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THE GLUEBALL MASS SPECTRUM IN SU(3) LATTICE GAUGE THEORY

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This talk summarizes recent calculations of the glueball mass spectrum that I have done in collaboration with F. Brandstaeter, A.S. Kronfeld and K.J.M. Moriarty.

1. INTRODUCTION

In this talk I will present new results[1] on the 0^{++} and 2^{++} glueball masses in the pure $SU(3)$ lattice gauge theory. The main progress in this work comes from using glueball operators which have been constructed[2] so as to improve the projection onto the low-lying glueball states and, consequently, the signal over noise ratio.

To compute the glueball mass spectrum one proceeds in two steps. First, one constructs color singlet, zero momentum operators $\Phi(t)$ localized at time t and belonging to some representation of the cubic group to project out the glueball states. In the second step one computes the correlation function

$$\begin{aligned} C(t) &= <\Phi(t)\Phi(0)> \\ &= Tr(T^{L_t - t}\Phi T^t \Phi) \\ &\overset{L_t \to \infty}{=} <0|\Phi(t)\Phi(0)|0> \\ &= \sum_{n=0}^{\infty} |<0|\Phi|n>|^2 e^{-m_n t}, \end{aligned} \tag{1}$$

where T is the transfer matrix and L_t is the temporal extent of the lattice. The mass of the ground state, $m \equiv m_1$, which we are particularly interested in, is then extracted from the asymptotic behavior of the correlation function.

For our further discussions it is useful to introduce the normalized correlation function

$$\Gamma(t) = \frac{C(t)}{C(0)}. \tag{2}$$

In the limit of large times and for $L_t \gg 2t$ this reduces to

$$\Gamma(t) \cong c e^{-mt}, \tag{3}$$

where the coefficient c denotes the projection onto the ground state ($0 \leq c \leq 1$). To be sure that one is extracting the true ground state, the correlation function must be computed at large times until the asymptotic exponential decay is displayed. This demands a sufficiently large projection c. Otherwise the signal will be lost in the noise before one gets there.

We have emphasized[2,3] that the projection, in particular its dependence on the lattice spacing a, is largely determined by the dimension of the operator Φ. For local operators, such as the plaquette, which have dimension four, one computes

$$c \propto a^5. \tag{4}$$

This dependence was also found[3] in the Monte Carlo data.[4] As a result, the projection went down to the

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level of a few per cent already at $\beta = 5.9$, and it was not possible to extend the calculations beyond that.

To remedy this situation, we have taken a theoretical approach and systematically constructed glueball operators of lower dimensions. It turns out that the best one can achieve is

$$c \propto a. \tag{5}$$

The operators that accomplish this derive from the inverse of the spatial covariant Dirac operator. They have been presented in refs. 2,3 and will not be repeated here. For a detailed discussion of their properties and the derivation of equ. (5) the reader is furthermore referred to ref. 1.

2. THE CALCULATION

This work extends our previous calculations[3] in two respects. First of all, we have done computations on larger lattices and for larger values of β. The objective is

- to compute the masses for various values of z (say $5 \leq z \leq 15$),
- to increase β until they fall on a universal curve,
- and finally to use Lüscher's equation,[5]

$$m(z) = m(\infty)(1 - G\frac{e^{-\frac{\sqrt{3}}{2}z}}{z}), \tag{6}$$

to extrapolate them to the infinite volume.

Here $z = m_{0^{++}} a L_s$ (assuming that $m_{0^{++}}$ is the lightest glueball mass), where L_s is the spatial size ot the lattice, and G parameterizes the triple-glueball coupling. Secondly, we have computed the glueball operators for all fermionic boundary conditions,

$$\chi(x + \hat{i}L_s) = \xi\chi(x), \xi = 1, e^{\pm 2\pi i/3}, \tag{7}$$

where χ is the one-component Grassmann field in the Kogut-Susskind action, and $\hat{i}$ is a unit vector in the spatial direction. As our covariant Dirac operator lives on spatial planes, this amounts to $3 \times 3 = 9$ different boundary conditions. This allows us to compute the string tension and the energy of all states of one and two units of electric flux at the same time. In the glueball mass calculations the operators are averaged over all posible boundary conditions.

In order to speed up the inversion of the Dirac operator, we have partially blocked the gauge field configurations and applied our method to the blocked lattice. The prescription is adopted from the factor-of-two renormalization group transformation, i.e. each spatial link is replaced by a link two lattice spacings in extent which represents the average color field of its spatial surroundings. This step is applied only once.

In the original formulation our method was restricted to states belonging to the representations A_1 and E. By blocking the gauge field configurations according to the $\sqrt{2}$ ($\sqrt{3}$) renormalization group transformation and applying the method to the resulting, nonorthogonal planes, we can also construct,[1] rather elegantly, glueball operators belonging to the representations A_2, T_1, T_2 (T_2).

3. RESULTS

So far we have reliable results only for the 0^{++} and 2^{++} glueball masses, and in the latter case only for the representation E. I will restrict myself to recent results obtained on larger lattices. The parameters are given in the following table:

$L_s^3 L_t$	β	sweeps
$14^3 20$	6.0	15000
$16^3 16$	6.0	18000
$16^3 16$	6.2	15000

The correlation functions are computed for a matrix M of 20×20 different masses of the Dirac operators ranging from 0.02 to 1.6. The glueball masses are then extracted from the correlation function

$$C = x^T M x, \tag{8}$$

where x is a vector which is fitted such that the statistical errors are minimal. It should be noted that our philosophy is very different from that of Michael and Teper[6] who optimize $C(1)/C(0)$. It requires a strong signal at larger time separations though.

In figs. 1-3 I have shown the resulting correlation functions of states of one unit of electric flux averaged over the spatial directions and of the 0^{++} and 2^{++} glueball operators on the 16^4 lattice at $\beta = 6.0$. The lowest energy in the one-unit-of-flux channel is Ka^2L_s, where K is the string tension. We obtain a useful signal up to $t = 6$ for the flux state and the 0^{++} glueball and up to $t = 5$ for the 2^{++} glueball. This has never been achieved before on lattices of this size and for $\beta \geq 6.0$. It is perhaps interesting to remark that the vector x chosen by the fitting routine does not nearly maximize $C(1)/C(0)$, and, vice versa, the vector x that maximizes $C(1)/C(0)$

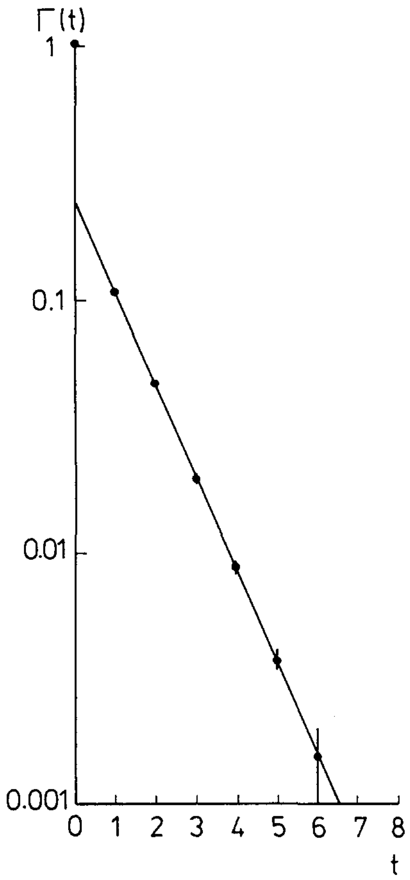

Figure 1: The correlation function of states of one unit of flux.

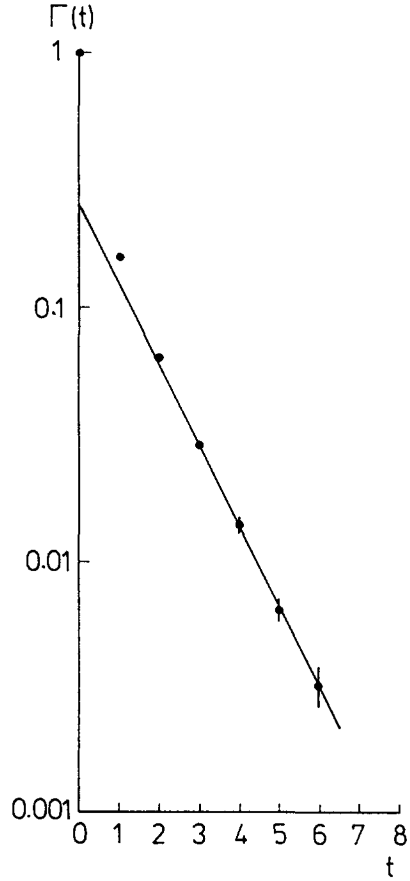

Figure 2: The 0^{++} correlation function.

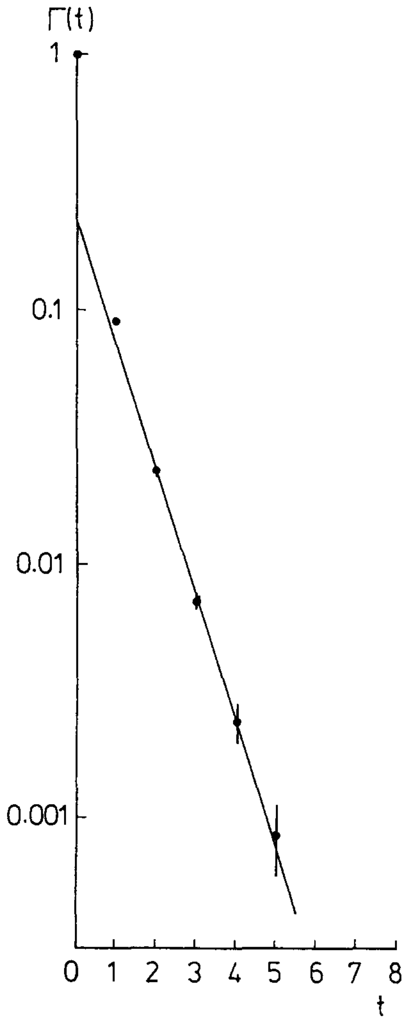

Figure 3: The 2^{++} correlation function.

does not guarantee a large signal over noise ratio.

The results for the glueball masses and the string tension are compiled in the following table:

$L_s^3 L_t$	β	$m_{0^{++}}a$	$m_{2^{++}}a$	$\sqrt{K}a$
$14^3 20$	6.0	0.63(3)	1.01(9)	
$16^3 16$	6.0	0.67(3)	1.10(7)	0.234(4)
$16^3 16$	6.2	0.48(4)	0.77(4)	0.180(2)

The correlation functions of the higher flux states are (still) too noisy to give any useful information on the corresponding energies beyond upper bounds.

In fig. 4 I have shown the obligatory z-plot. The solid circles represent our data, while the open circle is taken from the calculations in ref. 4 - which are of similar quality - and refers to the $10^3 20$ lattice at $\beta = 5.9$.

As far as the scalar glueball mass is concerned, our errors are small enough so that we can make a quantitative statement about the scaling behavior (for a comparison with other work see ref. 7). We find that $m_{0^{++}}$ obeys asymptotic scaling within $\approx 5\%$. It follows that the 0^{++} masses fall quite nicely on a universal curve $m_{0^{++}}(z)$. This allows us to extrapolate $m_{0^{++}}$ to the infinite volume. I have fitted Lüscher's equation (6) to this curve and obtain

$$G = 190 \pm 70. \tag{9}$$

The result is the solid curve shown in fig. 4. From this we conclude that $m_{0^{++}}(\infty) \approx m_{0^{++}}(10)$.

The 2^{++} mass is about 50% higher than the 0^{++} mass. Note, however, that in the extreme case the cutoff is only

$$a^{-1} = 0.9 m_{2^{++}}, \tag{10}$$

so that we may expect large $O(a^2)$ corrections.

If we extrapolate the string tension to the infinite volume by means of the equation[8]

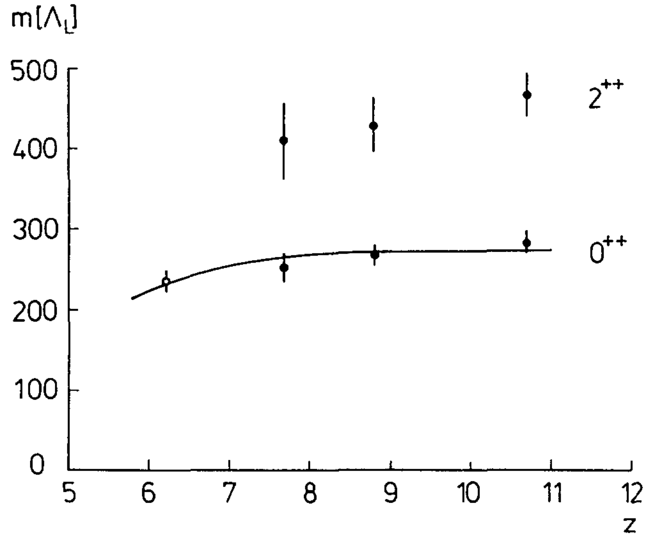

Figure 4: The z-plot.

$$K(L_s) = K(\infty) - \frac{\pi}{3L_s}, \qquad (11)$$

we find that the two values of $K(\infty)$ are also consistent with asymptotic scaling within the errors.

4. OUTLOOK

Before we can claim that the results for the 2^{++} glueball mass are relevant to the continuum limit, we have to increase β further, and this is what we plan to do in the future. This should also allow us to (reliably) compute the masses of some of the higher glueball states lying above the 2^{++}.

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