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CHARGED HIGGS BOSONS AND FLAVOR CHANGING Z-DECAYS

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Flavor changing Z^0-decays are analyzed within the standard model with two Higgs doublets. Taking into account constraints from the K^0–$\overline{K}^0$, D^0–$\overline{D}^0$ and B^0–$\overline{B}^0$ systems we obtain the maximal branching ratios for heavy–light quark–antiquark final states: $BR(Z \to \bar{t}c, t\bar{c}) < 5 \times 10^{-6}$ and $BR(Z \to b\bar{s}, \bar{b}s) < 2 \times 10^{-6}$.

1. Introduction

Today the standard model [1] of strong and electroweak interactions of Glashow, Weinberg and Salam (GWS) describes all known phenomena in elementary particle physics very well, but it will have to pass a lot of precision tests in future experiments at LEP and HERA. There is still an open question in this model: the nature of spontaneous breaking of the weak gauge symmetry. Experimentally, almost nothing is known about the Higgs sector. While the single neutral Higgs scalar in the GWS model is yet to be found, there is some speculation, whether the Higgs sector is to be enlarged or even replaced by a whole spectrum of dynamically generated states due to an additional strong interaction. At a less dramatic level there are some currently interesting models, which involve at least two Higgs doublets, such as left–right symmetric gauge theories, supersymmetric versions of the standard model or the Peccei–Quinn model.

In a model with two Higgs doublets there occurs besides two additional neutral scalars one charged Higgs scalar in the physical spectrum. Related to this charged Higgs boson is the appearance of the additional parameter v_1/v_2, which is the ratio of the vacuum expectation values (VEV's) of the two doublets. It enters the Yukawa couplings and can enhance the effect of the charged Higgs boson in various physical processes.

In particular rare processes are very interesting, because they often provide a window to particles which are too heavy to be produced directly. I will consider two neutral flavor changing processes. They are rare due to the GIM mechanism [2], which ensures that in the GWS model flavor changing neutral currents (FCNC) are naturally absent at tree level and suppressed even at one-loop level. This is also true for the standard model with two Higgs doublets, which is considered here.

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In sect. 2 experimental data on the neutral meson mixing in the K^0–$\overline{K}^0$, B^0–$\overline{B}^0$ and D^0–$\overline{D}^0$ systems are used to obtain bounds on the parameter v_1/v_2.

In sect. 3 the contributions of charged Higgs bosons to flavor changing Z^0-decays into a heavy–light quark–antiquark final state $Z \to \bar{b}s$ and $Z \to \bar{t}c$ are calculated. Using the bounds from sect. 2 the maximal branching ratios for these processes within a two Higgs doublet model are obtained.

A heavy–light final state will give the highest decay rate and have the clearest experimental signature, namely one fat, heavy jet opposite to a thin, light one or two jets and a hard lepton, if the heavy quark decays further semileptonically. The decay $Z \to t\bar{c}$ is an opportunity to produce a single t-quark with $m_t < m_Z < 2m_t$, which cannot be pair-produced.

Branching ratios for flavor changing Z^0-decays within the GWS model are much too small to be observed in the near future. For example the ratio for $Z \to \bar{t}c$ is at most 10^{-10} [3]. But they can in principle be substantially enhanced in extensions of the GWS model. So it is important to look for these decay modes at LEP, where one expects a large number of Z-bosons, about 10^7, per year. In this paper I will compute the maximal enhancement which can be achieved in an extension of the GWS model with a second Higgs doublet.

2. Bounds on v_1/v_2

2.1. YUKAWA COUPLINGS

In the GWS model the Yukawa couplings of the neutral Higgs bosons are flavor diagonal, because the Yukawa coupling matrices are proportional to the fermion mass matrices (they differ only by a factor v, the VEV of the Higgs field) and are diagonalized together with the last ones. This is no longer true for the two Higgs doublet model, but there are several possibilities to avoid FCNC (which are very small in nature) at tree level. A natural way to achieve this is to introduce an additional discrete symmetry, for example

$$\Phi_2 \leftrightarrow -\Phi_2, \qquad u_R \leftrightarrow -u_R$$

which makes the two Higgs doublets distinguishable and ensures that one Higgs doublet, Φ_2, couples only to the right-handed up-quarks u_R and the other one, Φ_1, only to the right-handed down-quarks [4]. By diagonalizing the fermion mass matrices and the Higgs mass matrix in the usual way one obtains [5] the Yukawa couplings of the physical charged Higgs field H^+ to the quark fields:

$$\mathscr{L}_{\mathrm{Yuk}\,H^+} = \frac{g}{\sqrt{2}} H^+ \left(\frac{m_r^{\mathrm{u}}}{m_{\mathrm{W}}} \frac{v_1}{v_2} \bar{u}_{\mathrm{R}r} V_{\mathrm{KM}rs} d_{\mathrm{L}s} + \frac{m_s^{\mathrm{d}}}{m_{\mathrm{W}}} \frac{v_2}{v_1} \bar{u}_{\mathrm{L}r} V_{\mathrm{KM}rs} d_{\mathrm{R}s} \right) + \mathrm{h.c.} \qquad (2.1)$$

$u_R = P_R u = \frac{1}{2}(1+\gamma_5)u$ is the Dirac spinor for the right-handed up-quarks, d_L the left-handed down-quarks, $r, s = 1,2,3$ are the family indices. V_{KM} is the Kobayashi–Maskawa (KM) matrix for which the following parametrization will be used [6]:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \tag{2.2}$$

with $s_i = \sin\theta_i$, $c_i = \cos\theta_i$. For simplicity one can replace c_2 and c_3 by one. The couplings (2.1) have the structure required by supersymmetry [5] and the Peccei–Quinn model [7] and will be used throughout this paper, if nothing else is said.

Another way is to couple only one Higgs doublet to the quark fields as in the GWS model and let the other one be without Yukawa couplings to the quarks. In this model one finds the following couplings of the physical H^+-boson:

$$\mathcal{L}_{\mathrm{Yuk}\,H^+} = \frac{g}{\sqrt{2}} H^+ \left(\frac{m_r^u}{m_W} \frac{v_1}{v_2} \bar{u}_{Rr} V_{KMrs} d_{Ls} - \frac{m_s^d}{m_W} \frac{v_1}{v_2} \bar{u}_{Lr} V_{KMrs} d_{Rs} \right) + \text{h.c.} \tag{2.3}$$

In both cases the new parameter v_1/v_2, the ratio of the VEV's of the two Higgs doublets, can enhance or suppress the strength of the Yukawa couplings.

2.2. NEUTRAL MESON MIXING

In pure QCD the two neutral meson states $|B^0\rangle$ and $|\bar{B}^0\rangle$ do not mix and are degenerate in mass. But if the weak interactions are turned on, they interact for example via the famous Gaillard–Lee box diagram [8] (fig. 1). The internal quarks i and j are the up-quarks u, c, t. Diagonalization of the effective weak hamiltonian describing this mixing leads to a mass splitting Δm_B^{WW} and to eigenstates $|B_{1,2}\rangle$, whose deviation from the CP eigenstates $|B^0\rangle \pm |\bar{B}^0\rangle$ can be parametrized by the CP violation parameter ε_B^{WW} [9]. The index WW means that this contribution results from the exchange of two W-bosons. "Long-distance" contributions are usually neglected. In the two Higgs doublet model there are the additional contributions in fig. 2, where one or both W-bosons are replaced by charged Higgs bosons.

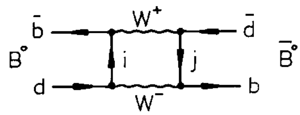

Fig. 1. B-meson mixing in the GWS model.

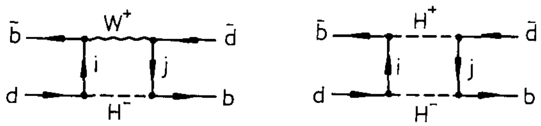

Fig. 2. Charged Higgs contributions.

Neglecting external momenta, which have the magnitude of the B-meson mass and are therefore small compared to the charged Higgs mass m_H, and the u-quark mass one obtains for these two contributions to Δm_B [10]:

$$\Delta m_B^{WH} = \tfrac{4}{3} G_F^2 \left(\frac{v_1}{v_2}\right)^2 \Big[(V_{cb}^* V_{cd})^2 m_c^4 \big(4 m_W^2 I_2(m_c) + I_3(m_c)\big) + 2 V_{cb}^* V_{cd} V_{tb}^* V_{td} m_c^2 m_t^2 \big(4 m_W^2 I_5(m_c, m_t) + I_6(m_c, m_t)\big) + (V_{tb}^* V_{td})^2 m_t^4 \big(4 m_W^2 I_2(m_t) + I_3(m_t)\big)\Big] f_B^2 B_B m_B ,$$

$$\Delta m_B^{HH} = \tfrac{2}{3} G_F^2 \left(\frac{v_1}{v_2}\right)^4 \Big[(V_{cb}^* V_{cd})^2 m_c^4 I_1(m_c) + 2 V_{cb}^* V_{cd} V_{tb}^* V_{td} m_c^2 m_t^2 I_4(m_c, m_t) + (V_{tb}^* V_{td})^2 m_t^4 I_1(m_t)\Big] f_B^2 B_B m_B . \quad (2.4)$$

G_F is the Fermi constant, the parameter f_B is the analogon to the pion decay constant f_π and of the same order of magnitude. B_B is the "bag parameter", the value $B_B = 1$ corresponds to the vacuum insertion approximation when calculating the hadronic matrix element of the effective weak hamiltonian. The integrals I_i are given in the appendix of ref. [11], in particular:

$$I_1(m) = \frac{1}{16\pi^2 m_H^2} \bar{I}_1\left(\frac{m^2}{m_H^2}\right),$$

$$\bar{I}_1(x) \equiv \frac{1+x}{(1-x)^2} + \frac{2x}{(1-x)^3} \ln x = 1 + O(x) . \quad (2.5)$$

There are QCD corrections to the results (2.4) [12], but they are small in the B-system and will be neglected. The results can be transferred to the K^0–$\bar{K}^0$ system by replacing B by K and b by s. For the D^0–$\bar{D}^0$ system one has to replace B by D, d by u and b by c, the loop internal quarks i and j are then the down-quarks d, s, b.

2.3. BOUNDS

There are now four experimental observables, Δm_K, ε_K, Δm_B and Δm_D, which can be used to obtain bounds on v_1/v_2. It turns out that Δm_K does not give a very stringent upper bound, because the leading contribution in eq. (2.4) for the K-system is only the m_c^4 term due to the KM matrix. Also ε_K is not a useful quantity, because there is no strong lower bound on the phase δ in the KM matrix within the two Higgs doublet model. Therefore Δm_B will be used. Δm_B was measured in 1987 by the ARGUS collaboration in $\Upsilon(4s)$ decays via the observable

$$r = \frac{N(B^0B^0) + N(\bar{B}^0\bar{B}^0)}{N(B^0\bar{B}^0)} \cong \frac{(\Delta m/\Gamma)^2}{2 + (\Delta m/\Gamma)^2}\,.$$

The experimental result reads [13]

$$3.1 \times 10^{-10}\,\mathrm{MeV} < \Delta m_B < 5.1 \times 10^{-10}\,\mathrm{MeV}\,. \tag{2.6}$$

This is much larger than the standard model prediction for a wide range of parameters. In order to obtain an upper bound for v_1/v_2, I will assume for simplicity that Δm_B^{HH} is the leading contribution to Δm_B^{exp} and that $v_1/v_2 \gg 1$. Then one has

$$\Delta m_B^{HH} < \Delta m_B^{exp}\,. \tag{2.7}$$

In Δm_B^{HH} [eq. (2.4)] only the leading term proportional to m_t^4 is retained.

Using the B^0 lifetime $\tau_B = (\Gamma(b \to u) + \Gamma(b \to c))^{-1}$ and the ratio $R = \Gamma(b \to u)/\Gamma(b \to c)$ one obtains a lower bound for the parameter s_2 in the GWS model [14]. $\tau_B = 1.16 \times 10^{-12}$ s and $R < 0.08$ [15] yield $s_2 > 0.01$, which can be used even in the two Higgs doublet model, since the corrections to τ_B and R due to the charged Higgs boson are small.

Putting all this together, and using $f_B = 160$ MeV, $m_B = 5275$ MeV and $B_B = 1/3$, one finds the bound:

$$\left(\frac{v_1}{v_2}\right)^2 < \frac{m_H}{m_t^2}\,\frac{\sqrt{24}\,\pi\sqrt{\Delta m_B/m_B}}{G_F|V_{td}|f_B\sqrt{B_B}}\,\bar{I}_1^{-1/2}\left(\frac{m_t^2}{m_H^2}\right)$$

$$< \frac{1.9\,\mathrm{TeV}}{m_t}\,\frac{m_H}{m_t}\,\bar{I}_1^{-1/2}\left(\frac{m_t^2}{m_H^2}\right). \tag{2.8}$$

The best lower bound for v_1/v_2, or rather upper bound for v_2/v_1, can be obtained from the D^0–$\bar{D}^0$ system, now under the assumption $v_2/v_1 \gg 1$. Transferring formula (2.4) gives

$$\Delta m_{\mathrm{D}}^{\mathrm{HH}} \simeq \frac{G_{\mathrm{F}}^2}{24\pi^2} f_{\mathrm{D}}^2 B_{\mathrm{D}} m_{\mathrm{D}} \frac{v_2^4}{v_1^4} \frac{1}{m_{\mathrm{H}}^2} \left| (V_{\mathrm{ub}}^* V_{\mathrm{cb}})^2 m_{\mathrm{b}}^4 + 2 V_{\mathrm{ub}}^* V_{\mathrm{cb}} V_{\mathrm{us}}^* V_{\mathrm{cs}} m_{\mathrm{b}}^2 m_{\mathrm{s}}^2 + (V_{\mathrm{us}}^* V_{\mathrm{cs}})^2 m_{\mathrm{s}}^4 \right| . \tag{2.9}$$

The lower bound for V_{ub} is very small (0.002), so only the leading m_{s}^4-term will be retained. Using $\Delta m_{\mathrm{D}}^{\mathrm{exp}} < 6.5 \times 10^{-10}$ MeV, $m_{\mathrm{D}} = 1864$ MeV [16] and $f_{\mathrm{D}}^2 B_{\mathrm{D}} \simeq f_{\mathrm{B}}^2 B_{\mathrm{B}}$ the bound $\Delta m_{\mathrm{D}}^{\mathrm{HH}} < \Delta m_{\mathrm{D}}^{\mathrm{exp}}$ gives

$$\left(\frac{v_2}{v_1} \right)^2 < \frac{m_{\mathrm{H}}}{0.57\ \mathrm{MeV}} . \tag{2.10}$$

For realistic m_{H}, which is at least 19 GeV [17], this bound seems to be unrealistically high. In fact perturbation theory was used to obtain this bound, so the theory should not be strongly interacting. Hence I will impose the condition

$$\alpha_{\mathrm{Yuk}} = \left(\frac{g}{\sqrt{2}} \frac{m_{\mathrm{b}}}{m_{\mathrm{W}}} \frac{v_2}{v_1} \right)^2 \Big/ 4\pi < 1 , \tag{2.11}$$

which yields

$$(v_2/v_1)^2 < 1.8 \times 10^5 . \tag{2.12}$$

This bound is stronger than that in eq. (2.10). A strongly interacting theory violating eq. (2.12) is not the subject of this paper.

3. Flavor changing decays of the Z-boson

3.1. STANDARD CONTRIBUTIONS

In the standard model there are no FCNC at tree level due to the GIM [2] mechanism. This means, that the tree lagrangian does not contain a $\mathrm{ZQ\bar{q}}$ vertex, where Q and q are quarks of different flavor. The main decay modes of the Z-boson are flavor diagonal: $\mathrm{Z} \to \ell\bar{\ell}, \mathrm{q\bar{q}}$ (ℓ: leptons). But the flavor changing Z-decay $\mathrm{Z} \to \mathrm{Q\bar{q}}$ does occur at the one-loop level via the Feynman diagrams (fig. 3). The internal quark i is a down(up)-quark, if the final state quarks Q and q are up(down)-quarks, summation about $i = 1, 2, 3$ is understood. Every W-vertex contains an element of the KM matrix, and this one-loop contribution is suppressed by the unitarity of the KM matrix, which is again a result of the GIM mechanism. The

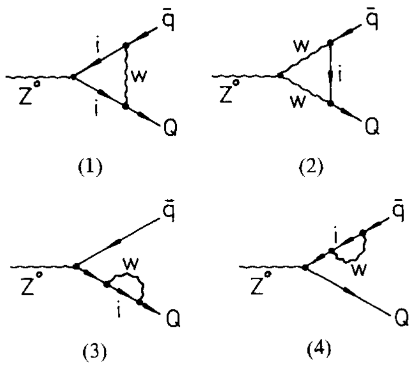

Fig. 3. Standard contributions to the flavor changing Z-decay.

branching ratios

$$\mathrm{BR} = \frac{\Gamma(\mathrm{Z} \to \mathrm{Q\bar{q}}) + \Gamma(\mathrm{Z} \to \mathrm{\bar{Q}q})}{\Gamma(\mathrm{Z} \to \mathrm{all})}$$

in the GWS model have been calculated for $\mathrm{Z} \to \mathrm{b\bar{s}}$ in refs. [18,19] and for $\mathrm{Z} \to \mathrm{t\bar{c}}$ in ref. [3]. The results are in fact very small:

$$\mathrm{BR}(\mathrm{Z} \to \mathrm{Top}) \sim 10^{-11}, \qquad \mathrm{BR}(\mathrm{Z} \to \mathrm{Bottom}) \sim 10^{-8}. \tag{3.1}$$

Even the amount of *CP* violation

$$\frac{\Gamma(\mathrm{Z} \to \mathrm{b\bar{s}}) - \Gamma(\mathrm{Z} \to \mathrm{\bar{b}s})}{\Gamma(\mathrm{Z} \to \mathrm{b\bar{s}}) + \Gamma(\mathrm{Z} \to \mathrm{\bar{b}s})}$$

in these decays has been calculated in refs. [20,21].

3.2. NON-STANDARD CONTRIBUTIONS

Any extension of the GWS model can in principle enhance these decay rates. The possibility of a fourth family of quarks and leptons was also discussed in refs. [18,19]. The contributions due to gluinos and scalar quarks in the loop within a supersymmetric model were first calculated in ref. [22].

In the two Higgs doublet model there are the additional contributions shown in fig. 4, where the W-boson is replaced by the charged Higgs boson, and they will now be calculated.

Since I am interested in the maximal possible enhancement which one can obtain within the two Higgs doublet model I will assume, that the main contributions are the non-standard ones and neglect the standard contributions and the interference terms between standard and Higgs boson contributions. This can be understood as a

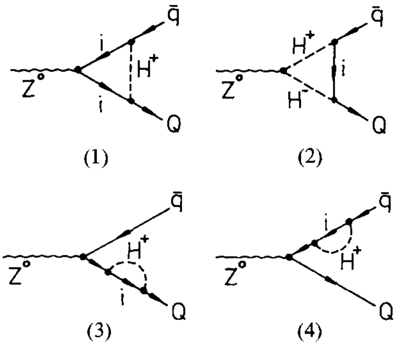

Fig. 4. Charged Higgs contributions.

formal expansion in v_1/v_2 for $Z \to b\bar{s}$ and in v_2/v_1 for $Z \to \bar{t}c$, since every H^+-vertex contains a factor $(v_1/v_2)(m_i/m_W)$, respectively $(v_2/v_1)(m_i/m_W)$. Only the terms $\sim (v_1/v_2)^4$, respectively $\sim (v_2/v_1)^4$ will be retained in the decay rate. From the sum over the internal quarks only the heaviest one will be considered for simplicity. So in the case $Z \to b\bar{s}$ the expected enhancement factor $(v_1/v_2)^4(m_t/m_W)^4$ is assumed to be large, whereas the factor $(v_2/v_1)^4(m_b/m_W)^4$ is assumed to be large when calculating $\Gamma(Z \to \bar{t}c)$.

3.3. RENORMALIZATION

Since the $ZQ\bar{q}$ vertex is computed to the lowest non-vanishing order in a renormalizable model, it does not get renormalized and the decay amplitude is expected to be finite. Generating counterterms in the usual way by multiplicative renormalization one can see, that the lowest non-vanishing order counterterms do not contribute to the flavor non-diagonal Z-decay.

Although the sum of the four diagrams in fig. 4 is finite, the single diagrams are divergent. Thus dimensional regularization was used to handle the divergent terms. The amplitudes are expanded in $\varepsilon = 4 - d$, keeping only pole and finite parts. When adding the four diagrams, the ε-poles have to cancel. For the matrix γ_5 I assume in d dimensions:

$$\{\gamma^\mu, \gamma_5\} = 0, \quad \gamma_5^2 = 1, \quad \gamma_5^\dagger = \gamma_5. \tag{3.2}$$

This assumption is justified, since the standard model with two Higgs doublets is anomaly free and the complete amplitude is finite [23].

3.4. CALCULATIONAL SCHEME

Keeping all five masses m_Z, m_H, m_i, m_Q, m_q non-zero leads to an algebraic explosion. A considerable simplification is achieved by neglecting the masses of the light quarks m_s, m_c and m_b. This is a good approximation, because the relevant scale in this process is given by the Z-boson mass m_Z or the charged Higgs mass m_H.

The calculation then proceeds as follows: From the Feynman rules one obtains analytic expressions for the Feynman diagrams (1)–(4) in fig. 4. The appearing d-dimensional momentum tensor integrals are expressed by external momenta and scalar integrals using Lorentz invariance as described in ref. [24]. A computer program for algebraic manipulations was used to check the results. These are simplified using Dirac algebra and the following mass shell conditions:

$$\bar{u}(p_Q, s_Q)\not{p}_Q = \bar{u}(p_Q, s_Q)m_Q, \qquad \not{p}_{\bar{q}}v(p_{\bar{q}}, s_{\bar{q}}) = -m_q v(p_{\bar{q}}, s_{\bar{q}}), \qquad \not{p}^2 = p^2 = m^2,$$

$$p_Z^\mu \varepsilon_\mu(p_Z, s_Z) = 0 \Rightarrow p_{\bar{q}}^\mu \varepsilon_\mu(p_Z, s_Z) = -p_Q^\mu \varepsilon_\mu(p_Z, s_Z). \tag{3.3}$$

The amplitude $M_{Q\bar{q}}$ is then expressed in the form

$$M_{Q\bar{q}} = C\bar{u}(p_Q, s_Q)((A_L P_L + A_R P_R)p_Q^\mu + B_L\gamma^\mu P_L + B_R\gamma^\mu P_R)v(p_{\bar{q}}, s_{\bar{q}})\varepsilon_\mu(p_Z, s_Z). \tag{3.4}$$

Here C is a constant, A_L, A_R, B_L and B_R are "form factors", $P_{L/R}$ is the left/right-handed projector, u and v are the free field Dirac spinors in momentum space, ε_μ is the Z polarization vector. The scalar integrals are expressed in terms of logarithms and Spence functions as described in ref. [25]. The decay rate is then given in the Z rest frame as

$$\Gamma(Z \to Q\bar{q}) = \frac{w\left(m_Z^2, m_Q^2, m_q^2\right)}{16\pi m_Z^3} \sum{}' |M_{Q\bar{q}}|^2, \tag{3.5}$$

where Σ' means the average over the polarization of the Z-boson and the sum over the polarization and color of the final state quarks Q and $\bar{q}$, w is the kinematic function

$$w(x, y, z) \equiv \left(x^2 + y^2 + z^2 - 2xy - 2yz - 2xz\right)^{1/2}.$$

3.5. CHECKS ON THE CALCULATION

Several checks of the results have been carried out: Cancellation of the poles in ε was verified at each stage of the calculation. Ward identities for the flavor changing Z-decay were found to be satisfied, although they are not very useful in the case $m_Q = m_q = 0$. The imaginary part of the amplitude $Z \to \bar{t}c$ due to diagram (2) in fig. 4 has been calculated via unitarity relations and was successfully compared with the direct result. Since from the sum over the internal quarks only one term (the heaviest one) is kept, the result is CP invariant: $\Gamma(\mathrm{Z}\to \mathrm{Q\bar{q}}) = \Gamma(\mathrm{Z}\to \mathrm{\bar{Q}q})$. This provides a nontrivial check for $\mathrm{Z}\to \mathrm{\bar{t}c}$, where $m_{\mathrm{Q}} \neq m_{\mathrm{q}}$.

3.6. RESULTS FOR $\mathrm{Z}\to \mathrm{b\bar{s}}$

In this case the quarks Q and q are the down-quarks b and s, whereas the internal quark i is the t-quark. When setting $m_{\mathrm{b}} = m_{\mathrm{s}} = 0$, only the form factor $B_{\mathrm{Lbs}} = B_{\mathrm{L}}(1) + B_{\mathrm{L}}(2) + B_{\mathrm{L}}(3) + B_{\mathrm{L}}(4)$ remains. The result of the lengthy calculation is

$$
\begin{aligned}
B_{\mathrm{L}bs} = {} & \left(\left(\tfrac{1}{2} - \tfrac{2}{3}s^2\right)m_{\mathrm{t}}^2 - \tfrac{2}{3}s^2\frac{\left(m_{\mathrm{t}}^2 - m_{\mathrm{H}}^2\right)^2}{m_{\mathrm{Z}}^2}\right) C_0\left(m_{\mathrm{t}}, m_{\mathrm{H}}, m_{\mathrm{t}}; 0, 0, m_{\mathrm{Z}}^2\right) \\
& + \left(\tfrac{1}{2} - s^2\right)\left(m_{\mathrm{t}}^2 + \frac{\left(m_{\mathrm{t}}^2 - m_{\mathrm{H}}^2\right)^2}{m_{\mathrm{Z}}^2}\right) C_0\left(m_{\mathrm{t}}, m_{\mathrm{H}}, m_{\mathrm{H}}; 0, m_{\mathrm{Z}}^2, 0\right) \\
& + \left[\left(\tfrac{1}{3}s^2\left(3\frac{m_{\mathrm{t}}^2}{m_{\mathrm{Z}}^2} - 2\frac{m_{\mathrm{H}}^2}{m_{\mathrm{Z}}^2} + \frac{1}{2}\frac{m_{\mathrm{t}}^4}{\left(m_{\mathrm{t}}^2 - m_{\mathrm{H}}^2\right)^2} + 1\right)\right.\right. \\
& \left. - \frac{1}{4}\frac{m_{\mathrm{t}}^4}{\left(m_{\mathrm{t}}^2 - m_{\mathrm{H}}^2\right)^2} - \frac{1}{2}\frac{m_{\mathrm{t}}^2}{m_{\mathrm{Z}}^2}\right] \ln\frac{m_{\mathrm{t}}^2}{m_{\mathrm{H}}^2} + \left(\frac{m_{\mathrm{t}}^2 - m_{\mathrm{H}}^2}{m_{\mathrm{Z}}^2} + \tfrac{1}{2}\right) \\
& \times \left[\tfrac{2}{3}s^2 \begin{cases} \sqrt{4m_{\mathrm{t}}^2/m_{\mathrm{Z}}^2 - 1}\left(\pi - 2\arctan\sqrt{4m_{\mathrm{t}}^2/m_{\mathrm{Z}}^2 - 1}\right), & m_{\mathrm{Z}} < 2m_{\mathrm{t}} \\ -\sqrt{1 - 4m_{\mathrm{t}}^2/m_{\mathrm{Z}}^2}\left(\ln\dfrac{1 - \sqrt{1 - 4m_{\mathrm{t}}^2/m_{\mathrm{Z}}^2}}{1 + \sqrt{1 - 4m_{\mathrm{t}}^2/m_{\mathrm{Z}}^2}} + i\pi\right), & m_{\mathrm{Z}} > 2m_{\mathrm{t}} \end{cases}\right. \\
& \left. + \left(\tfrac{1}{2} - s^2\right) \begin{cases} \sqrt{4m_{\mathrm{H}}^2/m_{\mathrm{Z}}^2 - 1}\left(\pi - 2\arctan\sqrt{4m_{\mathrm{H}}^2/m_{\mathrm{Z}}^2 - 1}\right), & m_{\mathrm{Z}} < 2m_{\mathrm{H}} \\ -\sqrt{1 - 4m_{\mathrm{H}}^2/m_{\mathrm{Z}}^2}\left(\ln\dfrac{1 - \sqrt{1 - 4m_{\mathrm{H}}^2/m_{\mathrm{Z}}^2}}{1 + \sqrt{1 - 4m_{\mathrm{H}}^2/m_{\mathrm{Z}}^2}} + i\pi\right), & m_{\mathrm{Z}} > 2m_{\mathrm{H}} \end{cases}\right] \\
& + \left(-\tfrac{1}{2} + \tfrac{1}{3}s^2\right)\left(\frac{m_{\mathrm{t}}^2 - m_{\mathrm{H}}^2}{m_{\mathrm{Z}}^2} - \frac{1}{4}\frac{m_{\mathrm{t}}^2 + m_{\mathrm{H}}^2}{m_{\mathrm{t}}^2 - m_{\mathrm{H}}^2}\right) - \tfrac{1}{2} + s^2. \qquad (3.6)
\end{aligned}
$$

The abbreviations s^2 and c_w stand for the sine squared and the cosine of the Weinberg angle. The scalar three-point function C_0 can be expressed in terms of the

Spence function

$$\mathrm{Sp}(x) \equiv -\int_0^1 \mathrm{d}t \frac{\ln(1-xt)}{t} \tag{3.7}$$

and is given by

$$C_0(m_1, m_2, m_1; 0, 0, p^2)$$

$$= C_0(m_2, m_1, m_1; 0, p^2, 0)$$

$$= \frac{1}{p^2}\left\{\mathrm{Sp}\left(\frac{y_0}{y_0 - y_1}\right) - \mathrm{Sp}\left(\frac{y_0 - 1}{y_0 - y_1}\right) - \sum_{i=2}^{3}\left[\mathrm{Sp}\left(\frac{y_0}{y_0 - y_i}\right) - \mathrm{Sp}\left(\frac{y_0 - 1}{y_0 - y_i}\right)\right]\right\}$$

$$\text{with } y_0 = \frac{m_1^2 - m_2^2}{p^2}, \qquad y_1 = \frac{m_2^2 - i\delta}{m_2^2 - m_1^2}, \qquad y_{\substack{2\\3}} = \tfrac{1}{2}\left(1 \pm \sqrt{1 - 4m_1^2/p^2 + i\delta}\right) \tag{3.8}$$

As expected, the divergent terms proportional to ε^{-1} have cancelled and do not appear in the result (3.6). The real and imaginary parts of B_{Lbs} have been plotted in figs. 5 and 6 as a function of the t-quark mass m_t for different values of the charged Higgs mass m_H. The imaginary part arises due to real particles in the loop on the mass shell. The threshold at $m_t = m_Z/2$ is clearly visible.

The decay rate reads

$$\Gamma(\mathrm{Z} \to \mathrm{b\bar{s}}) + \Gamma(\mathrm{Z} \to \mathrm{\bar{b}s}) = \frac{4m_Z}{16\pi}\left(\frac{g^3}{32\pi^2 c_w}\frac{m_t^2}{m_W^2}\frac{v_1^2}{v_2^2}|V_{ts}V_{tb}^*|\right)^2 |B_{\mathrm{Lbs}}|^2. \tag{3.9}$$

This leads to the branching ratio:

$$\mathrm{BR}(\mathrm{Z} \to \mathrm{Bottom}) \cong 2.5 \times 10^{-6} |V_{ts}|^2 \frac{m_t^4}{m_W^4}\frac{v_1^4}{v_2^4}\left|B_{\mathrm{Lbs}}(s^2, m_Z, m_H, m_t)\right|^2$$

$$\cong 6.2 \times 10^{-9} \frac{m_t^4}{m_W^4}\frac{v_1^4}{v_2^4}\left|B_{\mathrm{Lbs}}(s^2, m_Z, m_H, m_t)\right|^2. \tag{3.10}$$

For $\Gamma(\mathrm{Z} \to \text{all})$ a theoretical value of 2.6 GeV [26] was used, the other parameters have been taken from ref. [16]. $|B_{\mathrm{Lbs}}|^2$ is of the order 10^{-2} to 10^{-1}. The result is of the same form as the GWS model result [18], but aside from the enhancement factor

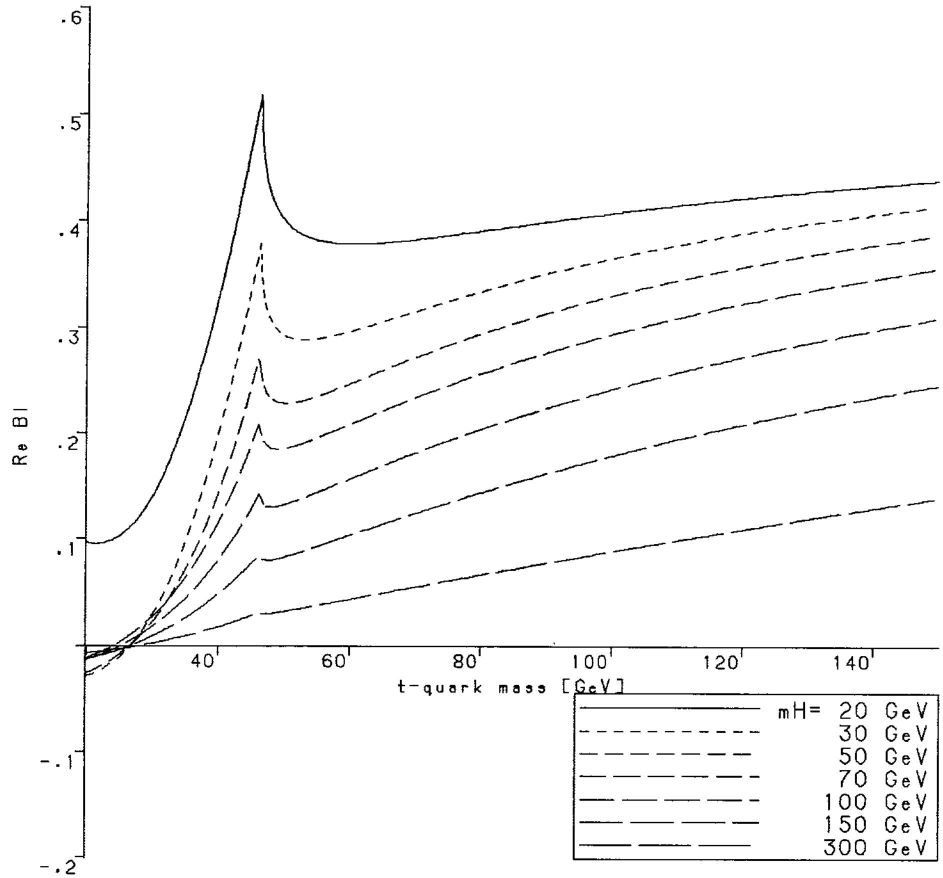

Fig. 5. The real part of B_{Lbs} for different m_{H}.

$(v_1/v_2)^4(m_t/m_W)^4$ it is about one or two orders of magnitude smaller. This deficit has to be compensated by the enhancement factor, before the charged Higgs contributions begin to dominate.

Using the bound (2.8) one obtains for the maximal possible branching within the two Higgs doublet model:

$$\mathrm{BR}(\mathrm{Z} \to \mathrm{Bottom}) < 3.4 \times 10^{-6} \frac{m_{\mathrm{H}}^2}{m_{\mathrm{W}}^2} |B_{\mathrm{Lbs}}|^2 \bar{I}_1^{-1}\left(\frac{m_t^2}{m_{\mathrm{H}}^2}\right). \tag{3.11}$$

Fig. 7 shows this maximal branching ratio as a function of m_t for different values of m_{H}.

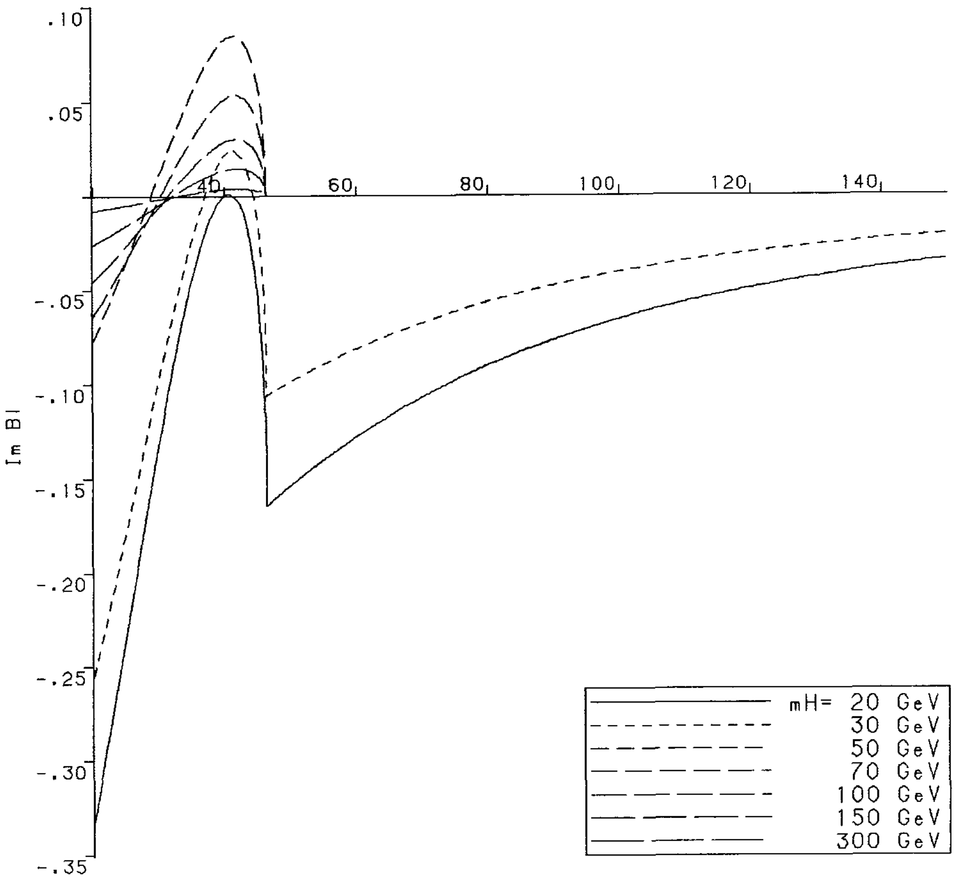

Fig. 6. The imaginary part of B_{Lbs} for different m_{H}.

3.7. RESULTS FOR $\mathrm{Z} \to \bar{\mathrm{t}}\mathrm{c}$

Now Q and q are the up-quarks c and t, the internal quark is the b-quark. After setting $m_{\mathrm{b}} = m_{\mathrm{c}} = 0$ only the formfactors $A_{\mathrm{Rtc}} = A_{\mathrm{R}}(1) + A_{\mathrm{R}}(2)$ and $B_{\mathrm{Ltc}} = B_{\mathrm{L}}(1) + B_{\mathrm{L}}(2) + B_{\mathrm{L}}(4)$ are different from zero. They are shown in table 1.

The required functions C_0 read

$$C_0\left(0, m, 0;\, p_1^2, 0,\, p_3^2\right)$$

$$= \frac{1}{p_3^2 - p_1^2} \sum_{i=1}^{2} \left(\mathrm{Sp}\left(\frac{y_0}{y_0 - y_i}\right) - \mathrm{Sp}\left(\frac{y_0 - 1}{y_0 - y_i}\right) - \mathrm{Sp}\left(\frac{y_0}{y_0 - z_i}\right) + \mathrm{Sp}\left(\frac{y_0 - 1}{y_0 - z_i}\right)\right)$$

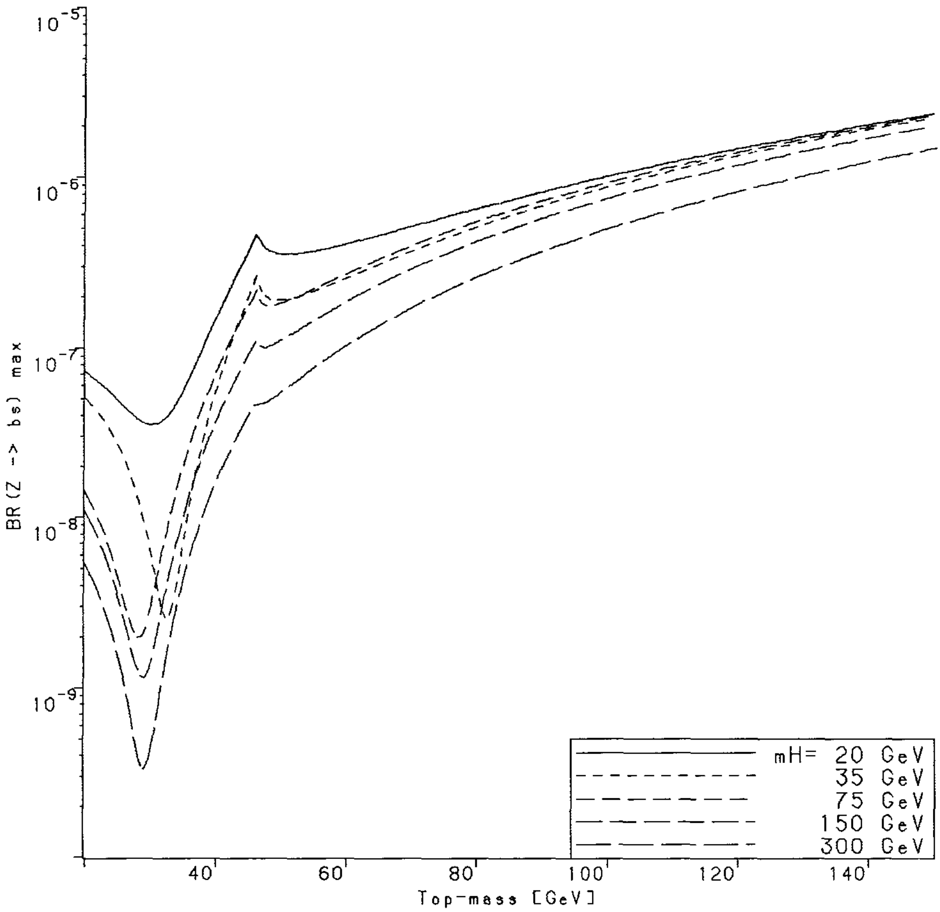

Fig. 7. The maximal branching ratio for Z → Bottom.

with $y_0 = \dfrac{m^2}{p_1^2 - p_3^2}, \qquad y_1 = \begin{cases} 1 + i\delta \\ 1 - i\delta \end{cases},$

$$y_2 = \begin{Bmatrix} m^2/p_1^2 - i\delta & p_1^2 > m^2 \\ m^2/p_1^2 + i\delta & p_1^2 < m^2 \end{Bmatrix}, \qquad \begin{aligned} z_1 &= 1 + i\delta \\ z_2 &= -i\delta \end{aligned}$$

$$C_0(0, m, m; p_1^2, p_2^2, 0)$$

$$= \frac{1}{p_1^2 - p_2^2} \sum_{i=1}^{2} \left(\mathrm{Sp}\left(\frac{y_0}{y_0 - y_i} \right) - \mathrm{Sp}\left(\frac{y_0 - 1}{y_0 - y_i} \right) - \mathrm{Sp}\left(\frac{y_0}{y_0 - z_i} \right) + \mathrm{Sp}\left(\frac{y_0 - 1}{y_0 - z_i} \right) \right)$$

TABLE 1
Form factors A_{Rtc} and B_{Ltc}.

$$
\begin{aligned}
A_{\mathrm{Rtc}} = \frac{2m_t}{\left(m_Z^2 - m_t^2\right)^2} \Bigg[& \tfrac{1}{3}s^2\left(2m_Z^2 m_H^2 + \frac{3m_Z^2 m_H^4}{m_Z^2 - m_t^2}\right) C_0\left(0, m_H, 0; m_t^2, 0, m_Z^2\right) \\
& - \left(\tfrac{1}{2} - s^2\right)\left(\frac{3m_H^4 m_Z^2}{m_Z^2 - m_t^2} - m_H^2\left(m_Z^2 + m_t^2\right)\right) C_0(0, m_H, m_H; m_t^2, m_Z^2, 0) \\
& + \left(\left(-\tfrac{1}{2} + \tfrac{4}{3}s^2\right)\frac{m_Z^2}{2} + \left(\tfrac{1}{2} - \tfrac{2}{3}s^2\right)\left(\frac{m_H^2\left(2m_Z^2 + m_t^2\right)}{m_Z^2 - m_t^2} - \frac{m_Z^2 m_H^2}{2m_t^2}\right)\right) \\
& \times \left(\left(\frac{m_H^2}{m_t^2} - 1\right)\ln\left|1 - \frac{m_t^2}{m_H^2}\right| + 1 + \begin{cases} 0 & m_t < m_H \\ i\pi\left(1 - m_H^2/m_t^2\right) & m_t > m_H \end{cases}\right) \\
& - \tfrac{1}{3}s^2\left(\frac{3m_H^2 m_Z^2}{m_Z^2 - m_t^2} + \frac{m_Z^2}{2}\right)\left(\ln\frac{m_H^2}{m_Z^2} + 1 + i\pi\right) \\
& - \left(\tfrac{1}{2} - s^2\right)\left(-\frac{m_Z^2}{2} + \frac{3m_H^2 m_Z^2}{m_Z^2 - m_t^2}\right) \\
& \times \left(- \begin{cases} \sqrt{4m_H^2/m_Z^2 - 1}\left(\pi - 2\arctan\sqrt{4m_H^2/m_Z^2 - 1}\right) & m_Z < 2m_H \\ -\sqrt{1 - 4m_H^2/m_Z^2}\left(\ln\frac{1 - \sqrt{1 - 4m_H^2/m_Z^2}}{1 + \sqrt{1 - 4m_H^2/m_Z^2}} + i\pi\right) & m_Z > 2m_H \end{cases} + 1 \right) \\
& - \tfrac{1}{2}\left(-\tfrac{1}{2} + \tfrac{4}{3}s^2\right)\left(m_Z^2 - m_t^2\right), \Bigg]
\end{aligned}
\qquad (1')
$$

$$
\begin{aligned}
B_{\mathrm{Ltc}} = \frac{1}{\left(m_Z^2 - m_t^2\right)^2} \Bigg[& \tfrac{1}{3}s^2 m_H^4 m_Z^2 C_0\left(0, m_H, 0; m_t^2, 0, m_Z^2\right) \\
& - \left(\tfrac{1}{2} - s^2\right)\left(m_H^2 m_t^2\left(m_t^2 - m_Z^2\right) + m_H^4 m_Z^2\right) C_0(0, m_H, m_H; m_t^2, m_Z^2, 0) \\
& + \Bigg(\tfrac{1}{3}s^2\left(\frac{\left(m_H^2 - m_t^2\right)\left(m_Z^2 - m_t^2\right)}{2} + m_t^2 m_H^2\right) - \tfrac{1}{2}\left(\tfrac{1}{2} - s^2\right)\Big(m_t^2\left(m_Z^2 - m_t^2\right) \\
& - m_H^2\left(m_Z^2 + m_t^2\right)\Big) + \left(\tfrac{1}{2} - \tfrac{2}{3}s^2\right)\frac{\left(m_H^2 - m_t^2\right)\left(m_Z^2 - m_t^2\right)^2}{2m_t^2}\Bigg) \\
& \times \left(\left(\frac{m_H^2}{m_t^2} - 1\right)\ln\left|1 - \frac{m_t^2}{m_H^2}\right| + 1 + \begin{cases} 0 & m_t < m_H \\ i\pi\left(1 - m_H^2/m_t^2\right) & m_t > m_H \end{cases}\right) \\
& + \tfrac{1}{3}s^2\left(\frac{m_Z^2\left(m_Z^2 - m_t^2\right)}{2} - m_H^2 m_Z^2\right)\left(\ln\frac{m_H^2}{m_Z^2} + 1 + i\pi\right) \\
& - \tfrac{1}{2}\left(\tfrac{1}{2} - s^2\right) m_Z^2\left(m_t^2 - m_Z^2 + 2m_H^2\right) \\
& \times \left(1 - \begin{cases} \sqrt{4m_H^2/m_Z^2 - 1}\left(\pi - 2\arctan\sqrt{4m_H^2/m_Z^2 - 1}\right) & m_Z < 2m_H \\ -\sqrt{1 - 4m_H^2/m_Z^2}\left(\ln\frac{1 - \sqrt{1 - 4m_H^2/m_Z^2}}{1 + \sqrt{1 - 4m_H^2/m_Z^2}} + i\pi\right) & m_Z > 2m_H \end{cases}\right)\Bigg] \\
& + \tfrac{1}{2}\left(\tfrac{1}{2} - \tfrac{4}{3}s^2\right).
\end{aligned}
\qquad (2')
$$

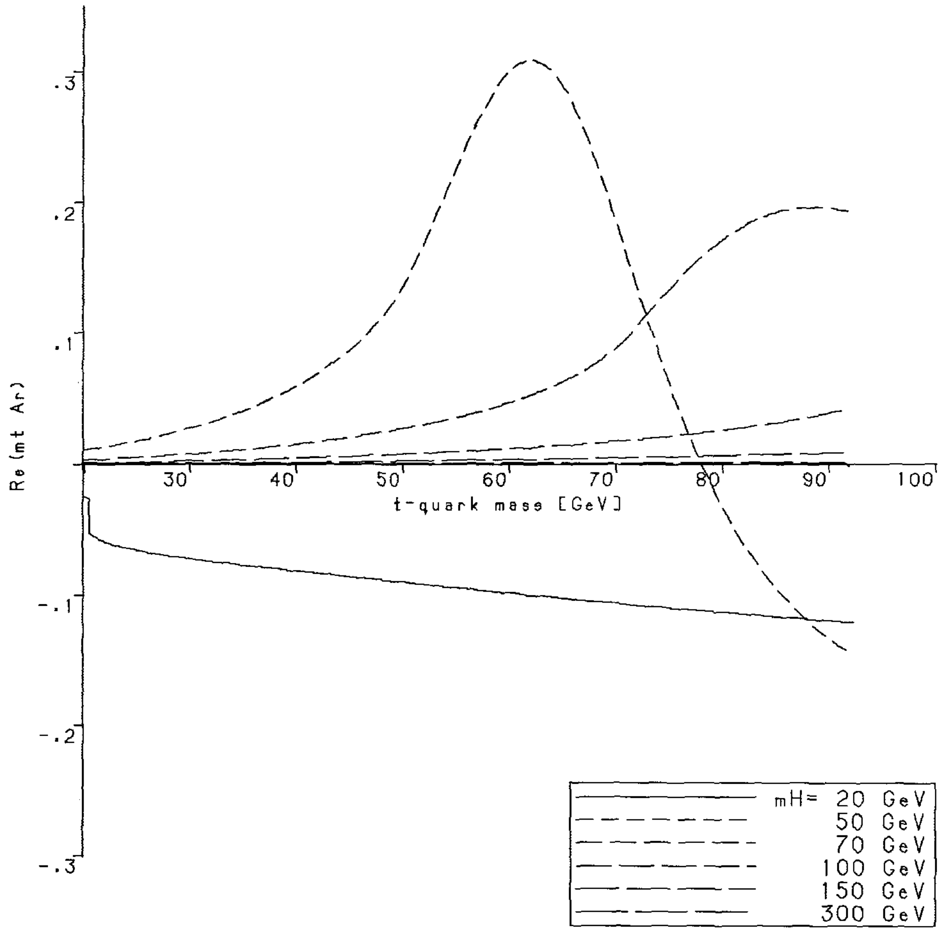

Fig. 8. The real part of $m_t A_{Rtc}$ for different m_H.

$$\text{with } y_0 = \frac{m^2}{p_2^2 - p_1^2}, \qquad y_{\substack{1\\2}} = \tfrac{1}{2}\left(1 \pm \sqrt{1 - 4m^2/p_2^2 + i\delta}\right), \qquad z_1 = \begin{cases} +i\delta \\ -i\delta \end{cases},$$

$$z_2 = \begin{Bmatrix} 1 - m^2/p_1^2 - i\delta & m^2 > p_1^2 \\ 1 - m^2/p_1^2 + i\delta & m^2 < p_1^2 \end{Bmatrix}. \tag{3.12}$$

Again the results (1′) and (2′) in table 1 do not contain any divergent term $\sim \varepsilon^{-1}$. The real and imaginary parts of $m_t \cdot A_{Rtc}$ and B_{Ltc} are shown in figs. 8–11. The

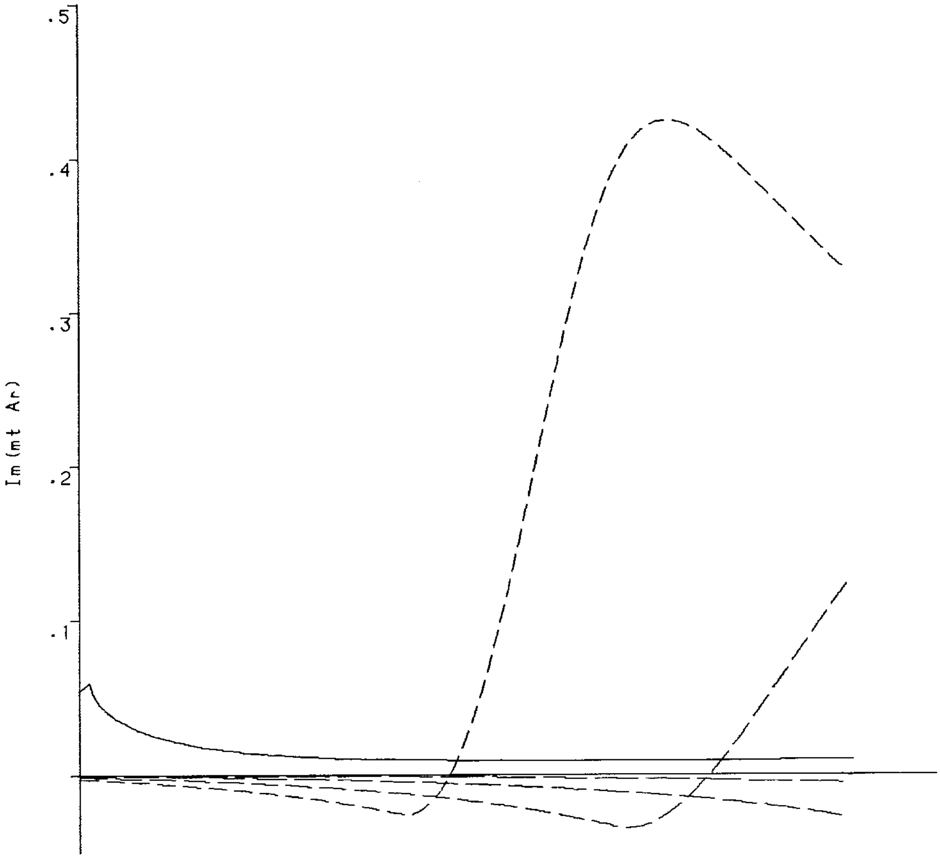

Fig. 9. The imaginary part of $m_t A_{Rtc}$.

decay rate is now

$$\Gamma(Z \to \bar{t}c) + \Gamma(Z \to t\bar{c}) = \frac{4m_Z}{16\pi}\left(\frac{g^3}{32\pi^2 c_w}\frac{m_b^2}{m_W^2}\frac{v_2^2}{v_1^2}|V_{cb}V_{tb}^*|\right)^2 F(s^2, m_Z, m_H, m_t) \tag{3.13}$$

with

$$F(s^2, m_Z, m_H, m_t) \equiv \left(1-\frac{m_t^2}{m_Z^2}\right)^2\left(\frac{1}{4}\left(1-\frac{m_t^2}{m_Z^2}\right)\left(\frac{m_Z^2-m_t^2}{2}|A_R|^2 - m_t(A_R^* B_L + B_L^* A_R)\right) + \left(1+\frac{m_t^2}{2m_Z^2}\right)|B_L|^2\right).$$

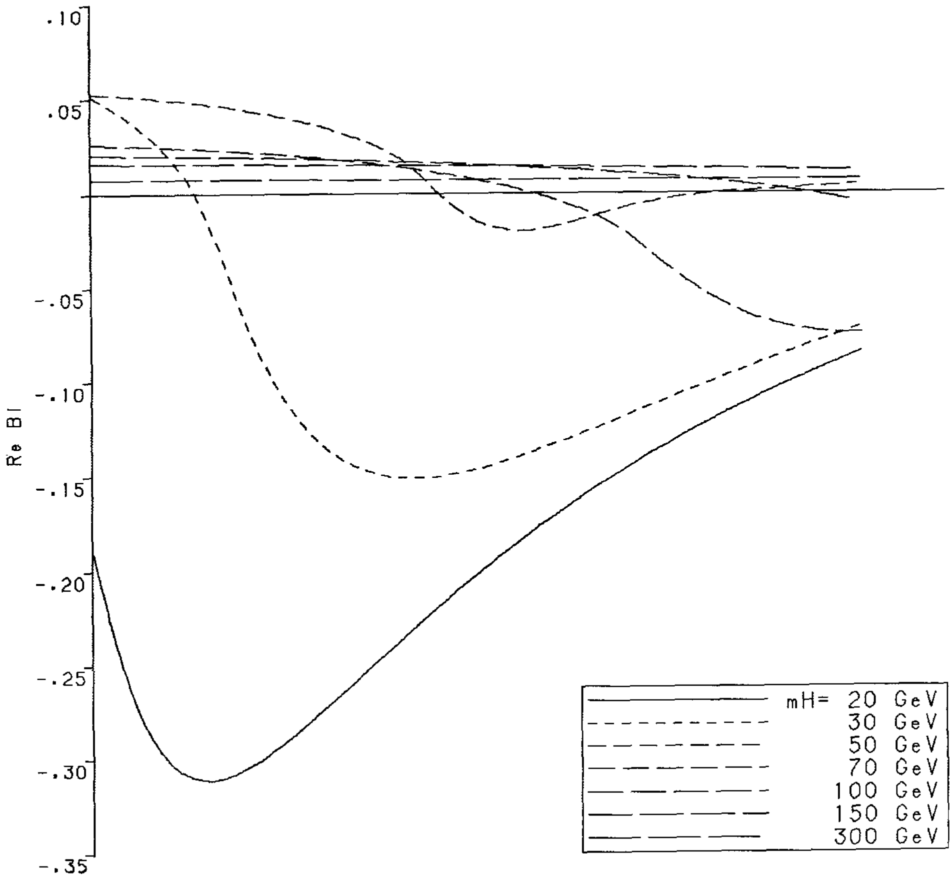

Fig. 10. The real part of B_{Ltc}.

This gives the branching ratio:

$$\mathrm{BR}(\mathrm{Z} \to \mathrm{Top}) \cong 2.5 \times 10^{-6} |V_{\mathrm{cb}}|^2 \frac{m_{\mathrm{b}}^4}{m_{\mathrm{W}}^4} \frac{v_2^4}{v_1^4} F\left(s^2, m_{\mathrm{Z}}, m_{\mathrm{H}}, m_{\mathrm{t}}\right)$$

$$\cong 6.7 \times 10^{-14} \frac{v_2^4}{v_1^4} F\left(s^2, m_{\mathrm{Z}}, m_{\mathrm{H}}, m_{\mathrm{t}}\right). \qquad (3.14)$$

F is in the range 10^{-3} to 10^{-1}. Aside from the enhancement factor $(m_{\mathrm{b}}/m_{\mathrm{W}})^4 (v_2/v_1)^4$ the result is of the same form and order of magnitude as the

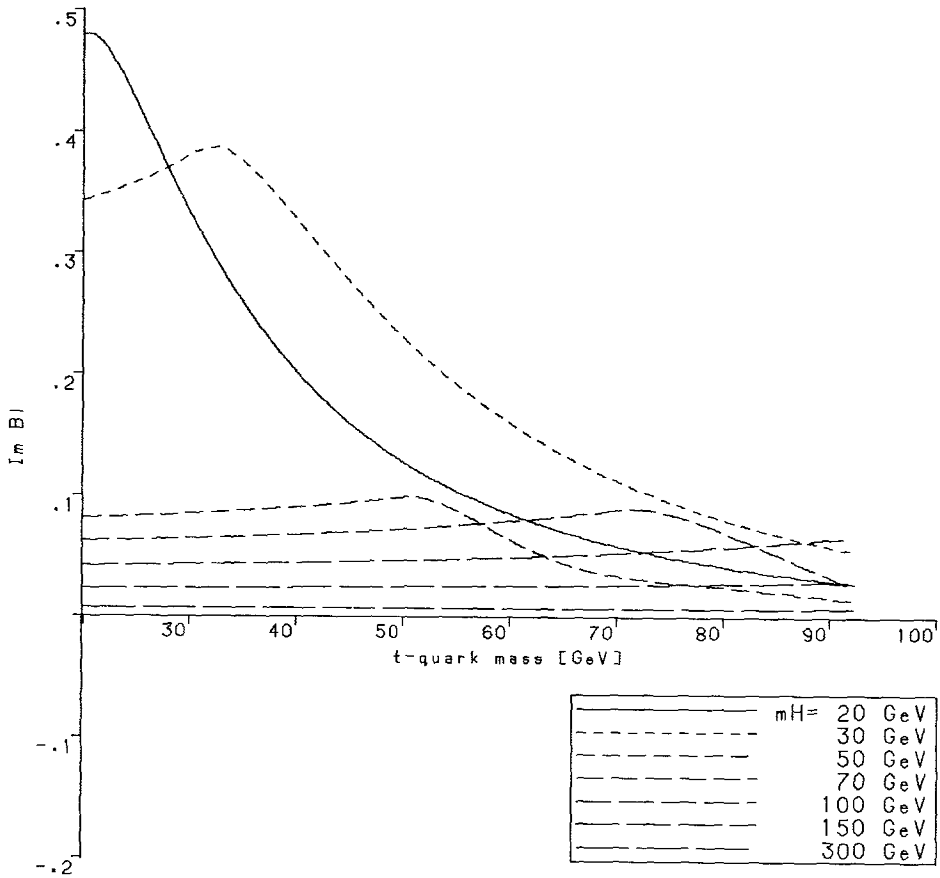

Fig. 11. The imaginary part of B_{Ltc}.

GWS model result. Use of the bound (2.10) leads to the maximal branching ratio:

$$\mathrm{BR}(\mathrm{Z} \to \mathrm{Top}) < 2.1 \times 10^{-3}\left(\frac{m_{\mathrm{H}}}{100\ \mathrm{GeV}}\right)^2 F\left(s^2, m_{\mathrm{Z}}, m_{\mathrm{H}}, m_{\mathrm{t}}\right). \tag{3.15}$$

Using the more sensible bound (2.12) one gets

$$\mathrm{BR}(\mathrm{Z} \to \mathrm{Top}) < 2.3 \times 10^{-5} F\left(s^2, m_{\mathrm{Z}}, m_{\mathrm{H}}, m_{\mathrm{t}}\right) \tag{3.16}$$

This maximal branching ratio is shown in fig. 12.

The imaginary part of the three-point function $C_0(0, m_{\mathrm{H}}, m_{\mathrm{H}}; m_{\mathrm{t}}^2, m_{\mathrm{Z}}^2, 0)$ and therefore the imaginary part of $A_{\mathrm{R}}(2)$ has a logarithmic singularity at $m_{\mathrm{H}}^2 = m_{\mathrm{t}}^2(1 - m_{\mathrm{t}}^2/m_{\mathrm{Z}}^2)$ for $m_{\mathrm{Z}} > 2m_{\mathrm{H}}$, because at this point all three propagators in the

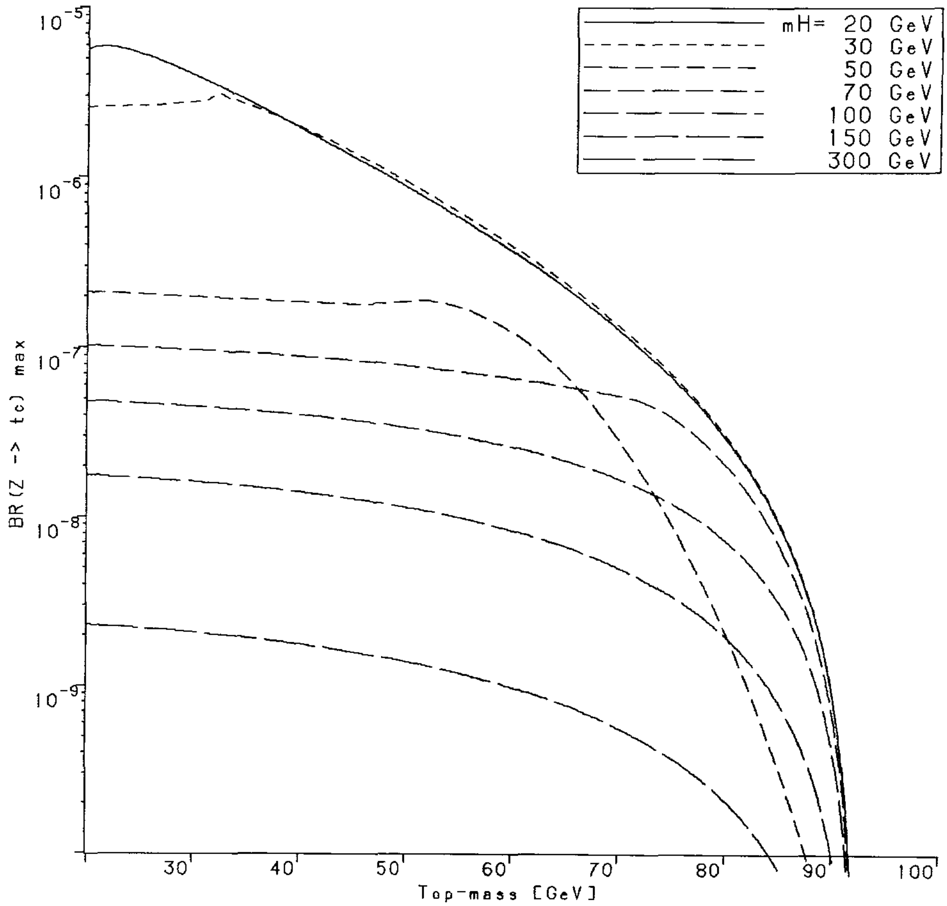

Fig. 12. The maximal branching ratio for Z → Top.

loop can be on the mass shell simultaneously. This is not a physical singularity, because the perturbation theory used to calculate the decay rate is not valid, when the decay rate becomes too large. So the region close to this point has to be excluded. In practice the involved particles have a natural line width, and there is no singularity, which will be discussed in more detail elsewhere.

3.8. OTHER TWO HIGGS DOUBLET MODELS

In the previous calculations the Yukawa couplings (2.1) were always used. But in a slightly different model one can also have the couplings (2.3) as mentioned before, which are throughout proportional to v_1/v_2. For the second and third family we have $m^u \gg m^d$, so the bound on v_1/v_2 is essentially due to the first term in eq. (2.3) and identical with the previous bound (2.8). Therefore this change of the model does not affect the process $Z \to b\bar{s}$, whereas $\Gamma(Z \to \bar{t}c)$ now contains no eventually very large enhancement factor $(v_2/v_1)^4$. The charged Higgs contributions to $\Gamma(Z \to \bar{t}c)$ are about a factor $(m_b/m_t)^2$ smaller than that to $\Gamma(Z \to b\bar{s})$.

There is also the possibility of having no natural neutral flavor conservation at tree level in the model, so that flavor changing Yukawa couplings of neutral Higgs bosons appear in the tree lagrangian. The bounds on these flavor non-diagonal couplings are very strong, because they contribute at tree level via neutral Higgs boson exchange to the neutral meson mixing. But in the decay of a physical Z-boson they occur at the one-loop level, and their contributions are small compared to the possible contributions of charged Higgs bosons, which we have calculated.

4. Summary and conclusions

The minimal extension of the GWS model with a second Higgs doublet is of phenomenological interest and is theoretically motivated by supersymmetry as well as by the Peccei–Quinn model.

Using experimental data for neutral meson mixing bounds for the parameter (v_1/v_2) were derived within the standard model with two Higgs doublets.

The contributions due to charged Higgs bosons to the flavor changing rare decays $Z \to \bar{t}c$ and $Z \to b\bar{s}$ were calculated in one-loop order neglecting the masses of the light quarks s, c, b.

Taking into account the bounds derived previously, one obtains for this model the maximal branching ratios:

$$\mathrm{BR}(Z^0 \to \mathrm{Top}) < 5 \times 10^{-6}, \qquad \mathrm{BR}(Z^0 \to \mathrm{Bottom}) < 2 \times 10^{-6}.$$

These are enhanced by factors of 10^4 and 10 compared to the maximal GWS model results [3, 18, 19]. The possible enhancement for $Z \to \bar{t}c$ is large, because the suppression of this process is very strong in the GWS model. Enhancement in one case excludes large enhancement in the other case.

The result is typically for models with charged Higgs bosons, whose Yukawa couplings are constrained by experimental data on neutral meson mixing.

With about 10^7 Z-bosons being produced per year at LEP the branching ratios above are on the point of being experimentally observable. Contrary to the rates predicted by the GWS model, which are unobservable, a charged Higgs scalar could in principle lead to a few flavor changing Z-decays at LEP. Observation of larger branching ratios would require a more dramatic change of the standard model beyond an extension of the Higgs sector.

I would like to thank W. Buchmüller for suggesting this topic and for many helpful discussions.

Numerical calculations and the plots have been made at the Regionales Rechenzentrum für Niedersachsen (RRZN), Hannover.

Note added in proof

After this manuscript was completed I became aware of two recent papers [27, 28], where similar results have been obtained for the decay $Z \to b\bar{s}$.

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