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CHARGED STATES AND ORDER PARAMETERS IN THE GEORGI-GLASHOW MODEL*

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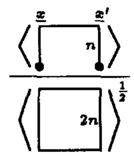
As an example for various generalizations of the vacuum overlap order parameter, we review gauge invariant two-point-functions and order parameters in the Georgi-Glashow model. From the discussion a surprising possibility emerges: the existence in the Higgs phase of Z_2-charged states.

The vacuum overlap order parameter (VOOP) was designed to replace the Wilson loop as an order parameter for gauge theories with matter fields in the fundamental representation[1]. In the case of scalar matter fields, it turned out to be a useful quantity for numerical investigations[2]. The VOOP is the scalar product with the vacuum (hence its name) of an appropriately constructed (energy regularized) candidate for a charged state. The charge is measured via Gauss' law, i.e. by measuring the total electric flux at infinity. In a nonabelian theory the electric field operators are not gauge invariant. The only gauge invariant electric flux is that associated with the gauge group center ("n-ality").

The original VOOP was defined as the limit $\underline{x}' \to \infty$, $n \geq c|\underline{x} - \underline{x}'|$ (c is some constant), of

$\rho(|\underline{x} - \underline{x}'|, n) =$  (1)

where the horizontal direction is one of the space directions, the vertical direction is Euclidean time, the strings in the numerator and denominator carry a path-ordered exponential of the gauge field in the fundamental representation, the dots stand for the fundamental matter fields at time zero, the "staple" in the numerator has height n (n is the energy regularization parameter), the rectangle in the denominator has height $2n$, and both have the points $\underline{x}$ and $\underline{x}'$ as their spatial projections (we shall use the convention that purely spatial quantities are underlined, but space-time ones are not). $\rho(|\underline{x} - \underline{x}'|, \infty)$ (best possible energy regularization) is a *gauge invariant* version of the matter field *two-point-function* (2PF) and has the following generic behaviour in the three typical regions of standard phase diagrams:

1. In a *free charge* phase (e.g. QED), $\rho(|\underline{x} - \underline{x}'|, \infty)$ behaves like a propagator of the charged particle (e.g. electron); for large distances $|\underline{x} - \underline{x}'|$ it goes to zero, i.e. the candidate for the charged states becomes orthogonal to the vacuum; the charge is not screened.

2. In a *Higgs mechanism* phase/region, assuming the matter field φ is scalar, $\rho(|\underline{x} - \underline{x}'|, \infty)$ behaves like a perturbative φ-2PF; for large distances it goes to a constant which is nothing else than the square of the "vacuum expectation value of the Higgs field"; the charge is screened.

3. In a *confinement* phase/region, and assuming that we call the matter field "quark", $\rho(|\underline{x} - \underline{x}'|, \infty)$ behaves at small distances like a perturbative quark propagator, but at some characteristic length scale

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hadronization sets in and for asymptotically large distances it goes to a nonzero constant (which is a nonperturbative effect); the charge is again screened, but by a different mechanism than in the Higgs phase.

In generalizing the VOOP, we have to consider situations in which we expect to have particle states that are charged with respect to something else than the center of the *original* gauge group G. One typical situation is that of a Higgs mechanism whereby the "unbroken" subgroup H is nontrivial. There we can ask the question of whether there exist free charges with respect to the center of H[3].

A different generalization is to ask whether there are charged states with respect to a nonabelian group, which may be either G or H. It is possible that the argument that such charged states cannot exist since there is no gauge invariant electric flux, is nothing more than a heuristic argument that hides our inability to come up with a better way to measure a nonabelian electric flux at infinity. One thing that can be done is to contruct a candidate for a state containing such a nonabelian flux, the contruction being analogous to that used to define the original VOOP, and then to use the vacuum overlap as a test of whether this state is in the vacuum Hilbert space or in a different superselection sector.

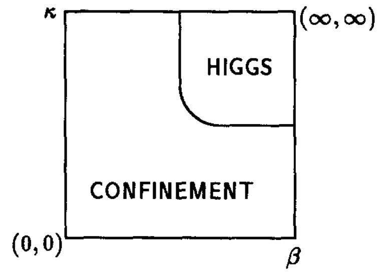
In order to define these generalizations, the first step is to decide what are the possible objects whose gauge invariant 2PFs we want to write down. The simplest possibility is to take those particles occuring in the perturbative description of a given phase, that are not neutral with respect to the "unbroken group" of that phase (for the neutral particles, the 2PFs are between gauge invariant local quantities and we have nothing new to say with respect to them). After having replaced the perturbative definition of a 2PF with the gauge invariant one, we can use nonperturbative methods to compute that 2PF in the other phases too. This procedure can be carried out for each phase/region, so that in the end we can compute simultaneously the particle 2PFs for all possible breakings and (hopefully) decide dynamically which is the realized situation.

In the end (see [3] for what "in the end" means), all generalizations of the VOOP are defined by putting in (1) something different on the strings, corresponding to the flux we wish to create, and something different at the endpoints, possibly composite objects in terms of the original fields, with the correct transformation properties (of course, both for the numerator and for the denominator, the quantities whose expectation value we take have to be gauge invariant).

We shall exemplify these ideas for the case of the Georgi-Glashow model[4], which is one of the simplest examples we can use. On a hypercubic lattice this model has the action[2,3]:

$$S = -\beta \sum_{p} \tfrac{1}{2} \mathrm{Tr}\, U(p) - \kappa \sum_{x,\mu} (\varphi_x, D^1(U_{x,\mu})\, \varphi_{x+\hat{\mu}}) \quad (2)$$

where the lattice gauge field $U_{x,\mu}$ is an $SU(2)$ matrix in the fundamental representation that lives on the link starting from x and going in the μ-direction, $U(p)$ stands for the product of U's around the plaquette p, φ_x is a real 3-component (i.e. adjoint) field with $|\varphi_x| = 1$ (for simplicity we take the φ^4 coupling λ to be infinite), D^1 denotes the adjoint representation and $\hat{\mu}$ denotes the unit vector in direction μ. This model has the following phase diagram[5,2]:



The Wilson loop of the U-fields is a good order parameter: it obeys an area law in the confining phase and a perimeter law in the Higgs phase. The reasons for this are described e.g. in [5]. In the confining phase, the electric flux between two fundamental sources at $\underline{x}$ and $\underline{x}'$ cannot be interrupted if there are no fundamental matter fields. As a consequence, the potential between these two sources grows linearly with the distance, which can be proven using the strong gauge coupling expansion. In the Higgs phase however, the vacuum can be viewed as a condensate of the φ-fields which screens the force between the two fundamental sources in a similar way to the screening of the force in a plasma.

The perturbative picture is the following. In the confining phase, which from the point of view of perturbation theory is a free charge phase, the particles are the charged φ's and the gluons. Both carry the adjoint representation of $G = SU(2)$. In the Higgs phase the particles are two vector bosons, to be called W's, a photon and a Higgs scalar. With respect to $H = U(1)$, the W's carry the charge ± 1, while the photon and the Higgs are neutral and will not be discussed further.

The φ-2PF is obtained by putting $D^1(U)$ on the strings and φ at the endpoints of (1) (and then, of course, taking $n \to \infty$). In the confinement phase it behaves like a propagator for short distances, but after fragmentation sets in it goes to a nonzero constant at large distances. For weak gauge couplings, the small distance behaviour can be computed in perturbation theory, at large distances however we can only use a hopping parameter expansion in κ or numerical simulations. In the Higgs phase, the φ-2PF can be computed in perturbation theory for all distances. It decreases monotonically to a nonzero value, which is a possible definition of the "Higgs expectation value". This definition can be used in nonperturbative calculations too.

The gluon-2PF is obtained by putting $\mathrm{ad}\, U = D^1(U)$ on the strings and $\mathrm{Tr}\,\tau_\alpha U(p_{\underline{x}})$ (and a similar object at $\underline{x}'$) at the endpoints; here τ_α are the Pauli matrices, $U(p_{\underline{x}})$ is the product of the four U's around the plaquette $p_{\underline{x}}$, starting at the point $\underline{x}$ in the time-zero hyperplane which is one of the corners of $p_{\underline{x}}$. This is nothing but the gluon B-field-2PF (spacelike $p_{\underline{x}}$) or the gluon E-field-2PF (timelike $p_{\underline{x}}$). Again, in the confining phase, the gluon-2PFs can be computed in perturbation theory for short distances alone. At large distances we probably can only use numerical simulations. If the coventional wisdom that there are no free gluons is true, the gluon 2PF should go for large distances to a nonzero constant, whose physical meaning is similar to that of the "gluon condensate" $< \mathrm{Tr} F_{\mu\nu}^2 >$. In the Higgs phase the gluon-2PF has not yet been computed in perturbation theory. Since we do not expect the gluons to be particles here, its asymptotic value should again be a nonzero constant.

We have seen that both order parameters that test the existence of nonabelian charge do not distinguish between the confining and the Higgs phase. We should however keep in mind the fact that, from the point of view of rigorous results, a phase with free gluons and charged φ's has not really been excluded, either in this particular model or on general grounds. For order parameters associated with the "unbroken group" of the Higgs phase the situation is however different. This group is Abelian, and we expect the corresponding generalizations of the VOOP to be zero in the Higgs phase (since there are charged W's) and nonzero in the other phase.

The W-2PF was first introduced in [3]. Let us define the group $H = U(1)$ as the stability group of the vector $\phi = (0\ 0\ 1)$, $H = \{h \in G \,|\, D^1(h)\,\phi = \phi\}$, and let us further define the "transformation to the unitary gauge" $v(\varphi)$ as a rotation around an axis in the x-y plane, the rotation angle being not larger than π, that obeys $D^1(v(\varphi))\,\varphi = \phi$ (up to a zero measure set where $v(\varphi)$ is discontinuous and has to be fixed by extra conditions, the solution of this equation is unique). Let us introduce the new link variable $W_{x,\mu} = v(\varphi_x) U_{x,\mu} v(\varphi_{x+\hat{\mu}})^{-1}$, which transforms as a gauge field under gauge transformations that are only in H. To define the W-2PF we put on the strings of (1) products of a function u of $W_{x,\mu}$ with the following transformation properties:

$$u(hW) = u(Wh) = \chi(h)\, u(W) \qquad (3)$$

for $h \in H$ and χ the charge-1 character of H (in our conventions the fundamental $U(1)$-charge is $\frac{1}{2}$). At the endpoints of (1) we put a function w of a spatial link $W_{x,\mu}$ (since the W particle has spin one) with the property:

$$w(hW) = \chi(h)\, w(W)\,, \quad w(Wh) = w(W) \qquad (4)$$

The simplest choices for u and w are $u = D^1_{11}$ and $w = D^1_{10}$ (D^j_{mn} are the irreducible representation matrix element functions). In the naïve continuum limit, we obtain the usual $U(1)$ gauge field and W field respectively. There is an ambiguity here[3]. We could take for u any linear combination with positive coefficients of the D^j_{11}. As far as we can see now, we have to live with this kind of ambiguity, which is another case where there is not a unique lattice regularization for one and the same continuum quantity (the charged W) – see [3] for more details. Notice that, although we took some ideas from the unitary gauge discussion, we never fix a gauge.

The W-2PF behaves in the Higgs phase like a W-propagator. It can be computed perturbatively. The large distance asymptotic value, i.e. the order parameter, is then zero. In the confinement phase, the W-2PF can be computed in the strong gauge coupling expansion. For large distances it goes to a nonzero constant. It is interesting to note that for large κ, close to the pure $U(1)$ gauge theory (the limit $\kappa \to \infty$), the finite distance behaviour of the W-2PF is similar to that of a gauge invariant matter-field-2PF in an ordinary $U(1)$ gauge theory with a charged matter field. For small κ we can also compute the W-2PF in the hopping parameter expansion. The interesting thing to note is that, modulo a factor of κ^4, it is almost equal to the gluon E-field-2PF.

We *conclude* that the generalized VOOP defined with the W-2PF is indeed a good order parameter, that is zero in the Higgs phase and nonzero in the confining phase.

Up to now all the fields whose gauge invariant 2PFs we constructed had trivial transformation properties under the center of $SU(2)$. Let us now couple an additional *heavy* 2-component (i.e. fundamental) matter field ψ. In principle ψ could be either bosonic or fermionic. Just for simplicity, let us assume that it is a scalar matter field of fixed length 1 (again this can be achieved by appropriately tuning parameters in the potential part of the action). The action now gets an extra piece:

$$S \mapsto S - \kappa' \sum_{x,\mu} \mathrm{Re}\,(\psi_x, U_{x,\mu}\psi_{x+\hat{\mu}}) \qquad (5)$$

(in this way we only have a 3-parameter phase diagram). The fact that ψ is a heavy field implies that the hopping parameter κ' is small[2]. For a small κ' we expect to have the same two phases as for $\kappa' = 0$. Of course, for $\kappa' \neq 0$ the Wilson loop of the U's obeys a perimeter law throughout the whole phase diagram. The W-2PF on the other hand still goes to zero in the Higgs phase and to a nonzero value in the confinement phase.

In the presence of the field ψ, the usual Higgs mechanism discussion predicts a $U(1)$-charge-$\frac{1}{2}$ particle η in the Higgs phase. Let us introduce the field $\eta_x = v(\varphi_x)\psi_x$. Under $h \in H$ its first component, η^+, has charge $+\frac{1}{2}$, while its second component, η^-, has charge $-\frac{1}{2}$. The gauge invariant η-2PF is defined by putting in (1) a product of $u_{\frac{1}{2}}(W) = D^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}}(W)$ on the strings, an η^+ at one endpoint and an η^- at the other endpoint. In the Higgs phase it behaves like a particle propagator. One method to derive this conclusion is perturbation theory; another is the small κ' hopping parameter expansion, supplemented by the known behaviour of the $u_{\frac{1}{2}}$-Wilson loops at $\kappa' = 0$ (perimeter law in the Higgs phase, area law in the confining phase – the degree of rigour to which this is known is the same as for the U-Wilson loops). In the confining phase, the expansion in κ' can be used to show that at large distances the η-2PF goes towards a nonzero constant.

The last 2PF we discuss is that for the field ψ itself. It is defined by the original version of (1), i.e. by putting fundamental representation U's on the strings and ψ's at the endpoints. Using the small κ' expansion, it follows that at large distances the ψ-2PF goes to zero in the Higgs phase and to a nonzero constant in the confinement phase. A zero value indicates the existence of a charged state. As opposed to the η particle, in the Higgs phase the ψ is neutral under $U(1)$, since we tried to create an electric flux of a type that is screened (see [3] for more details). On the other hand, the ψ transforms nontrivially under the center Z_2 of the original gauge group. So we may have constructed a state carrying a center charge alone! There are no other examples in the literature of such a state in models with nonabelian (not to speak of continuous) gauge groups.

A computation, again in the small κ' expansion, of the ratio of expectation values of the total (multiplicative) Z_2-electric flux at infinity in the candidate state for a charged ψ and in the vacuum, gives 1 in the confining phase and -1 in the Higgs pase. This gives us confidence that, as far as we can trust the small κ' expansion, the conclusion about the Z_2-charged state is correct.

Apparently, there is a contradiction in this discussion of Z_2-charged states. For a given closed surface, the exponentials of the (additive) total-$U(1)$-flux operators form a $U(1)$ group. The total-Z_2-flux operator, together with the identity, is a subgroup thereof. How can it be that for the candidate of a Z_2-charged state the subgroup is represented nontrivially while the whole group is represented trivially? At least as presented here, this is a fake contradiction. As opposed to the ratios of flux expectation values, the total flux operators are not well defined when the surface becomes infinite. The reason is that particle-antiparticle fluctuations across the surface wash out the flux measurement. These operators need to be regularized, but for the regularized versions we have a priori no reason to believe that the relevant copy of Z_2 is still a subgroup of the relevant copy of $U(1)$.

As a next step have to compute the ψ-2PF in the Higgs phase using perturbation theory. We also have to investigate whether the $U(1)$-charged η particle is also Z_2-charged. Finally the relationship between the $U(1)$ and the Z_2 flux operators requires a more detailed treatment.

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