5+E_4=10$ and therefore yields $\frac{5}{8}$ baryons or antibaryons. This observation gives a qualitative explanation of the larger baryon to pion ratio in quasi-isotropic Υ decays, as compared

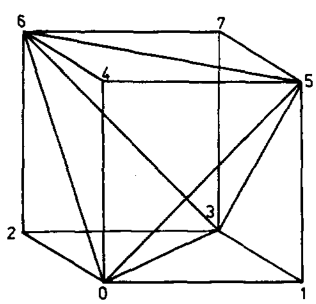

Fig. 3. Subdivision of a 3-cube into 5-tetrahedra: [0, 1, 3, 5], [0, 2, 3, 6], [0, 4, 5, 6], [3, 5, 6, 7], [0, 3, 5, 6].

[#5] We note that the use of this minimal set of simplices is a strong physics assumption because a priori it is possible to connect the tetrahedron (7) with all eight vertices of a hypercube with i_4+1.

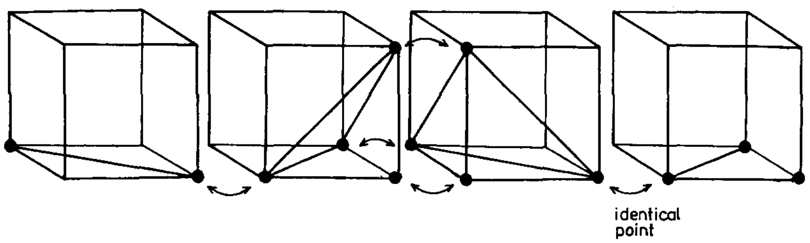

Fig. 4. A configuration of eight excited vertices that has $E_5+E_4=4$ and hence yields $\frac{1}{4}$ baryon or antibaryon on the average.

to continuum events dominated by two-jet $\bar{q}q$ final states [#6].

Asymptotically we expect the number of baryons to be proportional to the number of gluons produced in high energy collisions. However, at the presently accessible energies the number of gluons is still relatively small. Therefore we expect, as indicated by the results of table 1, that the number of baryons will increase more strongly with energy than the number of gluons. This is confirmed by data [3]. Finally, from the number of gluons given by eq. (3) and the probabilities to form a baryon from table 1 we can predict the number of baryon and antibaryon production in e^+e^- events. We obtain ≈ 0.52 (1.2) baryons or antibaryons at $\sqrt{s}=10$ (30) GeV which fits well to the observed values of 0.58 (1.18) [3]. Obviously, we consider these numbers only as order-of-magnitude predictions since we are still several steps away from a fully-fledged baryon fragmentation model.

So far we have only considered the two-flavour case where U takes values in SU(2), and have not touched on the three-flavour case where U takes values in SU(3). The geometry of SU(3) is considerably more complicated than SU(2). Homotopy classes of SU(3) can be computed by exploiting the fact that locally $SU(3) \approx S^5 \times SU(2)$ [22]. However, in this reduction of the structure group, geodesics are not mapped on geodesics, so the probability of finding a non-zero SU(3) winding number is not the same as the probability of finding a non-zero SU(2) winding number. Thus a further investigation in SU(3) is necessary. Notice, though, that numerically the topological susceptibility of gauge fields on a lattice in SU(2) and SU(3) are quite similar [23]. Hence we conjecture that the extension to SU(3) will not greatly increase the baryon-number probabilities.

This is just one of many improvements needed to convert our topological approach into a fully-fledged fragmentation model. Other important steps are the combination of this philosophy with a perturbative QCD Monte Carlo for parton showers and the establishment of a criterion for deciding when two cells are close enough to be connected in a single simplex. A combined perturbative and chiral non-perturbative QCD Monte Carlo should also allow one to study the momentum distribution of baryons, which we would expect to be more similar to the momentum distribution of the original partons than to that of mesons. Although these and many other issues remain unsolved, we hope that our topological philosophy will stimulate sufficient new thinking about baryon production to result in a new calculational method for fragmentation models.

[#6] An unwarranted extrapolation of this approach could even explain the now discredited Centauro events. If in a dense configuration of "cool" partons our assumption of a minimal set of simplices would not be valid the number of simplices could greatly exceed the number of vertices. This would lead to events dominated by baryon–antibaryon production.

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