Z. Phys. C – Particles and Fields 43, 287–300 (1989)

Zeitschrift für Physik C Particles and Fields

Jet masses in e^+e^- annihilation

J. del Peso[1], L. Labarga[2,*] F. Barreiro[3,*]

[1] Universidad Autónoma, E-28049 Madrid, Spain
[2] University of California at Santa Cruz, Santa Cruz, CA, USA
[3] DESY, D-2000 Hamburg, Federal Republic of Germany

Received 16 December 1988

Abstract. We calculate heavy (M_H^2/s) and light (M_L^2/s) jet masses up to $O(\alpha_s^2)$ in perturbative Quantum Chromodynamics (QCD). The sensitivity of these quantities to radiative corrections, quark masses, fragmentation effects as well as their infrared stability properties, are investigated and compared to those exhibited by other jet measures. A comparison with recent Mark II data is also presented.

1 Introduction

Particular attention has been paid in the last years to testing Quantum Chromodynamics in e^+e^- annihilations to hadrons. For this purpose, it has been advocated the need to measure quantities which are i) insensitive to radiative corrections ii) infrared stable and iii) not sensitive to fragmentation effects [1]. The asymmetric component of the energy–energy correlations (AEEC) has been found to best satisfy these requisites [2]. In particular its energy variation is very slow [3], similar to that exhibited by the total cross section $\sigma_T(e^+e^- \to \gamma \to hadrons)$ and compatible with that expected in perturbation theory. This is in contrast to the behaviour of most other jet measures exhibiting large power corrections with the exception of the jet mass difference [4]. Moreover it has been shown that the end point of the M_H^2/s distribution, if suitably defined, is fixed to all orders in perturbation theory [5]. This fact should allow a reasonable comparison between QCD predictions and data in the tail of the M_H^2/s distribution. These results have prompted us to undertake a systematic study of the properties exhibited by jet masses in perturbative QCD, as well as the influence of fragmentation effects upon them.

* Fullbright Fellow, on leave from Universidad Autónoma de Madrid, Spain
** Alexander von Humbolt Fellow, on leave from Universidad Autónoma de Madrid, Spain

Among the several definitions of M_H^2 and M_L^2 which can be treated, we will consider two: the first one is the manifestly Lorentz invariant definition according to which the final state particles are divided into two groups (jet 1 and jet 2) such that the sum of the squared invariant masses of the two groups is minimal (mass criterion, MAC), M_H^2 (M_L^2) being the larger of the two (resp. smaller). Apart from the Lorentz invariance, such definition has the advantage that M_H^2/s is bounded to all orders in perturbation theory by 1/3 [5]. Therefore, at sufficiently high energy, there is no region of the perturbative tail of the M_H^2/s distribution in which higher-order terms dominate. The second definition differs from the above one in the method of separation of two jets (or hemispheres). In this case we will make use of the plane perpendicular to the thrust axis [6] (thrust criterion, THC).

In perturbative QCD, and up to $O(\alpha_s^2)$, the jet masses receive contributions from the following processes:

$$e^+e^- \to q\bar{q} \tag{1}$$

$$e^+e^- \to q\bar{q}g \tag{2}$$

$$e^+e^- \to q\bar{q}gg,\, q\bar{q}q\bar{q} \tag{3}$$

so that in the case of massless quarks and gluons, M_L^2/s receives only contributions from second order processes. To $O(\alpha_s)$ we note that $M_H^2/s = 1 - T$, T denoting thrust. To this order, one then has [7]

$$\left\langle \frac{M_H^2}{S} \right\rangle = \langle 1 - T \rangle = 1.05 \frac{\alpha_s}{\pi} \tag{4}$$

$$\left\langle \frac{M_L^2}{S} \right\rangle = 0. \tag{5}$$

Including corrections to $O(\alpha_s^2)$ one finds [5]

$$\left\langle \frac{M_H^2}{S} \right\rangle = 1.05 \frac{\alpha_s}{\pi} + 6.9 \left(\frac{\alpha_s}{\pi} \right)^2 \tag{6}$$

$$\langle 1-T\rangle = 1.05\frac{\alpha_s}{\pi}+9\left(\frac{\alpha_s}{\pi}\right)^2 \quad (7)$$

$$\left\langle \frac{M_H^2}{S}-\frac{M_L^2}{S}\right\rangle = 1.05\frac{\alpha_s}{\pi}+2.9\left(\frac{\alpha_s}{\pi}\right)^2. \quad (8)$$

Notice that second order corrections to the jet mass difference $\langle M_H^2/S - M_L^2/S\rangle$ partly cancel.

At the fragmentation level and because of the substraction procedure, non-perturbative effects for the jet mass difference are expected to be supressed for 2-jet like events.

The aim of this paper is to calculate second order corrections not to the average values but to the complete differential distribution for the heavy jet mass, the light jet mass and the jet mass difference, taking as well into account resolution effects in the spirit of Sterman and Weinberg [8] and Fabricius *et al.* [9]. We will also investigate the sensitivity of these quantities to the fragmentation process and present a comparison with recent MARK II data. Our results therefore extend those presented in [5] and agree with them in those topics of overlap.

2 Perturbative calculations

To the lowest non-trivial order we have

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\frac{M_H^2}{S}}=\frac{\alpha_s}{\pi}T\left(\frac{M_H^2}{S}\right);\quad \frac{1}{\sigma_0}\frac{d\sigma}{d\frac{M_L^2}{S}}=0 \quad (9)$$

where σ_0 is the cross section for $e^+e^-\to q\bar{q}$, i.e. $\sigma_0=(4\pi\alpha^2/3S)\Sigma Q_f^2$, and $T(M_H^2/S)$ is a scalar function whose explicit form is given in [7].

In order to calculate second order corrections we follow the Monte Carlo technique described in [10] which has also been applied to a calculation of second order corrections to energy-energy correlations [2]. It makes use of the ERT virtual corrections to the 3-partons final states [11]. The finite 4-parton piece is obtained by integrating the matrix elements given in [12] over the appropriate phase space, after substracting the pole terms [11, 13]. Four partons final states not surviving a $y_{\min}=\min(y_{ij})$ cut with $y_{ij}=|p_i+p_j|^2/Q^2$ and p_i being the ith-parton's 4-momentum, are treated as 3-jet events after recombining those two partons giving rise to the smallest in-

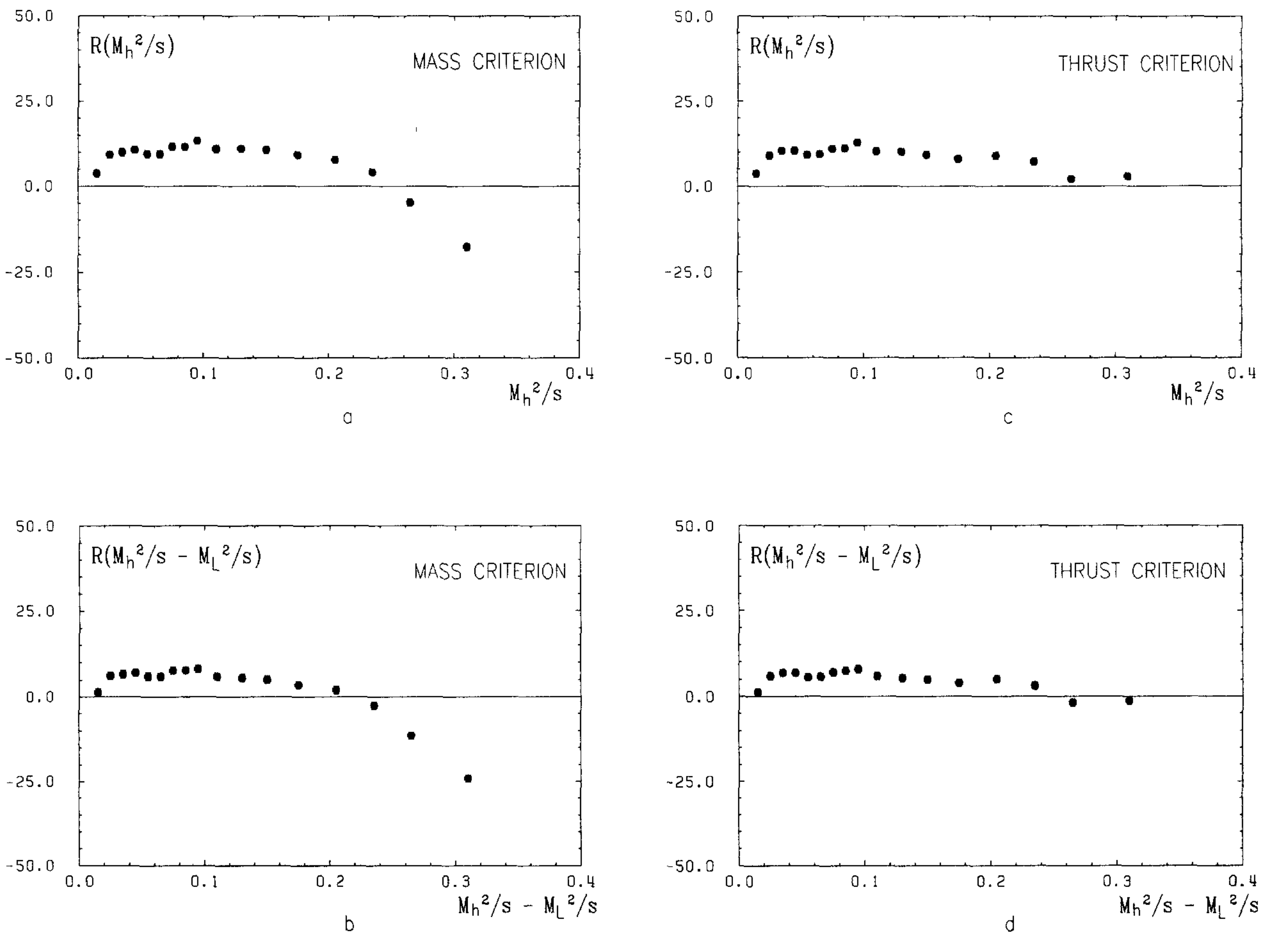


Fig. 1 a–d. Correction function R, defined in (10), for the heavy jet mass and jet mass difference using both jet-separation criteria

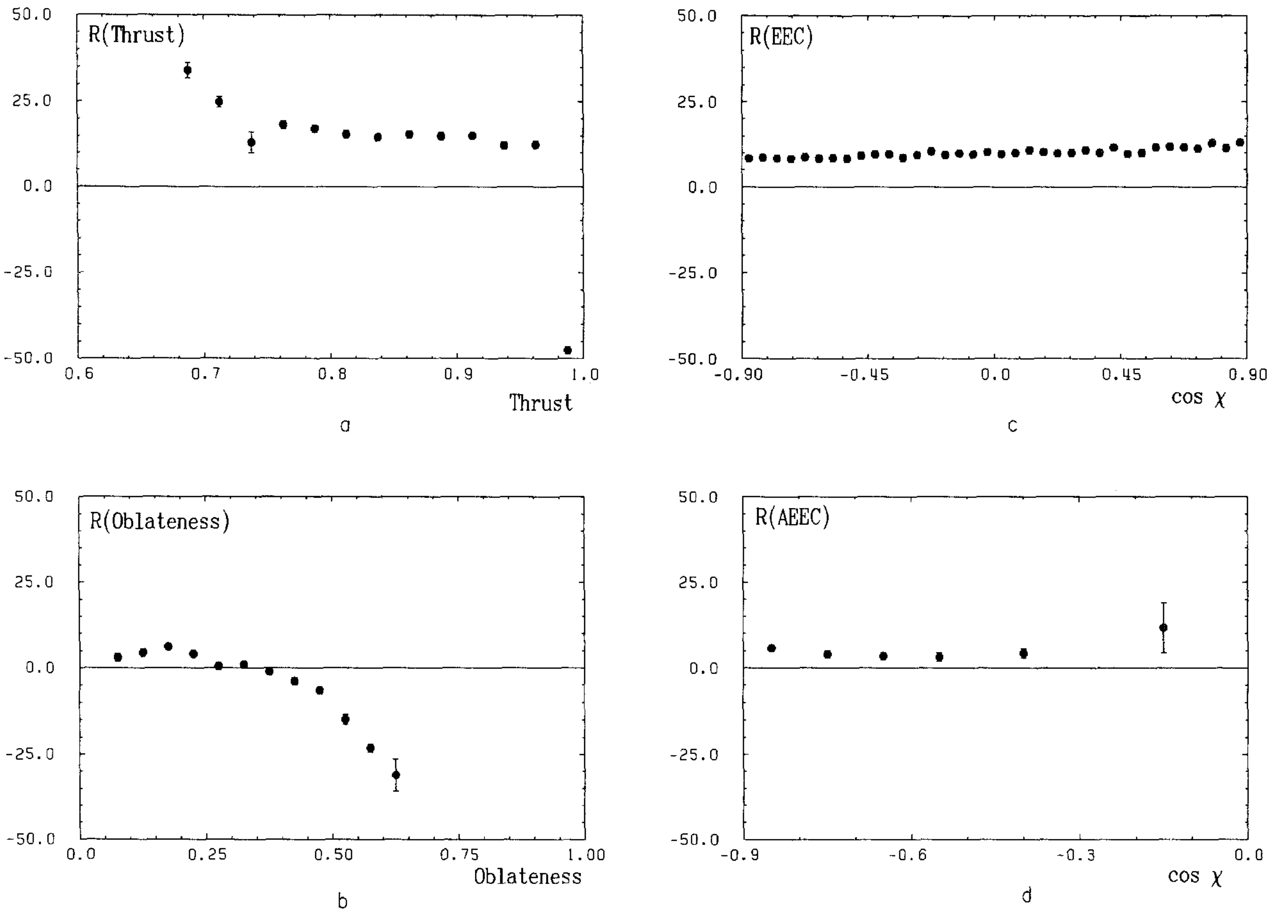


Fig. 2 a–d. Correction functions R forthrust, oblateness, EEC and AEEC

Table 1. Second order correction function for $M^2 = M_H^2$, $M_H^2 - M_L^2$

	MAC		THC	
$\frac{M^2}{S}$	$R\left(\frac{M_H^2}{S}\right)$	$R\left(\frac{(M_H^2 - M_L^2)}{S}\right)$	$R\left(\frac{M_H^2}{S}\right)$	$R\left(\frac{(M_H^2 - M_L^2)}{S}\right)$
0.01–0.02	3.70 ± 0.27	1.34 ± 0.26	3.71 ± 0.30	1.23 ± 0.29
0.02–0.03	9.24 ± 0.31	6.21 ± 0.30	9.04 ± 0.35	5.92 ± 0.34
0.03–0.04	9.91 ± 0.32	6.68 ± 0.31	10.40 ± 0.37	6.90 ± 0.36
0.04–0.05	10.62 ± 0.35	7.06 ± 0.35	10.44 ± 0.42	6.83 ± 0.41
0.05–0.06	9.45 ± 0.38	6.04 ± 0.37	9.25 ± 0.43	5.57 ± 0.43
0.06–0.07	9.44 ± 0.43	5.96 ± 0.42	9.34 ± 0.50	5.79 ± 0.50
0.07–0.08	11.58 ± 0.46	7.73 ± 0.46	11.05 ± 0.55	7.05 ± 0.54
0.08–0.09	11.48 ± 0.62	7.76 ± 0.61	11.11 ± 0.74	7.47 ± 0.74
0.09–0.10	13.29 ± 0.51	8.18 ± 0.50	12.79 ± 0.51	7.93 ± 0.51
0.10–0.12	10.80 ± 0.34	5.95 ± 0.33	10.22 ± 0.39	5.90 ± 0.39
0.12–0.14	11.05 ± 0.39	5.56 ± 0.39	10.14 ± 0.44	5.38 ± 0.44
0.14–0.16	10.66 ± 0.41	5.04 ± 0.41	9.24 ± 0.48	4.91 ± 0.48
0.16–0.19	9.14 ± 0.39	3.50 ± 0.39	8.00 ± 0.44	3.92 ± 0.43
0.19–0.22	7.72 ± 0.40	2.00 ± 0.40	9.00 ± 0.48	5.04 ± 0.47
0.22–0.25	4.01 ± 0.47	-2.65 ± 0.46	7.23 ± 0.56	3.11 ± 0.55
0.25–0.28	-4.80 ± 0.74	-11.40 ± 0.74	2.14 ± 0.93	-1.86 ± 0.92
0.28–0.34	-17.74 ± 0.82	-24.10 ± 0.80	2.85 ± 1.04	-1.22 ± 1.03

variant mass pair. Their contribution is then integrated over the appropriate three-jet phase space.

With this technique we are able to obtain for $M^2 = M_H^2$, $M_H^2 - M_L^2$, correction functions so that the result to $O(\alpha_s^2)$ can be written as follows,

$$\frac{1}{\sigma_T}\frac{d\sigma}{d\frac{M^2}{S}} = \frac{\alpha_s(Q^2)}{\pi}T\left(\frac{M^2}{S}\right)\left\{1+\frac{\alpha_s(Q^2)}{\pi}R\left(\frac{M^2}{S}\right)\right\}, \tag{10}$$

where σ_T is the total cross section to $O(\alpha_s^2)$, i.e. $\sigma_T = \sigma_0(1+\alpha_s/\pi+(1.98-0.116n_f)(\alpha_s/\pi)^2)$. In QCD, M_L^2 receives only second and higher order contributions, therefore one would be able to investigate the corresponding correction function only once third order contributions have been calculated.

To cancel divergencies we need a regulator, which in this case we take to be $y_{\min}$. We used the value $y_{\min}^{\text{reg}} = 10^{-3}$ and checked that for values below it, our correction functions defined in (10) are stable.

The correction functions, for the two criteria discussed above, are shown in Fig. 1. The fact that, as indicated in (8), $\langle (M_H^2 - M_L^2)/S \rangle$ is less affected to $O(\alpha_s^2)$ corrections than $\langle M_H^2/S \rangle$ in (6) is not fortuitous, i.e. due to the cancellation of opposite sign correction factors, but the consequence of a much better overall behaviour of the former.

From Fig. 1 and (10) we find that second order contributions to M_H^2 and $M_H^2 - M_L^2$ are $\sim 40\%$ and 20% respectively at $\sqrt{s} = 44$ GeV and corresponding to $\Lambda_{\overline{MS}} = 0.1$ GeV. The results of our calculations shown in Fig. 1 are also presented in tabular form, Table 1.

For the sake of comparison we show in Fig. 2 those correction functions obtained for a variety of jet measures like thrust, oblateness, the energy–energy correlations (EEC) and its associated asymmetry (AEEC). If one would have to rate these measures according to their sensitivity to $O(\alpha_s^2)$ corrections one would therefore say that $(M_H^2 - M_L^2)/S$ and the AEEC receive the smallest contributions in second order, thrust the largest while oblateness, the energy-energy correlations (EEC) and the heavy jet mass are lying in between. The coefficients of the $(\alpha_s/\pi)^2$ terms in the expression for the mean values, (6–8), could therefore serve as figures of merit.

The perturbative expansion for quantities like AEEC, M_H^2/S and $(M_H^2 - M_L^2)/S$ seems to be converging fast enough so as to allow quantitative QCD tests. On the other hand the light jet mass receives non-zero contribution in perturbative QCD only to $O(\alpha_s^2)$. Therefore we are not in a position to judge how fast the perturbative expansion in this variable is converging. In order to get an approximate idea of the importance of higher order corrections to jet masses we show in Fig. 3 both the heavy and light jet masses in a LLA approximation. This has been implemented using *B.* Webber [14] Monte Carlo, once fragmentation effects are switched off. We used the parameters proposed, in order to reproduce the data, by Webber [14]. (See also [20]). The prediction for M_L^2/S to $O(\alpha_s^2)$ lies much below the LLA expectation, thus indicating the importance of $O(\alpha_s^3)$ and higher corrections for this variable. This is in contrast with the situation for the M_H^2/S distribution.

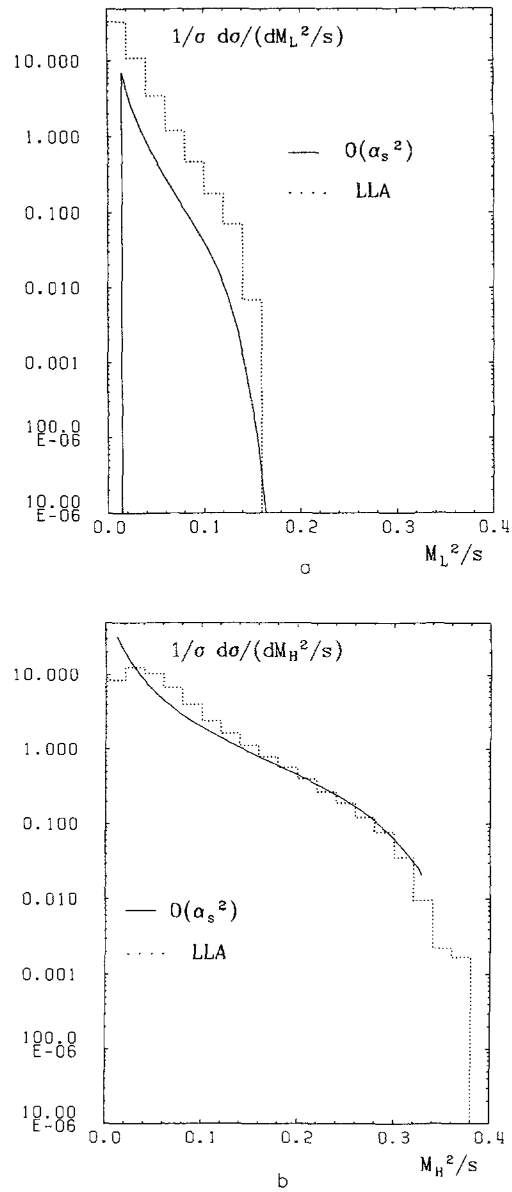


Fig. 3 a, b. Comparison between the $O(\alpha_s^2)$ and LLA cross sections for the light and heavy jet masses. The LLA was implemented using *B.* Webber Monte Carlo without fragmentation effects and with the parameters: $\Lambda_{\text{LLA}} = 0.25$ GeV, $Q_0 = 0.6$ GeV

We now turn to a discussion of the infrared stability properties of the jet masses. This amounts to recalculate the cross sections discussed above giving an angular width to a parton, thus defining when soft partons are not resolvable. The procedure we follow is the same described above, since the virtual corrections in [11] are cast in a form particularly suited to

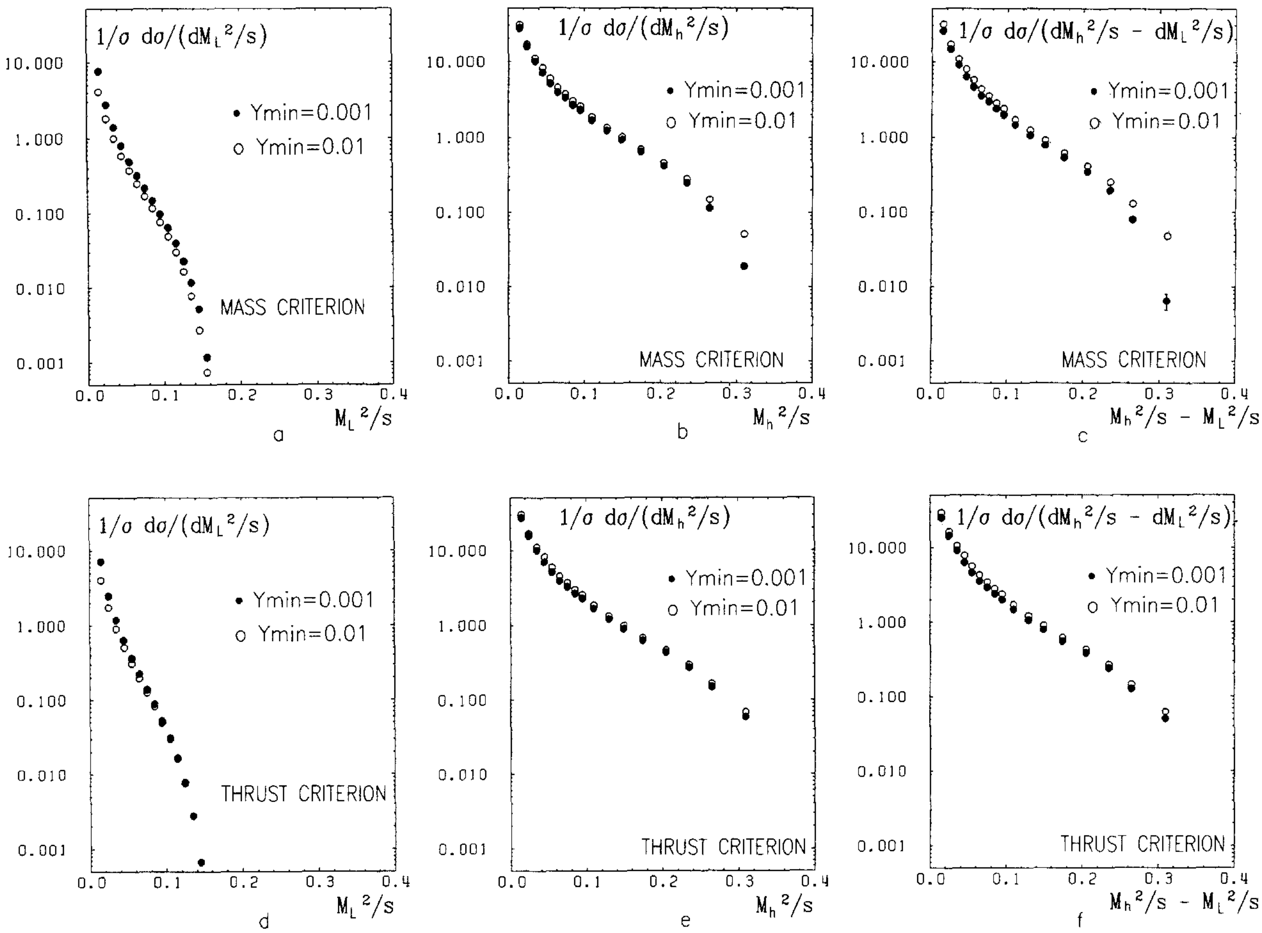


Fig. 4 a–f. The complete $O(\alpha_s^2)$ cross section $d\sigma/d(M^2/s)$, $M^2 = M_L^2, M_H^2, M_H^2 - M_L^2$, for $y_{\min} = 0.001$ and 0.01 and for both jet-separation criteria

be used in a Monte Carlo programme thus allowing to impose any resolution criteria.

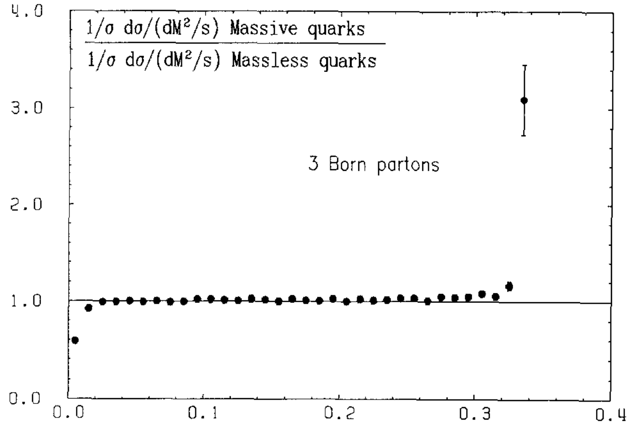
Particular attention has to be paid to the type of resolution parameters one chooses, namely (ε, δ) cuts a la Sterman–Weinberg [8], or mass cuts. In general one could argue that (ε, δ) type resolution parameters are best suited when one is looking at angular distributions or correlations. In particular, it has been argued that already to $O(\alpha_s^2)$ the AEEC is stable upon (ε, δ) resolution cut, but not upon mass cuts [12, 2].

When studying jet masses, the situation is reversed and we find it more appropriate to introduce $y_{\min}$ cuts [9]. For illustrative purposes we have chosen the value $y_{\min} = 0.01$, which is equivalent at the highest energies available at PETRA to 4–5 GeV mass cut.

The complete $O(\alpha_s^2)$ cross section $d\sigma/d(M^2/S)$, $M^2 = M_L^2$, M_H^2, $M_H^2 - M_L^2$ for $y_{\min} = 0.001$ and 0.01 are shown in Fig. 4. It should be noted that these cross sections correspond to $\sqrt{s} = 44\,\text{GeV}$ and $\Lambda_{\overline{MS}} = 0.1\,\text{GeV}$. The numerical values are given in Tables 2, 3, 4.

Notice that for large masses the distributions obtained using mass or thrust criterion differ. We attribute this fact to the uncertainty in assigning low energetic gluons to a particular hemisphere. When the mass criterion is used, they play an active part in the selection of the jets, which leads to smaller heavy jet masses and jet mass difference, and larger light jet masses than for the thrust method. This effect will be enhanced once the hadronization of the partons has taken place. As expected, both sets of distributions in fig. 4 approach each other with increasing $y_{\min}$ cut.

For the sake of completeness we have also investigated the effect of including quark masses on the jet mass calculations. Of course we do this in the approximation of keeping mass term corrections only for the Born diagrams associated to processes (2) and (3). As already discussed in [2] this amounts to stating that the cancellation of divergences between the $O(\alpha_s^2)$ virtual 3-jet cross section and the soft 4-jet cross sections are done in the limit of massless quarks. The mass correction terms for Born three-jets are taken from [15] and those for Born four jets from [12]. The ratio between the Born $O(\alpha_s)$ and $O(\alpha_s^2)$ cross sections with quark mass effects to those obtained in the limit $m_q \to 0$ are shown in Fig. 5a, 5b. It is seen that quark mass effects are therefore small but for very low or very large jet masses, see also Table 5.



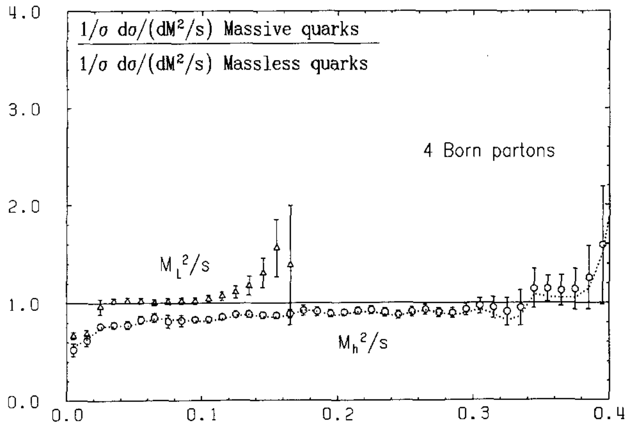


Fig. 5 a, b. Mass corrections to the 3 and 4 Born partons cross sections $d\sigma/(d(M^2/s))$, $M^2 = M_L^2$, M_H^2, $M_H^2 - M_L^2$, for $\sqrt{s} = 44\,\text{GeV}$ and $\Lambda_{\overline{MS}} = 0.1\,\text{GeV}$

3 Non-perturbative contributions to jet masses

In the absence of an understanding of strong interaction dynamics at large distances, any discussion of the metamorphoses of partons into hadrons is model dependent. In order to estimate the importance of fragmentation effects we will consider two extreme models: the independent jet fragmentation model (IJF), originally developed by Field and Feynman for two-jet final states [16] and further improved by Hoyer et al. [17] and Ali et al. [18] to consider $O(\alpha_s)$ and $O(\alpha_s^2)$ effects respectively, and the string fragmentation model developped by the LUND group [19]. In the former, partons fragment independent of each other, while in the latter, final state hadrons are the result of successive breakups of colour strings.

Irrespective of the model we use for fragmentation, we find an effect similar to that discussed in the previous section. Low energetic particles appearing in the hadronization process play different roles for both jet-separation methods under study. One could argue that in the case of the mass criterion, low energetic particles at large angle with respect to the parton direction of flight have an influence in the determination of the minimum massive jets. This is not the case for the thrust criterion where the jet axis remains almost unchanged from the original direction of the parton. For this jet-separation criterion, the differences among distributions before and after fragmentation come from the intrinsic transverse momentum of the hadrons and not from the reconstruction of the jet axes. We consider in Fig. 6 a sample of 3-jet events generated at $\sqrt{s} = 34\,\text{GeV}$ and fragmenting independently and plot $(1/\sigma_T)$ $(d\sigma/d(M_H^2/s - M_L^2/s))$ and $(1/\sigma_T)\,(d\sigma/d(M_H^2/s))$ before and after the hadronization of the partons and for the two jet-separation methods under consideration. We observe that the fragmenta-

Table 2. $O(\alpha_s^2)$ cross section for the light jet mass, $(1/\sigma_T)(\,d\sigma)/(M_L^2/S)$, using two jet separation criteria (MAC and THC) and two $y_{\min}$ values

$\frac{M^2}{S}$	$\frac{1}{\sigma_T}\frac{d\sigma}{M_L^2/S}$ MAC $y_{\min} = 0.001$	MAC $y_{\min} = 0.010$	THC $y_{\min} = 0.001$	THC $y_{\min} = 0.010$
0.00–0.01	–	90.60 ± 0.42	–	91.01 ± 0.48
0.01–0.02	7.651 ± 0.031	4.089 ± 0.014	7.138 ± 0.033	4.023 ± 0.016
0.02–0.03	2.784 ± 0.012	1.8312 ± 0.0061	2.480 ± 0.012	1.7428 ± 0.0068
0.03–0.04	1.4077 ± 0.0064	0.9971 ± 0.0035	1.1865 ± 0.0063	0.9051 ± 0.0037
0.04–0.05	0.8024 ± 0.0043	0.5857 ± 0.0021	0.6379 ± 0.0038	0.5124 ± 0.0022
0.05–0.06	0.4907 ± 0.0029	0.3744 ± 0.0015	0.3657 ± 0.0024	0.3115 ± 0.0015
0.06–0.07	0.3167 ± 0.0020	0.2466 ± 0.0010	0.2248 ± 0.0016	0.1968 ± 0.0010
0.07–0.08	0.2154 ± 0.0020	0.1683 ± 0.0008	0.1390 ± 0.0011	0.1256 ± 0.0007
0.08–0.09	0.1471 ± 0.0012	0.1167 ± 0.0006	0.0892 ± 0.0008	0.0829 ± 0.0005
0.09–0.10	0.0975 ± 0.0009	0.0763 ± 0.0005	0.0528 ± 0.0005	0.0500 ± 0.0004
0.10–0.11	0.0643 ± 0.0008	0.0492 ± 0.0003	0.0310 ± 0.0003	0.0299 ± 0.0003
0.11–0.12	0.0399 ± 0.0005	0.0301 ± 0.0003	0.0167 ± 0.0002	0.0163 ± 0.0002
0.12–0.13	0.0228 ± 0.0004	0.0165 ± 0.0002	0.0077 ± 0.0001	0.0076 ± 0.0001
0.13–0.14	0.0117 ± 0.0003	0.0077 ± 0.0001	0.0028 ± 0.0001	0.0028 ± 0.0001
0.14–0.15	0.0052 ± 0.0003	0.0027 ± 0.0001	0.0007 ± 0.0001	0.0007 ± 0.0001

Table 3. $O(\alpha_s^2)$ cross section for the heavy jet mass, $(1/\sigma_T)(d\sigma/M_H^2/S)$, using two jet separation criteria (MAC and THC) and two y_{min} values

$\frac{M^2}{S}$	$\frac{1}{\sigma_T}\frac{d\sigma}{M_H^2/S}$ MAC $y_{min}=0.001$	MAC $y_{min}=0.010$	THC y_{min} 0.001	THC $y_{min}=0.010$
0.00–0.01	–	-0.3126 ± 0.0012	–	-0.3113 ± 0.0017
0.01–0.02	27.45 ± 0.24	30.31 ± 0.35	27.40 ± 0.26	30.09 ± 0.39
0.02–0.03	15.80 ± 0.13	16.72 ± 0.17	15.77 ± 0.15	16.71 ± 0.20
0.03–0.04	10.021 ± 0.087	11.00 ± 0.11	10.06 ± 0.10	11.07 ± 0.12
0.04–0.05	7.044 ± 0.066	8.212 ± 0.077	7.008 ± 0.077	8.215 ± 0.091
0.05–0.06	5.135 ± 0.052	5.961 ± 0.057	5.137 ± 0.061	5.953 ± 0.066
0.06–0.07	3.930 ± 0.046	4.544 ± 0.049	3.922 ± 0.054	4.548 ± 0.058
0.07–0.08	3.325 ± 0.039	3.721 ± 0.041	3.268 ± 0.047	3.692 ± 0.049
0.08–0.09	2.676 ± 0.043	3.008 ± 0.044	2.650 ± 0.051	2.990 ± 0.052
0.09–0.10	2.307 ± 0.029	2.568 ± 0.030	2.283 ± 0.029	2.559 ± 0.031
0.10–0.12	1.691 ± 0.015	1.874 ± 0.016	1.666 ± 0.017	1.862 ± 0.018
0.12–0.14	1.243 ± 0.013	1.358 ± 0.013	1.217 ± 0.014	1.334 ± 0.015
0.14–0.16	0.928 ± 0.010	1.014 ± 0.010	0.897 ± 0.012	0.992 ± 0.012
0.16–0.19	0.640 ± 0.007	0.6878 ± 0.0070	0.6213 ± 0.0077	0.6751 ± 0.0078
0.19–0.22	0.4147 ± 0.0048	0.4550 ± 0.0049	0.4295 ± 0.0056	0.4630 ± 0.0057
0.22–0.25	0.2476 ± 0.0037	0.2810 ± 0.0038	0.2721 ± 0.0044	0.2939 ± 0.0044
0.25–0.28	0.1131 ± 0.0037	0.1467 ± 0.0038	0.1474 ± 0.0046	0.1622 ± 0.0047
0.28–0.34	0.0188 ± 0.0016	0.0511 ± 0.0017	0.0586 ± 0.0020	0.0683 ± 0.0020

Table 4. $O(\alpha_s^2)$ cross section for the jet mass, $(1/\sigma_T)(d\sigma/(M_H^2-M_L^2)/S)$, using two jet separation criteria (MAC and THC) and two y_{min} values

$\frac{M^2}{S}$	$\frac{1}{\sigma_T}\frac{d\sigma}{(M_H^2-M_L^2)/S}$ MAC $y_{min}=0.001$	MAC $y_{min}=0.010$	THC $y_{min}=0.001$	THC $y_{min}=0.010$
0.00–0.01	–	1.406 ± 0.011	–	4.27 ± 0.06
0.01–0.02	25.37 ± 0.23	30.91 ± 0.35	25.22 ± 0.25	29.18 ± 0.39
0.02–0.03	14.50 ± 0.13	16.73 ± 0.17	14.42 ± 0.15	16.24 ± 0.20
0.03–0.04	9.152 ± 0.085	10.86 ± 0.11	9.127 ± 0.097	10.56 ± 0.12
0.04–0.05	6.384 ± 0.065	8.017 ± 0.077	6.341 ± 0.076	7.848 ± 0.090
0.05–0.06	4.660 ± 0.051	5.768 ± 0.057	4.621 ± 0.060	5.636 ± 0.066
0.06–0.07	3.558 ± 0.045	4.362 ± 0.049	3.543 ± 0.053	4.289 ± 0.057
0.07–0.08	2.996 ± 0.039	3.552 ± 0.041	2.928 ± 0.046	3.450 ± 0.049
0.08–0.09	2.420 ± 0.042	2.855 ± 0.044	2.400 ± 0.051	2.808 ± 0.052
0.09–0.10	2.017 ± 0.028	2.423 ± 0.030	2.007 ± 0.029	2.368 ± 0.030
0.10–0.12	1.477 ± 0.015	1.740 ± 0.015	1.475 ± 0.017	1.717 ± 0.018
0.12–0.14	1.065 ± 0.013	1.244 ± 0.013	1.062 ± 0.014	1.220 ± 0.015
0.14–0.16	0.791 ± 0.010	0.924 ± 0.010	0.791 ± 0.012	0.904 ± 0.012
0.16–0.19	0.541 ± 0.007	0.6228 ± 0.0070	0.5498 ± 0.0076	0.6163 ± 0.0078
0.19–0.22	0.3471 ± 0.0047	0.4145 ± 0.0049	0.3828 ± 0.0056	0.4258 ± 0.0057
0.22–0.25	0.1950 ± 0.0036	0.2523 ± 0.0037	0.2397 ± 0.0044	0.2672 ± 0.0044
0.25–0.28	0.0800 ± 0.0037	0.1300 ± 0.0038	0.1275 ± 0.0046	0.1453 ± 0.0046
0.28–0.34	0.0064 ± 0.0016	0.0481 ± 0.0017	0.0507 ± 0.0020	0.0623 ± 0.0020

tion process leads to a noticeable change of the shape of the partonic cross-section when using the mass criterion. In contrast, when the thrust axis is used to divide the event into two hemispheres, the induced changes are less important. Considering the better behaviour observed with fragmentation, we will use the thrust criterion along the rest of this section.

In order to quantify the relative importance of fragmentation effects in different jet topologies, we start with the contribution of two-jet events, where there is no low order perturbative QCD part at all. We use the limited-p_T model of Field and Feynman [16]. In

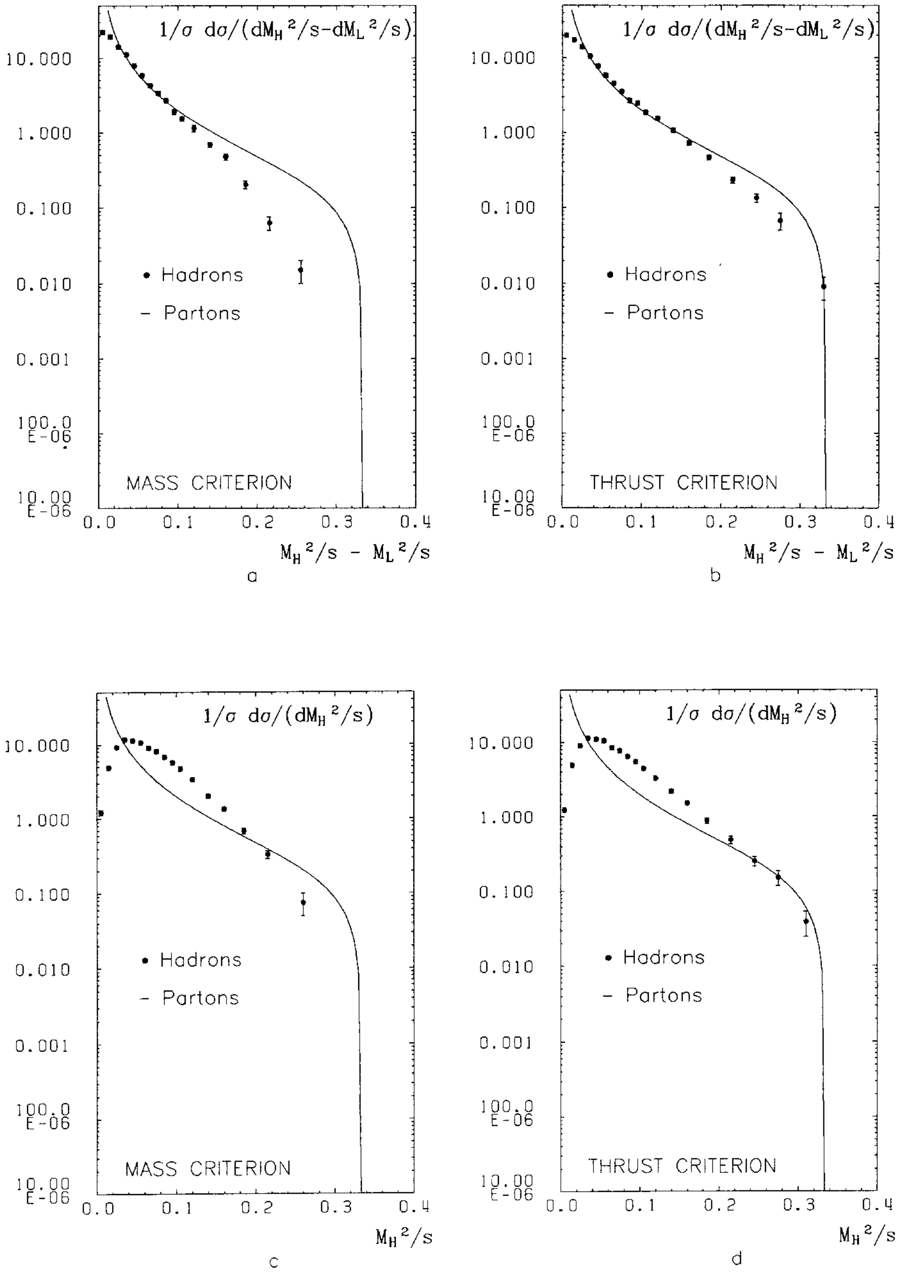


Fig. 6 a–d. $(1/\sigma_T)(d\sigma/d(M_H^2/s - M_L^2/s))$ and $(1/\sigma_T)$ $(d\sigma/d(M_H^2/s))$ before and after the hadronization process at $\sqrt{s} = 34\,\mathrm{GeV}$ for both jet-separation criteria under study. A sample of 3 jets fragmenting independently has been used

Fig. 7 we show the distributions for the light, heavy jet mass and jet mass difference at $\sqrt{s} = 22$, 44 and 90 GeV. As expected, the heavy jet mass receives a more important contribution at larger masses than the jet-mass difference. It is clear from Fig. 7 that at the highest energies available at PETRA, the fragmentation effects due to the dominant two jet topology are negligible above $\Delta M^2/S \simeq 0.15$, 0.1 for $M = M_H$, $M_H - M_L$ respectively.

We investigate fragmentation-induced deviations from the QCD result for events with not only two but also three and four jets by studying the following quantity:

$$\Delta\left(\frac{M^2}{S}\right) = \frac{\left.\dfrac{1}{\sigma_T}\dfrac{d\sigma}{d\dfrac{M^2}{S}}\right|_{\mathrm{QCD}} - \left.\dfrac{1}{\sigma_T}\dfrac{d\sigma}{d\dfrac{M^2}{S}}\right|_{FR}}{\left.\dfrac{1}{\sigma_T}\dfrac{d\sigma}{d\dfrac{M^2}{S}}\right|_{\mathrm{QCD}}} \qquad (11)$$

where the subscripts QCD and FR denote parton $(O(\alpha_s^2))$ and hadron level respectively. We will refer to that quantity as "fragmentation effects", which includes higher than second order corrections. We consider both the independent fragmentation scheme

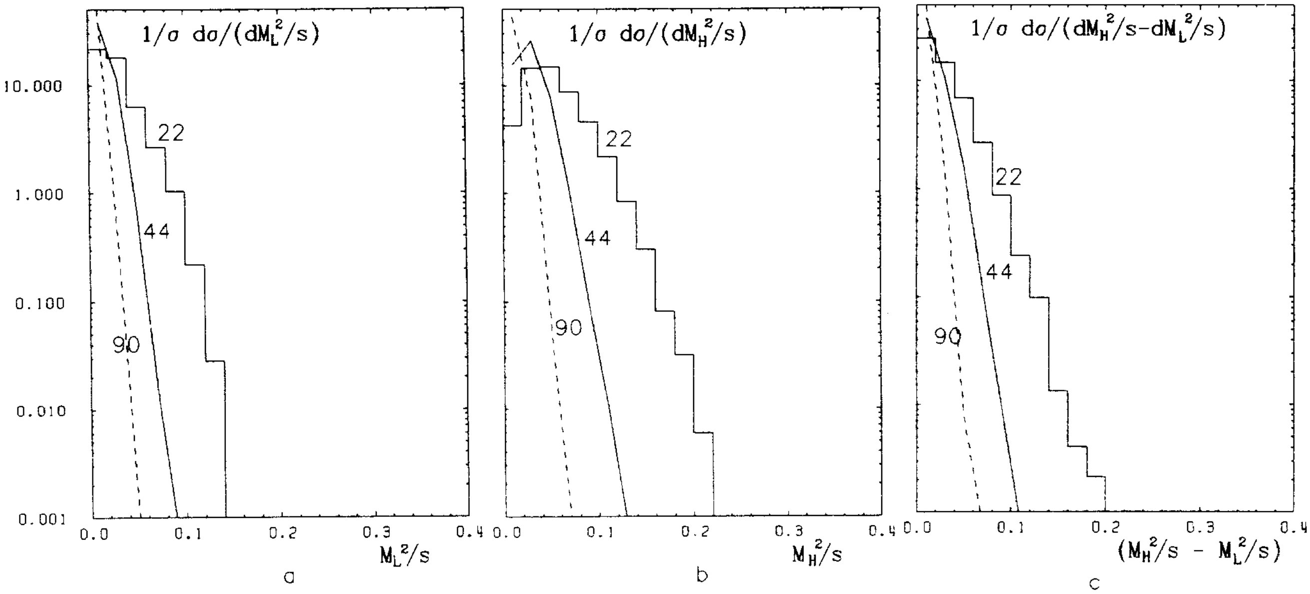


Fig. 7 a–c. Predictions of the Field and Feynman model for two-jet contribution to the light and heavy jet masses and the jet mass difference distributions at $\sqrt{s} = 22$, 44 and 90 GeV

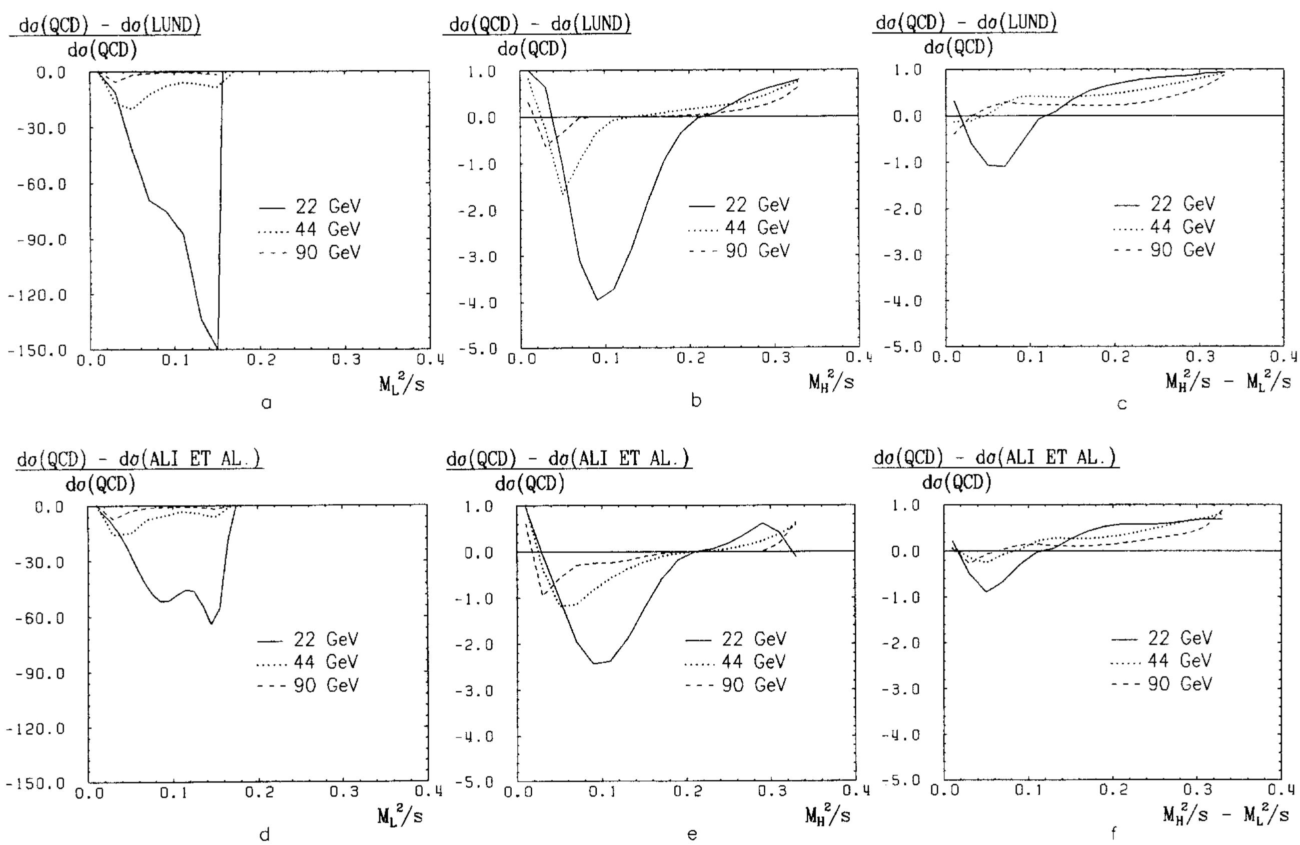


Fig. 8 a–f. The fragmentation contribution, as defined in (11), to the light and heavy jet masses and to the jet mass difference estimated with the Ali et al. (IJF) and Lund models for $\sqrt{s} = 22, 44$ and 90 GeV

Table 5. $C(M^2)=(d\sigma/(dM^2/S)(\text{massive partons}))/(d\sigma/(dM^2/S)$ (massless partons))

	3 Born's partons	4 Born's partons	
$\frac{M^2}{S}$	$C(M_H^2)$	$C(M_H^2)$	$C(M_L^2)$
0.00–0.01	0.589 ± 0.012	0.525 ± 0.065	0.668 ± 0.032
0.01–0.02	0.920 ± 0.006	0.619 ± 0.057	0.689 ± 0.043
0.02–0.03	0.990 ± 0.007	0.766 ± 0.033	0.963 ± 0.077
0.03–0.04	0.9917 ± 0.0075	0.777 ± 0.030	1.015 ± 0.020
0.04–0.05	0.9995 ± 0.0078	0.777 ± 0.040	1.019 ± 0.023
0.05–0.06	0.9905 ± 0.0080	0.831 ± 0.038	1.018 ± 0.025
0.06–0.07	1.0002 ± 0.0083	0.851 ± 0.044	1.002 ± 0.025
0.07–0.08	0.9875 ± 0.0084	0.814 ± 0.065	1.012 ± 0.027
0.08–0.09	0.9944 ± 0.0087	0.824 ± 0.057	1.021 ± 0.030
0.09–0.10	1.0197 ± 0.0091	0.833 ± 0.030	1.021 ± 0.032
0.10–0.11	1.0175 ± 0.0092	0.833 ± 0.027	1.044 ± 0.036
0.11–0.12	1.0095 ± 0.0094	0.860 ± 0.025	0.073 ± 0.045
0.12–0.13	1.0024 ± 0.0095	0.884 ± 0.028	1.113 ± 0.062
0.13–0.14	1.0206 ± 0.0098	0.886 ± 0.028	1.178 ± 0.096
0.14–0.15	1.015 ± 0.010	0.878 ± 0.029	1.31 ± 0.15
0.15–0.16	1.000 ± 0.01	0.872 ± 0.032	1.56 ± 0.29
0.16–0.17	1.022 ± 0.010	0.889 ± 0.042	1.39 ± 0.61
0.17–0.18	1.008 ± 0.011	0.924 ± 0.051	–
0.18–0.19	1.003 ± 0.011	0.912 ± 0.046	–
0.19–0.20	1.023 ± 0.011	0.890 ± 0.033	–
0.20–0.21	0.999 ± 0.011	0.901 ± 0.034	–
0.21–0.22	1.022 ± 0.012	0.916 ± 0.036	–
0.22–0.23	1.009 ± 0.012	0.925 ± 0.040	–
0.23–0.24	1.016 ± 0.013	0.903 ± 0.041	–
0.24–0.25	1.030 ± 0.013	0.877 ± 0.041	–
0.25–0.26	1.030 ± 0.014	0.909 ± 0.049	–
0.26–0.27	1.008 ± 0.014	0.933 ± 0.050	–
0.27–0.28	1.047 ± 0.016	0.903 ± 0.046	–
0.28–0.29	1.036 ± 0.017	0.898 ± 0.050	–
0.29–0.30	1.048 ± 0.019	0.933 ± 0.060	–
0.30–0.31	1.080 ± 0.023	0.969 ± 0.079	–
0.31–0.32	1.051 ± 0.028	0.953 ± 0.107	–
0.32–0.33	1.162 ± 0.045	0.91 ± 0.14	–
0.33–0.34	3.09 ± 0.36	0.95 ± 0.18	–
0.34–0.35	–	1.15 ± 0.20	–
0.35–0.36	–	1.14 ± 0.13	–
0.36–0.37	–	1.13 ± 0.16	–
0.37–0.38	–	1.14 ± 0.21	–
0.38–0.39	–	1.25 ± 0.32	–
0.39–0.40	–	1.59 ± 0.60	–

by Ali et al. [18] and the string model by the Lund group [19]. We have calculated $\Delta(M^2/s)$ at $\sqrt{s}=22$, 44 and 90 GeV for $M^2=M_L^2$, M_H^2 and $M_H^2-M_L^2$. The results are shown in Fig. 8. Both models show the same trend with increasing energy, but their predictions differ quantitatively. By looking at the 44 GeV curves one can see that, in the first half of the spectrum $\sim$(0.03–0.15), the fragmentation effects partially cancel for the jet mass difference, while they are still large for both light and heavy jet masses. In contrast, in the second half of the spectrum, $\sim$(0.15–0.3), the heavy jet mass is little affected by fragmentation effects, while at the order we are considering, the jet mass difference is still influenced by the behaviour of the light jet mass.

4 Comparison with experiment

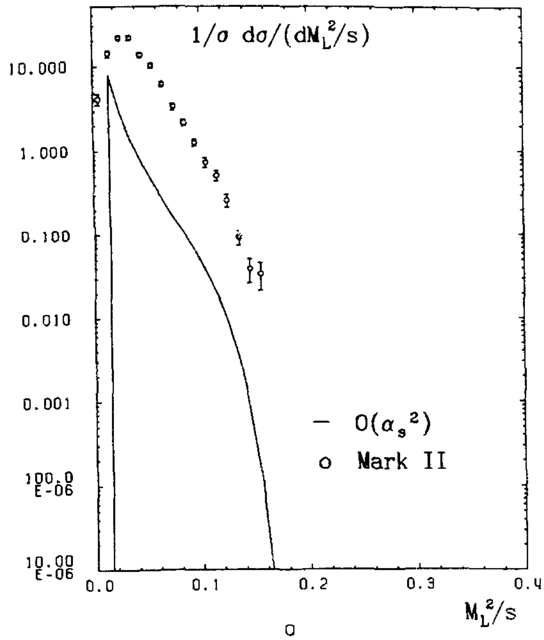
We would like now to compare the QCD predictions worked out in previous sections with recent experimental data published by the MARK II Collaboration [20].

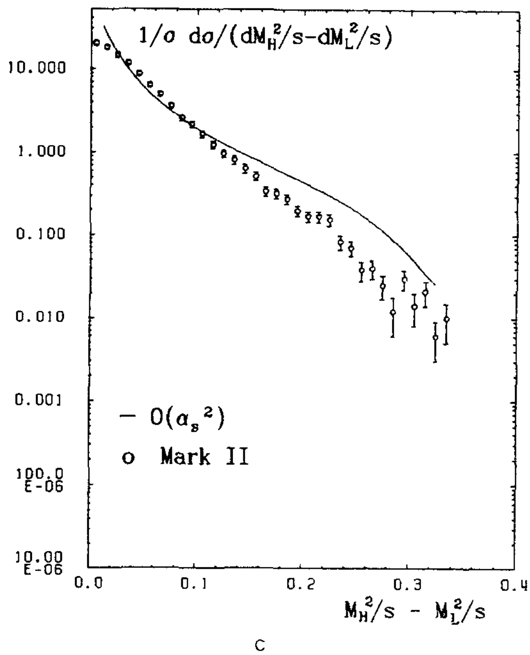
In Fig. 9 we show the MARK II data on M_H^2/s, M_L^2/s, and $(M_H^2-M_L^2)/s$, obtained at a c.m. energy of $\sqrt{s}=29$ GeV, together with our bare QCD predictions for a value of $\Lambda_{\overline{MS}}=0.1$ GeV, which corresponds to $\alpha_s(\sqrt{s}=29\text{ GeV})=0.127$. As one would expect from the discussion in the preceeding section, the bare QCD prediction is in rough agreement with the heavy jet mass (jet mass difference) in the second half (resp. first half) of the spectrum. On the other hand, the bare QCD prediction for the light jet mass lies much below the experimental data.

In order to describe the experimental distribution for M_H^2/S, M_L^2/S and $(M_H^2-M_L^2)/S$, over their whole range of variation, we need to convolute our bare QCD predictions to $O(\alpha_s^2)$ and $y_{\min}=0.01$ with model dependent fragmentation schemes for quarks and gluons. As already discussed above, we will consider two extreme fragmentation pictures, IJF [18] and LUND [19]. The values for the parameters used in our fragmentation models are summarized in Table 6. These parameters have been tuned in order to describe the gross features of the data obtained by the TASSO collaboration [21]. In Fig. 10, we present the MARK II data, together with the results of a fit (over the complete mass range) with our QCD plus fragmentation model where the value of α_s, determining the ratios of 3/2 and 4/3 jets, was left as a free parameter. The agreement between data and both models is satisfactory. Values obtained for α_s and the QCD scale $\Lambda_{\overline{MS}}$ are given in Table 7 together with the corresponding χ^2/NDF, obtained in the fitting procedure.

The following comments are in order:

- The errors quoted are of a statistical nature and do not take into account correlations between fragmentation parameters and $\Lambda_{\overline{MS}}$
- Values obtained within the LUND scheme are systematically higher than those obtained within the the IJF model
- The highest α_s values are obtained from fitting the light mass distribution. One can understand this fact by looking at Fig. 3a, where higher order corrections, estimated in the LLA, are shown to be large and "positive", and Fig. 9a, where the α_s^2 perturbative calculation is shown to be much below the data. This is in contrast with the corresponding situation for M_H^2/S, see figs. 3b and 9b.
- The values of α_s obtained from fitting the heavy jet mass, M_H^2/S, in a given fragmentation scheme, are higher than those obtained from fitting the jet mass difference, $(M_H^2-M_L^2)/S$. We interpret that as a consequence of the underestimation of the light jet mass at the perturbative level, as previously discussed.





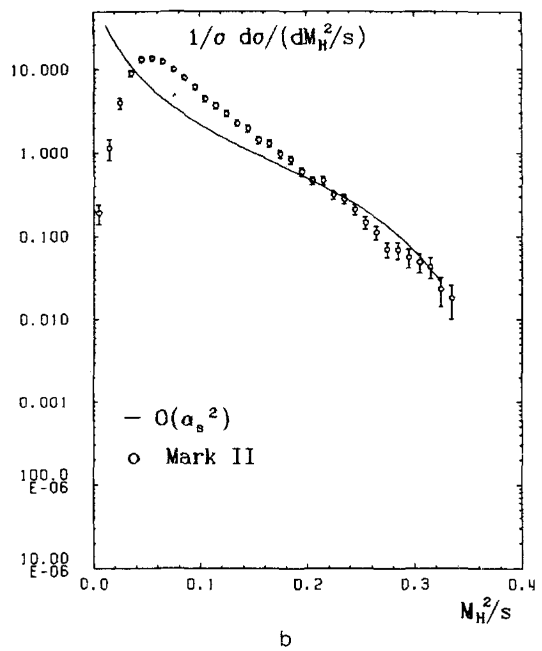


Fig. 9 a–c. Comparison between the $O(\alpha_s^2)$ perturbative calculation ($\Lambda_{\overline{MS}} = 0.1$ GeV) and the MARK II data at 29 GeV, for M_L^2/S, M_H^2/S, and $(M_H^2 - M_L^2)/S$

Table 6. Fragmentation parameters used in the IJF and LUND models

LUND V5.2	
A fragmentation-function parameter	0.116
B fragmentation-function parameter	0.252
σ_q parameter of the Gaussian p_t	0.407
IJF	
A fragmentation-function parameter	0.350
σ_q of the Gaussian p_t for $q \to$ hadrons	0.355
$\sigma_{gq\bar{q}}$ of the Gaussian p_t for gluon splitting $g \to q\bar{q}$	0.320

Hence, the fact that these systematic effects are larger than the statistical errors quoted is due to:

- different sensitivity of different jet measures to high order corrections and,
- our lack of understanding of the fragmentation process.

We would like to remark that the values for $\Lambda_{\overline{MS}}$ obtained from M_H^2/S are, according to the preceeding discussion, the most reliable ones, and fall in fair agreement with those obtained from $R = (\sigma(e^+e^- \to \text{hadrons}))/(\sigma(e^+e^- \to \mu^+\mu^-))$ [22] and the AEEC [23, 24].

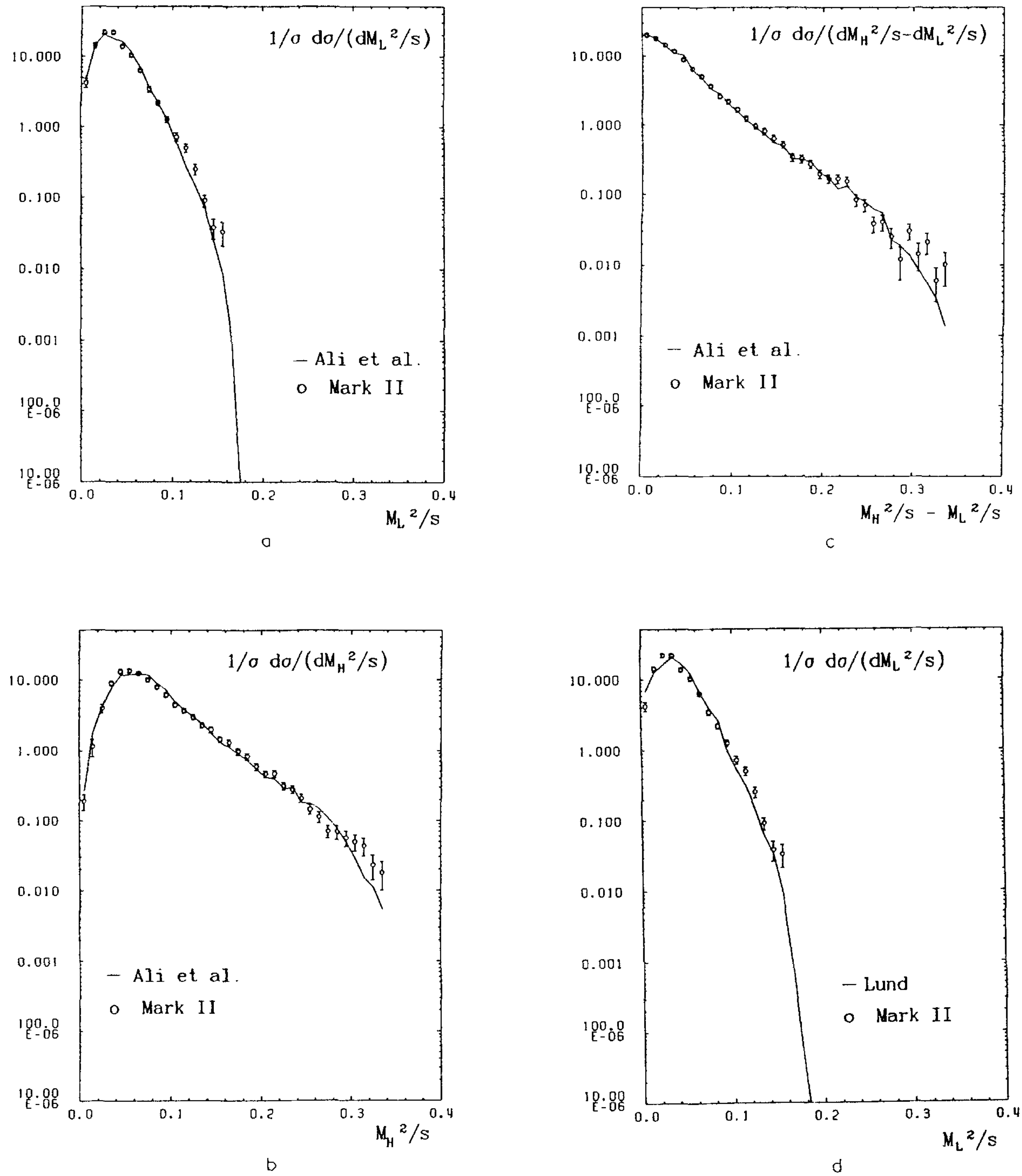
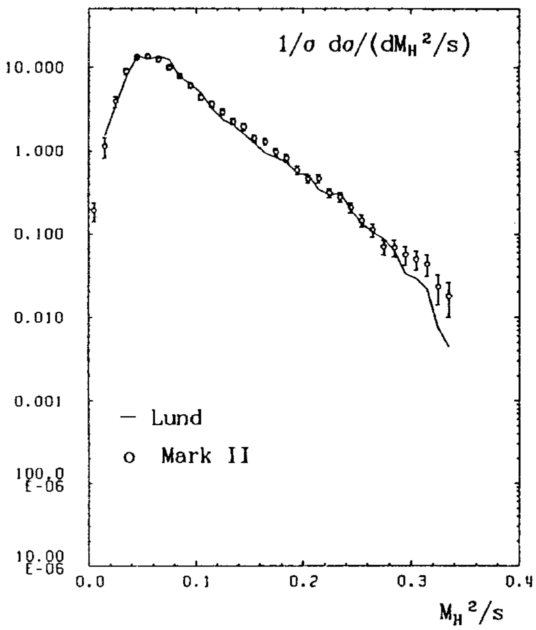


Fig. 10 a–f. Fits of the IJF and LUND models incorporating exact $O(\alpha_s^2)$ QCD matrix elements to the MARK II data for M_L^2/S, M_H^2/S and $(M_H^2 - M_L^2)/S$. The only free parameter was α_s. The fragmentation parameters were tuned in order to describe the TASSO data and are summarized in Table 6

5 Conclusions

We have calculated heavy and light jet masses, as well as their difference, M_H^2/S, M_L^2/S, and $(M_H^2 - M_L^2)/S$, to $O(\alpha_s^2)$ in perturbation theory. As far as second order corrections are concerned, the latter shows a behaviour similar to that exhibited by the asymmetric energy-energy correlation function. Second order corrections to the heavy jet mass are somewhat larger, comparable to those affecting the energy–energy correlation itself and smaller than those calculated for thrust. Higher than second order corrections, estimated in the LLA, are found to be important for the M_L^2 distribution.

Fragmentation effects, as defined in (11) are found



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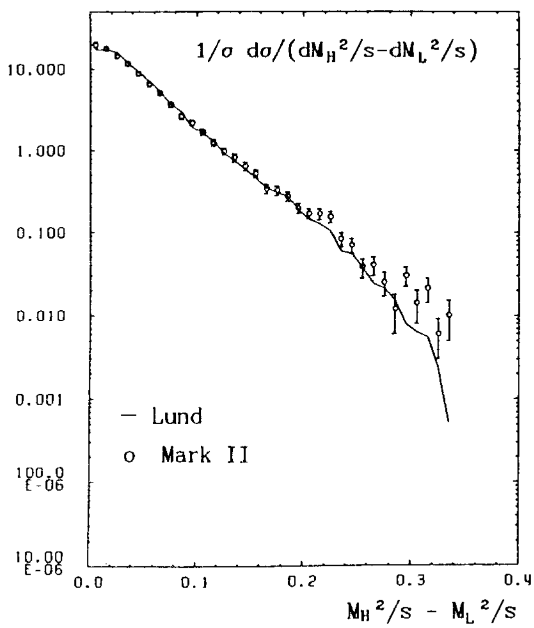

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Fig. 10 e, f

to be particularly important for the light jet mass, while they are comfortably small for the heavy jet mass in the perturbative tail.

We have compared recent MARK II data on jet masses with bare QCD and QCD plus fragmentation predictions. We found that the data are reasonably

Table 7. Values of α_s from fits to the jet mass distributions of MARK II data, using the independent jet fragmentation model (IJF) and the LUND model with exact $O(\alpha_s^2)$ corrections

Model	Measure	α_s	Λ_{MS}(GeV)	χ^2/NDF
IJF	$\frac{M_H^2}{S}-\frac{M_L^2}{S}$	0.116 ± 0.002	0.058 ± 0.006	2.2
IJF	$\frac{M_H^2}{S}$	0.131 ± 0.002	0.124 ± 0.010	2.5
IJF	$\frac{M_L^2}{S}$	0.149 ± 0.003	0.246 ± 0.028	3.3
LUND	$\frac{M_H^2}{S}-\frac{M_L^2}{S}$	0.137 ± 0.003	0.161 ± 0.016	2.1
LUND	$\frac{M_H^2}{S}$	0.145 ± 0.003	0.212 ± 0.017	2.6
LUND	$\frac{M_L^2}{S}$	0.160 ± 0.004	0.352 ± 0.044	4.5

described by both the IJF and the LUND model incorporating exact $O(\alpha_s^2)$ corrections. Measurements of M_H^2/S provide an alternative way of extracting the QCD coupling constant α_s yielding, $\alpha_s(\sqrt{s}=29\,\text{GeV}) = 0.131 \pm 0.002$, in the IJF, and $\alpha_s(\sqrt{s}=29\,\text{GeV}) = 0.145 \pm 0.003$ for LUND.

Acknowledgements. We would like to thank our colleagues at Madrid, Santa Cruz and Hamburg for many helpful discussions. We warmly thank Dr. A. Ali for his advice and a critical reading of the manuscript. This study was supported by CICYT, Spain.

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