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QCD duality calculation of the B-parameter for $N=1$ supergravity-induced local operators

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Abstract. QCD duality sum rules are applied to the calculation of the B-parameter for $N=1$ supergravity-induced local operators. Using the chiral realization of local operators, the duality region has been found and the tree-level coupling constant determined. The result is in full accordance with a related calculation of the B-parameter in the standard model by Pich and de Rafael and support it. The result is an order of magnitude smaller than the one obtained by using vacuum saturation. Consequences of softened constraints are discussed.

I Introduction

Supersymmetric (SUSY) theories [1] present a very appealing extension of the standard model (SM). On the one hand, they are mathematically less divergent and, because of underlying supersymmetry, they are more constrained. On the other hand, in the local version, i.e. supergravity, they are a very promising candidate for the unification of the SM with gravity. Last but not least, SUSY is the unavoidable content of superstring theories.

The most intriguing prediction of SUSY is the existence of superparticles, which have presently escaped experimental detection. Most of these are presumably heavy particles with masses already constrained either by experiment (direct searches) or by almost perfect agreement of the SM with experiment. Being very heavy, they are difficult to detect and a viable alternative is to look for the effects of heavy superpartners on theoretical predictions.

The predictive power of supersymmetric models is largely constrained once supersymmetry breaking is considered. The most appealling way to break SUSY softly appears to be through the hidden sector of supergravity. Requiring the possible minimal number of effective fields in low-energy theory defines the minimal model of supergravity.

Low-energy supergravity effects have been studied by a number of authors [2–3]. Dugan et al. [4] have studied the minimal $N=1$ supergravity in the 'general' version, with only one restriction on the soft operators (at $\mu = M_{\text{Planck}}$):

$$M_{ab}^2 = m_{3/2}^2 \delta_{ab}, \tag{1}$$

where M_{ab} are scalar masses and $m_{3/2}$ is the gravitino mass, which appears to be of the order of 100 GeV.

The most stringent constraints on supergravity parameters come from the virtual (loop) effects of superpartners in the kaon system (CP violation, ε'/ε, rare decays, etc.). Among a class of supersymmetric graphs studied by Dugan et al., of particular interest are those leading to effective operators with mixed L–R helicities, i.e. with the structure typical of penguin graphs in the SM. Their existence sets the most stringent constraints on supergravity parameters.

Once heavy fields are integrated out, the effective hamiltonian contains only light-quark fields, u, d, s, and the problem is reduced to the calculation of matrix elements of composite quark operators, i.e. one has to deal with a typical QCD calculation. The calculations of these matrix elements are largely influenced by our lack of knowledge concerning the QCD confinement. The standard estimate, vacuum saturation, which (without fierzing) corresponds exactly to the large-N_c limit, is known to be rather uncertain. This is especially pronounced for operators with mixed L–R helicities. The same is true for calculation in phenomenological

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quark models. A few new, rather sofisticated approaches have recently been addressed to this problem: QCD lattice calculations [5], the large-N_c approach [6] and the QCD duality approach [7–10]. In this letter we propose to calculate the matrix element of the new induced $\Delta S=2$ operators θ_i in the framework of QCD duality sum rules, first used by Pich and de Rafael [7] to estimate the B parameter in the standard model and later extended to a number of different problems [8–10].

We organize the paper as follows. In Sect. II we discuss the effective $\Delta S=2$ operators induced by $N=1$ supergravity in its general version and the vacuum-saturation value of their matrix elements. In Sect. III we apply the duality sum rules to the calculation of these matrix elements. In Sect. IV we discuss the results obtained. Finally, in Sect. V we give a short conclusion.

II Supergravity-induced local operators

The local $\Delta S=2$ operators with the L–R current structure are generated by the strong superbox graph (Fig. 1) which leads to the effective operators

$$\mathscr{H}_{\text{eff}}=\tfrac{1}{120}\alpha_3^2\frac{M_{\bar{S}D}^2M_{\bar{D}S}^{*2}}{m_{3/2}^6}f\left(\frac{M_3^2}{m_{3/2}^2}\right)(\theta_1-3\theta_2), \tag{2}$$

where

$$\begin{Bmatrix}\theta_1\\ \theta_2\end{Bmatrix}=\bar{s}_L^i\gamma_\mu d_L^j\bar{s}_R^k\gamma^\mu d_R^l\begin{Bmatrix}\delta^{il}\delta^{jk}\\ \delta^{ij}\delta^{kl}\end{Bmatrix} \tag{3}$$

and

$$f(x)=20\int_0^1 d\zeta\zeta^3(1-\zeta)[\zeta+x(1-\zeta)]^{-3},\quad f(1)=1. \tag{4}$$

The notation for the mass parameters in (2) is explained in [4].

The contribution of the above operators to kaon processes sets the most stringent limit on the quantity $\tilde{\xi}_{U,D}$ which enters in a cubic gauge-invariant polynomial in the complex scalar fields. From the K_L-K_S mass difference one has at the Planck scale

$$\xi_U=m_{3/2}A\lambda_U+\tilde{\xi}_U \tag{5}$$

and the same for ξ_D with $U\to D$. In the general version of the model considered by Dugan et al., A may have an imaginary part and $\tilde{\xi}$, although small, may be nonzero.

The matrix elements of the operators θ_1 and θ_2 can be calculated in the vacuum-sturation approximation. One uses the Fierz reshuffling of quark fields and the well-known relations

$$\langle\bar{K}^0|\bar{s}\gamma_5 d|0\rangle=-i\frac{f_Km_K^2}{m_s+m_d}$$

$$\langle 0|\bar{s}\gamma_5 d|K^0\rangle=-i\frac{f_Km_K^2}{m_s+m_d}. \tag{6}$$

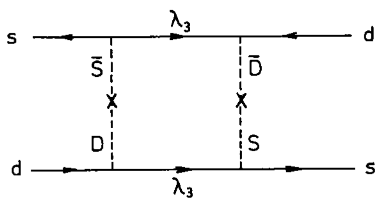

Fig. 1. Supersymmetric box $\Delta S=2$ diagram. Full lines represent fermions and dashed lines scalars. Crosses on lines represent mass insertions

The results follow straightforwardly:

$$\langle\bar{K}^0|\theta_1|K^0\rangle=\tfrac{1}{2}f_K^2m_K^2\left[-\frac{1}{3}-2\left(\frac{m_K}{m_s+m_d}\right)^2\right],$$

$$\langle\bar{K}^0|\theta_2|K^0\rangle=\tfrac{1}{2}f_K^2m_K^2\left[-1-\frac{2}{3}\left(\frac{m_K}{m_s+m_d}\right)^2\right]. \tag{7}$$

These results differ from those obtained by Dugan et al. in the sign of the second terms in (7). This trivial error, however, changes the results essentially. Numerically, with $m_s+m_d\sim 200\,\text{MeV}$, the result of Dugan et al. gives positive value for matrix elements. The corrected formulae (7), however, give *negative* values for the respective matrix elements. This completely changes the character of the derived constraints. Before going into details, we want to estimate the above matrix elements using the QCD duality approach. We shall keep the usual definition of B parameter in units of $\frac{1}{2}f_K^2m_K^2(1+1/N_c)$. Thus we define

$$\langle\bar{K}^0|\theta_i|K^0\rangle=\tfrac{1}{2}f_K^2m_K^2\left(1+\frac{1}{N_c}\right)B_{\theta_i}. \tag{8}$$

With this definition, the vacuum saturation values are $B_{\theta_1}=-9.6$ and $B_{\theta_2}=-4$. For the reason we shall explain later, we first proceed to the calculation of the θ_2 operator.

III QCD duality sum rules

The basic idea is to combine the information one gets from the chiral realization of the effective weak hamiltonian with its short-distance behavior. It happens via duality which is studied using finite-energy sum rules (FESR's).

A chiral realization of the effective weak hamiltonian can be studied in the framework of chiral perturbation theory (ChPT) in the sense of the Weinberg–Gasser–Leutwyler program [11, 12]. The latter represents a meaningful low-energy field theory which has advantages over the standard approaches in many cases. Its importance as an alternative approach is especially evident in processes where our lack of knowledge of the true QCD confinement is blurring our view of electroweak interactions [13, 14]. The Weinberg program [11] has been successfully realized by Gasser and Leutwyler [12] in the strong and elctromagnetic sector and has been put forward recently by Ecker et al. [15], showing the powerfulness of ChPT in rare-kaon processes, where a complicated interplay between short- and long-distance dynamics is especially pronounced.

To proceed, one needs the chiral representation of the operators θ_i. The operator θ_2 is a composite operator which in the large-N_c limit reduces to the product of two bare weak currents with the L–R structure. Their respective chiral realization can be obtained by gauging the strong lagrangian locally:

$$(V-A)_\mu = i\frac{f^2}{2}\,U\partial_\mu U^\dagger,$$

$$(V+A)_\mu = i\frac{f^2}{2}\,U^\dagger\partial_\mu U. \tag{9}$$

Here U is the unitary matrix field

$$U(\phi) = \exp i\frac{2}{f}\Phi, \tag{10}$$

where Φ is the usual 3×3 matrix containing the pseudo-Goldstone bosons, π's, K's and η_8. The field U enters the strong lagrangian which to order p^2 reads

$$\mathscr{L}^{(2)}_{\text{strong}} = \frac{f^2}{8}\operatorname{tr}(\partial_\mu U\partial^\mu U^\dagger) + v\operatorname{tr}(\mathscr{M}U + U^\dagger\mathscr{M}), \tag{11}$$

where $\mathscr{M}$ is the quark mass matrix and v is the quark condensate, related to quark and meson masses.

The operator θ_2, being the composite operator, would have an overall constant g_{θ_2} whose value cannot be obtained from ChPT alone.

$$\theta_2^{\text{chiral}} = g_{\theta_2}\frac{1}{3}\left(\frac{f_K^2}{f_\pi^2}\right) : \left(i\frac{f_\pi^2}{2}U\partial_\mu U^\dagger\right)_{23}\left(i\frac{f_\pi^2}{2}U^\dagger\partial^\mu U\right)_{23} :. \tag{12}$$

We have normalized the operator θ_2^{chiral} in such a way that if it were of the $(V-A)\times(V-A)$ type, the constant $g_{\theta_2} = B_{\theta_2}$ would correspond exactly to the B-parameter of the $K^0-\bar{K}^0$ mixing in the standard model. This enables one to keep a close analogy with the calculation of the B-parameter by Pich and de Rafael [7]. Defined in such a way, both B_{SM} in the SM and B_{θ_2} in our case have the same absolute value in the large-N_c limit, $|B_{SM}| = |B_{\theta_2}| = 3/4$.

Next, we have to study the behavior of the two-point function

$$\psi_{\theta_2}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T(\theta_2(x)\theta_2^\dagger(0))|0\rangle, \tag{13}$$

both in ChPT and QCD. In ChPT, the presence of final-state interactions would lead to the formation of resonances—their influence will be taken into account by modulating them using the Breit–Wigner form.

The spectral function in ChPT then reads*

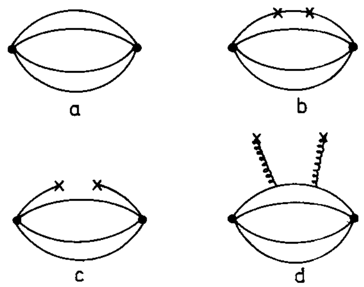

Fig. 2. QCD calculation of the two-point function: **a** asymptotic diagram with chiral quarks, **b** mass corrections, **c** quark condensate corrections, **d** gluon condensate corrections

$$\begin{aligned}
\frac{1}{\pi}\operatorname{Im}\psi_{\theta_2}(t) &= g_{\theta_2}^2\frac{1}{16\pi^2}\frac{1}{18}\left(\frac{f_K^2}{f_\pi^2}\right)^2\left\{\int_{t_{10}}^{(\sqrt{t}-\sqrt{t_{20}})^2} dt_1\right. \\
&\cdot\int_{t_{20}}^{(\sqrt{t}-\sqrt{t_1})^2} dt_2\,\lambda^{1/2}(1, t_1/t, t_2/t) \\
&\cdot\sum_h\left[(t_1+t_2-t)^2\frac{1}{\pi}\operatorname{Im}\Pi_h^{(0)}(t_1)\frac{1}{\pi}\operatorname{Im}\Pi_h^{(0)}(t_2)\right. \\
&+ 2\lambda(t,t_1,t_2)\frac{1}{\pi}\operatorname{Im}\Pi_h^{(0)}(t_1)\frac{1}{\pi}\operatorname{Im}\Pi_h^{(1)}(t_2) \\
&+ [(t_1+t_2-t)^2 + 8t_1t_2]\frac{1}{\pi}\operatorname{Im}\Pi_h^{(1)}(t_1) \\
&\left.\left.\cdot\frac{1}{\pi}\operatorname{Im}\Pi_h^{(1)}(t_2)\right] + \Sigma(t)\right\},
\end{aligned} \tag{14}$$

where $h = VV, AA$, i.e. $\operatorname{Im}\Pi_h^{(0,1)}$ is either $\operatorname{Im}\Pi_{VV}^{(0,1)}$ or $\operatorname{Im}\Pi_{AA}^{(0,1)}$. Here, contrary to the case of $\mathcal{O}_{\Delta S=2}$ in SM, only the products $VV\times VV$ and $AA\times AA$ survive.

Next, we shall calculate the spectral function ψ_{θ_2} in QCD. It receives contributions from the asymptotic diagrams (Fig. 2a), mass corrections (2b) and quark (2c) and gluon condensates (2d). Radiative corrections are not calculated for reasons to be discussed later. The spectral function then reads

$$\frac{1}{\pi}\operatorname{Im}\psi_{\theta_2}^{\text{QCD}}(t) = \frac{1}{\pi}\operatorname{Im}\psi_{\theta_2}^{\text{asym}}(t)\left[1 - \frac{130}{3}\frac{\bar{m}_s^2}{t} + \frac{80}{3}(16\pi^2)\frac{m_s\langle\bar{q}q\rangle}{t^2} + \cdots\right], \tag{15}$$

where $\bar{m}_s$ is the running quark mass, $\langle\bar{q}q\rangle$ is the quark condensate and the gluon condensate does not contribute in this case. The asymptotic spectral function is given by

$$\frac{1}{\pi}\operatorname{Im}\psi_{\theta_2}^{\text{asym}}(t) = \frac{1}{20}\frac{1}{(16\pi^2)^3}t^4. \tag{16}$$

Compared with $(1/\pi)\operatorname{Im}\psi_{\mathcal{O}_{\Delta S=2}}^{\text{asym}}(t)$, which was calcu-

* The notation is explained in [7]

lated by Pich and de Rafael [7], one finds that

$$\frac{1}{\pi}\operatorname{Im}\psi_{\theta_2}^{\text{asym}}(t)=\frac{3}{8}\frac{1}{\pi}\operatorname{Im}\psi_{\mathcal{O}_{\Delta S=2}}^{\text{asym}}(t). \qquad (17)$$

This relation, of course, is no longer valid once the mass and condensate corrections are included.

Duality constraints can be easily written in terms of FESR's. We need two sum rules to fix s_0, which is the onset of the asymptotic QCD continuum. These are

$$\mathscr{R}_0=\int_{4m_K^2}^{s_0} dt\,\frac{1}{\pi}\operatorname{Im}\psi_{\theta_2}(t),$$

$$\mathscr{R}_1=\int_{4m_K^2}^{s_0} dt\,t\,\frac{1}{\pi}\operatorname{Im}\psi_{\theta_2}(t). \qquad (18)$$

Once s_0 is fixed, any of the above sum rules leads to the value of g_{θ_2}. The ratio of the sum rules does not depend on g_{θ_2} and may be used to fix s_0

$$r=\frac{6}{5s_0}\frac{\mathscr{R}_1}{\mathscr{R}_0}. \qquad (19)$$

We want to stress that the ratio r in (19) is a very sensitive test of duality which

(i) in ChPT does not depend on the unknown coupling g_{θ_2}, and

(ii) in QCD does not depend on missing leading-logarithmic and finite α_s corrections in (15), which are canceled in the ratio (19).

IV Results and discussion

The results are plotted in Fig. 3, both in ChPT and QCD. The departure from the asymptotic QCD prediction (the dashed line) is due to mass and condensate corrections, which are obviously important in restoring the duality. The dots are the values of r in ChPT, and the solid line is the full QCD result. The duality region is clearly established in the range $8 \leqq s_0 \leqq 11.5\,\text{GeV}^2$. The QCD curve asymptotically approaches the dashed line for high s_0. For $s_0 \leqq 8\,\text{GeV}^2$, the mass and condensate corrections become large and break the expansion. On the other hand, ChPT values depart from the QCD curve after $s_0 \geqq 11.5\,\text{GeV}^2$, indicating the necessity of including higher resonances.

Any of the sum rules in (18) leads to the value of g_{θ_2} once s_0 has been determined. In fact, since we have found the duality region for the range $8 \leqq s_0 \leqq 11.5\,\text{GeV}^2$, any value of s_0 in this range lead to the same value of g_{θ_2}. Therefore, g_{θ_2} plotted as a function of s_0 should show plateau behavior in the duality region. This is shown in Fig. 4. The duality plateau is clearly seen in the range $8 \leqq s_0 \leqq 11.5\,\text{GeV}^2$. To reduce possible errors in establishing the duality and plateau regions, we employ the following 'theoretical cuts':

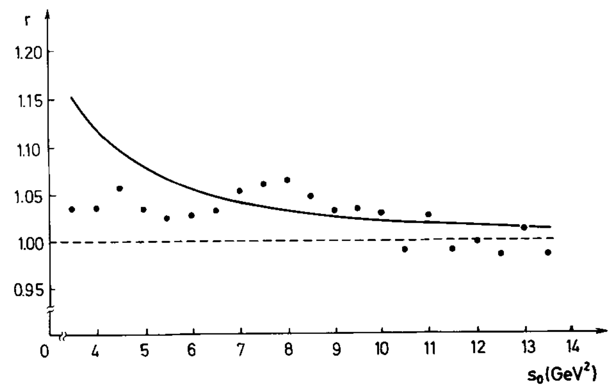

Fig. 3. The ratio r is plotted versus s_0. The dots represent the behavior obtained in ChPT. The continuous line is the full QCD result which approaches the asymptotic freedom behavior—the dashed line—at large s_0 values

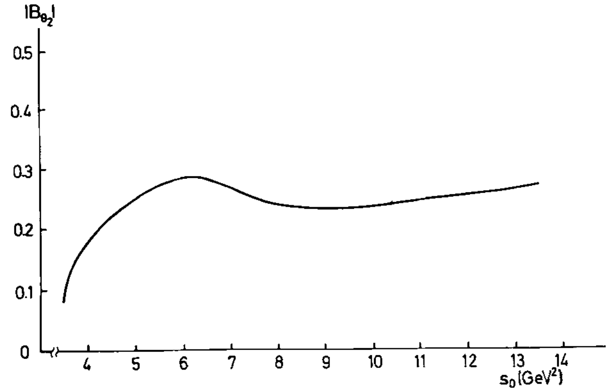

Fig. 4. Results for the B_{θ_2} parameter as a function of s_0

(i) We establish duality region in Fig. 3;

(ii) we use the sum rule $\mathscr{R}_0$ in (18) to find the plateau behavior which should appear in the duality region established in Fig. 3;

(iii) we apply procedure (ii) to the sum rule $\mathscr{R}_1$ and demand that the values of g_{θ_2} obtained by (ii) and (iii) agree within a few percent.

Applying the above criteria, we find that they are really satisfied. The values for g_{θ_2} obtained using the sum rules $\mathscr{R}_0$ and $\mathscr{R}_1$ agree within 2%. Then, the value of $|B_{\theta_2}|$ is centered arround $|B_{\theta_2}|=\frac{1}{4}$. In the following, we discuss the errors coming from the inherent approximations we are using.

To the order we are working on there are no logarithmic or finite α_s corrections. The effective hamiltonian (2) corresponds in the SM to the effective hamiltonian $\mathscr{H}_{\Delta S=2}$ with the Wilson coefficient $\eta=1$, i.e. without leading-logarithmic corrections. However, the result for B_{SM} scales as $(\alpha_s(M_W^2)/\alpha_s(s_0))^{2/9}$ which is rather modest, since the scale s_0 is much higher than the usual choice of the renormalization scale μ. Also, finite α_s corrections influence the curve for B in such a way that the plateau is slightly translated either to higher or to lower values. We shall take these effects into account by increasing the error bars, which in the calculation of Pich and de Rafael [7] amount to $\sim 25\%$. Our final result is then

$$|B_{\theta_2}| = 0.25 \pm 0.15. \tag{20}$$

Our calculation nicely confirms the validity of the QCD duality approach and clearly shows that in the calculation of Pich and de Rafael duality and plateau are not accidental. Our input parameters (condesates, masses, resonances) are the same as in [7]. However, because of the different helicity structure, the ChPT spectral function (14) is 'crippled', i.e. the terms $\Pi_{VV}\Pi_{AA}$ are absent. This difference with regard to [7] has had to be compensated by different QCD corrections if the whole approach makes any sense. Our results show that this is really the case. Changes in the ChPT spectral function are in one-to-one correspondence with adequate changes in QCD spectral functions, as required by duality.

There are some principal difficulties in determining the matrix element of the operator $\theta = \theta_1 - 3\theta_2$. The QCD duality approach determines only the absolute value of the B parameter and separate determination of B_{θ_1} would not be very useful. However, the operator θ_1 is of higher order in ChPT with respect to θ_2 and is expected to be significantly smaller. Therefore, we employ the *Ansatz* from [10], where penguins (also of higher order in ChPT) are accounted for as a correction to the QCD spectral function and the ChPT spectral function has been kept unchanged. Effectively, this means that the tree-level renormalized g_8 has been slightly changed. As long as the corrections are modest, this can be considered as a safe approximation. In our case, we obtain the following spectral function for θ:

$$\frac{1}{\pi}\operatorname{Im}\psi_\theta^{\mathrm{QCD}}(t) = \frac{8}{20}\frac{1}{(16\pi^2)^3}t^4\left[1 - \frac{110}{3}\frac{\bar{m}_s^2}{t} + \frac{40}{3}(16\pi^2)\frac{m_s\langle\bar{q}q\rangle}{t^2} - \frac{5}{8}(16\pi^2)\frac{\langle\alpha_s/\pi F^2\rangle}{t^2}\right]. \tag{21}$$

Again, we look for the duality and plateau regions. We have found that both regions correspond in the range to the regions found in the case of the operator θ_2. The value of $|B_\theta|$ agrees within a few percent with $3|B_{\theta_2}|$. Our conclusion is that the θ_1 contribution can be neglected to a very good approximation.

Using our estimate (20) of the matrix element of the θ_2 operator and the experimental values for Δm_K and ε, we obtain the following constraints on $M_{\bar{S}D}^2$ and $M_{\bar{D}S}^{*2}$:

$$\operatorname{Re}\left(\frac{M_{\bar{S}D}^2 M_{\bar{D}S}^{*2}}{m_{3/2}^4}\right) \leqq 4.5 \times 10^{-5}\frac{m_{3/2}^2}{M_W^2}\left[f\left(\frac{M_3^2}{m_{3/2}^2}\right)\right]^{-1} \tag{22}$$

$$\operatorname{Im}\left(\frac{M_{\bar{S}D}^2 M_{\bar{D}S}^{*2}}{m_{3/2}^4}\right) \leqq 3 \times 10^{-7}\frac{m_{3/2}^2}{M_W^2}\left[f\left(\frac{M_3^2}{m_{3/2}^2}\right)\right]^{-1}. \tag{23}$$

These constraints are rather sensitive to the variation of the ratio $M_3/m_{3/2}$. If we take $M_3 = m_{3/2}$ as it was done in [4], we find that (22) and (23) are weaker by a factor of 5 than the corresponding constraints of Dugan et al.*

The scalar mass insertions $M_{\bar{S}D}^2$ and $M_{\bar{D}S}^{*2}$ also appear in the $\Delta S = 1$ transition dipole moment operators [4] yielding the effective hamiltonian

$$\mathscr{H}_{\mathrm{eff}} = \frac{1}{32\pi}\frac{\alpha_3 g_s}{m_{3/2}}\frac{M_3}{m_{3/2}}\mathscr{F}\left(\frac{M_3^2}{m_{3/2}^2}\right) \cdot\left[\frac{M_{\bar{S}D}^2}{m_{3/2}^2}\bar{s}_R\sigma_{\mu\nu}F^{\mu\nu}d_L + \frac{M_{\bar{D}S}^{*2}}{m_{3/2}^2}\bar{s}_L\sigma_{\mu\nu}F^{\mu\nu}d_R\right], \tag{24}$$

with

$$\mathscr{F}(x) = 12\mathscr{J}(x) + \frac{4}{3}\frac{1}{x^2}\mathscr{J}\left(\frac{1}{x}\right),$$
$$\mathscr{J}(x) = \int_0^1 d\zeta(1-\zeta)^2[\zeta + x(1-\zeta)]^{-2},$$
$$\mathscr{F}(1) = 1 + \tfrac{1}{9}. \tag{25}$$

This effective hamiltonian could give a significant contribution to the $\Delta I = \frac{1}{2}$ amplitude in the $K \to \pi\pi$ decay. Using, for example, the bag model estimate** of the above transition moment operators [16], we find the transition dipole moment contribution (in GeV)

$$a_{1/2}^{\mathrm{trans\,mom}} = 1.2 \times 10^{-5}\left(\frac{100\,\mathrm{GeV}}{m_{3/2}}\right) \cdot\operatorname{Re}\left(\frac{M_{\bar{S}D}^2}{m_{3/2}^2} + \frac{M_{\bar{D}S}^{*2}}{m_{3/2}^2}\right)\frac{M_3}{m_{3/2}}\mathscr{F}\left(\frac{M_3^2}{m_{3/2}^2}\right). \tag{26}$$

The constraints (22) and (23) indicate that the imaginary parts of $M_{\bar{S}D}^2$ and $M_{\bar{D}S}^{*2}$ are much smaller than the real parts. Thus, neglecting the imaginary parts and assuming further that $\operatorname{Re} M_{\bar{S}D} \approx \operatorname{Re} M_{\bar{D}S}$, we obtain from (22) and (26)

$$a_{1/2}^{\mathrm{trans\,mom}} \leqq 20 \times 10^{-8}\frac{M_3}{m_{3/2}}\mathscr{F}\left(\frac{M_3^2}{m_{3/2}^2}\right)\left[f\left(\frac{M_3^2}{m_{3/2}^2}\right)\right]^{-1/2}, \tag{27}$$

which has to be compared with the experimental value

$$a_{1/2}^{\mathrm{exp}} = 27 \times 10^{-8}\,\mathrm{GeV}. \tag{28}$$

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* The constraint (22) in [4] should be larger by a factor of 2

** The bag model estimate may be rather crude for the operators containing gluon fields. Unfortunately, the QCD duality approach is very difficult to apply here because of the huge α_s corrections, typical of $\Delta I = 1/2$ processes [10]. The work on this problem is in progress

V. Conclusions

Our results show that there is no matrix-element enhancement as speculated by Dugan et al. [4]. On the contrary, there is suppression of the θ_2 matrix element by a factor of 16 with respect to the vacuum-saturation result. Nevertheless, we find no dramatic change in the constraints of the mass parameters given by Dugan et al. One reason is the already mentioned error in the Fierz rearrangement which partly compensates for the suppression. Equation (27) opens an interesting, although perhaps unrealistic, speculation that a great deal of yet unexplained $\Delta I = 1/2$ enhancement could be attributed to the transition dipole effective operators induced by the extended, supergravity model. The mass relation $M_3/m_{3/2}$ is yet unknown. If we assume $M_3 \geqq m_{3/2}$, then the transition moment contribution (27) could be as large as the experimental value (28). This should rather be interpreted either as an overestimate of the bag model (cf. second footnote on previous page) or that the true constraints are in reality much stronger than the constraints derived.

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