FIVE-JET PRODUCTION IN e^+e^- ANNIHILATION

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We investigate the five-jet production processeses $^+e^- \to q\bar{q}3g$ and $e^+e^- \to 2q2\bar{q}g$ in lowest order QCD perturbation theory. We give results for the integrated cross section as a function of the mass resolution cut as well as for thrust and acoplanarity distributions.

A now well known feature of electron–positron annihilation at high energies is the appearance of jets in the final states. Recently, experimental results for the production of multijet events have been presented [1]. They provide an interesting test of higher order QCD calculations. The full gauge structure of QCD only shows up in second of higher order perturbation theory, where the three- and four-gluon vertex come in. In second order ($O(\alpha_s^2)$) one is led to four-jet final states enabling the test of the three-gluon vertex. The four-gluon vertex, however, only enters in third order ($O(\alpha_s^3)$), where the production of five jets is possible.

The rate of two and three-jet events is in rather good agreement with predictions of $O(\alpha_s^2)$ perturbation theory [1]. The four-jet production rate, however, is underestimated in $O(\alpha_s^2)$, if compared to data, by roughly 50% (or more, depending on the used resolution cut) [1]. This deficiency of four-jet like events in the $O(\alpha_s^2)$ theory can partly be attributed to the neglect of contributions with more than four partons in the final state which, of course, occur only in higher than second order. The other part comes from genuine higher order corrections to the four-jet rate. So, to solve the problems encountered in connection with the four-jet rate the calculation of $O(\alpha_s^3)$ contributions is called for. A first step into this direction is the calculation of the tree diagrams in third order. In addition, the cross section for the production of five partons is of interest by itself. At PETRA energies the five-jet rate, as measured by the JADE and TASSO collaborations, is not larger than 1%, depending on the resolution cut. This makes a detailed study of the production characteristics of these events rather difficult. At higher energies, in the range of the forthcoming accelerators LEP and SLC, one will obtain data at much smaller cutoffs, for which the five-jet rate is appreciable.

In this letter we present first results for the five-jet production cross section based on tree level diagrams for various resolution cuts together with some examples for thrust and acoplanarity distributions. Throughout our analysis we assumed five quarks which we considered to be massless.

To order α_s^3 the five-jet cross section is given by the two sets of diagrams shown in fig. 1 corresponding to the final (jet) states

$$e^+e^- \to q(p_1)\bar{q}(p_2)g(p_3)g(p_4)g(p_5)\,, \tag{1}$$

$$e^+e^- \to q(p_1)\bar{q}(p_2)q(p_3)\bar{q}(p_4)g(p_5)\,. \tag{2}$$

Several methods for the calculation of the cross section are available by now [#1]. We applied essentially the same conventional method as for the four-jet cross section done several years ago [4]. In this approach the calculation is, though lengthy, straightforward. The cross section is proportional to the hadron tensor as defined in ref. [4]. To obtain the cross

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[#1] See ref. [2] for calculational procedures based on helicity amplitudes and ref. [3] for supersymmetric techniques.

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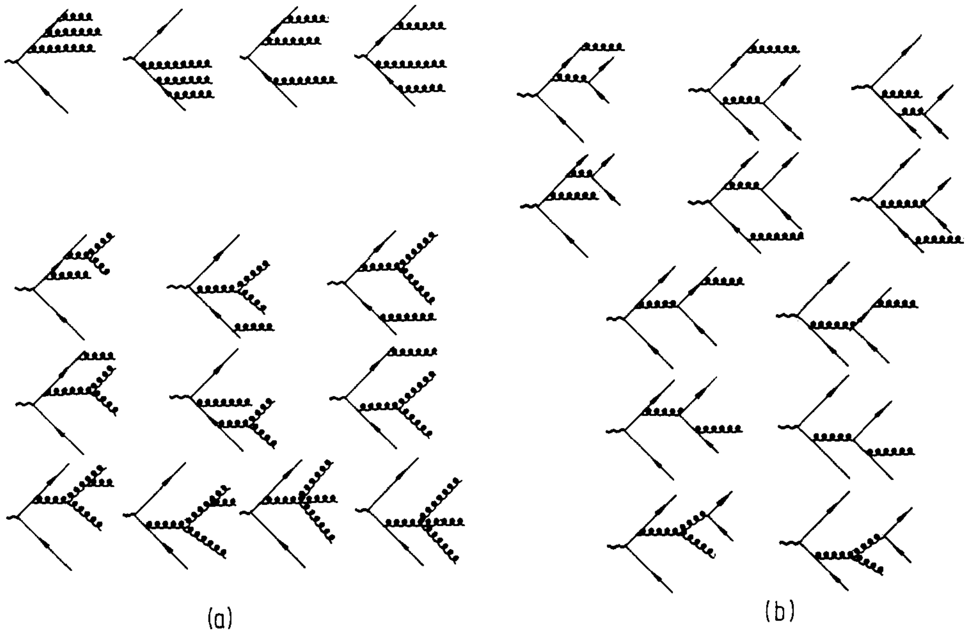

Fig. 1. Feyman diagrams for the production of five partons. (a) Production of $q\bar{q}3g$: all necessary permutations of the gluon lines have to be included. (b) Production of $2q2\bar{q}g$: all permutations of the quark and of the antiquark lines have to be included.

section integrated over angles with respect to the beam direction we must take the trace of this hadron tensor. The necessary Dirac traces have been calculated using REDUCE 3. The result is expressed by the invariants $y_{ij}=2p_ip_j/q^2$ where $\sqrt{q^2}$ is the total CM energy. Details will be presented elsewhere. The subsequent integration over quark, antiquark and gluon momenta has been performed using the Monte Carlo program FOWL. To avoid the infrared and collinear regions present in these tree graph cross sections we applied the usual invariant mass resolution cut. With this cut only those events are accepted for which all $y_{ij}\geqslant y$.

In fig. 2 we show $\sigma_{5\text{-jet}}$ as a function of y in the range of y between 0.01 and 0.04, normalized to the zeroth order cross section $\sigma_0 4\pi\alpha^2\Sigma_f Q_f^2/q^2$, using a coupling constant $\alpha_s=0.2$. Since the cross section is proportional to α_s^3 this result can easily be transformed to other values of the coupling constant. With $\alpha_s=0.2$ and $y=0.02$ the five-jet rate is 0.50%. In a logarithmic plot it increases roughly linearly with decreasing y. Compared to the experimental values for the five-cluster rate reported in ref. [1], it is smaller by a factor of 3. Five-cluster events, however, are already present in the $O(\alpha_s^2)$ perturbative model in which the

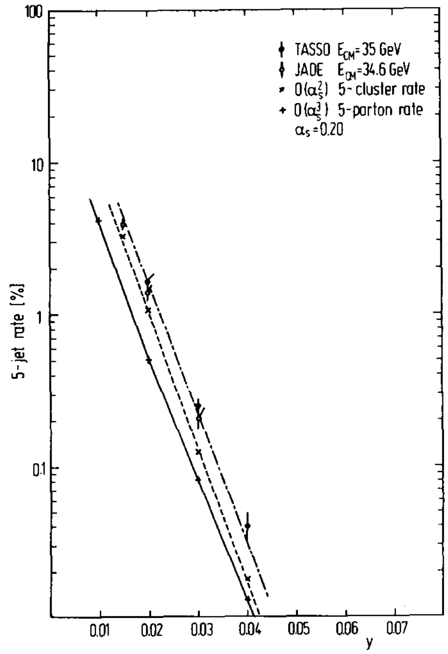

Fig. 2. Five-jet rate versus invariant mass cutoff y. (a) Five-parton tree calculation: full line. (b) Five-cluster rate coming from four partons: dashed line. (c) Sum of (a) and (b): dash-dotted line, compared to the experimental data of ref. [1].

fragmentation of quarks and gluons into hadrons is built in. These contributions have been calculated by the JADE and TASSO collaborations [1]. In fig. 2 they are shown as the points labelled $O(\alpha_s^2)$. If we add these contributions to our result for $\sigma_{5\text{-jet}}/\sigma_0$, we obtain very good agreement with the five-cluster multiplicity as measured by the JADE and TASSO collaboration [1]. In this comparison we neglected the 5% change coming from the normalization of our result with respect to σ_0 instead of σ_{tot}. Furthermore we must keep in mind that $\sigma_{5\text{-jet}}/\sigma_0$ is the cross section for the production of five partons. This is modified if hadronization of these partons is taken into account, in particular part of its contribution will also influence the four-cluster rate. In principle, these effects can be calculated. We expect that they will not change our comparison with data by more than 30%. We also remark that the assumed value of α_s is somewhat larger than usually deduced from jet analysis in the PETRA energy range [1]. This increase of α_s can be justified with the fact that a similar increase is needed to bring the calculated $O(\alpha_s^2)$ four-cluster rate into agreement with the data [5].

The main contribution of $\sigma_{5\text{-jet}}(y)/\sigma_0$ as given in fig. 2 comes from the qq̄3g final state. The 2q2q̄g final state amounts to approximately 14% (with five flavours). This ratio is larger than what one obtains in the four-jet case when comparing the qq̄2g contribution with the 2q2q̄ contribution [4]. In process (1), the amount of the QED type diagrams (the first four diagrams in fig. 1), which is proportional to the colour factor C_F^3, is 45% for $y=0.02$. This means that the genuine QCD type contributions, which are proportional to N_C, are much larger in the five-jet cross section than in the four-jet cross section, where the N_C contributions are smaller, approximately 12% of the C_F^2-term in the range of y considered in this paper [6].

In fig. 3 we have plotted the thrust distributions for $y=0.04$ and $y=0.02$. Their shapes are similar to the corresponding distributions for four jets except reduced in magnitude [6]. The acoplanarity distributions for the same values of y are shown in fig. 4. All these distributions have been calculated with $\alpha_s=0.2$. The minima for large T and small A are caused by the resolution cut y.

In conclusion we find that $\sigma_{5\text{-jet}}$ with a cutoff $y=0.02$ is about 0.5% of the total cross section and is

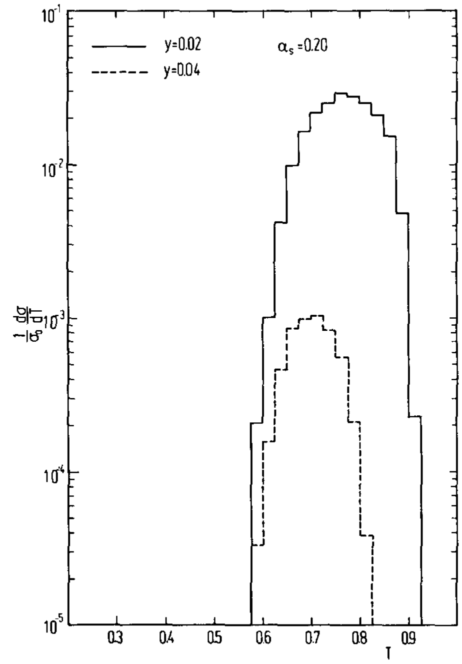

Fig. 3. Thrust distribution of the five-parton production for $y=0.02$ and $y=0.04$.

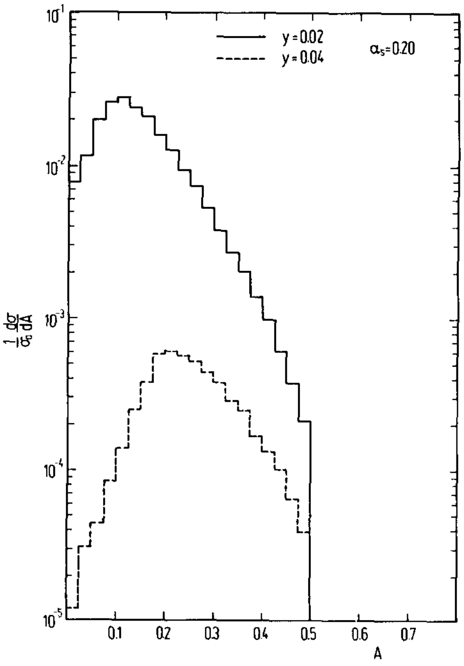

Fig. 4. Acoplanarity distribution of the five-parton production for $y=0.02$ and $y=0.04$.

in reasonable agreement with the measured five-cluster rate at 35 GeV. Compared to the four-jet cross section the non-abelian contributions are much larger. This means that in the five-jet case the abelian contributions to the $q\bar{q}3g$ production do not dominate. This should allow a much better test of the non-abelian nature of QCD than it was possible with four jets.

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