RARE DECAY $B \to K^* \gamma$ IN THE STANDARD MODEL

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The decay rate of $B \to K^*(890)\gamma$ is estimated in the framework of QCD sum rules combined with vector meson dominance. We obtain: $\Gamma(B \to K^* \gamma)/\Gamma(b \to s\gamma) = 0.28 \pm 0.11$. As a byproduct we find that $\Gamma(B \to Q\gamma) \simeq \Gamma(B \to K^* \gamma)$.

The rare, flavour-changing, B-meson radiative decays $B \to K_i^* \gamma$, with $K_i^* = K^*(890)$, Q(1400), etc., have been identified as important tests of the higher order corrections in the standard model (for a recent review see e.g. ref. [1]). At the quark level these decays are expected to be controlled mainly by the $b \to s\gamma$ [2] electromagnetic penguin operator, which for $m_s \ll m_b$ can be written as

$$\mathscr{H}_{\text{eff}} = C m_b \epsilon^\mu \bar{s} \sigma_{\mu\nu} q^\nu b_R . \quad (1)$$

In eq. (1) $b_R = \frac{1}{2}(1+\gamma_5)b$, and the constant C contains the dependence on the Cabibbo–Kobayashi–Maskawa angles and the charm and top quark masses. The important point in this case is that QCD corrections lift the Glashow–Iliopoulos–Maiani suppression and lead to an order of magnitude enhancement of the branching ratio $B(b \to s\gamma)$ [3,4]. Indeed, for $m_t \leqslant M_W$ and neglecting m_c, the parameter C can be expressed as [3]

$$C = (G_F/\sqrt{2})(e/4\pi^2) V_{ts}^* V_{tb} \times [F_2(m_t) + \tfrac{4}{3}(\alpha_s/\pi) \ln(m_t^2/m^2)], \quad (2)$$

where $m \simeq m_b$ is the typical hadronic scale of the process, and the function $F_2(m_t) \simeq 0.1$–0.25 for m_t in the range $m_t \simeq 45$–100 GeV. This leads to $B(b \to s\gamma) \simeq (1.4$–$4.0) \times 10^{-4}$ for the same range of values of m_t.

The task of estimating the exclusive hadronic radiative mode is rather difficult and, obviously, depends on the particular choice of hadronization model. Some attempts have been made in the framework of the constituent quark model (CQM). Depending on the choice of the hadronic wavefunction the predictions e.g. for $\Gamma(B \to K^* \gamma)/\Gamma(b \to s\gamma)$ span the range (5–40)% [5–7].

In this note we discuss an estimate of $\Gamma(B \to K^* \gamma)$ and of $\Gamma(B \to Q\gamma)$ in the framework of QCD sum rules [8] combined with vector meson dominance (VMD). The main motivation and underlying ideas of this approach have been discussed recently [9,10] in connection with charm and beauty semileptonic decays. Since this method leads to quite reasonable

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results there and some parallel may be drawn between semileptonic and flavour-changing radiative decays, we feel that its application to the latter should be reasonably reliable.

According to eq. (1) the amplitude for $B(p)\to K^*(k,\eta)\gamma(q,e)$ can be written as

$$A(B\to K^*\gamma)=e_\mu(q)\langle K^*(k,\eta)|J^\mu_{\rm eff}|B(p)\rangle, \qquad (3)$$

with

$$J^\mu_{\rm eff}=Cm_b\bar{s}\tfrac{1}{2}\sigma^{\mu\nu}(1+\gamma_5)q_\nu b, \qquad (4)$$

and where the operator $\bar{s}\sigma_{\mu\nu}q^\nu b$ may be interpreted as the divergence of the tensor current $J_{\mu\nu}=\bar{q}\sigma_{\mu\nu}q$ [11]. The matrix element (3) involves *a priori* two hadronic form factors. However, since $\bar{q}\sigma_{\mu\nu}\gamma_5 q$ is not independent from $\bar{q}\sigma_{\mu\nu}q$ there is a general relation between these two form factors. We concentrate then on the matrix element of $\bar{s}\sigma_{\mu\nu}b$ and include at the end the contribution from the other term. In the framework of VMD we can write

$$\langle K^*(k,\eta)|\tfrac{1}{2}\bar{s}\sigma_{\mu\nu}q^\nu b|B(p)\rangle=i\epsilon_{\mu\nu\rho\sigma}\eta^\nu p^\rho k^\sigma F_1(q^2)$$
$$=\tfrac{1}{2}(M_{B^*_s}/f_T)\epsilon_\mu[M^2_{B^*_s}/(M^2_{B^*_s}-q^2)]\,G_{B^*_sBK^*}\mathscr{F}(q^2), \qquad (5)$$

where

$$\langle 0|\bar{s}\sigma_{\mu\nu}b|B^*_s(q,\epsilon)\rangle=(M_{B^*_s}/f_T)(\epsilon_\mu q_\nu-\epsilon_\nu q_\mu), \qquad (6)$$

defines the coupling constant f_T, and

$$G_{B^*_sBK^*}=ig_{B^*_sBK^*}\epsilon_{\alpha\beta\gamma\delta}k^\alpha\eta^\beta q^\gamma\epsilon^\delta, \qquad (7)$$

with $g_{B^*_sBK^*}$ being the strong coupling constant having mass dimension M^{-1}. In eq. (5) the form factor $\mathscr{F}(q^2)$ accounts for potential corrections to VMD, presumably arising from B^*_s radial excitations.

We proceed to estimate the *leptonic* decay constant f_T in eq. (6) with the aid of QCD sum rules for a two-point function involving the operator $\bar{s}\sigma_{\mu\nu}b$. To this end we follow essentially the same procedure as in ref. [10] for the estimate of the $B^*_{u,d}$ leptonic decay constant corresponding to the vector current operator $V_\mu=\bar{u}\gamma_\mu b$. In brief, the strategy there was to start from the two-point function

$$\Pi_{\mu\nu}(q)=i\int d^4x\,e^{iq\cdot x}\langle 0|T(V_\mu(x)V^\dagger_\nu(0))|0\rangle$$
$$=-(g_{\mu\nu}q^2-q_\mu q_\nu)\Pi^{(1)}(q^2)+q_\mu q_\nu\Pi^{(0)}(q^2), \qquad (8)$$

and compute the Hilbert moments at $q^2=0$

$$\phi_n(0)\equiv[1/(n+1)!](d/dq^2)^{(n+1)}q^2\Pi^{(1)}(q^2)|_{q^2=0}$$
$$=\frac{1}{\pi}\int\frac{ds}{s^{n+1}}\,\mathrm{Im}\,\Pi^{(1)}(s). \qquad (9)$$

The LHS of eq. (9) admits a well-defined short-distance QCD expansion in α_s and in inverse powers of the (current) b-quark mass m_b, i.e.

$$\phi_1(0)=(3/32\pi^2)m_b^{-2}[1+O(\alpha_s)]+\phi_1(0)|_{NP}, \qquad (10)$$

$$\phi_2(0)=(1/40\pi^2)m_b^{-4}[1+O(\alpha_s)]+\phi_2(0)|_{NP}, \qquad (11)$$

where the $\phi_n(0)|_{NP}$ stand for the non-perturbative contributions from quark and gluon condensates (almost negligible in this channel [10]), and the explicit $O(\alpha_s)$ terms are known [12]. Parametrizing the hadronic spectral function appearing on the RHS of eq. (9) by the lowest B* pole plus a continuum, approximated by the asymptotic freedom expansion, allows for a determination of the B* leptonic decay constant and mass; the latter from the ratio of eqs. (10) and (11). Actually, since the location of the continuum threshold s_0 is not well known, it is safer to use the experimental value of M_{B^*} to fix s_0 and then predict the coupling constant.

Following closely the above procedure we consider now the two-point function

$$\Pi_{\mu\nu\alpha\beta}(q)=i\int d^4x\,e^{iq\cdot x}\langle 0|T(J_{\mu\nu}(x)\,J^\dagger_{\alpha\beta}(0))|0\rangle$$
$$=P^{(-)}_{\mu\nu\alpha\beta}\Pi^{(-)}(q^2)+P^{(+)}_{\mu\nu\alpha\beta}\Pi^{(+)}(q^2), \qquad (12)$$

where $J_{\mu\nu}(x)=\bar{s}(x)\sigma_{\mu\nu}b(x)$, and

$$P^{(-)}_{\mu\nu\alpha\beta}=(1/q^2)\times(g^{\mu\alpha}q^\nu q^\beta+g^{\nu\beta}q^\mu q^\alpha-g^{\mu\beta}q^\nu q^\alpha-g^{\nu\alpha}q^\mu q^\beta), \qquad (13)$$

$$P^{(+)}_{\mu\nu\alpha\beta}=P^{(-)}_{\mu\nu\alpha\beta}+(g_{\mu\beta}g_{\nu\alpha}-g_{\mu\alpha}g_{\nu\beta}). \qquad (14)$$

$P^{(-)}$ and $P^{(+)}$ above are orthogonal projectors of intermediate states with $J^P=1^-$ and $J^P=1^+$, respectively, so that the B^*_s meson contributes to the spectral function $\mathrm{Im}\,\Pi^{(-)}$ in eq. (12). Proceeding as outlined above, and evaluating the quark loop asymptotic freedom expansion, we obtain the following Hilbert moment sum rules:

$$\phi_1(0)=(1/16\pi^2)m_b^{-2}\,[1+O(\alpha_s)]+\phi_1(0)|_{NP}$$
$$=\frac{1}{\pi}\int\frac{ds}{s^3}\,\mathrm{Im}\,\Pi^{(-)}(s), \quad (15)$$

$$\phi_2(0)=(3/160\pi^2)m_b^{-4}\,[1+O(\alpha_s)]+\phi_2(0)|_{NP}$$
$$=\frac{1}{\pi}\int\frac{ds}{s^4}\,\mathrm{Im}\,\Pi^{(-)}(s). \quad (16)$$

When compared with eqs. (10), (11), the above sum rules indicate only a slight change in the value of the short-distance coefficients. Such a change can be easily compensated by a slightly different choice of the asymptotic freedom threshold leading to the same value of the B*-mass. Notice that, as emphasized in ref. [10], the accuracy of this method cannot resolve the small SU(3) mass splitting between $B^*_{u,d}$ and B^*_s; consequently we are making the approximation $m_s/m_b\simeq 0$. All things considered, we may then safely translate the results of ref. [10] to the present case and predict

$$f_T=\sqrt{2}\times(22\pm4). \quad (17)$$

Proceeding to the strong coupling constant $g_{B^*_sBK^*}$ entering eq. (7), its determination lies outside the realm of two-point function QCD sum rules. A rough estimate may be obtained by taking the SU(3) rotated value of the coupling constant $g_{B^*_dB\rho^0}$ which was estimated in ref. [10], in which case we would find

$$g_{B^*_sBK^*}\simeq\sqrt{2}\times 11\ \mathrm{GeV}^{-1}. \quad (18)$$

Finally, the corrections to single-pole dominance, accounted for by the form factor $\mathscr{F}(q^2)$ in eq. (5), are expected at the level of 60% [10], i.e.

$$\mathscr{F}(0)\simeq 0.40\pm0.05. \quad (19)$$

Such a large correction should not come as a surprise, given the large extrapolation involved in going from $q^2=M^2_{B^*}$ to $q^2=0$, and given the fact that already in the case of ρ-dominance $\mathscr{F}(0)\simeq 0.80$ from experiment ($g_{\rho\pi\pi}/f_\rho|_{EXP}=1.22\pm0.03$).

We should point out that the estimate of $g_{B^*B\rho}$, together with the QCD sum rule value of the B^*_d leptonic coupling and the form factor $\mathscr{F}(0)$ above, leads to a prediction [10] for the (vector) semileptonic B→ρ transition in good agreement with the CQM estimate of ref. [13]. Substituting the above results (17)–(19) in eq. (5), we obtain

$$F_1(0)=\tfrac{1}{2}(M_{B^*_s}/f_T)g_{B^*BK^*}\mathscr{F}(0)=0.5\pm0.1. \quad (20)$$

An independent determination of $F_1(0)$ may be obtained by starting from the (covariant) VMD expression (5) and using the naive CQM in the following way. The coupling constant f_T in eq. (6) may be related to the hadronic S-wave function at the origin through

$$(M^3_{B^*_s}/f_T)\epsilon_\mu\equiv\langle 0|\bar{s}\sigma_{\mu\nu}q^\nu b|B^*_s(q,\epsilon)\rangle$$
$$=2(M_{B^*_s})^{3/2}[\sqrt{3}|\psi(0)_{B^*_s}|]\epsilon_\mu. \quad (21)$$

Analogously,

$$(M^2_{B^*_s}/\sqrt{2}\gamma_{B^*_s})\epsilon_\mu\equiv\langle 0|\bar{s}\gamma_\mu b|B^*_s(q,\epsilon)\rangle$$
$$=2(M_{B^*_s})^{1/2}[\sqrt{3}|\psi(0)_{B^*_s}|]\epsilon_\mu, \quad (22)$$

from which it follows that

$$f_T=\sqrt{2}\,\gamma_{B^*_s}. \quad (23)$$

Notice that $\gamma_{B^*_s}$ is expected to scale as

$$\gamma_{K^*}/\gamma_{B^*_s}=(M_{K^*}/M_{B^*_s})^{3/2}\ |\psi(0)_{B^*_s}/\psi(0)_{K^*}|. \quad (24)$$

Turning to the strong coupling constant $g_{B^*_sBK^*}$ it can be related to $g_{B^*_sBV}$, where V has the quantum numbers of ūs, through VMD for the K* meson, i.e.

$$(1/\sqrt{2}\,\gamma_{K^*})M_B\,g_{B^*_sBK^*}=\sqrt{2M_B}\sqrt{2M_{B^*_s}}\ g_{B^*_sBV}\,. \quad (25)$$

Now, in the non-relativistic CQM one has

$$g_{B^*_sBV}=\tfrac{1}{2}(1/m_b+1/m_s), \quad (26)$$

which gives

$$g_{B^*_sBK^*}=(1/m_b+1/m_s)(M_{B^*_s}/M_B)^{1/2}\sqrt{2}\,\gamma_{K^*}, \quad (27)$$

hence, the result for $F_1(0)$ in eq. (20) is

$$F_1(0)=(M_{K^*}/M_B)^{1/2}M_{K^*}$$
$$\times(1/m_b+1/m_s)\ \tfrac{1}{2}|\psi(0)_{B^*_s}/\psi(0)_{K^*}|\,\mathscr{F}(0)$$
$$\simeq 0.40. \quad (28)$$

The numerical value above has been obtained using conventional values for the constituent quark masses, and assuming the S-wave functions to scale as $\psi(0)\sim\mu$, where μ is the reduced q$\bar{\text{q}}$ mass. It is rewarding to find that two different methods lead essentially to the same value for the form factor, especially on account of the various unavoidable approximations involved.

Computing the decay rate one obtains

$$\Gamma(\mathrm{B}\to\mathrm{K}^*\gamma)=(1/32\pi)m_\mathrm{b}^2|C|^2[(M_\mathrm{B}^2-M_{\mathrm{K}^*}^2)^3/M_\mathrm{B}^3]$$
$$\times[|F_1(0)|^2+4|F_2(0)|^2], \qquad (29)$$

where $F_2(0)$ is the form factor associated to the $\bar{s}\sigma_{\mu\nu}\gamma_5 b$ piece of the effective current. One can easily show, from $\sigma^{\mu\nu}\gamma_5=-\mathrm{i}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$, that $|F_2(q^2)|=\frac{1}{2}|F_1(q^2)|$. Normalizing to the inclusive rate we finally predict

$$R\equiv\Gamma(\mathrm{B}\to\mathrm{K}^*\gamma)/\Gamma(\mathrm{b}\to\mathrm{s}\gamma)=0.28\pm0.11. \qquad (30)$$

Turning to the decay $\mathrm{B}\to\mathrm{Q}(1400)\gamma$, the matrix elements in the present framework are analogous to the ones for $\mathrm{B}\to\mathrm{K}^*\gamma$. The only difference is that since Q(1400) is an axial-vector ($J^P=1^+$), the roles of the currents $\bar{s}\sigma_{\mu\nu}b$ and $\bar{s}\sigma_{\mu\nu}\gamma_5 b$ are exchanged. Hence, defining

$$\langle \mathrm{Q}(k,\eta)|\bar{s}\tfrac{1}{2}\sigma_{\mu\nu}q^\nu b|\mathrm{B}(p)\rangle$$
$$=[\eta_\mu(M_\mathrm{B}^2-M_\mathrm{Q}^2)-(p+k)_\mu(q\cdot\eta)]G_2(q^2), \qquad (31)$$

with $G_2=\frac{1}{2}G_1$, where G_1 is now the form factor associated to $\sigma_{\mu\nu}q^\nu\gamma_5$, the analogue of eq. (20) is

$$G_2(0)=\tfrac{1}{2}(M_{\mathrm{B}_\mathrm{S}^*}/f_\mathrm{T})f_{\mathrm{B}^*\mathrm{BQ}}\,\mathscr{F}(0). \qquad (32)$$

The constant $f_{\mathrm{B}^*\mathrm{BQ}}$ above is the strong, S-wave, B*BQ coupling, whose order of magnitude may be found by scaling the light-quark analogue $f_{\mathrm{A}_1\rho\pi}\simeq0.5/f_\pi\gtrsim5.5$ GeV^{-1} [14]. In this case we obtain $G_2(0)\simeq0.20$, which leads to $\Gamma(\mathrm{B}\to\mathrm{Q}\gamma)\simeq\Gamma(\mathrm{B}\to\mathrm{K}^*\gamma)$.

We wish to point out in closing that we are aware of the phenomenological consequences of the relatively large branching ratio obtained here, eq. (30). However, one should keep in mind that form factor models, the present one being no exception, are unavoidably affected by somewhat large uncertainties which become compounded at the time of estimating decay rates. An improvement of the present experimental upper limit $B(\mathrm{B}\to\mathrm{K}^*\gamma)<2.4\times10^{-4}$ [15] will be most welcome in order to test the various theoretical approaches within and beyond the standard model.

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