TOWARDS A BETTER QUANTITATIVE UNDERSTANDING OF THE SU(2) HIGGS MODEL

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We describe the results of a high-statistics simulation of the SU(2) Higgs model at $\lambda=\infty$, $\beta=2.4$, 2.7 and 3.0, on lattices of sizes ranging between $8^3\times 16$ and $14^3\times 28$. The measured quantities are the Higgs and W masses, the potential between two static sources from which the renormalized gauge coupling is extracted, and the vacuum overlap order parameter which defines the Higgs expectation value in a gauge invariant way. Strong finite-size effects are observed. They seem to come under control for the larger lattices, at least at $\beta=2.7$. Renormalized tree level relations are found to hold remarkably well at $\beta=2.7$ and 3.0.

1. Introduction

Nonperturbative studies of the SU(2) gauge theory with a scalar matter field in the fundamental representation, usually referred to as the SU(2) Higgs model, have been fruitful both from a phenomenological and from a more theoretical point of view. The model is a laboratory for investigating the Higgs mechanism in the standard model. Among the best tools available for performing nonperturbative studies are lattice Monte Carlo (MC) simulations.

On a hypercubic lattice, the SU(2) Higgs model is given by the action [#1]

$$S=-\beta\sum_{\mathrm{p}}\tfrac{1}{2}\mathrm{Tr}\,U(\mathrm{p})-\kappa\sum_{x,\mu}2\,\mathrm{Re}(\varphi_x,U_{x,\mu}\varphi_{x+\hat{\mu}})+\sum_x(\varphi_x,\varphi_x)+\lambda\sum_x[(\varphi_x,\varphi_x)-1]^2, \qquad (1)$$

where the lattice gauge field $U_{x,\mu}$ is an SU(2) matrix in the fundamental representation that lives on the link starting from x and going in the μ-direction ($\mu=0, 1, 2, 3$), $U(\mathrm{p})$ stands for the product of U's around the plaquette p, φ_x is a complex two-component scalar field, and $\hat{\mu}$ denotes the unit vector in direction μ. As usual, the continuum bare parameters g (gauge coupling), λ_c (the subscript c means "continuum"), and m_c^2 (the coefficient of the φ_c^2 term in the lagrangian), are replaced by the parameters β, λ, and κ [1] (a is the lattice spacing):

$$g^2=4/\beta,\quad \lambda_c=\lambda/\kappa^2,$$

$$m_c^2=\frac{1-2\lambda-8\kappa}{\kappa a^2}, \qquad (2)$$

the lattice scalar field is rescaled: $\varphi_x=a\kappa^{-1/2}\varphi_c(x)$.

There have been numerous previous MC studies of this model. First the phase diagram was explored [1]. Later nonlocal quantities were computed in the Higgs region: masses in refs. [2–4], Wilson loops and the renormalized gauge coupling in refs. [2,5,4], and the vacuum overlap order parameter (VOOP) [6] in ref. [5]. While providing a coherent overall picture, these "first-generation" studies tended to either underestimate or neglect the dependence of the results on the lattice size. Moreover, at $g>0$ no thorough quantitative comparison between MC results and perturbation theory has been done. since the model is of interest for obtaining real physical results, like an upper bound on the Higgs mass [7,2,4,8–10], it is worthwhile to perform a more precise study, that concen-

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trates on a relatively small portion of the three-parameter phase diagram (the previous investigations are an invaluable guide to finding a suitable region). In performing such a "second-generation" study we set ourselves the following goals:
- to compare simulation results with predictions from renormalized perturbation theory;
- to estimate the magnitude of finite size effects (FSE) for the lattice sizes accessible to simulations;
- to show how to use the VOOP as a gauge invariant version of the "Higgs vacuum expectation value" (VEV).

Notice that we will not deal with the confining region at all here, which, due to huge autocorrelations, is much more difficult to handle numerically [11].

As a first step in comparing simulation results with perturbative predictions, we try to determine the validity range for tree level relations between the quantities that we measure. In this paper we discuss two such relations: the Yukawa form for the potential between two static sources, obtained in the one-W-exchange approximation, and the formula relating the W mass to the renormalized values of the gauge coupling and of the VEV.

Perturbation theory is an expansion around $\beta=\infty$, $\lambda=0$, and $\kappa=\frac{1}{8}$. The farther away from this point we find agreement between perturbative and nonperturbative results, the better our check on the accuracy of perturbation theory will be. One of the consequences of the *triviality* of the SU(2) Higgs model, for which a lot of evidence exists [8], is that the renormalized φ^4 coupling λ_{ren} is smaller than the bare coupling λ. It has been checked nonperturbatively for the case of the O(4) pure matter theory at $\beta=\infty$ [9,10] that this effect is strong enough for renormalized perturbative relations to hold even at $\lambda=\infty$ (which is also where the upper bound on the Higgs mass is obtained). In view of the arguments in refs. [7,8] there is a good chance that this is true for the full model too. We thus decided to run our simulations at $\lambda=\infty$.

The choice of values for β is related to our ability to control the FSE reliably. At *large* values of β (small gauge couplings g), where tree level relations hold best, correlation lengths are large and critical slowing down decreases the accuracy of MC simulations; for the lattice sizes accessible today, FSE can easily become an insurmountable problem. At *small* values of β the reverse is true: tree level relations are not well obeyed, but correlation lengths are small, so that simulations are easier to perform; we can expect that FSE are relatively harmless here.

Unfortunately, in contrast to pure matter theories [12], a theory of the FSE has not yet been worked out for the SU(2) Higgs model. We therefore have no choice but to adopt an empirical approach [11,13]. First we try to get a numerical handle on the FSE at small enough values of β by performing very accurate simulations on lattices of increasing sizes, until the changes in the measured quantities become smaller than the statistical errors. Then we go to larger values of β in a controlled way, i.e. in small steps. It is not a priori clear whether there is a window in coupling constant space where both FSE can be overcome and perturbative predictions can be checked.

Early on in our investigation [11] it became clear that, for lattice sizes well accessible with the present computing resources, the FSE are large, even at the fairly small $\beta=3.0$ (this value of β is fairly small when compared to $8\leqslant\beta\leqslant10$, which corresponds to a realistic value for the renormalized weak coupling – at least at large Higgs masses [2,4]). We observed large discrepancies between values am_{W} and am_{H} for the W and Higgs masses measured on an $8^3\times16$ versus those on a $12^3\times24$ lattice. In order to get control over the FSE we could either increase the lattice size further, or go to even smaller values of β. We decided to start with the latter, taking $\beta=2.4$ and 2.7. Then we also considered different lattice sizes at $\beta=3.0$.

The interest in studying the VOOP comes from the fact that it is not possible to measure the conventionally defined VEV if the gauge is not fixed, which is the case in most simulations. Besides, the usual VEV is a gauge dependent quality. We would like however to test perturbative relations between physical, i.e. gauge invariant quantities. In tree level relations, where this gauge dependence is not felt, the VOOP and the conventional VEV are identical. It was therefore proposed [6] to use the VOOP as an alternative definition of the VEV.

As seen from the results of ref. [5], there are a few pitfalls in performing numerically the limits necessary for determining the value of the VOOP. We show here how to avoid such problems. Moreover, we test some of the theoretical expectations [6] for the VOOP. In particular, we check that two different methods to compute it give the same result on large

enough lattices (N.B. not in a free charge phase). Comparing the results obtained with the two methods provides a useful estimate of the FSE.

In section 2 we shall give the technical details of our simulation. Then, in section 3, we describe the quantities we have measured. In section 4 the results are presented. Finally we state our conclusions in section 5.

2. The Monte Carlo simulations

As described in the introduction, we chose to do our simulations in the Higgs region of the phase diagram, at $\lambda=\infty$, $\beta=2.4$, 2.7 and 3.0. The W mass am_{W}, which at fixed λ and β does not vary strongly with κ [2–4], always turned out to be smaller than 0.5. On the other hand, for λ and β fixed, the Higgs mass am_{H} is a rapidly increasing function of κ [2–4]. Our choice of values for κ was restricted by two considerations. If we increase κ too much, am_{H} becomes larger than 1 and we can no longer assume that the lattice model is an approximation of the continuum model (notice that 1 is a somewhat arbitrary cut; notice also we do not have this problem for am_{W}). If we decrease κ too much, we get too close to the first order [1,2] Higgs-confinement transition, and on a finite lattice there will be flip-flops between the two phases. Since we were interested here in the properties of the system inside the Higgs phase and not in studying the transition itself, we decided to avoid the situations where flip-flops occur.

We have performed calculations on lattices of size $L_{\mathrm{s}}^3\times L_{\mathrm{t}}=8^3\times 16$, $8^3\times 24$, $10^3\times 24$, $12^3\times 24$ and $14^3\times 28$, with 15 000 thermalization and 90 000–120 000 measurement sweeps for each data point. We employed Creutz's heat bath algorithm [14], slightly modified for full (and easy) vectorization [15], which in a comparison at $\beta=2.4$ needed about three times less sweeps and computer time than a six-hit Metropolis algorithm in order to obtain the same statistical precision.

We were very careful with the error analysis. By the central limit theorem, the averages of the measured physical quantities over a long time series are approximately gaussian random variables. The gaussian distribution is characterized by a *covariance matrix*. One way to estimate it is to partition the time series into bins and compute the covariance matrix for the bin averages. As the bin size is increased, this bin average covariance matrix divided by the number of bins converges to the covariance matrix for the whole-run averages. In computing functions of the measured quantities or in fitting them, we did the error analysis using the whole covariance matrix.

The more widespread procedure of only considering the variances (i.e. the diagonal of the covariance matrix) may produce numerical mistakes. It normally results in smaller values of the reduced χ^2, which may *mislead* us to declare a fit to be good when in fact this was not the case. What often happens is that by only using the variances to compute the χ^2, we take too many data points for the fit. This alters the value of the fitted quantity and (typically) reduces the estimated error. On the other hand, for exactly the same data the error estimated with the full covariance matrix is often smaller than when using the variances alone. In practice we did find a number of cases where the values of the fitted quantities and/or of the errors varied by more than one standard deviation between the two procedures. Besides, when testing the stability of fits with respect to leaving out data points (e.g. varying the smallest and the largest distances used to fit the masses from euclidean two-point functions), we found better consistency while using the whole covariance matrix.

3. The measured quantities

The Higgs mass am_{H} and the W mass am_{W} were obtained from euclidean time correlations of zero three-momentum combinations of the usual link quantities [2–4,11,13], fitted with a periodic exponential decay.

The potential $V_{\mathrm{W}}(R)$ $(1\leqslant R\leqslant L_{\mathrm{s}}/2)$ between two fundamental representation *static* sources was determined from $R\times T$ Wilson loops $W(R, T)$ by fitting

$$W(R, T)=C_{\mathrm{W}}(R)\exp[-V_{\mathrm{W}}(R)T] \qquad (3)$$

$(1\ll T\leqslant L_{\mathrm{s}}/2)$. Then V_{W} was fitted with a Yukawa potential

$$V_{\mathrm{W}}(R)=-\tfrac{3}{4}\alpha_{\mathrm{ren}}Y(R, am_{\mathrm{W}})+2E_{\mathrm{q}}, \qquad (4)$$

for which we took both the continuum version $Y_{\mathrm{cont}}(R, am_{\mathrm{W}})=\exp(-am_{\mathrm{W}}R)/R$, and the lattice

version [4,3]. This potential is a first perturbative approximation (one-W-exchange, no loops) and cannot be expected to fit the data well for all couplings and for the full range of R. If the fit is good, then $\alpha_{\rm ren}=g^2_{\rm ren}/4\pi$ with $g_{\rm ren}$ the renormalized gauge coupling, and $E_{\rm q}$ is the energy of a single fundamental static source. Depending on the version of the Yukawa potential used, we will write $E_{\rm q,cont}$ and $\alpha_{\rm ren,cont}$, or $E_{\rm q,latt}$ and $\alpha_{\rm ren,latt}$. Since the fits are too insensitive to changes in the W mass, we fixed $am_{\rm W}$ at the value obtained from the correlations.

The static source energy $E_{\rm q}$ can also be determined in two more ways: from a fit to the expectation values of gauge invariant timelike lines,

$$G_1(T)\equiv\left\langle \,\Big|_{T}\right\rangle = C_1\,\exp(-E_{\rm q,line}T) \tag{5}$$

$(1 \ll T \leqslant L_{\rm s}/2)$, or directly from the timelike Polyakov loop $P_{\rm t}\equiv G_1(L_{\rm t})$,

$$E_{\rm q,Pol}=-(1/L_{\rm t})\,\ln(P_{\rm t}). \tag{6}$$

In general we expect the finite-size corrections for the different determinations of the same quantity $E_{\rm q}$ to be different. In addition, the fitted values of $E_{\rm q,Pol}$ and $E_{\rm q,line}$ on the one hand and those of $E_{\rm q,cont}$ and $E_{\rm q,latt}$ on the other hand can differ because the corrections to the Yukawa form of the potential are not really negligible.

The VEV, $v=\langle\varphi^{\rm c}\rangle$ ($\varphi^{\rm c}$ is the continuum Higgs field), cannot be defined beyond tree level unless the gauge is fixed. A gauge invariant alternative is to consider the VOOP [6]. Let us define

$$G_2(R,T)\equiv\left\langle \overset{R}{\sqcap}_{T}\right\rangle, \tag{7}$$

and

$$\rho(R,T)\equiv G_2(R,T)/W(R,2T)^{1/2}. \tag{8}$$

The VOOP is obtained from $\rho(R,T)$ in the limit $R, T\to\infty$, taken such that T grows at least linearly in R. For all gauges it is related to v at tree level by

$$\text{VOOP}\equiv\rho(\infty,\infty)=(av)^2/\kappa. \tag{9}$$

Beyond tree level, the wave function renormalization of the VOOP is needed, but has not yet been determined.

The quantity $\rho(R,T)$ is the vacuum overlap of a candidate for a charge–anticharge state with bounded energy [6]. If T is increased at fixed R, the energy of this state decreases and therefore its vacuum overlap increases. On the other hand, if R is increased at fixed T, $\rho(R,T)$ decreases, similarly to the decrease of a usual two-point function as the distance is increased. Thus if R and T are increased simultaneously, strange things may happen, as noticed in refs. [1,5]. A consistent way to determine the VOOP is to take the limit $T\to\infty$ first while keeping R fixed. We can do this for the numerator and the denominator separately: we fit the Wilson loops with (3) and the "staples" (7) with

$$G_2(R,T)=C_2(R)\,\exp[-V_2(R)T]. \tag{10}$$

$V_{\rm W}$ and V_2 represent the *same* potential, fitted from two different quantities. If the fits (3) and (10) are both good and after checking that $V_{\rm W}=V_2$, we have

$$\rho(R,\infty)=C_2(R)/C_{\rm W}(R)^{1/2}. \tag{11}$$

$\rho(R,\infty)$ is a gauge invariant alternative to the Higgs field two-point function. As R becomes large, it stabilizes to a value that is the VOOP.

It has been suggested theoretically [6] that in a phase without free charges the VOOP is equal to the constant C_1 obtained from the fit (5)

$$\rho(\infty,\infty)=C_1. \tag{12}$$

This is a numerically easier way to determine the VOOP. As in the case of the source energy, having more than one method to determine a given quantity provides a useful check on the magnitude of the FSE and on the consistency of the whole numerical approach.

The tree level result for the W mass is

$$(am_{\rm W})^2=2\pi\kappa\alpha_{\rm ren}\rho(\infty,\infty). \tag{13}$$

We shall denote by $am_{\rm W,cont}$ and $am_{\rm W,latt}$ the results obtained for the RHS of (13) using the *measured* (i.e. fitted) values for $\alpha_{\rm ren,cont}$ and $\alpha_{\rm ren,latt}$, respectively, and those for the VOOP. In order to test the validity of the tree level relation, we shall compare the values of the W mass obtained using (13) to the *directly measured* ones.

A tree level relation can also be used to *define* the renormalized φ^4 coupling:

$$\lambda^{\rm c}_{\rm ren}=(am_{\rm H})^2/4\kappa\rho(\infty,\infty). \tag{14}$$

4. Results

Our analysis of the MC results is summarized in tables 1–3. All numbers except those followed by an asterisk are results of fits where both $\chi^2 < 1$ (see section 2) and an asymptotic value had clearly been reached. For all fits we also checked for insensitivity of the results against variations in the range of distances considered. The last three lines in the tables were obtained using the tree level relations (13) and (14).

From the point of view of the numerical analysis, the data at $\beta=2.7$ turned out to be the easiest to handle. At $\beta=2.4$ the analysis was more difficult because the data were noisier (this is due to the stronger gauge coupling). At $\beta=3.0$ some difficulties were caused by the smallness of the W mass.

Let us start the discussion of the results with the *masses*. For $\beta=2.4$ and 2.7 the general trend is that the changes in the masses are rather drastic between $L_s=8$ and 10, but between $L_s=10$ and 12 they are not significant. Actually for $\beta=2.4$ the data are also consistent with a drop in the W mass as L_s goes from 10 to 12. This would mean that the FSE are stronger at $\beta=2.4$ than at $\beta=2.7$, which is somewhat surprising, since we would naively expect the FSE to be smaller where the masses are larger, i.e. at $\beta=2.4$. However, as pointed out to us by I. Montvay, we can also expect to have stronger FSE for the W mass if the gauge coupling is stronger, and this effect may well win in our case. The FSE are most likely under control at $\beta=2.7$. For our largest value of β, $\beta=3.0$, the FSE for am_W do not seem to be completely under control yet.

Within error bars, the W mass decreases (or becomes constant) with L_s for all sets of couplings we considered. For the Higgs mass however, this is only true in two cases, where for the larger lattices the Higgs particle is *stable* ($am_H < 2am_W$). In a number of cases am_H is below the $2am_W$ threshold for $L_s=8$, but when L_s is increased am_W goes down, am_H goes up, and the combined effect is strong enough to make the Higgs a *resonance*.

Unfortunately there is no theoretical prediction for the influence of a threshold on the FSE. In order to predict what happens in the thermodynamic limit, we have to rely on the apparent stabilization of the results as L_s is increased. For the large lattices then the standard picture [1–3] emerges: for fixed β, am_H increases rapidly with κ, whereas am_W stays almost constant. For κ close to the transition (almost as close as we could get without having flip-flops between the two phases), both masses seem to stay finite, which

Table 1
Collected results for $\beta=2.4$ for various lattice sizes.

	$\kappa=0.390$			$\kappa=0.400$		
	$8^3\times24$	$10^3\times24$	$12^3\times24$	$8^3\times24$	$10^3\times24$	$12^3\times24$
am_W	0.47(2)	0.41(2)	0.38(2)	0.47(1)	0.44(1)	0.40(2)*
am_H	0.80(2)	0.55(6)*	0.67(4)	0.91(3)	0.96(3)	0.94(3)
$\alpha_{ren,cont}$	0.56(12)*	0.61(7)*	0.58(4)*	0.64(4)	0.58(3)	0.51(8)
$\alpha_{ren,latt}$	0.51(11)*	0.53(6)*	0.50(4)*	0.60(3)	0.51(3)	0.42(7)
$E_{q,line}$	0.312(3)	0.298(2)	0.301(2)*	0.288(2)	0.286(1)	0.282(1)*
$E_{q,Pol}$	0.306(9)	0.301(4)	0.297(3)	0.290(6)	0.281(3)	0.279(2)
$E_{q,cont}$	0.283(8)*	0.293(5)*	0.293(4)*	0.287(2)	0.281(1)	0.280(3)
$E_{q,latt}$	0.270(5)*	0.283(2)*	0.284(3)*	0.268(1)	0.276(1)	0.275(2)
VOOP	0.172(14)	0.168(4)	0.171(4)	0.216(7)	0.215(9)	0.215(12)
C_1	0.183(3)	0.170(3)	0.178(2)*	0.228(3)	0.231(1)	0.226(1)*
$am_{W,cont}$	0.48(7)*	0.50(3)*	0.49(2)*	0.59(3)	0.56(3)	0.52(6)
$am_{W,latt}$	0.46(7)*	0.47(3)*	0.46(2)*	0.57(3)	0.53(3)	0.48(5)
λ^c_{ren}	2.4(3)	1.2(3)	1.7(3)	2.4(3)	2.6(3)	2.6(3)

Table 2
Collected results for $\beta=2.7$ for various lattice sizes.

	$\kappa=0.365$			$\kappa=0.370$		
	$8^3\times24$	$10^3\times24$	$12^3\times24$	$8^3\times24$	$10^3\times24$	$12^3\times24$
am_W	0.41(2)	0.32(2)	0.33(2)	0.40(2)	0.31(1)	0.31(1)
am_H	0.59(2)	0.53(3)	0.53(3)	0.56(8)	0.73(2)	0.74(2)
$\alpha_{ren,cont}$	0.297(6)	0.39(2)*	0.36(2)	0.296(4)	0.37(2)	0.37(2)
$\alpha_{ren,latt}$	0.279(4)	0.36(2)	0.34(1)	0.276(4)	0.34(2)	0.32(1)
$E_{q,line}$	0.267(6)*	0.251(5)	0.243(3)	0.257(4)	0.242(2)	0.235(2)
$E_{q,Pol}$	0.277(4)	0.258(2)	0.243(1)	0.256(3)	0.241(1)	0.234(1)
$E_{q,cont}$	0.219(1)	0.228(2)*	0.228(1)	0.218(1)	0.225(2)	0.224(1)
$E_{q,latt}$	0.206(2)	0.218(3)	0.220(1)	0.210(1)	0.216(1)	0.217(1)
VOOP	0.103(6)	0.105(4)	0.105(2)	0.133(4)	0.133(3)	0.131(5)
C_1	0.101(6)*	0.104(4)	0.103(3)	0.138(5)	0.137(2)	0.135(1)
$am_{W,cont}$	0.27(1)	0.31(1)*	0.29(1)	0.30(1)	0.34(1)	0.33(1)
$am_{W,latt}$	0.26(1)	0.29(1)	0.29(1)	0.29(1)	0.32(1)	0.31(1)
λ^c_{ren}	2.3(3)	1.9(3)	1.9(3)	1.6(5)	2.7(2)	2.8(3)

Table 3
Collected results for $\beta=3.0$ for various lattice sizes.

	$\kappa=0.354$		$\kappa=0.355$	$\kappa=0.357$	$\kappa=0.360$		
	$12^3\times24$	$14^3\times28$	$12^3\times24$	$12^3\times24$	$8^3\times16$	$10^3\times24$	$12^3\times24$
am_W	0.24(1)	0.22(1)	0.25(1)	0.26(1)	0.37(2)	0.30(2)	0.26(1)
am_H	0.53(3)	0.54(1)	0.50(3)	0.58(3)	0.63(6)	0.77(6)	0.73(3)
$\alpha_{ren,cont}$	0.254(18)	0.288(5)	0.292(6)	0.287(5)		0.276(11)	0.279(11)
$\alpha_{ren,latt}$	0.234(14)	0.255(4)	0.259(6)	0.253(6)		0.252(8)*	0.245(10)
$E_{q,line}$	0.213(2)	0.203(1)	0.205(3)	0.200(2)		0.205(1)	0.197(2)
$E_{q,Pol}$	0.207(1)	0.2004(7)	0.2045(8)	0.1992(6)		0.203(1)	0.1964(9)
$E_{q,cont}$	0.188(2)	0.1924(5)	0.1906(5)	0.1889(5)		0.185(2)	0.187(1)
$E_{q,latt}$	0.182(1)	0.1857(4)	0.1831(5)	0.1817(2)		0.177(1)*	0.180(1)
VOOP	0.085(6)*	0.071(6)	0.084(3)	0.097(5)		0.113(7)	0.120(3)
C_1	0.087(1)	0.0828(8)	0.089(3)	0.101(2)		0.122(1)	0.119(2)
$am_{W,cont}$	0.22(2)	0.22(2)	0.24(2)	0.25(2)		0.27(2)	0.27(2)
$am_{W,latt}$	0.21(2)	0.21(2)	0.22(2)	0.24(2)		0.26(2)*	0.26(2)
λ^c_{ren}	2.4(4)	2.8(4)	2.1(3)	2.4(4)		3.4(5)	3.1(3)

supports the general belief that the transition is first order.

The *Yukawa fits* were generally good for $\beta=2.7$ and $\beta=3.0$. At $\beta=2.4$ the same was true for the higher value of κ only.Notice that the errors are smaller for larger β. The fitted values differ between the continuum and the lattice version of the potential by $\leqslant 15\%$ for α_{ren} and by $\leqslant 5\%$ for E_q. Again, the results for $L_s=8$ differ by a lot from the results on the larger lattices, which are stable within error bars. The only exception is at $\beta=3.0$, $\kappa=0.354$, which shows that also for the Yukawa fits the FSE are not completely under control at our largest β.

The renormalization effects are stronger for stronger gauge couplings: the ratio between the renormalized and the bare value of α is around 2.5 at $\beta=3.0$, around 3 at $\beta=2.7$, and around 4 at $\beta=2.4$.

We have determined the static source energy also independently of the potential calculation, using (5) and (6). The results for $E_{q,line}$ and $E_{q,Pol}$ agree very well with each other in all cases except at $\beta=3.0$, $\kappa=0.354$. For the values of E_q determined in this way, the FSE at $\beta=2.7$ and 3.0 seem to be somewhat larger than from the potential determination. There is, however, no contradiction in this. The former are obtained from properties of a state containing one source, the latter from a two-source state.

Although the results for $E_{q,line}$ and $E_{q,Pol}$ on one side and those for $E_{q,cont}$ and $E_{q,latt}$ on the other side approach one another as L_s increases, it seems that a small discrepancy would persist in the thermodynamic limit. The discrepancy is larger for $E_{q,latt}$, although in principle the correct tree level form for the potential is the *lattice* Yukawa form. This sheds some doubt on the reliability of the Yukawa fits. Furthermore, at large distances, close to $L_s/2$, the statistical errors for the potential are larger, so these data points have less influence in determining the outcome of the fit. It is however at large distances that the correct value of E_q is determined.

Let us assume that the corrections to the tree level formulas cannot be completely neglected. The gauge coupling would then *run*, and since it is asymptotically free it would be stronger for the potential at large distances. To compensate for this, $E_{q,latt}$ would have to increase too, i.e. to move towards $E_{q,line}$ and $E_{q,Pol}$. One should however not forget that this is a small effect, and one can trust that the Yukawa fit results are accurate to within at least 10–20%.

Some of the most important results of our analysis concern the VOOP and the tree level relation for am_W.

As opposed to the quantities discussed up to now, there are, within error bars, no significant FSE for the VOOP (with the exception of $\beta=3.0$, $\kappa=0.354$). The theoretical prediction (12) that the two different ways of determining the VOOP should give the same result, are well supported by the data. As expected, the VOOP increases with κ. The number of Monte Carlo sweeps needed for a good determination of the VOOP is considerably smaller than e.g. for the masses.

At $\beta=2.7$ and 3.0 and for $L_s \geq 10$, the *renormalized tree level relation* (13) is fulfilled remarkably well, to within about 5%. This is all the more noteworthy as the ratio $\alpha_{ren}/\alpha_{bare}$ is not at all small. Notice that the FSE in $am_{W,cont}$ and $am_{W,latt}$, i.e. in the W mass *computed* from (13), are small. This is partly due to the fact that we take the square root of the RHS of (13), but there may also be compensations between the FSE for α_{ren} and for the VOOP (as suggested by the results at $\beta=3.0$, $\kappa=0.354$). At $\beta=2.4$, $\kappa=0.4$ however, (13) is no longer well obeyed, the discrepancy being about 25% ($\kappa=0.39$ cannot be considered since the Yukawa fits are not good).

Another indication for the applicability of *renormalized* perturbation theory are the values of λ_{ren}^c, which stay well below the unitarity bound $\lambda_{ren}^c \approx 5$ [10].

5. Conclusions and outlook

We performed high-statistics simulations of the lattice SU(2) Higgs model at $\lambda=\infty$ for three values of β, with the aim of studying finite-size effects (FSE) and then comparing the numerical results with perturbation theory.

The FSE, the greatest numerical obstacle, are most likely under control on the largest lattices employed at $\beta=2.7$. We have thus found a suitable starting point for gaining a quantitative numerical understanding of the model. Somewhat surprisingly, the FSE seem to increase at $\beta=2.4$. There, and also at $\beta=3.0$, where the FSE seem to be coming under control now, simulations on larger lattices are necessary.

We showed how to determine the VOOP in a Higgs phase reliably from finite lattice simulations. Thus we have a method of computing the VEV nonperturbatively.

The potential between two static sources agrees well with a Yukawa form at $\beta=2.7$ and 3.0, but clearly deviates from this form at $\beta=2.4$.

The renormalized tree level relation between the W mass, the gauge coupling (obtained from the Yukawa fits), and the VEV (obtained from the VOOP), holds remarkably well at $\beta=2.7$ and 3.0. Moreover, in the range $0.5 \leqslant am_H \leqslant 1$ considered by us, the values of λ_{ren}^c turned out to be well below the unitarity bound and quite similar to those from the calculations [9,10] that show the triviality of the O(4)

model. Sizeable deviations from tree level behaviour occur, on the other hand, at $\beta=2.4$.

It will be very interesting to see to what extent discrepancies between the nonperturbatively obtained data and tree level predictions can be reduced by one-loop corrections, calculated on the finite lattice, and, of course, renormalization group improved. In particular, if the potentials can then be fitted well at $\beta=2.4$, we would have a highly nontrivial test for the accuracy of perturbation theory.

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