EXPERIMENTAL INVESTIGATION OF THE ENERGY DEPENDENCE OF THE STRONG COUPLING STRENGTH

JADE Collaboration

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The energy dependence of the relative production rate of three-jet events is studied in hadronic e^+e^- annihilation events at centre of mass energies between 22 and 46.7 GeV. Three-jet events are defined by a jet finding algorithm which is closely related to the definition of resolvable jets used in $O(\alpha_s^2)$ perturbative QCD calculations, where the relative production rate of three-jet events is roughly proportional to the size of the strong coupling strength. The production rates of three-jet events in the data decrease significantly with increasing centre of mass energy. The experimental rates, which are independent of fragmentation model calculations, can be directly compared to theoretically calculated jet production rates and are in good agreement with the QCD expectations of a running coupling strength. The hypothesis of an energy independent coupling constant can be excluded with a significance of four standard deviations.

Quantum Chromodynamics (QCD) is the renormalizable gauge theory of the strong interaction between quarks and gluons [1]. The theory is asymptotically free, i.e. at small distances or at high momentum transfers the coupling strength decreases and the quarks and gluons are quasi-free [2]. Asymptotic freedom is a characteristic property of non-abelian gauge theories which include, as for gluons in the case of QCD, the self-coupling of the

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gauge bosons. QCD perturbation theory has been proven to successfully describe many phenomena of particle reactions at high momentum transfers, where the couplings are sufficiently small [3]. The most characteristic features of QCD, however, the existence of the gluon selfcoupling or the energy dependence of the coupling strength, have not yet been established experimentally [3].

In this letter we present the results of an experimental investigation of the energy dependence of the strong coupling strength. The analysis is based on the multihadronic event sample taken with the JADE detector [4] at the PETRA e^+e^- storage ring in the centre of mass energy range of 22 GeV to 46.7 GeV.

In second order perturbation theory, the energy dependence of α_s is calculated to be

$$\alpha_s(Q^2)=12\pi[b_0\ln(Q^2/\Lambda^2_{\overline{\mathrm{MS}}}) + (b_1/b_0)\ln\ln(Q^2/\Lambda^2_{\overline{\mathrm{MS}}})]^{-1}\,, \qquad (1)$$

where $b_0=(33-2N_f)$, $b_1=(918-114N_f)$ and N_f is the number of quarks flavours produced. In e^+e^- annihilation, Q^2 is usually chosen to be the centre-of-mass energy of the hadronic system ($Q^2\equiv E^2_{\mathrm{CM}}$), and $\Lambda_{\overline{\mathrm{MS}}}$ is the QCD scale parameter which has to be determined by experiment.

Although α_s has been determined in many different analyses during the past few years (for reviews see refs. [3,5,6]), it has not yet been possible to verify experimentally whether or not α_s decreases with increasing energy, due to relatively large systematic errors which typically must be taken into account. Phenomenological models are usually used to describe the nonperturbative hadronization process introducing an intrinsic uncertainty in the extraction of α_s from experimental observables [7].

In view of these systematic uncertainties, the energy dependence of experimental observables which are closely related to theoretical calculations is studied in this paper, rather than extracting individual values of α_s at different energies. Such an observable is e.g. the relative production rate of three-jet events, $R_3=\sigma_{3\text{-jet}}/\sigma_{\mathrm{tot}}$, where σ_{tot} is the total hadronic cross section in e^+e^- annihilation and $\sigma_{3\text{-jet}}$ is the corresponding cross section of three-jet production. In second order perturbative QCD, R_3 is a simple function of α_s:

$$R_3=C_1\alpha_s+C_2\alpha_s^2\,. \qquad (2)$$

C_1 and C_2 denote the first-order term and the second-order virtual correction term, respectively, and can be precisely calculated.

Within perturbative QCD calculations, the cross sections for n-parton final states are singular for vanishing parton energies and for the case where two partons are collinear in space (infrared and collinear singularities). After the assumed fragmentation of partons into jets of hadrons, such configurations are however practically indistinguishable from $(n-1)$-parton events. Therefore a resolution cut-off is introduced in order to calculate finite production cross sections of n-parton events, where n indicates 2, 3 and 4 in second-order perturbation theory. The detailed choice of resolution criteria is, to a certain amount, arbitrary. A commonly used method is to require the square of the scaled invariant mass of any pair of partons i and j,

$$y_{ij}=M^2_{ij}/E^2_{\mathrm{CM}}\,, \qquad (3)$$

to satisfy the relation

$$y_{ij}\geqslant y_{\min}\,, \qquad (4)$$

where $y_{\min}$ is the cut-off parameter defining *resolvable* partons. Within a given renormalisation scheme (e.g. the $\overline{\mathrm{MS}}$ scheme), C_1 and C_2 in eq. (2) are constants for fixed values of $y_{\min}$, and the energy dependence of three-parton event rates is only determined by the energy evolution of α_s.

The algorithm introduced to define resolvable jets on parton level (eqs. (3), (4)) can easily be adopted to define jets in measured hadronic events. Such a jet finding algorithm was developed and extensively used in earlier studies of multijet event production rates [8,9]. The algorithm works as follows:

In each hadronic event, the squares of the scaled pair masses,

$$y_{kl}=M^2_{kl}/E^2_{\mathrm{vis}}\,, \qquad (5)$$

are calculated for all pairs of particles k and l, where E_{vis} is the visible energy of the event (charged particles are assumed to be pions and neutral particles to be photons). The two particles i and j with the smallest invariant pair mass are replaced by a pseudoparticle or "cluster" with four-momentum (p_i+p_j). This procedure is repeated until the pair masses of all particle or pseudoparticle pair-combinations exceed a certain threshold value y_{cut}:

$$y_{kl} \geqslant y_{\text{cut}} , \tag{6}$$

and the remaining clusters are called "jets". To calculate the pair mass M_{kl}, we use the expression

$$M_{kl}^2 = 2E_k E_l (1 - \cos \Theta_{kl}) . \tag{7}$$

Studies with Monte Carlo generated events show that this choice of M_{kl} provides close agreement between jet- and parton-multiplicities at comparable values of y_{cut} (the experimental cutoff in the jet finding algorithm) and y_{min} (the QCD cutoff parameter for massless partons in the perturbative QCD calculations). While this agreement has been demonstrated in ref. [8] for fixed centre-of-mass energy as a function of y_{cut}, an analysis of energy-dependent effects requires that the agreement also holds as a function of energy for fixed values of y_{cut}.

To demonstrate that this requirement is fulfilled, the ratio of reconstructed three-jet event rates and original three-parton rates,

$$q = \frac{R_3(\text{reconstructed jets})}{R_3(\text{partons})} \tag{8}$$

is studied, as a function of energy, in model calculations for various values of $y_{\text{cut}} = y_{\text{min}}$. Ideally, this ratio should be close to one and should be constant over the entire energy range. The ratio q is shown in fig. 1 for $y_{\text{cut}} = y_{\text{min}} = 0.08$ and CM energies between 20 GeV and 60 GeV, using the Lund QCD shower model [10], based on leading logarithmic approximations [#1], and the second-order QCD Lund string model [10] (Lund MA), based on the complete second-order QCD matrix elements of Gottschalk and Shatz [11]. [#2]. The QCD- and fragmentation parameters of these models, presently the most successful models as far as the general description of the data is concerned [13], are described in footnote 3. The results of this study, however, do not depend on the detailed choice of parameter values.

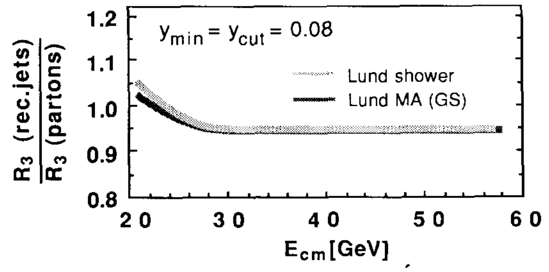

Fig. 1. The ratio q of reconstructed three-jet and three-parton event rates for different QCD and fragmentation model calculations.

As can be seen in fig. 1, the ratio q is close to unity and constant for centre-of-mass energies above 28 GeV for both models. This behaviour has been verified to persist up to energies of 100 GeV, provided that no new generation of heavy and yet unknown particles will be produced in this energy range. Below 28 GeV a slight increase in q is seen, as is expected when the minimum required pair mass of jets gets so low that effects of fluctuations in the nonperturbative fragmentation region or the decays of heavy particles become noticeable ($y_{\text{cut}} = 0.08$ corresponds to $M_{\text{cut}} = 6.2$ GeV at $E_{\text{CM}} = 22$ GeV). For higher values of y_{cut}, q starts to deviate from the same constant value at lower energies than seen in fig. 1: At $y_{\text{cut}} = 0.12$ the constant region extents down to 22 GeV, while at $y_{\text{cut}} = 0.04$ q is constant above 31 GeV. Thus, within the energy region where q is seen to be flat, the jet algorithm does not introduce any energy-dependent effects by itself and is therefore well suited for an investigation of the energy dependence of α_s from experimental jet production rates. Moreover, since q is close to unity, experimental jet rates can directly be compared with parton rates calculated in perturbative QCD, and no further fragmentation model calculations are needed to derive conclusions from the

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[#1] The shower model produces multi-parton final states with minimum invariant pair masses down to 1 GeV. Therefore, the parton multiplicities for the desired values of y_{min} have been reconstructed using the same jet algorithm as for the final hadrons. It has been verified that this procedure is equivalent to terminating the parton generation at the higher cutoff values of $y_{\text{min}} = y_{\text{cut}}$.

[#2] The calculations are incomplete; updated calculations by ref. [12].

[#3] Version 6.3 is used with a coherent shower evolution where the Q^2 scale in the theoretical expression for α_s, $\alpha_s = 12\pi/[23 \times \ln(Q^2/\Lambda_{\text{LLA}}^2)]$, is $z(1-z)m^2$. The scale parameter is $\Lambda_{\text{LLA}} = 0.31$ GeV and the shower cut-off scale is $Q_0 = 1$ GeV. The fragmentation parameters are $a = 0.45$, $b = 0.9$ GeV^{-2} and $\sigma_q = 0.33$ GeV [10]. Note that Λ_{LLA} cannot directly be compared to the scale parameter $\Lambda_{\overline{\text{MS}}}$ used in first or second-order perturbative QCD calculations. For the second-order QCD matrix element model (Lund MA), the fragmentation parameters are as described in ref. [8]. In order to maintain a constant phase-space for the fragmentation region at all centre of mass energies, the *invariant mass* of the QCD cutoff parameter, $M_{\text{min}}^2 = E_{\text{cm}}^2 y_{\text{min}}$, is kept constant ($M_{\text{min}} = 6$ GeV) rather than y_{min} itself. The QCD scale parameter is $\Lambda_{\overline{\text{MS}}} = 200$ MeV.

data. Experimental jet rates presented in this analysis will *not* be corrected for the 6% deviation of q from unity observed in the figure [#4].

In the following, the energy dependence of three-jet event production rates will be investigated, using the data taken with the JADE detector at PETRA between 1979 and the end of 1985 at centre-of-mass energies of 22 GeV, 29.0–36.7 GeV and 38.0–46.7 GeV. The JADE detector allows homogeneous measurements of both charged-particle momenta and photon energies in more than 90% of the solid angle. In this analysis, both charged and neutral particles with momenta exceeding 100 MeV/c and 150 MeV/c, respectively, are used. A description of the detector, the trigger conditions and the basic selection of hadronic events is given in ref. [4].

In order to further reject events with hard initial-state photon radiation or significant particle losses around the beam pipe, the total missing momentum of an event, $|\Sigma \boldsymbol{p}_i|$, is required not to exceed 30% of the e^+e^- centre-of-mass energy and the angle θ_T of the event thrust axis [14] with the beam direction to satisfy the relation $|\cos\theta_T| < 0.8$. After these cuts, which are applied in addition to those described in ref. [4], the background of events originating from two-photon scattering processes or τ-lepton pair production has been calculated to be less than 0.1% and 1%, respectively. Table 1 shows the integrated luminosities and the total number of events which enter the analysis.

Table 1
Luminosities and the number of hadronic events.

$\langle E_{CM} \rangle$ (GeV)	$\int L dt$ (pb^{-1})	Number of events
22	2.69	1666
30	3.28	843
34.6	71.3	13617
38	10.2	1950
44	43.1	6636

The n-jet event rates of the data at different centre of mass energies and for different values of y_{cut} are shown in table 2. The results are not explicitly corrected for the effects of detector resolution, acceptance and initial-state photon radiation, since model calculations show that the sum of these corrections are of negligible size: Due to fact that E_{vis} rather than E_{CM} is used to calculate the y_{kl} and that, by the event selection chosen, only well-contained events without hard initial-state photon radiation are analysed, the sum of the correction factors deviates from one by less than ±2%.

The three-jet event rates from the table are also shown in fig. 2, together with the theoretical predictions of three-parton event production rates as calculated from eq. (2) using the complete second-order perturbative QCD matrix elements from Gottschalk and Shatz (GS) [11] and from Kramer and Lampe (KL) [15]. The factors C_1 and C_2 (eq. (2)) computed from the results of KL are listed in table 3. The QCD scale parameter, $\Lambda_{\overline{MS}}$, has been adjusted to the three-jet rates of the data at $y_{cut} \geqslant 0.06$, resulting in values of 210 MeV and 205 MeV for GS and KL, respectively.

The two sets of theoretical calculations shown in fig. 2 provide an equally good description of the data at the higher values of y_{cut}, but result in slightly different values of $\Lambda_{\overline{MS}}$ and show different results for the lowest value of y_{cut}. Such differences may be explained [16] by the different choice of three-jet variables which are used to perform the cancellation of infrared and collinear singularities between higher order virtual and real corrections [11,15], but have no practical impact on our study of the energy dependence of the coupling strength. Within each set of theoretical calculations (GS or KL) and the statistical fitting error of $\Delta(\Lambda_{\overline{MS}}) = \pm 13$ MeV, the values of $\Lambda_{\overline{MS}}$ determined for individual values of y_{cut} ($y_{cut} \geqslant 0.06$) agree with each other.

The theoretical curves fit the data both in absolute normalisation and in the energy dependence for *all* values of y_{cut} larger than 0.04. This is certainly no trivial point, since there is only one free parameter ($\Lambda_{\overline{MS}}$) in the theory. Only for $y_{cut} = 0.04$ does the theory overestimate R_3 at the higher PETRA energies.

[#4] This deviation may be explained by the fact that phenomenological string fragmentation models tend to pull the original jet axes together and thereby soften the multijet structure of an event during fragmentation. This is, however, a kinematical effect and is not predicted by perturbative QCD. The question whether experimental jet rates should be corrected by these 6% cannot conclusively be answered, but is of minor importance for an analysis which concentrates on studying energy-dependent effects in jet production.

Table 2
n-jet event rates of the data at different centre of mass energies obtained with the jet finding algorithm, for different values of y_{cut} (number in % of the total hadronic cross section).

y_{cut}	Rate	E_{CM} (GeV)				
		22	30	34.6	38	44
0.03	R_2	33.7 ± 1.2	40.2 ± 1.7	45.6 ± 0.5	46.0 ± 1.1	49.6 ± 0.6
	R_3	52.5 ± 1.2	50.5 ± 1.7	46.7 ± 0.5	46.3 ± 1.1	44.6 ± 0.6
	R_4	12.9 ± 0.8	9.0 ± 1.0	7.5 ± 0.2	7.4 ± 0.6	5.7 ± 0.3
	R_5	1.0 ± 0.2	0.24 ± 0.17	0.21 ± 0.04	0.29 ± 0.12	0.09 ± 0.04
0.04	R_2	43.6 ± 1.2	50.8 ± 1.7	56.1 ± 0.4	56.4 ± 1.1	58.7 ± 0.6
	R_3	49.7 ± 1.2	44.8 ± 1.7	40.2 ± 0.4	40.1 ± 1.1	38.5 ± 0.6
	R_4	6.60 ± 0.62	4.39 ± 0.70	3.75 ± 0.17	3.58 ± 0.41	2.80 ± 0.20
0.05	R_2	52.6 ± 1.2	61.1 ± 1.7	63.4 ± 0.4	63.5 ± 1.1	65.7 ± 0.6
	R_3	43.9 ± 1.2	36.8 ± 1.7	34.6 ± 0.4	34.7 ± 1.1	32.6 ± 0.6
	R_4	3.54 ± 0.46	42.1 ± 0.45	2.02 ± 0.13	1.74 ± 0.29	1.61 ± 0.16
0.06	R_2	61.4 ± 1.2	66.8 ± 1.6	69.0 ± 0.4	68.4 ± 1.0	71.4 ± 0.6
	R_3	37.0 ± 1.2	32.0 ± 1.6	29.8 ± 0.4	30.7 ± 1.0	27.8 ± 0.6
	R_4	1.50 ± 0.30	1.19 ± 0.38	1.14 ± 0.10	0.92 ± 0.21	0.83 ± 0.11
0.08	R_2	72.5 ± 1.2	76.9 ± 1.5	77.7 ± 0.4	77.0 ± 0.9	79.8 ± 0.5
	R_3	27.1 ± 1.2	23.1 ± 1.5	22.0 ± 0.4	22.7 ± 0.9	20.1 ± 0.5
	R_4	0.42 ± 0.16	0.36 ± 0.21	0.31 ± 0.05	0.29 ± 0.11	0.14 ± 0.05
0.10	R_2	80.6 ± 1.0	83.9 ± 1.3	84.6 ± 0.3	83.0 ± 0.9	85.2 ± 0.4
	R_3	19.4 ± 1.0	16.1 ± 1.3	15.4 ± 0.3	17.0 ± 0.9	14.8 ± 0.4
0.12	R_2	87.0 ± 0.9	88.5 ± 1.1	88.8 ± 0.3	87.7 ± 0.7	89.6 ± 0.4
	R_3	13.0 ± 0.9	11.5 ± 1.1	11.2 ± 0.3	12.3 ± 0.7	10.4 ± 0.4
0.14	R_2	90.6 ± 0.7	91.3 ± 0.9	92.0 ± 0.2	90.3 ± 0.7	92.5 ± 0.3
	R_3	9.4 ± 0.7	8.7 ± 0.9	8.0 ± 0.2	9.7 ± 0.7	7.5 ± 0.3

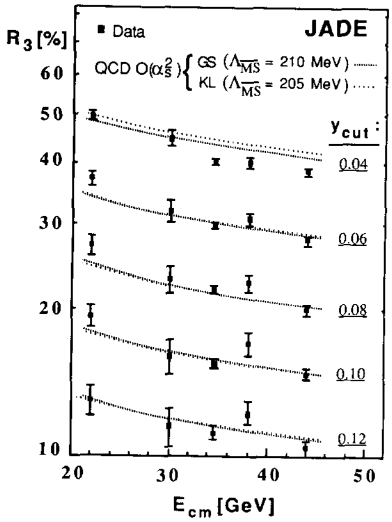

Fig. 2. Three-jet event rates at different centre of mass energies for various values of y_{cut}, together with the direct predictions of the complete second-order perturbative QCD calculations of Gottschalk and Shatz (GS) and of Kramer and Lampe (KL).

This effect can be explained by the earlier observation [8,9,17] that $O(\alpha_s^2)$ calculations underestimate the production rate of four-jet events. In this paper, $\Lambda_{\overline{\mathrm{MS}}}$ has been adjusted at $y_{\mathrm{cut}} \geqslant 0.06$, where almost no four-jet events are resolved separately, so that, by the normalization $\sum R_n = 100\%$, at lower values of y_{cut} discrepancies in R_4 also affect the description of R_3. If the sum of three- and four-jet production rates is analysed instead of R_3, theory provides a good description of the data also for $y_{\mathrm{cut}} = 0.04$.

Table 3
$O(\alpha_s^2)$ QCD factors C_1 and C_2 (see eq. (2)), from Kramer and Lampe [15].

$y_{\min}$	C_1	C_2
0.04	2.266	6.390
0.06	1.516	4.675
0.08	1.074	3.384
0.10	0.783	2.456
0.12	0.578	1.785

The good agreement between the data and the QCD expectations can be quantified in terms of χ^2. For this, the 22 GeV data points are not taken into consideration, since at this low energy fragmentation effects may introduce an additional energy dependence (see fig. 1). Using all data above 22 GeV, the χ^2 between the data and the QCD curves is always about 1 per degree of freedom for y_{cut} larger than 0.04 (e.g. $\chi^2=3.3$ and 3.6 for 3 d.o.f. at $y_{\text{cut}}=0.06$ and 0.08, respectively). Assuming an energy independent coupling strength (and therefore expecting a constant rate of three-jet events at all energies[#5]) leads to $\chi^2=12.4/3$ d.o.f. and 12.3/3 d.o.f. at $y_{\text{cut}}=0.06$ and 0.08, respectively. Both these values correspond to a four-standard deviation effect in favour of the running coupling strength. At higher values of y_{cut} the significance reduces to less than three standard deviations, due to the smaller number of three-jet events and larger statistical errors.

An additional way to investigate the reliability of the analysis and its theoretical interpretation is to study the observed ratio r of three-jet event production rates, $r=R_3(E_{\text{CM}})/R_3(E'_{\text{CM}})$, at two different centre-of-mass energies. Within the QCD calculations, this ratio is expected to be roughly proportional to $\alpha_s(E_{\text{cm}})/\alpha_s(E'_{\text{CM}})$ (c.f. eq. (2)), which should be less than unity for $E_{\text{CM}}>E'_{\text{CM}}$. More important for this test, r is expected to be independent of the common resolution cut-off y_{cut}. In order to verify whether the data are consistent with these expectations, the ratio r of the data at $E_{\text{CM}}=44$ GeV and $E'_{\text{CM}}=34.6$ GeV is presented in fig. 3 for y_{cut} ranging from 0.015 to 0.140, together with the QCD $O(\alpha_s^2)$ predictions (eq. (2)) of Kramer and Lampe [15] for three different values of $\Lambda_{\overline{\text{MS}}}$. Within the statistical errors, the experimental values of r are significantly smaller than one and are consistent with being independent of y_{cut} for $y_{\text{cut}}\geqslant 0.040$. Note that the errors are correlated since all values of r are calculated using the same data sample. This "self-consistency" check of the data shows that the results of this study do not depend on the detailed value of y_{cut} (provided it is chosen to be large enough) and are in good agreement with the expectations of perturbative QCD and the energy dependence of α_s.

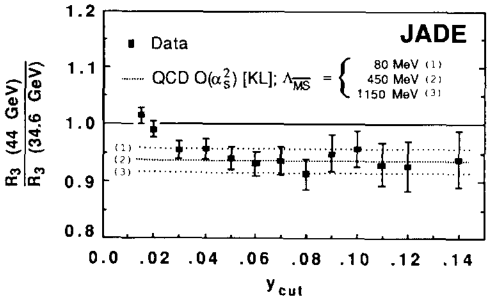

Fig. 3. The ratio of three-jet event rates observed at $E_{\text{CM}}=44$ GeV and 34.6 GeV.

The ratio r also provides the possibility to determine $\Lambda_{\overline{\text{MS}}}$ from the *energy dependence* of the three-jet production rates alone (rather than from absolute or relative event production *cross sections* as has been done in the past [3,5,6]). Averaging r in the region of $y_{\text{cut}}>0.04$ and taking the statistical error from the data point at $y_{\text{cut}}=0.06$ results in a mean value of $\langle r\rangle=0.935\pm0.023$. This corresponds, if compared to the second-order QCD calculations of Kramer and Lampe [15], to $\Lambda_{\overline{\text{MS}}}=(450^{+700}_{-370})$ MeV and to $\alpha_s=0.154\pm0.038$ at $E_{\text{CM}}=44$ GeV. This is, within the errors, consistent with $\Lambda_{\overline{\text{MS}}}=(205\pm13)$ MeV determined from the absolute *normalisation* of R_3 shown in fig. 2. Note that the determination of $\Lambda_{\overline{\text{MS}}}$ from r, although the error is rather large, does not depend on fragmentation model calculations and avoids the problem of the overall normalisation uncertainty between three-jet and three-parton rates discussed when presenting fig. 1.

Recently, preliminary results of this analysis together with data from the MARK-II and AMY collaborations [8], as well as an analysis by TASSO [7] using the same method of jet reconstruction as in this paper, have been reported. These results are in good agreement with our data and the QCD expectations presented above.

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[#5] The test presented in fig. 1 has been repeated with model calculations based on a constant value of α_s at all energies, resulting in the same behaviour of q as for the QCD models. This justifies to compare the data with a constant value of R_3 to verify the hypothesis of $\alpha_s=$ const.

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