Effective Lagrangian for axion emission from SN 1987A

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We study issues connected to the emission of axions from SN 1987A. For these purposes we construct an effective Lagrangian to describe the interaction of axions with nucleons and analyze, particularly, the relevance of its dimension-6 axion-pion-nucleon interaction terms. We find that these terms do not contribute, in the nonrelativistic limit appropriate to axion emission from SN 1987A, thereby reconfirming various extant calculations.

The nucleon-nucleon-axion bremsstrahlung process is considered to be the dominant mechanism by which axions could be emitted from SN 1987A (Ref. 1). This process has been studied in the one-pion-exchange (OPE) approximation employing effective interaction techniques. Apart from the issue whether this OPE approximation suffices—a matter which is discussed thoroughly in Ref. 2—there is some confusion in the literature regarding the magnitude of the cross section for this process,[3] which is connected with whether one should use a pseudoscalar or a pseudovector axion coupling to nucleons. This matter was discussed in Ref. 1 and analyzed in some detail in a recent paper of Choi, Kang, and Kim.[4] By straightforward calculations it was shown, within the OPE approximation and in the nonrelativistic (NR) limit, that it is the same to consider derivative couplings for both the pion and the axion or to take one derivative and one pseudoscalar coupling, irrespective of which of them corresponds to the pion or the axion.[4,5] If one takes both couplings as pseudoscalar, Raffelt and Seckel[1] noticed that an additional interaction term of the type $\bar{N}a\pi N$ must appear, to obtain the same result as in the other cases. It has also been pointed out by one of us[6] and in Ref. 4, where different interaction Lagrangians are quite carefully analyzed, that dimension-6 terms of the form $\bar{N}(\partial a)\pi N$, $\bar{N}a(\partial\pi)N$ could give additional contributions to neutron-proton-axion bremsstrahlung in the OPE approximation.

Although the results given in Ref. 1, which have been refined by the recent calculation of Brinkmann and Turner,[7] are basically correct, no *ab initio* theoretical discussion has been given to explain why this is so. The purpose of this Brief Report is to remedy this situation. In particular, we shall show that the calculations in Refs. 1 and 7 have all forgotten a dimension-6 term. This contact term is part and parcel of the axion-pion-nucleon interaction and must really be considered for the neutron-proton scattering process. However, it turns out that in the nonrelativistic limit of interest, this dimension-6 term does not contribute to the squared matrix element. Hopefully, our discussion should clarify this problem once and for all.

We recall[6,8] that the axion couples derivatively to the current connected with the $U(1)_{PQ}$ symmetry, but that it has also a nonderivative coupling to the associated $SU(3)\times U(1)$ anomalies. This is equivalent to the original Yukawa coupling of axions with quarks, as both schemes are related through a chiral local transformation. However, to be able to use current-algebra methods, the total coupling of axions with quarks needs to be reexpressed in an alternative way, through its derivative coupling with the Bardeen-Tye nonanomalous current.[9] If one considers, for simplicity, only the u and d quarks as light quarks, this current is given by

$$\tilde{J}_{\mu PQ} = v_{PQ}\partial_\mu a + \lambda_3 \tfrac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) + \lambda_0 \tfrac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) \ . \tag{1}$$

Here λ_3 and λ_0 are parameters that depend on the Peccei-Quinn (PQ) charges and on the masses of the quarks and v_{PQ} is the scale parameter associated with the spontaneous breaking of $U(1)_{PQ}$. One can use this current as the starting point for a current-algebra computation of axion bremsstrahlung in nucleon-nucleon scattering. However, to derive the interactions of axions with light hadrons it is much easier, and more instructive, to consider an effective Lagrangian technique,[6,8] rather than to do current algebra.

To construct the correct Lagrangian that describes the interactions between axions and light hadrons, we begin by considering a $U(2)\times U(2)$ chiral-invariant Lagrangian, describing the strong interactions of pions and η mesons with nucleons,[10] to which we add the axion kinetic energy:

$$\mathcal{L}_{\rm chiral} = i\bar{N}\gamma_\mu\partial^\mu N - m\bar{N}N + g_{\pi NN}\bar{N}\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}N + f^2_{\pi NN}\bar{N}\gamma_\mu\boldsymbol{\tau}\cdot(\partial_\mu\boldsymbol{\pi}\times\boldsymbol{\pi})N + g_{\eta NN}\bar{N}\gamma_\mu\gamma_5\partial_\mu\eta N - \tfrac{1}{4}F^2_\pi {\rm Tr}(\partial_\mu U^\dagger\partial^\mu U) - \tfrac{1}{2}\partial_\mu a\,\partial^\mu a \ . \tag{2}$$

Here U is the 2×2 matrix

$$U = \exp\left[i\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}+\eta}{F_\pi}\right] , \tag{3}$$

while the various coupling constants obey the relations[10]

$$g_{\pi NN} \equiv \frac{f}{m_\pi} \simeq \frac{1}{m_\pi}, \quad f_{\pi NN} = \frac{1}{2F_\pi} \ . \tag{4}$$

In this formalism, which is essentially that discussed in

Ref. 8, the interactions of axions with hadrons arise only through mass-breaking terms. We shall detail these terms for the specific case of the Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) axion model,[11] but the general procedure is trivially generalizable. These mass-breaking terms must respect the original symmetry properties of the underlying Lagrangian of quarks and Higgs fields. Thus, for the DFSZ axion case,[6] one must add to Eq. (2) a mass-breaking term for the meson matrix U, which must be invariant under the $U(1)_{PQ}$-symmetry transformations:

$$a \to a + \alpha v_{PQ} ,\qquad U \to U \begin{bmatrix} \exp(iX_1\alpha) & 0 \\ 0 & \exp(iX_2\alpha) \end{bmatrix} . \tag{5}$$

Here $X_1 = 2v_2^2/v^2$ and $X_2 = 2v_1^2/v^2$ are related to the vacuum expectation value of the Higgs fields $\langle \Phi_i \rangle = v_i/\sqrt{2}$ and $v = (v_1^2 + v_2^2)^{1/2}$. Thus, $X_1 + X_2 = 2$. This mass-breaking term reads

$$\mathcal{L}_{\text{mass breaking}} = -\frac{c}{2}\text{Tr}(UAM + M^\dagger A^\dagger U^\dagger) , \tag{6}$$

where

$$M = \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix} , \tag{7}$$

$$A = \begin{bmatrix} \exp\left[\dfrac{-iX_1 a}{v_{PQ}}\right] & 0 \\ 0 & \exp\left[\dfrac{-iX_2 a}{v_{PQ}}\right] \end{bmatrix} . \tag{8}$$

In the above $c = F_\pi^2 m_\pi^2/(m_u + m_d)$ is related to the scale of the spontaneous chiral-symmetry breaking.

The Lagrangian of Eq. (6) takes into account only the Yukawa-type interactions where axions are coupled to light quarks. To it we must add also a term which reflects the heavy-quark contribution to the PQ anomaly. Furthermore, the total Lagrangian should also have a piece which mimics the effects of the strong anomaly contribution, associated with the $U(1)_A$ current. Both anomalies are proportional to $F\tilde{F}$ and for our effective Lagrangian they will produce an additional mass term for a combination of the axion a and the η (Ref. 8). This term corresponds, essentially, to the mass term for the physical η meson. For the DFSZ model, one finds[6]

$$\mathcal{L}_{\text{anomaly}}^{\text{eff}} = -\tfrac{1}{2}m_0^2 \left[\eta + \frac{F_\pi}{v_{PQ}}(N_g - 1)a\right]^2 , \tag{9}$$

where $m_0 \simeq m_\eta$, and N_g is the number of fermion generations.

In the presence of the mass-breaking term, Eq. (6), and the anomaly term, Eq. (9), all the neutral bosons acquire masses. After the diagonalization of the neutral-mass Lagrangian, the axion field contains a small admixture of the physical π^0 and η fields. One finds

$$\begin{aligned} a &\simeq a_{\text{phys}} - \frac{\lambda_3 F_\pi}{v_{PQ}}\pi^0_{\text{phys}} - \frac{\lambda_0 F_\pi}{v_{PQ}}\eta_{\text{phys}} \\ &\equiv a_{\text{phys}} - \epsilon_{a\pi}\pi^0_{\text{phys}} - \epsilon_{a\eta}\eta_{\text{phys}} , \\ \pi^0 &\simeq \pi^0_{\text{phys}} + \epsilon_{a\pi}a_{\text{phys}} , \\ \eta &\simeq \eta_{\text{phys}} + \epsilon_{a\eta}a_{\text{phys}} , \end{aligned} \tag{10}$$

where $\lambda_0 = (1 - N_g)$ and $\lambda_3 = (X_1 - X_2)/2 - N_g(m_d - m_u)/(m_d + m_u)$. These are precisely the parameters which enter in the Bardeen-Tye current.[9] Indeed, the current of Eq. (1), with the usual identifications made for the quark currents in terms of the mesons fields of the effective Lagrangian, may be rewritten as

$$\begin{aligned} \tilde{J}_{\mu PQ} &= v_{PQ}\partial_\mu a + \lambda_3 F_\pi \partial_\mu \pi^0 + \lambda_0 F_\pi \partial_\mu \eta \\ &\simeq v_{PQ}\partial_\mu a_{\text{phys}} . \end{aligned} \tag{11}$$

That is, the Bardeen-Tye current contains only the physical axion field.

Through the mixing of axions with the mesons fields, we can now easily obtain the coupling of axions to nucleons. Using the above relations [Eq. (10)], one can directly pick up from Eq. (2) the relevant terms for the process in question. For ease of notation, henceforth, we drop the subscript phys, so that below, a, π^0, and η are what we denoted as a_{phys}, π^0_{phys}, and η_{phys}, previously. The resulting Lagrangian is

$$\begin{aligned} \mathcal{L}_{NN\to aNN} = {} & i\bar{N}\gamma_\mu\partial^\mu N - m\bar{N}N + g_{\pi NN}\bar{N}\gamma_\mu\gamma_5[\tau^+\partial^\mu\pi^- + \tau^-\partial^\mu\pi^+ + \tau^0(\partial^\mu\pi^0 + \epsilon_{a\pi}\partial^\mu a)]N \\ & + if_{\pi NN}^2\bar{N}\gamma_\mu\{\tau^+[\partial^\mu\pi^-(\pi^0 + \epsilon_{a\pi}a) - (\partial^\mu\pi^0 + \epsilon_{a\pi}\partial^\mu a)\pi] \\ & \qquad + \tau^-[(\partial^\mu\pi^0 + \epsilon_{a\pi}\partial^\mu a)\pi^+ - \partial^\mu\pi^+(\pi^0 + \epsilon_{a\pi}a)] + \tau^0(\pi^-\partial^\mu\pi^+ - \pi^+\partial^\mu\pi^-)\}N \\ & + g_{\eta NN}\bar{N}\gamma_\mu\gamma_5(\partial^\mu\eta + \epsilon_{a\eta}\partial^\mu a)N \\ \equiv {} & i\bar{N}\gamma_\mu\partial^\mu N - m\bar{N}N + \mathcal{L}_{\pi NN} + \mathcal{L}_{\eta NN} + \mathcal{L}_{\pi\pi NN} + \mathcal{L}_{aNN} + \mathcal{L}_{\pi aNN} , \end{aligned} \tag{12}$$

where $\mathcal{L}_{\pi NN}$, $\mathcal{L}_{\eta NN}$, $\mathcal{L}_{\pi\pi NN}$, and $\mathcal{L}_{aNN}$ are the interaction Lagrangians which give the pion-nucleon-nucleon, η-nucleon-nucleon, pion-pion-nucleon-nucleon, and the axion-nucleon-nucleon vertices, respectively, while $\mathcal{L}_{\pi aNN}$ contains the pion-axion-nucleon-nucleon interaction. Note that this latter term arises as a result of the π^0-a mixing, from the $\bar{N}N\pi\partial\pi$ interaction in the original Lagrangian. This interaction is dictated by chiral symmetry[10] and so the dimension-6 interaction $\mathcal{L}_{\pi aNN}$ is unavoidable, as long as $\epsilon_{a\pi}\neq 0$. In Eq. (12), furthermore, again essentially because of chiral symmetry, we have pseudovector derivative couplings for both the pion and the axion. The coupling constants of the axion with protons and neutrons are seen to be

$$\begin{aligned} g_{app} &= g_{\pi NN}\epsilon_{a\pi} + g_{\eta NN}\epsilon_{a\eta} , \\ g_{ann} &= -g_{\pi NN}\epsilon_{a\pi} + g_{\eta NN}\epsilon_{a\eta} . \end{aligned} \tag{13}$$

In Fig. 1 we display the two extra diagrams coming from the axion-pion-nucleon-nucleon terms of Eq. (12), which should be also taken into account for the axion-proton-neutron bremsstrahlung process. These extra contributions give rise to the matrix element

$$\begin{aligned} M &= \frac{8mg_{\pi NN}f^2_{\pi NN}\epsilon_{a\pi}}{(\mathbf{p}_3-\mathbf{p}_1)^2+m^2_\pi}\bar{u}(p_3)\gamma_5 u(p_1)\bar{u}(p_4)\gamma_\mu p^\mu_5 u(p_2) \\ &\quad +(p_1\leftrightarrow p_2)(p_3\leftrightarrow p_4) \\ &\equiv M^a + M^b , \end{aligned} \tag{14}$$

where p_i, $i=1,2$ are the four-momenta of the initial nucleons, p_j, $j=3,4$ are those of the final nucleons, and p_5 is the axion momentum. Within the nonrelativistic approximation, which is the appropriate limit for the energies involved in axion emission in stars, a straightforward calculation shows that $M^a M^{b*}$ and $M^b M^{a*}$ vanish identically, while the momentum dependence of $|M^a|^2$ and $|M^b|^2$ is proportional to $(|\mathbf{p}_i-\mathbf{p}_j|^2E^2_5)/(|\mathbf{p}_i-\mathbf{p}_j|^2+m^2_\pi)^2$. These contributions are therefore suppressed relative to those arising from the usual axion bremsstrahlung graphs considered in the literature,[1,7] coming from the $\mathcal{L}_{aNN}$ term in Eq. (12), which lead to a squared matrix element $|M|^2=O(|\mathbf{p}_i-\mathbf{p}_j|^4/(|\mathbf{p}_i-\mathbf{p}_j|^2+m^2_\pi)^2)$. Moreover, all the interference terms between the diagrams of Fig. 1 and those where the axion is emitted from the nucleon legs may be also neglected, since the only appreciable contributing terms cancel among each other. This means that the recent comprehensive analysis of Brinkmann and Turner[7] is correct, even though, technically, they have omitted the dimension-6 operator in $\mathcal{L}_{NN\to NNa}$ in their considerations.

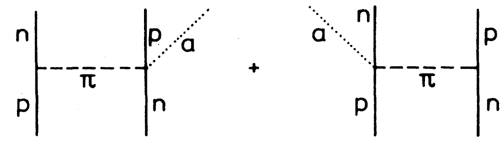

FIG. 1. Feymann diagrams for the axion-proton-neutron bremsstrahlung process, arising from the axion-pion-nucleon interaction terms $\mathcal{L}_{a\pi NN}$ in the effective Lagrangian.

The fact that no extra contributions to the squared matrix element results from the axion-pion-nucleon interaction Lagrangian, in the above pseudovector-pseudovector coupling scheme, may also be deduced from a different analysis. One can perform a chiral transformation in Eq. (12) to have a pseudoscalar axion-nucleon coupling and a pseudovector derivative pion-nucleon coupling, or vice versa. After this is done, different expressions for the dimension-6 term appear. The axion-pion-nucleon interaction term in Eq. (12), using the fact that due to the vector-current conservation $\partial_\mu j^\mu = \partial_\mu \bar{N}\gamma^\mu N = 0$ a term containing $j^\mu\partial_\mu(a\pi)$ can be neglected, can be written as

$$\mathcal{L}_{\pi aNN} = 2if^2_{\pi NN}\epsilon_{a\pi}\bar{N}\gamma_\mu a(\tau^+\partial^\mu\pi^- - \tau^-\partial^\mu\pi^+)N . \tag{15}$$

By performing the following chiral transformation one can change the derivative axion coupling to a pseudoscalar one:

$$N \to \exp[i\gamma_5 a(g_{\eta NN}\epsilon_{a\eta}+\tau^0 g_{\pi NN}\epsilon_{a\pi})]N . \tag{16}$$

The transformed Lagrangian reads, neglecting terms of order ϵ^2_{ak} (with $k=\eta,\pi$),

$$\begin{aligned} \mathcal{L}'_{NN\to aNN} &= i\bar{N}\gamma_\mu\partial^\mu N - m\bar{N}N - 2im\bar{N}\gamma_5 a(g_{\eta NN}\epsilon_{a\eta}+g_{\pi NN}\epsilon_{a\pi}\tau^0)N + g_{\pi NN}\bar{N}\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\pi}N + g_{\eta NN}\bar{N}\gamma_\mu\gamma_5\partial^\mu\eta N \\ &\quad + if^2_{\pi NN}\bar{N}\gamma_\mu[\tau^+(\pi^0\partial^\mu\pi^- - \pi^-\partial^\mu\pi^0)+\tau^-(\pi^+\partial^\mu\pi^0-\pi^0\partial^\mu\pi^+)+\tau^0(\pi^-\partial^\mu\pi^+ - \pi^+\partial^\mu\pi^-)]N \\ &\quad + 2if^2_{\pi NN}\epsilon_{a\pi}\bar{N}\gamma_\mu a(\tau^+\partial^\mu\pi^- - \tau^-\partial^\mu\pi^+)N - 2ig^2_{\pi NN}\epsilon_{a\pi}\bar{N}\gamma_\mu\tau^0 a(\tau^+\partial^\mu\pi^- + \tau^-\partial^\mu\pi^+)N \\ &\equiv i\bar{N}\gamma_\mu\partial^\mu N - m\bar{N}N + \mathcal{L}_{\pi NN} + \mathcal{L}_{\eta NN} + \mathcal{L}_{\pi\pi NN} + \mathcal{L}'_{aNN} + \mathcal{L}'_{\pi aNN} . \end{aligned} \tag{17}$$

It is easy to see that the new pion-axion-nucleon interaction Lagrangian in Eq. (17) can be rewritten as that of Eq. (15), but with the replacement $f^2_{\pi NN}\to f'^2_{\pi NN} = f^2_{\pi NN} - g^2_{\pi NN}$:

$$\mathcal{L}_{\pi aNN} \to \mathcal{L}'_{\pi aNN} = \mathcal{L}_{\pi aNN}(f^2_{\pi NN}\leftrightarrow f'^2_{\pi NN}) . \tag{18}$$

The same calculation as before shows that no extra contribution to the squared matrix elements follows from $\mathcal{L}'_{\pi aNN}$.

We know already[4,5] that, neglecting the dimension-6 terms, the same matrix element appears for the process $nn\to nna$, $pp\to ppa$, independently of the type of pion and axion couplings one uses, as long as at least one of them is a pseudovector one. The above can be easily proved also for the $np\to npa$ process, where different values of the axion couplings to protons and neutrons appear. The equivalence of the pseudovector-pseudovector and pseudovector-pseudoscalar amplitudes provides a direct way of inferring the vanishing contribution of the pion-axion-nucleon interaction Lagrangians in these coupling schemes. Since the Lagrangian of Eq. (12), $\mathcal{L}_{NN\to aNN}$, after the chiral transformation Eq. (16), yields the new Lagrangian $\mathcal{L}'_{NN\to aNN}$ of Eq. (17), it is clear that both Lagrangians must be physically equivalent. Hence, their contributions to the squared matrix element must be the same. From what we just said, the contribution of $\mathcal{L}_{\pi NN}+\mathcal{L}_{aNN}$ in Eq. (12) to the matrix elements is the same as that of $\mathcal{L}_{\pi NN}+\mathcal{L}'_{aNN}$ in Eq. (17). Moreover, Eq. (18) informs us that both pion-axion-nucleon interaction Lagrangians are the same, but they differ by a redefinition of the coupling constant. It follows, therefore, that within the NR approximation we are considering, these terms necessarily must give a vanishing contribution to the matrix element. This is the conclusion we arrived at also by direct calculation. As a last point, we remark that, if one considers a chiral transformation to obtain pseudoscalar couplings for both the pion and the axion, the above equivalence between different couplings does not exist any more. Then an extra term must necessarily contribute to the total squared matrix element, as observed by Raffelt and Seckel.[1]

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