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THE DECAY $K^+ \to \pi^+ X$ IN $SU(2) \times U(1) \times U'(1)$ GAUGE THEORIES

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In the framework of $SU(2) \times U(1) \times U'(1)$ gauge models we investigate the decay $K^+ \to \pi^+ + X$, where X is the light gauge boson corresponding to the $U'(1)$ group. Contributions of the annihilation and spectator diagrams are taken into account. We discuss possibilities of the experimental observation of the X-bosons in the decays $K^+ \to \pi^+ + X$. It is shown that if the X boson is the carrier of a new long-range interaction ("the fifth force"), then the width of the process $K^+ \to \pi^+ X$ is extremely small.

1. Introduction

At present, the standard $SU(3) \times SU(2) \times U(1)$ model is in a good agreement with all the data. However, a number of theoretical drawbacks of the model (the problems of hierarchy, the number of families, the unification of interactions) require going beyond its framework (the grand unified theories, technicolour, supersymmetry, superstrings, etc.). Quite often, the low-energy symmetry group proves to be larger than $SU(3) \times SU(2) \times U(1)$. The simplest and at the same time most frequent case is the enlargement of the standard group with an additional U(1) factor. This possibility is often realized in grand unified theories [1], supersymmetric models [2], and superstring theories [3]. Recently, the interest in such kinds of models has been kindled by possible indications for the existence of a new long-range interaction (the "fifth force") [4]. So, the important problem is the theoretical study of the properties of new U(1) gauge bosons and the investigation of the possibilities of their experimental detection.

Since the masses of such bosons in general are not predicted by the theory, one should extensively use the experimental data in order to obtain information on the possible masses of these bosons.

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If these bosons were sufficiently light, their effects could be searched for in the e^+e^- annihilation, deep-inelastic scattering, the measuring of muon and electron anomalous magnetic moments, in the quarkonia decays, etc. In ref. [5] it was shown that one of the effective methods of searching for new light gauge bosons is to look for the decay $\pi^0 \to \gamma +$ "nothing" (by "nothing" we mean neutral unobservable particles). A number of limits on the mass and the coupling constant of new U(1) bosons was obtained in ref. [6].

In a number of cases rather strong constraints on the properties of such bosons can be obtained from the existing data on the decay $K^+ \to \pi^+ +$ "nothing", which is a usual place to look for new light unobservable particles. A detailed investigation of this process is inspired also by the fact that a rather sensitive experiment in search of this decay is taking place in the Brookhaven National Laboratory [7]. It is expected that the decay $K^+ \to \pi^+ +$ "nothing" will be detected when its branching ratio exceeds $\sim 10^{-10}$. It is worth noting that the width of this decay in the framework of the standard model is [8]

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) \simeq 10^{-11}. \tag{1}$$

Therefore, the positive result of the BNL experiment will mark the observation of new light undetectable particles. So it seems to be very interesting to know whether new U(1) bosons can play the role of "nothing" in this process.

In this paper we investigate the decay $K^+ \to \pi^+ + X$, where X is the gauge boson corresponding to the extra U′(1) group, possessing an arbitrary mass ($m_X < m_K - m_\pi$) and coupling to a quark current. The most general type of the interaction lagrangian is discussed in sect. 2. The effective sdX vertex is calculated in sect. 3. In sect. 4 we obtain the contribution of the annihilation diagrams to the matrix element of the decay $K^+ \to \pi^+ X$ using PCAC for the K meson. Conclusions and an outlook are presented in sect. 5.

2. Coupling of the X boson to quarks

We consider the gauge theory based on the group SU(2) × U(1) × U′(1) (strong interactions are not taken into account). We assume that the quark and lepton sectors of the theory are identical to those of the standard model, and for simplicity we restrict ourselves to considering only the first two quark generations:

$$L_1 = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}_L, \qquad L_2 = \begin{pmatrix} c \\ -d\sin\theta + s\cos\theta \end{pmatrix}_L.$$

$$u_R, c_R, s_R, d_R.$$

We wish to find the most general form of the lagrangian of the interaction of the X boson and quarks. It turns out that the U′(1) charges of the doublets L_1 and L_2 and of the right-singlets cannot be independent.

Because of the mixing between different generations, the Higgs sector of the theory has to contain terms of the following type:

$$\mathscr{L}_H = f_1 \bar{L}_1 H s_R + f_2 \bar{L}_2 H s_R + f_3 \bar{L}_1 H d_R + f_4 \bar{L}_2 H d_R + \dots, \tag{2}$$

where H is the Higgs doublet giving mass to down quarks (it does not matter which Higgs boson gives masses to up quarks and leptons*.

In order that the first two terms in eq. (2) be U′(1) gauge invariant, the U′(1) charges of the left doublets L_1 and L_2 have to be equal. From the gauge invariance of the first and third terms one can deduce that the U′(1) charges of the right singlets s_R and d_R are equal as well. In the same manner one can prove the equality of the charges of the right singlets u_R and c_R. Thus the coupling of the X boson to quarks depends on three parameters: G_L which is the U′(1) charge of the left doublets $L_{1,2}$, G_R^u and G_R^d which are the U′(1) charges of the right up and down quarks respectively. So the most general form of the interaction lagrangian of the X boson and quarks reads

$$\mathscr{L}_{int} = g_X \left\{ G_L \sum_{\text{up, down}} \bar{q}_L \gamma_\mu q_L + G_R^u \sum_{\text{up}} \bar{q}_R \gamma_\mu q_R + G_R^d \sum_{\text{down}} \bar{q}_R \gamma_\mu q_R \right\} X_\mu . \tag{3}$$

Note that the couplings of the X boson to the s and d quarks are the same. So it proves impossible to build any model based on the group SU(2) × U(1) × U′(1) in which the X boson would couple to the vector current of the baryon hypercharge. (Such a model, in which such a X boson was called a hyperphoton, was proposed in ref. [9] in order to explain gravitational anomalies in geophysical data.)

Let us consider once more the quark mass term $\bar{L} H q_R$. One can see that if $G_L \neq G_R$, then the Higgs doublet H has nonzero U′(1) charge. In that case the symmetries SU(2) × U(1) and U′(1) are broken down at the same time. The physical Z and X bosons will be superpositions of the fields X^0 and Z^0 entering the interaction lagrangian.

If there is only one Higgs doublet H in the theory then in order to make the X boson massive, one should introduce an extra Higgs field ϕ which is a singlet under SU(2) × U(1). It is its vacuum expectation value that determines the value of the physical X-boson mass:

$$m_X^2 = \tfrac{1}{2} g_X^2 G_\phi^2 \langle \phi \rangle^2, \qquad m_Z^2 = \tfrac{1}{2}\left(g^2 + g_X^2 G_H^2 + g'^2 \right) \langle H \rangle^2 . \tag{4}$$

* One can see that one must have at least three nonzero constants f_i in eq. (2) in order to get mixing.

The fields of physical bosons are

$$X = \frac{2g_X G_H}{\sqrt{g^2 + g'^2}} Z^0 + X^0, \qquad Z = Z^0 - \frac{2g_X G_H}{\sqrt{g^2 + g'^2}} X^0 \tag{5}$$

In deriving eqs. (4) and (5) it was assumed that $m_X \ll m_Z$ and $g_X \ll g, g'$.

The gauge invariance of the mass terms of the up and down quarks gives

$$G_H = G_L - G_R^d = G_R^u - G_L \tag{6}$$

Hence, in the case where the same Higgs doublet gives mass to both up and down quarks, the coupling of the X boson to quarks is determined by the two parameters G_R^u and G_R^d, while $G_L = \frac{1}{2}(G_R^u + G_R^d)$.

We do not investigate restrictions on the quark U′(1) charges which can be imposed by the requirement of the absence of the triangle anomalies, because it always can be respected by adjusting U′(1) charges in the lepton sector or by introducing new heavy fermions.

In the case of the nonminimal Higgs sector one is not obliged to introduce an extra Higgs singlet. For instance, in the case of two Higgs doublets H_1 and H_2 which give masses to up and down quarks respectively (as is the case in supersymmetric theories), the X boson acquires the mass

$$m_X^2 = \tfrac{1}{2} g_X^2 \left(G_{H_1} + G_{H_2}\right)^2 \frac{\langle H_1\rangle^2 \langle H_2\rangle^2}{\langle H_1\rangle^2 + \langle H_2\rangle^2}. \tag{7}$$

In the case when $\langle H_1\rangle \approx \langle H_2\rangle = 250$ GeV we obtain

$$\alpha_X = \frac{g_X^2}{4\pi} \approx \left(\frac{m_X}{250\ \text{GeV}}\right)^2, \tag{8}$$

which is a rather strong limit on α_X in the case of sufficiently light X bosons.

In many theories the mixing between Higgs doublets, $\varepsilon_{ij} H_1^i H_2^j$, plays a rather important role. When it is absent, there usually appear almost massless pseudoscalar (axion) and scalar bosons which are forbidden by the experiment. If this mixing is present then $G_{H_1} + G_{H_2} = 0$. From eq. (7) we conclude that in order to make the X boson massive, in this case one has to introduce again an extra Higgs field, singlet under SU(2) × U(1). As in the first case here we have $G_L = \frac{1}{2}(G_R^u + G_R^d)$.

In what follows we shall restrict ourselves to the theory, the Higgs sector of which consists of one doublet and one singlet. In this case we are free from the rather stringent constraint (8).

3. Calculation of the effective sdX vertex

In the calculation of the effective sdX vertex in general we shall follow the method of ref. [10], where the effective sdZ vertex was calculated in detail.

First, let us note the following circumstance. When the X boson is massless, the decay $K^+ \to \pi^+ X$ is forbidden kinematically by angular momentum conservation. Indeed, in the K-meson restframe the π meson and X boson produced in the decay fly away in the opposite directions along the same line. Since the X boson is massless, the projection of its spin on this line equals ± 1. The projections of the orbital angular momentum of the π meson and the X boson on this line are equal to zero. Since in the initial state we had the scalar (K meson), the process is forbidden. Hence, the width of the decay $K^+ \to \pi^+ X$ should be proportional to the X-boson mass

$$\mathrm{BR}(K^+ \to \pi^+ X) \sim \left(\frac{m_X}{m_K} \right)^2 . \tag{9}$$

So the bosons with extremely small masses, which could be carriers of a new long-range interaction, give negligibly small contributions to the width of the decay $K^+ \to \pi^+ +$ "nothing".

When calculating the effective sdX vertex one may assume that the s and d quarks are on their mass shell and their masses are equal to zero. Accounting for the quark masses leads to small corrections $\mathrm{BR} \to \mathrm{BR}\,[1 + O(m_s^2/m_W^2)]$

We shall make all calculations in the framework of the four-quark model. Besides we shall keep only the leading terms in the powers of m_c^2/m_W^2, m_X^2/m_W^2.

Now let us come to the detailed calculation of the sdX^0 vertex. The process $S(k) \to d(p)X(q)$ is described by two types of diagrams depicted in fig. 1. Diagram (a) corresponds to the sum of six diagrams presented in fig. 2. Self-energy diagrams marked with black circles in fig. 1b are described by the three diagrams of fig. 3. We choose the renormalization scheme in fig. 3 such that the counterterm (c) completely cancels the contribution of the diagrams (a) and (b) in the non-diagonal sd transition. This sd counterterm gives rise to the counterterm of the form $S_L \gamma_\mu d_L X_\mu$ which is depicted in fig. 2f (for more details see ref. [10]). So the effective sdX_0 vertex is described by fig. 1a only.

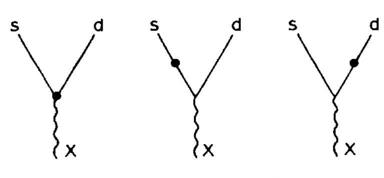

Fig. 1.

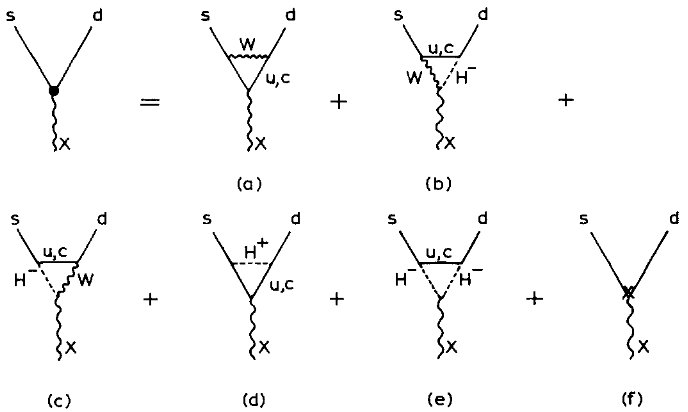

Fig. 2.

All calculations in this section were carried out in the R_ξ gauge, in which the propagator of the W boson reads

$$\Delta_{\mu\nu} = \frac{-i}{p^2 - m_W^2}\left(g_{\mu\nu} + (\xi - 1) p_\mu p_\nu / \left(p^2 - \xi m_W^2\right)\right)$$

The expressions for each diagram 2a–2f of fig. 2 are

$$\Gamma_\mu^{2a} = A_\mu \left\{ 2\left(G_R^u - G_L^u\right) \frac{m_c^2}{m_W^2} \ln \frac{m_c^2}{m_W^2} + \left(G_L^u - 2G_R^u\right) \frac{m_c^2}{m_W^2} \right.$$

$$\left. + \left(2G_L^u - \tfrac{1}{2}G_R^u\right) \frac{m_c^2}{m_W^2} \ln \xi + 4G_L^u \frac{q^2}{m_W^2} \int_0^1 dx\, x(1-x) \ln \frac{m_c^2 - x(1-x) q^2}{m_u^2 - x(1-x) q^2} \right\}, \quad (10)$$

$$\Gamma_\mu^{2c} + \Gamma_\mu^{2b} = A_\mu G_{H^-} \frac{m_c^2}{m_W^2} \left(1 + \frac{3 \ln \xi}{\xi - 1}\right), \quad (11)$$

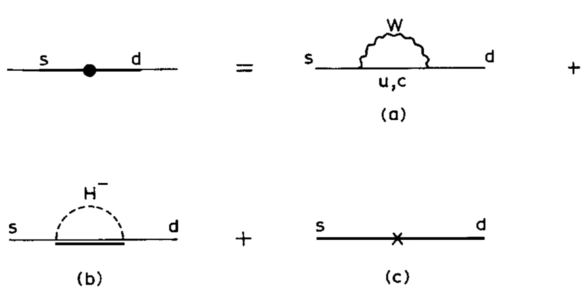

Fig. 3.

where

$$A_\mu = i\frac{g^2 g_X}{32\pi^2}\sin\theta_c\cos\theta_c\gamma_\mu P_L, \qquad P_L = \frac{1-\gamma_5}{2}$$

The contributions of the diagrams 2d, 2e and the contribution of that part of the diagram 2f which corresponds to the counterterm for the diagram in fig. 3b, cancel each other completely, as is the case for the Z^0 boson

$$\Gamma_\mu^{2d} + \Gamma_\mu^{2e} + \Gamma_\mu^{2f(H)} = 0 \tag{12}$$

The remaining part of the diagram 2f, which corresponds to the counterterm appearing due to the W-boson exchange (fig. 3a), reads

$$\Gamma_\mu^{2f(W)} = A_\mu G_L^d\left(1 + \tfrac{3}{2}\ln\xi\right) \tag{13}$$

Let us note that in the specific case of the effective sdZ^0 vertex these results agree with those obtained in ref. [10].

It was stressed above that in the case under consideration, the U′(1) charges obey the following equations:

$$G_L^u = G_L^d \equiv G_L = \tfrac{1}{2}\left(G_R^u + G_R^d\right),$$
$$G_{H^+} = G_L - G_R = -G_{H^-} \tag{14}$$

Hence we obtain the following expression for the effective sdX^0 vertex:

$$\Gamma_\mu^X = i\frac{g^2 g_X}{32\pi^2}\sin\theta_c\cos\theta_c\gamma_\mu P_L\left\{\left(G_R^u - G_L\right)\left(3 - 2\ln\frac{m_W^2}{m_c^2} + \frac{7-\xi}{2(\xi-1)}\ln\xi\right)\frac{m_c^2}{m_W^2}\right.$$
$$\left. + 4G_L\frac{q^2}{m_W^2}\int_0^1 dx\, x(1-x)\ln\frac{m_c^2 - x(1-x)q^2}{m_u^2 - x(1-x)q^2}\right\}. \tag{15}$$

The effective sdZ^0 vertex has the form

$$\Gamma_\mu^Z = i\frac{g^2\sqrt{g^2+g'^2}}{32\pi^2}\sin\theta_c\cos\theta_c\gamma_\mu P_L\frac{m_c^2}{m_W^2}\left\{-\tfrac{3}{2} + \ln\frac{m_W^2}{m_c^2} - \frac{7-\xi}{4(\xi-1)}\ln\xi\right\}. \tag{16}$$

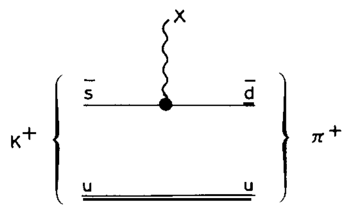

Fig. 4.

From eqs. (15) and (16), taking into account the mixing (5) and eq. (14) for the U′(1) charges, one gets the following expression for the sdX vertex:

$$\Gamma_\mu = i\frac{g_X}{\pi^2}\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\tfrac{1}{2}\left(G_R^u + G_R^d\right)q^2\bar{s}_L\gamma_\mu d_L$$

$$\times\int_0^1 dx\,x(1-x)\ln\frac{m_c^2 - q^2x(1-x)}{m_u^2 - q^2x(1-x)}. \tag{17}$$

It is worth noting that when there is no mixing between the X and Z bosons (i.e. the X boson couples only to a vector quark current) the expression (17) for the effective sdX vertex does not change.

Now we have everything to compute the matrix element of the decay $K^+(k) \to \pi^+(p)X(q)$ corresponding to the spectator diagram in fig. 4.

$$M = \varepsilon_\mu\langle K^+|\Gamma_\mu|\pi^+\rangle$$

$$= i\frac{g_X}{\pi^2}\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\tfrac{1}{2}\left(G_R^u + G_R^d\right)m_X^2\tfrac{1}{2}f_+^+\left(m_X^2\right)$$

$$\times(p+k)_\mu\varepsilon_\mu\int_0^1 dx\,x(1-x)\ln\frac{m_c^2 - x(1-x)m_X^2}{m_u^2 - x(1-x)m_X^2}, \tag{18}$$

where ε_μ is the polarization vector of the X boson, $f_+^+(q^2)$ is the form factor of the $K^+ \to \pi^+$ transition:

$$\langle K^+(k)|\bar{s}_L\gamma_\mu d_L|\pi^+(p)\rangle = f_+^+(q^2)(p+k)_\mu + f_-^+(q^2)q_\mu. \tag{19}$$

Since $\varepsilon q = 0$, the second term in eq. (19) does not contribute to the matrix element. To a good accuracy the form factor $f_+^+(m_X^2)$ equals [11]

$$f_+^+\left(m_X^2\right) \simeq \frac{1}{\sqrt{2}}\left(1 + 0.03\frac{m_X^2}{m_\pi^2}\right). \tag{20}$$

Note that the matrix element of the decay $K^+ \to \pi^+ X$ turns out to be proportional to m_X^2 in agreement with the qualitative arguments presented in the beginning of this section.

It is well known that in the limit $m_X \to 0$ (when $g_X \to 0$) the X boson behaves like the corresponding Goldstone boson G*. It is interesting to note that although the decay $K^+ \to \pi^+ G$ is not kinematically forbidden, its matrix element nevertheless vanishes (see appendix B).

4. Contribution of the annihilation diagrams

In this section we calculate the contribution to the matrix element corresponding to the annihilation diagrams. We assume that the X boson couples to the quark current of the general form

$$J_\mu = J_\mu^V + J_\mu^A = g_V^q \bar{q}\gamma_\mu q + g_A^q \bar{q}\gamma_5\gamma_\mu q .$$

As above we shall suppose that the X boson interacts with s and d quarks in the same way.

The matrix element of the decay $K^+(k) \to \pi^+(p)X(q)$ may be written as

$$M = \varepsilon_\mu M_\mu ,$$

where ε_μ is the polarization vector of the X boson and

$$M_\mu = \int d^4x \, e^{-iqx} \langle K^+ | T J_\mu(x) \mathscr{L}(0) | \pi^+ \rangle . \tag{21}$$

Here $\mathscr{L}$ is the effective lagrangian of the K–π transition. Without strong corrections it has the following form:

$$\mathscr{L} = -\frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \bar{s}\gamma_\nu(1-\gamma_5) u \bar{u}\gamma_\nu(1-\gamma_5)\, d . \tag{22}$$

Let us begin with the calculation of the contribution (21) due to the s-quark vector current. In this case the matrix element of the decay is

$$M_\mu = -i\frac{G_F}{\sqrt{2}} g_V^s \sin\theta_c \cos\theta_c \int d^4x \, e^{-iqx} \langle K^+ | T \bar{s}\gamma_\mu s(x)$$
$$\times \bar{s}\gamma_\nu(1-\gamma_5) u \bar{u}\gamma_\nu(1-\gamma_5)\, d(0) | \pi^+ \rangle .$$

* See, e.g. the second entry of ref. [2].

Using the standard vacuum dominance assumption one gets

$$M_\mu = G_F g_V^s f_\pi \sin\theta_c \cos\theta_c p_\nu \int d^4x\, e^{-iqx} \langle K^+ | T \bar{s}\gamma_\mu s(x) \bar{s}\gamma_\nu\gamma_5 u(0) | 0\rangle .$$

In the following calculations of the matrix elements we shall restrict ourselves to the leading terms in the limit of vanishing squared momenta of K and π mesons; K^2, $p^2 \to 0$. (This means that we apply the PCAC hypothesis to the entire mesonic SU(3) octet. The accuracy of our results will correspond to the accuracy of the hadron SU(3) symmetry. So, the expected error will be about 20%.) From the kinematics of the decay one deduces that in this limit all momenta are tending to zero. Hence one obtains

$$M_\mu \simeq -\frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \frac{f_\pi}{f_K} m_s p_\nu T_{\mu\nu} ,$$

where

$$T_{\mu\nu} = i \int d^4x\, d^4y\, e^{iqx - iky} \langle 0 | T \bar{s}\gamma_\mu s(x) \bar{s}\gamma_\nu\gamma_5 u(0) \bar{u}\gamma_5 s(y) | 0\rangle .$$

In the calculation of the matrix element M_μ we shall use the following method. Using the Lorentz invariance M_μ can be written as

$$M_\mu = a p_\mu + b q_\mu .$$

The value $T_{\mu\nu}$ can be calculated from the triangle diagram depicted in fig. 5. The coefficient b turns out to be finite. To render the coefficient a finite we use the following Ward identity:

$$q_\mu M_\mu = i G_F \sqrt{2} \sin\theta_c \cos\theta_c f_\pi f_K (pk) . \tag{23}$$

(The detailed derivation of this formula is contained in Appendix A.) Finally one

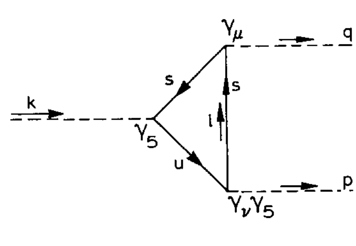

Fig. 5.

gets

$$M_\mu^s = i\frac{G_F}{\sqrt{2}} g_V^s \sin\theta_c \cos\theta_c$$

$$\times \frac{f_\pi}{f_K}\left\{ p_\mu \frac{1}{pq}\left[2f_K^2(kp) - \frac{q^2}{4\pi^2}\left(\tfrac{2}{3}pq - \tfrac{1}{2}p^2\right)\right] + q_\mu \frac{1}{4\pi^2}\left[\tfrac{2}{3}pq - \tfrac{1}{2}p^2\right]\right\}. \quad (24)$$

Reasoning in the same manner in the case of d quark and changing identity (23) to

$$q_\mu M_\mu^d = -iG_F\sqrt{2}\sin\theta_c \cos\theta_c f_\pi f_K(kp) \quad (25)$$

one obtains

$$M_\mu^d = -i\frac{G_F}{\sqrt{2}} g_V^d \sin\theta_c \cos\theta_c$$

$$\times \frac{f_\pi}{f_K}\left\{ q_\mu \frac{1}{2\pi^2}\left[\tfrac{3}{2}k^2 - pk\right] - k_\mu \frac{1}{kq}\left[2f_\pi^2(kp) + \frac{q^2}{2\pi^2}\left(\tfrac{3}{2}k^2 - pk\right)\right]\right\}. \quad (26)$$

And now let us find out the contribution of the coupling of the X boson to the current $\bar{u}\gamma_\mu u$. Since the u quark is contained both in K^+ and in π^+ mesons, the corresponding matrix element can be represented as the sum

$$M_\mu^u = M_{1\mu}^u + M_{2\mu}^u,$$

where

$$M_{1\mu}^u = -\frac{G_F}{\sqrt{2}} g_V^u \sin\theta_c \cos\theta_c \int d^4x\, e^{-iqx} \langle K^+ | T\bar{u}\gamma_\mu u(x)\bar{s}\gamma_\nu(1-\gamma_5)u(0)|0\rangle$$

$$\times \langle 0|\bar{u}\gamma_\nu(1-\gamma_5)\, d|\pi^+\rangle$$

$$= -\frac{G_F}{\sqrt{2}} g_V^u \sin\theta_c \cos\theta_c \frac{f_\pi}{f_K} m_s p_\nu T_{1\mu\nu},$$

$$T_{1\mu\nu} = i\int d^4x\, d^4y\, e^{iqx - iky} \langle 0|T\bar{u}\gamma_\mu u(x)\bar{u}\gamma_5 s(y)\bar{s}\gamma_\nu\gamma_5 u(0)|0\rangle$$

and

$$M_{2\mu}^{\mathrm{u}} = -\frac{G_{\mathrm{F}}}{\sqrt{2}} g_{\mathrm{V}}^{\mathrm{u}} \sin\theta_{\mathrm{c}} \cos\theta_{\mathrm{c}} \langle K^+ | \bar{s}\gamma_\nu (1-\gamma_5) u | 0\rangle$$

$$\times \int \mathrm{d}^4x\, \mathrm{e}^{iqx} \langle 0 | T \bar{u}\gamma_\mu u(x) \bar{u}\gamma_\nu (1-\gamma_5)\, d(0) | \pi^+\rangle$$

$$= \frac{G_{\mathrm{F}}}{\sqrt{2}} g_{\mathrm{V}}^{\mathrm{u}} \sin\theta_{\mathrm{c}} \cos\theta_{\mathrm{c}} \frac{f_{\mathrm{K}}}{f_\pi} (m_{\mathrm{u}} + m_{\mathrm{d}}) k_\nu T_{2\mu\nu}$$

$$T_{2\mu\nu} = i \int \mathrm{d}^4x\, \mathrm{d}^4y\, \mathrm{e}^{iqx+ipy} \langle 0 | T \bar{u}\gamma_\mu u(x)\, \bar{d}\gamma_5 u(y) \bar{u}\gamma_\nu\gamma_5\, d(0) | 0\rangle .$$

When computing the $M_{1\mu}^{\mathrm{u}}$ one cannot neglect the u-quark mass as compared to the s-quark mass, because in the PCAC limit this results in the appearance of divergent integrals.

Let us show the explicit form of $p_\nu T_{1\mu\nu}$ and $k_\nu T_{2\mu\nu}$ which will be useful for us later:

$$p_\nu T_{1\mu\nu} = 12 m_{\mathrm{s}} \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{l_\mu [2lp + pq] + q_\mu lp - p_\mu [l^2 + lq]}{\left[(l+q)^2 - m_{\mathrm{u}}^2\right]\left[(l-p)^2 - m_{\mathrm{s}}^2\right]\left[l^2 - m_{\mathrm{s}}^2\right]}, \tag{27}$$

$$k_\nu T_{2\mu\nu} = 12 m_{\mathrm{q}} \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{-2 l_\mu kp + q_\mu k^2 + k_\mu \left[l^2 - 2lq\right]}{\left[(l+q)^2 - m_{\mathrm{q}}^2\right]\left[(l-k)^2 - m_{\mathrm{q}}^2\right]\left[l^2 - m_{\mathrm{q}}^2\right]}, \tag{28}$$

In the last equation we assumed

$$m_{\mathrm{u}} \simeq m_{\mathrm{d}} \simeq m_{\mathrm{q}} .$$

Following the same procedure as for s and d quarks and using the identities (see appendix A)

$$q_\mu M_{1\mu}^{\mathrm{u}} = -iG_{\mathrm{F}}\sqrt{2} \sin\theta_{\mathrm{c}} \cos\theta_{\mathrm{c}} f_\pi f_{\mathrm{K}} (kp) g_{\mathrm{V}}^{\mathrm{u}},$$

$$q_\mu M_{2\mu}^{\mathrm{u}} = iG_{\mathrm{F}}\sqrt{2} \sin\theta_{\mathrm{c}} \cos\theta_{\mathrm{c}} f_\pi f_{\mathrm{K}} (kp) g_{\mathrm{V}}^{\mathrm{u}}, \tag{28a}$$

we get

$$M_{1\mu}^{u} = i\frac{G_F}{\sqrt{2}} g_V^u \sin\theta_c \cos\theta_c \frac{f_\pi}{f_K}\left\{ q_\mu \frac{1}{4\pi^2}\left[\tfrac{1}{2}p^2 - pq\left(\ln\frac{m_s^2}{m_u^2} - \frac{5}{6}\right)\right]\right.$$

$$\left. - p_\mu \frac{1}{pq}\left[2f_K^2(kp) + \frac{q^2}{4\pi^2}\left(\tfrac{1}{2}p^2 - pq\left(\ln\frac{m_s^2}{m_u^2} - \frac{5}{6}\right)\right)\right]\right\}, \tag{29}$$

$$M_{2\mu}^{u} = i\frac{G_F}{\sqrt{2}} g_V^u \sin\theta_c \cos\theta_c \frac{f_\pi}{f_K}\left\{ q_\mu \frac{1}{2\pi^2}\left[kp - \tfrac{3}{2}k^2\right] + k_\mu \frac{1}{kq}\left[2f_\pi^2(kp) - \frac{q^2}{2\pi^2}\left(kp - \tfrac{3}{2}k^2\right)\right]\right\}. \tag{30}$$

Now let us proceed to the calculation of the contribution to the matrix element due to the intersection of the X boson with axial quark currents. Note that in this case we cannot use the Ward identity as in the case above, because in general the K and π mesons have no definite axial charge.

We start with the case of the coupling of the X boson to the current $\bar{u}\gamma_5\gamma_\mu u$. As in the case of the vector coupling we decompose the matrix element into the sum:

$$M_\mu = M_{1\mu} + M_{2\mu},$$

where

$$M_{1\mu} = -\frac{G_F}{\sqrt{2}} g_A^u \sin\theta_c \cos\theta_c \int d^4x\, e^{iqx} \langle K^+ | T\bar{u}\gamma_5\gamma_\mu u(x)\bar{s}\gamma_\nu(1-\gamma_5)u(0)|0\rangle$$
$$\times \langle 0|\bar{u}\gamma_\nu(1-\gamma_5)\, d|\pi^+\rangle$$
$$= \frac{G_F}{\sqrt{2}} g_A^u \sin\theta_c \cos\theta_c \frac{f_\pi}{f_K} m_s p_\nu L_{1\mu\nu},$$

$$L_{1\mu\nu} = i\int d^4x\, d^4y\, e^{iqx+ipy} \langle 0|T\bar{u}\gamma_5\gamma_\mu u(x)\, \bar{d}\gamma_5 u(y)\bar{u}\gamma_\nu d(0)|0\rangle,$$

$$M_{2\mu} = -\frac{G_F}{\sqrt{2}} g_A^u \sin\theta_c \cos\theta_c \int d^4x\, e^{iqx} \langle K^+ |\bar{s}\gamma_\nu(1-\gamma_5)u|0\rangle$$
$$\times \langle 0|T\bar{u}\gamma_5\gamma_\mu u(x)\bar{u}\gamma_\nu(1-\gamma_5)\, d|\pi^+\rangle$$
$$= -\frac{G_F}{\sqrt{2}} g_A^u \sin\theta_c \cos\theta_c \frac{f_K}{f_\pi} 2m_q k_\nu L_{2\mu\nu},$$

$$L_{2\mu\nu} = i\int d^4x\, d^4y\, e^{iqx+iky} \langle 0|T\bar{u}\gamma_5\gamma_\mu u(x)\, \bar{d}\gamma_5 u(y)\bar{u}\gamma_\nu d(0)|0\rangle.$$

The value $p_\nu L_{1\mu\nu}$ is

$$p_\nu L_{1\mu\nu} = 3\int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}\,\gamma_5(\hat{l}-\hat{p}+m_s)\hat{p}(\hat{l}+m_u)\gamma_\mu\gamma_5(\hat{l}+\hat{q}+m_u)}{[(l-p)^2-m_s^2][l^2-m_u^2][(l+q)^2-m_u^2]}.$$

Using eq. (27) one gets

$$p_\nu(L_{1\mu\nu}-T_{1\mu\nu}) = -6m_u\int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{Tr}(\hat{l}-\hat{p}+m_s)\hat{p}\gamma_\mu(\hat{l}+\hat{q}+m_u)}{[(l-p)^2-m_s^2][(l+q)^2-m_u^2][l^2-m_u^2]}$$

$$\sim \mathrm{O}\left(\frac{m_u}{m_s}\right).$$

So, within our accuracy we obtain $p_\nu L_{1\mu\nu} \simeq p_\nu T_{1\mu\nu}$. Therefore,

$$M_{1\mu} = -i\frac{G_F}{\sqrt{2}} g_A^u \sin\theta_c \cos\theta_c \frac{f_\pi}{f_K}\left\{ q_\mu \frac{1}{4\pi^2}\left[\tfrac{1}{2}p^2 - pq\left(\ln\frac{m_s^2}{m_u^2}-\frac{5}{6}\right)\right]\right.$$
$$\left. -p_\mu \frac{1}{pq}\left[2f_K^2(kp) + \frac{q^2}{4\pi^2}\left(\tfrac{1}{2}p^2 - pq\left(\ln\frac{m_s^2}{m_u^2}-\frac{5}{6}\right)\right)\right]\right\}. \quad (31)$$

To compute $M_{2\mu}$ we proceed in the same way. The difference $k_\nu(L_{2\mu\nu} - T_{2\mu\nu})$ turns out to be finite, and taking eq. (28) into account one gets

$$k_\nu(L_{2\mu\nu}-T_{2\mu\nu}) = \frac{i}{4\pi^2}\frac{1}{m_q}\left[-2q_\mu kq + k_\mu(k^2+q^2)\right].$$

Using eqs. (28) and (30) we arrive at

$$M_{2\mu} = -i\frac{G_F}{\sqrt{2}} g_A^u \sin\theta_c \cos\theta_c$$
$$\times \frac{f_\pi}{f_K}\left\{ q_\mu \frac{-q^2}{4\pi^2} + k_\mu \frac{1}{kq}\left[2f_\pi^2 kp - \frac{q^2}{2\pi^2}\left(kp - \tfrac{3}{2}k^2\right) - \frac{1}{2\pi^2}(kq)(k^2+q^2)\right]\right\}.$$
$$(32)$$

Now let the X boson couple to the current $\bar{s}\gamma_5\gamma_\mu s$. The matrix element to be computed is

$$M_\mu = -\frac{G_F}{\sqrt{2}} g_A^s \sin\theta_c \cos\theta_c \int d^4x\, e^{iqx} \langle K^+ | T \bar{s}\gamma_5\gamma_\mu s(x)$$

$$\times \bar{s}\gamma_\nu(1-\gamma_5) u(0)|0\rangle\langle 0|\bar{u}\gamma_\nu(1-\gamma_5)\, d|\pi^+\rangle$$

$$= \frac{G_F}{\sqrt{2}} g_A^s \sin\theta_c \cos\theta_c \frac{f_\pi}{f_K} m_s p_\nu L_{\mu\nu},$$

$$L_{\mu\nu} = i \int d^4x\, d^4y\, e^{iqx - iky} \langle 0 | T \bar{s}\gamma_5\gamma_\mu s(x) \bar{s}\gamma_\nu u(0) \bar{u}\gamma_5 s|0\rangle. \tag{33}$$

As in the case of the vector current $\bar{s}\gamma_\mu s$, we represent the matrix element as the sum:

$$M_\mu = a p_\mu + b q_\mu.$$

The coefficient b may be extracted directly from the corresponding Feynman diagram. It proves to be

$$b = \frac{i}{4\pi^2} \frac{1}{m_s} \left[\tfrac{1}{2}p^2 - \tfrac{2}{3}pq\right].$$

The coefficient a is divergent. To obtain a one should use the corresponding Ward identity.

Consider the following field transformation:

$$u \to e^{i\alpha\gamma_5} u, \qquad s \to e^{-i\alpha\gamma_5} s.$$

The corresponding current J_μ^5 and the charge Q read

$$J_\mu^5 = \bar{s}\gamma_5\gamma_\mu s - \bar{u}\gamma_5\gamma_\mu u,$$

$$Q(K^+) = 0, \qquad Q|0\rangle = 0. \tag{34}$$

Note that J_μ^5 conserves in the PCAC limit. Consider the matrix element $C_{\mu\nu}$:

$$C_{\mu\nu} = \int d^4x\, e^{iqx} \langle K^+ | T J_\mu^5(x) \bar{s}\gamma_\nu(1-\gamma_5) u(0)|0\rangle. \tag{35}$$

It is shown in appendix A that

$$q_\mu C_{\mu\nu} = 0.$$

Inserting eq. (34) into (35) we can see that

$$q_\mu M_\mu^{\rm s} = q_\mu M_\mu^{\rm u},$$

where $M_\mu^{\rm s}$ and $M_\mu^{\rm u}$ are defined by the formulae (31) and (33), respectively. Using eq. (31) we conclude that the Ward identity which we need reads

$$q_\mu M_\mu = iG_{\rm F}\sqrt{2}\sin\theta_{\rm c}\cos\theta_{\rm c} f_\pi f_{\rm K}(kp).$$

So the final expression for M_μ is

$$M_\mu = i\frac{G_{\rm F}}{\sqrt{2}} g_{\rm A}^{\rm s}\sin\theta_{\rm c}\cos\theta_{\rm c}$$
$$\times\frac{f_\pi}{f_{\rm K}}\left\{q_\mu\frac{1}{2\pi^2}\left[\tfrac{1}{2}p^2-\tfrac{1}{3}pq\right]-p_\mu\frac{1}{pq}\left[2f_{\rm K}^2(kp)+\frac{q^2}{2\pi^2}\left(\tfrac{1}{2}p^2-\tfrac{1}{3}pq\right)\right]\right\}. \quad (36)$$

Finally, let us consider the interaction of the X boson with the current $\bar{d}\gamma_5\gamma_\mu d$. The matrix element we are looking for is

$$M_\mu = -\frac{G_{\rm F}}{\sqrt{2}} g_{\rm A}^{\rm d}\sin\theta_{\rm c}\cos\theta_{\rm c}\int {\rm d}^4x\, {\rm e}^{iqx}\langle {\rm K}^+|\bar{s}\gamma_\nu(1-\gamma_5)u|0\rangle$$
$$\times\langle 0|T\bar{d}\gamma_5\gamma_\mu d(x)\bar{u}\gamma_\nu(1-\gamma_5)\, d|\pi^+\rangle$$
$$= -\frac{G_{\rm F}}{\sqrt{2}} g_{\rm A}^{\rm d}\sin\theta_{\rm c}\cos\theta_{\rm c}\frac{f_{\rm K}}{f_\pi}2m_{\rm q}K_\nu L_{\mu\nu},$$

$$L_{\mu\nu} = i\int {\rm d}^4x\,{\rm d}^4y\,{\rm e}^{iqx+ipy}\langle 0|T\bar{d}\gamma_5\gamma_\mu d(x)\bar{u}\gamma_\nu d(0)\,\bar{d}\gamma_5 u(y)|0\rangle.$$

Comparing the contribution of the current $\bar{d}\gamma_5\gamma_\mu d$ with that of the current $\bar{d}\gamma_\mu d$, one can show that their difference is finite and is equal to

$$k_\nu(L_{\mu\nu}-T_{\mu\nu}) = -\frac{i}{2\pi^2}\frac{1}{m_{\rm q}}\left[k_\mu kq + q_\mu(k^2+q^2)\right].$$

Finally we get

$$M_\mu = i\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\frac{f_K}{f_\pi}\left\{q_\mu\frac{1}{2\pi^2}\left[\tfrac{1}{2}k^2 - q^2 - pq\right]\right.$$

$$\left. - k_\mu\frac{1}{kq}\left[2f_\pi^2(kp) + \frac{q^2}{2\pi^2}\left(\tfrac{3}{2}k^2 - kp\right) + \frac{1}{2\pi^2}kq\left(k^2 + q^2\right)\right]\right\}. \quad (37)$$

Thus, we have computed all the annihilation diagrams. Before we write down the final answer it is worth making two remarks.

First, all calculations were made in the approximation of the hadron SU(3) symmetry, so we should put $f_\pi \simeq f_K \simeq f = 100$ MeV. Moreover, we used the PCAC limit. Therefore we have to keep in the final expression only those terms which are proportional to the lowest power of the momenta.

Thus, the answer is (we do not write down terms proportional to q_μ since they do not contribute to the matrix element M)

(1) $g_V^u \neq g_V^d$

$$M_\mu = i\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c 2\left(g_V^d - g_V^u - g_A^d + g_A^u\right)p_\mu f^2 q^2\frac{kp}{(pq)(kq)} \quad (38)$$

(2) $g_V^u = g_V^d$ (in this case $g_A^{u;d} = 0$)

$$M_\mu = i\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c g_V^u q^2\frac{1}{4\pi^2}\left(\ln\frac{m_s^2}{m_u^2} - \frac{3}{2}\right)p_\mu. \quad (39)$$

Hence, for the physical X boson we obtain

(1) $G_R^u \neq G_R^d$ (in this case there is mixing between X and Z bosons). From eq. (38) the contributions of the X and Z bosons are

$$M_\mu^X = ig_X\left(G_R^d - G_R^u\right)\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c f^2 q^2\frac{kp}{(pq)(kq)}p_\mu,$$

$$M_\mu^Z = -i\sqrt{g^2 + g'^2}\cos^2\theta_W\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c f^2 q^2\frac{kp}{(pq)(kq)}p_\mu.$$

Taking eq. (5) into account we get the final result for the contribution of the annihilation diagrams:

$$M_\mu = ig_X\left(G_R^d - G_R^u\right)\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_c f^2 q^2\frac{kp}{(pq)(kq)}\left(1 + \cos^2\theta_W\right)p_\mu. \quad (40)$$

(2) $G_R^u = G_R^d$ (in this case the mixing between X and Z bosons is absent). Directly from eq. (39) we get

$$M_\mu = i g_X G_R^u \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c q^2 \frac{1}{4\pi^2}\left[\ln\frac{m_s^2}{m_u^2} - \frac{3}{2}\right] p_\mu . \tag{41}$$

Summing up the contributions of the spectator and annihilation diagrams we can write down the complete answer for the matrix element of the decay $K^+ \to \pi^+ X$:

(1) $G_R^u \neq G_R^d$

$$M = i g_X \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \left\{ \tfrac{1}{2}\left(G_R^u + G_R^d\right) m_X^2 \tfrac{1}{2} f_+^+\left(m_X^2\right)(p+k)_\mu \int_0^1 dx\, x(1-x) \right.$$
$$\left. \times \ln\frac{m_c^2 - m_X^2(1-x)x}{m_u^2 - m_X^2(1-x)x} + p_\mu\left(G_R^d - G_R^u\right) f^2 m_X^2 \frac{kp}{(pq)(kq)}\left(1+\cos^2\theta_W\right)\right\} \varepsilon_\mu . \tag{42}$$

(2) $G_R^u = G_R^d = G_R$

$$M = i g_X G_R \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c m_X^2 \left\{ \tfrac{1}{2} f_+^+\left(m_X^2\right)(p+k)_\mu \int_0^1 dx\, x(1-x) \right.$$
$$\left. \times \ln\frac{m_c^2 - m_X^2 x(1-x)}{m_u^2 - m_X^2 x(1-x)} + p_\mu \frac{1}{4\pi^2}\left(\ln\frac{m_s^2}{m_u^2} - \frac{3}{2}\right)\right\} \varepsilon_\mu . \tag{43}$$

The width of the decay is

$$\Gamma_\mu = \frac{1}{8\pi m_K^2} |\boldsymbol{p}_\pi|\, |M|^2 , \tag{44}$$

where

$$|\boldsymbol{p}_\pi| = \frac{1}{2m_K}\sqrt{\left(m_K^2 + m_\pi^2 - m_X^2\right)^2 - 4m_K^2 m_\pi^2}$$

and M is defined by eq. (42) if $G_R^u \neq G_R^d$, and by eq. (43) if $G_R^u = G_R^d$.

5. Conclusions and outlook

Let us consider the numerical results. Using eqs. (42), (43) and (44) we can find the width of the decay $K \to \pi X$ which proves to depend rather strongly on the

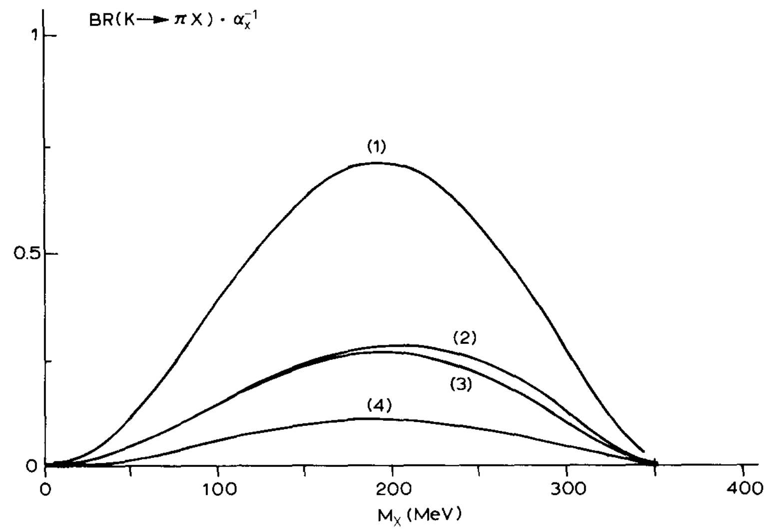

Fig. 6. The dependence of the branching ratio of the decay $K^+ \to \pi^+ X$ on the X-boson mass with various choices of U′(1) charges of right-handed quarks: (1) $G_R^u = 1$, $G_R^d = -1$, (2) $G_R^u = 1$, $G_R^d = 1$, (3) $G_R^u = 0$, $G_R^d = 1$, (4) $G_R^u = 1$, $G_R^d = 0$.

choice of the U′(1) charges of right quarks (G_R^u and G_R^d). Fig. 6 presents the behaviour of the branching ratio BR($K \to \pi X$) assuming various X-boson masses and various choices of G_R^u and G_R^d. One can see that in all four cases the maximal value of the branching ratio lies in the region about $m_X \simeq 200$ MeV.

When $G_R^u = G_R^d = 1$, one may take $\mathrm{BR}(K \to \pi X)_{max} \sim \alpha_X$ in order to get a rough estimate. So if $\alpha_X \geq 10^{-9}$–10^{-10} it is possible to observe X bosons in the BNL experiment by looking for the decay $K^+ \to \pi^+ +$ "nothing". But due to the strong dependence of BR($K^+ \to \pi^+ X$) on the values of the U′(1) charges of quarks and on the X-boson mass, it seems to be impossible to extract a model-independent upper bound on α_X from the existing data on the decay $K^+ \to \pi^+ +$ "nothing".

As an example let us consider the supersymmetric Fayet model [2] and try to get limits on the mass of the corresponding X boson (it is usually called the U boson). In this model the coupling constant of the U boson is

$$\alpha_U \simeq 3 \times 10^{-7} \left(\frac{m}{1\ \mathrm{GeV}} \right)^2 .$$

It is reasonable to suppose that the presence of two Higgs doublets in this model will insignificantly alter the obtained results. Then if $m_U \lesssim 300$ MeV we get

$$\mathrm{BR}(K \to \pi U) \lesssim 3 \times 10^{-8} .$$

So whatever the mass of the U boson, its existence does not contradict the experimental data on K-meson decays. (Obviously, other experiments can impose independent limits on m_U. For instance, from the beam-dump experiments one can conclude that the mass region 1 MeV $< m_U <$ 7 MeV is forbidden [2].)

It is worth noting as well that if the X boson is a superlight gauge boson corresponding to a new long-range interaction ("the fifth force"), then the rate of the decay $K \to \pi X$ is extremely small.

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Appendix A

In this appendix we present the detailed derivation of the Ward identities used in the text (eqs. (23), (25) and (28a)) [11]. Let us start with eq. (21):

$$M_\mu = \int d^4x\, e^{iqx} \langle K^+ | T J_\mu(x) \mathscr{L}(0) | \pi^+ \rangle , \tag{A.1}$$

where $\mathscr{L}$ is the effective lagrangian of the $K \to \pi$ transition. Without gluon corrections it has the form

$$\mathscr{L} = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \bar{s}\gamma_\nu(1-\gamma_5) u \bar{u}\gamma_\nu(1-\gamma_5)\, d . \tag{A.2}$$

Let the X boson interact with a vector current J_μ. In the lowest order of the weak interaction $\partial_\mu J_\mu = 0$. Therefore, from eq. (A.1) we obtain

$$\begin{aligned} q_\mu M_\mu &= i \int d^4x\, e^{-iqx} \partial_\mu \langle K^+ | T J_\mu(x) \mathscr{L}(0) | \pi^+ \rangle \\ &= i \int d^3x\, e^{iq\cdot x} \langle K^+ | [J_0(x) \mathscr{L}(0)] | \pi^+ \rangle . \end{aligned} \tag{A.3}$$

Since $q_\mu M_\mu$ is the Lorentz scalar, it may be calculated in the frame where $q = 0$. We define the charge in the standard way as

$$Q = \int d^3x J_0(x, 0) .$$

So from eq. (A.3) one gets

$$q_\mu M_\mu = i\langle K^+ | [Q, \mathscr{L}(0)] | \pi^+ \rangle = i(Q(K^+) - Q(\pi^+)) \langle K^+ | \mathscr{L}(0) | \pi^+ \rangle .$$

Assuming the vacuum dominance we obtain

$$q_\mu M_\mu = -i(Q(K^+) - Q(\pi^+)) G_F \sqrt{2} f_K f_\pi \sin\theta_c \cos\theta_c (kp) . \tag{A.5}$$

Formula (A.5) is the basic point for all further reasoning. Let the X boson couple to the current $\bar{s}\gamma_\mu s$. Since the charge corresponding to this current is the strangeness, we have $Q(K^+) = 1$, $Q(\pi^+) = 0$. So

$$q_\mu M_\mu^s = iG_F \sqrt{2} f_K f_\pi (kp) \sin\theta_c \cos\theta_c . \tag{A.6}$$

This is just the Ward identity used in sect. 4 (eq. (23)).

If the X-boson couples to the current $\bar{d}\gamma_\mu d$ then $Q_d(K^+) = 0$, $Q_d(\pi^+) = -1$ and

$$q_\mu M_\mu^d = -iG_F \sqrt{2} f_K f_\pi \sin\theta_c \cos\theta_c (kp) . \tag{A.7}$$

This is just eq. (25).

Let the X boson interact with the U-quark vector current. Since the U quark is contained both in K^+ and π^+ mesons, the matrix element can be decomposed into the sum

$$M_\mu^u = M_{1\mu}^u + M_{2\mu}^u .$$

For the $\bar{u}\gamma_\mu u$ current the charges are $Q_u(K^+) = Q_u(\pi^+) = 1$. When the X boson couples to the U quark contained in the K^+ meson, one gets from eq. (A.5)

$$q_\mu M_{1\mu} = -iG_F \sqrt{2} \sin\theta_c \cos\theta_c f_\pi f_K (kp) .$$

When the X boson couples to the U quark contained in the π^+-meson, then eq. (A.5) gives

$$q_\mu M_{2\mu} = iG_F \sqrt{2} \sin\theta_c \cos\theta_c (kp) . \tag{A.8}$$

Eqs. (A.6)–(A.8) are the Ward identities (23), (25) and (28a) of sect. 4.

Appendix B

It is well known that in local gauge theories vector bosons acquire mass eating up corresponding Goldstone bosons. So one may think that if the masses of the vector bosons tend to zero, then these bosons should behave like Goldstone bosons. The transverse polarization of the vector bosons is decoupled and the behaviour of the gauge bosons in this limit ($m_X \to 0$, $g_X \to 0$, $m_X/g_X = \text{const.}$) is determined by their longitudinal polarizations.

Now when the X bosons behave like the Goldstone boson, it is worthwhile confirming the correctness of eq. (17) with the help of independent calculations. That means to show that if $m_X \to 0$ then $\Gamma_\mu = 0$.

Let us define the conserved current J_μ in the Goldstone case (for more details see, e.g. ref. [2]):

$$J_\mu = v\sqrt{2}\ \partial_\mu \tilde{u} + J_\mu'(\text{leptons} + \text{quarks}). \tag{B.1}$$

Since $\partial_\mu J_\mu = 0$ we obtain

$$\Box \tilde{u} = -\frac{1}{v\sqrt{2}}\ \partial_\mu J_\mu'. \tag{B.2}$$

The interaction of the X^0 and Z^0 bosons with quark currents is (for up quarks)

$$\frac{g_X}{2}\left\{\sum_{\text{up}} \bar{q}\gamma_\mu\left[G_L + G_R^u + (-G_L + G_R^u)\gamma_5\right]q\right\}X_\mu^0$$
$$+\tfrac{1}{2}\sqrt{g^2 + g'^2}\sum_{\text{up}}\bar{q}\gamma_\mu(g_V + g_A\gamma_5)qZ_\mu^0, \tag{B.3}$$

where G_L, G_R^u are the U′(1) charges of the left doublet and right singlet, g_V and g_A are the vector and axial coupling constants of the Z^0 boson in the standard model.

Using eq. (5) we obtain that the interaction of the physical X boson with up quark has the form:

$$\frac{1}{1 + \left(2g_X G_H/\sqrt{g^2 + g'^2}\right)^2}\left\{\frac{g_X}{2}\sum_{\text{up}}\bar{q}\gamma_\mu\left[G_L + G_R^u + (-G_L + G_R^u)\gamma_5\right]q\right.$$
$$\left. + 2g_X G_H \sum_{\text{up}}\bar{q}\gamma_\mu(g_V + g_A\gamma_5)q\right\}X_\mu. \tag{B.4}$$

Hence one gets

$$\partial_\mu J_\mu'' = \frac{g_X}{1 + \left(2g_X G_H/\sqrt{g^2 + g'^2}\right)^2}\sum_{\text{up}} m_q \bar{q}\gamma_5 q\{-G_L + G_R^u - G_H\} = 0. \tag{B.5}$$

So the matrix element of the decay $K^+ \to \pi^+ +$ Goldstone equals zero. Therefore, if the X boson is massless the decay $K^+ \to \pi^+ X$ is forbidden and we obtain eq. (17).

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