ASYMPTOTIC FERMIONS IN TWO-DIMENSIONAL CHIRAL GAUGE THEORIES

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The gauge-invariant formulation of two-dimensional chiral U (N) gauge theories is analysed, for the special value of the regularization dependent parameter a where a close relation with the vector gauge theory is known to exist. We show using the canonical quantization of the model that the theory, which consists of free right fermions and interacting left fermions at the classical level, is equivalent to an interacting vector gauge theory plus free massless Dirac fermions at the quantum level. Furthermore, we show that, after redefinition of the charge value, this equivalence also holds in the abelian case for values of the parameter $a>1$. We also discuss the origin of the anomalous dimension at small distances found using the gauge-variant formulation of the abelian theory.

In the last years, there has been a lot of progress in the understanding of the behaviour of two-dimensional anomalous gauge theories. Jackiw and Rajaraman [1] demonstrated that in spite of the absence of gauge invariance the chiral abelian gauge model is a consistent and unitary theory provided the arbitrary parameter a, that appears in the regularization, does not take values lower than one. Later it was shown that in the path integral quantization procedure [2], the anomaly is absorbed while the group parameter of gauge transformations appears as a new dynamical field, not present at the classical level [3]. In this framework, the original formulation of the chiral theory may be seen as a particular gauge-fixing condition, the so-called unitary gauge [4]. More progress has been made in the quantization of the abelian model, using path integral methods and the bosonization procedure [5]. The theory admits a complete solution [6]. The spectrum consists of a massless and a massive state. The massive state is pseudoscalar and is related to the electromagnetic field strength $\mathscr{F}^{\mu\nu}$ in a similar way to the massive state of the Schwinger model [7]. In the gauge-variant formulation of the theory, it was also demonstrated that asymptotic left-handed fermions exist in the theory [8]. Moreover, it was shown, via the analysis of the propagator of the left-handed fermion, that the theory has an anomalous dimension at small distances, in contradistinction with the behaviour of fermions in the Schwinger model where asymptotic freedom takes place. These results show that the chiral theory has new and interesting properties in comparison with the Schwinger model.

However, some evidence of a close relation between the vector theory and the $a=2$ case of the chiral theory has been found. It was noticed that the gauge-invariant propagator of the left fermions and the gauge-invariant effective action of the gauge fields have the same expression in the chiral theory as in the vector theory [9,10]. In the general nonabelian case, similar results were found in ref. [11]. There, we proved that the structure of the constraints, the equal-time gauge current algebra, the gauge-invariant effective action of the gauge fields and the gauge-invariant propagator of the left fermions coincide with the ones obtained in the vector theory. Similar results were obtained in the path integral framework, using the fermionization of the Wess–Zumino field [10]. In this letter we will examine the canonical quantization of the U (N) theory in the $a=2$ case and of the abelian chiral theory in a more general case, namely $a>1$. We will show that a clear understanding of the theory can be obtained, once the hamiltonian is written in terms of currents. In addition, the origin of the above-mentioned anomalous dimension

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of the chiral theory at small distances will be discussed.

Let us first recall the results found in ref. [11]. The lagrangian density of the gauge-invariant formulation of the theory in its bosonized version [12,13] reads

$$\mathscr{L}=\mathrm{Tr}\bigg(-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{ae^2A_\mu A^\mu}{8\pi}+\frac{\partial_\mu g\partial^\mu g^{-1}}{8\pi} -\mathrm{i}e\frac{g^{-1}\partial^\mu g(g_{\mu\nu}+\epsilon_{\mu\nu})A^\nu}{4\pi}\bigg)+\mathscr{L}_{\mathrm{WZ}}(g) +\mathrm{Tr}\bigg(\partial_\mu\tilde{g}\partial^\mu\tilde{g}^{-1}\frac{(a-1)}{8\pi}-\mathrm{i}e\frac{\epsilon_{\mu\nu}\tilde{g}\partial^\nu\tilde{g}^{-1}A^\mu}{4\pi} -\mathrm{i}e\frac{\tilde{g}\partial_\mu\tilde{g}^{-1}A^\mu(a-1)}{4\pi}\bigg)+\mathscr{L}_{\mathrm{WZ}}(\tilde{g}), \quad (1)$$

where the same notation as in ref. [11] is used ($\epsilon_{01}=1$). Here the first part of the action is the bosonized version of the gauge-variant formulation, while the $\tilde{g}$ dependent part is obtained directly from the partition function of the bosonized theory considered in refs. [14,15], upon applying the Faddeev–Popov gauge fixing procedure in the same way as in ref. [3]. Both parts of the bosonic lagrangian have a Wess–Zumino term that we have symbolized with $\mathscr{L}_{\mathrm{WZ}}$.

From now on we will concentrate on the case $a=2$. Following the Dirac quantization procedure [14–16] it can be shown that there is a set of first-class constraints in the theory, namely

$$\pi_0^a=\omega_1^a, \quad (2)$$

$$(\mathscr{D}_1\pi_1+eJ_0)^a=\omega_2^a, \quad (3)$$

where ω_2^a is the Gauss law and J_μ is the gauge current. The first-class structure of the constraints is a reflection of the gauge invariance of the theory at the quantum level. The hamiltonian of the theory reads

$$H=\int \mathrm{d}x^1\bigg\{\tfrac{1}{2}\pi_1^a\pi_1^a-A_0^a\omega_2^a+\mathrm{Tr}\bigg[-2\pi(\hat{\tilde{\pi}}^{\mathrm{T}}\tilde{g}\hat{\tilde{\pi}}^{\mathrm{T}}\tilde{g}) +\frac{1}{8\pi}\partial_1\tilde{g}\partial_1\tilde{g}^{-1}-2\pi(\hat{\pi}^{\mathrm{T}}g\hat{\pi}^{\mathrm{T}}g) +\frac{\partial_1 g\partial_1 g^{-1}}{8\pi}+eA_1\bigg(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\frac{\mathrm{i}g^{-1}\partial_1 g}{4\pi}\bigg) +eA_1\bigg(\mathrm{i}\tilde{g}\hat{\tilde{\pi}}^{\mathrm{T}}-\mathrm{i}\frac{\tilde{g}\partial_1\tilde{g}^{-1}}{4\pi}\bigg)+e^2\frac{A_1^2}{2\pi}\bigg] +\lambda_1^a\omega_1^a+\lambda_2^a\omega_2^a\bigg\}, \quad (4)$$

where π_0, π_1, $\hat{\pi}_{ij}$ and $\hat{\tilde{\pi}}_{ij}$ are the conjugate momenta of A_0, A_1, g_{ij} and $\tilde{g}_{ij}$, respectively. The physical states are defined as those annihilated by the first-class constraints of the theory. The spatial and temporal component of the gauge current J_μ have a simple expression in terms of the conjugate momenta, namely

$$J_0=\bigg(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\frac{\mathrm{i}}{4\pi}g^{-1}\partial_1 g\bigg)+\bigg(-\mathrm{i}\tilde{g}\hat{\tilde{\pi}}^{\mathrm{T}}+\frac{\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1}}{4\pi}\bigg), \quad (5)$$

$$J_1=\bigg(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\mathrm{i}\frac{g^{-1}\partial_1 g}{4\pi}\bigg)-\bigg(-\mathrm{i}\tilde{g}\hat{\tilde{\pi}}^{\mathrm{T}}+\frac{\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1}}{4\pi}\bigg) +\frac{eA_1}{\pi}. \quad (6)$$

The equal-time gauge current algebra can be easily obtained from here, by using the nontrivial Poisson brackets of the conjugate momenta $\hat{\pi}_{ij}$ and $\hat{\tilde{\pi}}_{ij}$ [14]. The equal-time current algebra is given by [11]

$$[J_0^a(x),J_0^b(y)]_{\mathrm{t}}=\mathrm{i}f_{abc}J_0^c(x)\delta(x^1-y^1), \quad (7)$$

$$[J_0^a(x),J_1^b(y)]_{\mathrm{t}}=\mathrm{i}f_{abc}J_1^c(x)\delta(x^1-y^1) +\frac{\mathrm{i}\mathscr{D}_1^{ab}\delta(x^1-y^1)}{\pi}, \quad (8)$$

$$[J_1^a(x),J_1^b(y)]_{\mathrm{t}}=\mathrm{i}f_{abc}J_0^c(x)\delta(x^1-y^1), \quad (9)$$

or, defining $J_\pm=\tfrac{1}{2}(J_0\pm J_1)$, by

$$[J_\pm^a(x),J_\pm^b(y)]_{\mathrm{t}}=\mathrm{i}f_{abc}J_\pm^c(x)\delta(x^1-y^1) \pm\frac{\mathrm{i}\mathscr{D}_1^{ab}\delta(x^1-y^1)}{2\pi}, \quad (10)$$

$$[J_+^a(x),J_-^b(y)]_{\mathrm{t}}=0. \quad (11)$$

In ref. [11] we continued the quantization by going to the unitary gauge. This allowed us to prove the equivalence of this formulation with the gauge variant formulation. In this work we will keep the first-class structure of the constraints, because in this way it is easy to compare our results with the ones obtained in ref. [11] for the vector theory. The hamiltonian, eq. (4), may be expressed in terms of the gauge currents J_- and J_+ as

$$H=\int \mathrm{d}x^1\{[\tfrac{1}{2}\pi_1^a\pi_1^a-A_0^a\omega_2^a+\pi(J_+^aJ_+^a+J_-^aJ_-^a)+\lambda_1^a\omega_1^a+\lambda_2^a\omega_2^a]+[\pi(G_+^aG_+^a+G_-^aG_-^a)]\}\,. \quad (12)$$

In the above we have introduced the currents G_+ and G_-, whose expression in terms of the fields and their conjugate momenta is

$$G_+=\left(\mathrm{i}\hat{\tilde{\pi}}^{\mathrm{T}}\tilde{g}-\frac{\mathrm{i}}{4\pi}\tilde{g}^{-1}\partial_1\tilde{g}\right), \quad (13)$$

$$G_-=\left(-\mathrm{i}g\tilde{\pi}^{\mathrm{T}}+\frac{\mathrm{i}g\partial_1 g^{-1}}{4\pi}\right). \quad (14)$$

The hamiltonian is positive, when acting on physical states. It should be noticed that the currents $G_\pm$ do not depend explicitly on the gauge fields. These new currents commute with π_1, with the gauge currents and with the constraints of the theory. That is to say, they are gauge singlets. On the other hand, they satisfy the following algebra:

$$[G_\pm^a(x),G_\pm^b(y)]_t=\mathrm{i}f_{abc}G_\pm^c(x)\delta(x^1-y^1)\pm\frac{\mathrm{i}\delta^{ab}\partial_1\delta(x^1-y^1)}{2\pi}, \quad (15)$$

$$[G_+^a(x),G_-^b(y)]=0\,, \quad (16)$$

which is the algebra of free fermionic currents [12]. The hamiltonian is given as a sum of two parts, that commute with each other. We will call the first expression between square brackets in eq. (12) H_1 and the second expression between square brackets H_2. The time evolution of the currents $G_\pm$ is given only by H_2. Taking into account the current algebra, eqs. (15),(16), we see that the second part of the hamiltonian is nothing but the hamiltonian of a conformal invariant theory of free massless fermions, given in the Sugawara form.

A complete understanding of the theory is obtained if we remember the results obtained in ref. [11] for the vector U(N) theory. There we showed that the equal-time gauge current algebra and the structure of constraints of the vector theory are identical to the ones we obtained in the chiral theory, eqs. (2), (3), (10), (11). The hamiltonian of the vector theory may be given in terms of the gauge current as

$$H=\int \mathrm{d}x^1[\tfrac{1}{2}\pi_1^a\pi_1^a-A_0^a\omega_2^a+\pi(J_+^aJ_+^a+J_-^aJ_-^a)+\lambda_1^a\omega_1^a+\lambda_2^a\omega_2^a]\,. \quad (17)$$

This expression coincides with the one we obtained for the first part of the hamiltonian of the chiral theory, H_1. Moreover, the equal-time commutator of the gauge currents with π_1 and with the constraints of the theory also coincides with the one obtained in the vector U(N) theory. From the structure of the constraints, the equal-time algebra of currents and the expression of the hamiltonian we can conclude that the quantum theory of the nonabelian $a=2$ chiral U(N) theory in two dimensions is equivalent to a vector U(N) theory, plus free massless Dirac fermions [#1].

This result can be extended to the general $a>1$ case of the abelian theory. In this case the hamiltonian [11] of the theory is given by eq. (12) when a redefinition of the charge in the Gauss law is made, namely

$$\omega_2=\partial_1\pi_1+e\frac{aJ_0}{2\sqrt{a-1}}\,. \quad (18)$$

Moreover, after the above redefinition of the charge, the current algebra and the equal-time commutator of the gauge and free currents with π_1 and the constraints are the same as the ones in the $a=2$ chiral theory. The gauge and free currents in terms of the conjugate momenta read [#2]

$$J_0=\frac{\sqrt{4(a-1)}}{a}\left[\left(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\frac{\mathrm{i}}{4\pi}g^{-1}\partial_1 g\right)+\left(-\mathrm{i}\tilde{g}\hat{\tilde{\pi}}^{\mathrm{T}}+\frac{\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1}}{4\pi}\right)\right], \quad (19)$$

[#1] Indications of this equivalence have been given previously, by totally different methods, in ref. [10].

[#2] In the abelian theory the following relations are fulfilled: $g=\mathrm{e}^{\mathrm{i}\theta}$, $\mathrm{i}\hat{\pi}^{\mathrm{T}}g=\mathrm{i}g\hat{\pi}^{\mathrm{T}}=\pi_\theta$, $\mathrm{i}g\partial_1 g^{-1}=-\mathrm{i}g^{-1}\partial_1 g=\partial_1\theta$. Similar relations are fulfilled for $\tilde{g}=\mathrm{e}^{\mathrm{i}\alpha}$.

$$J_1=\frac{\sqrt{4(a-1)}}{a}\left[\left(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\mathrm{i}\frac{g^{-1}\partial_1 g}{4\pi}\right)\right.$$
$$-\left(-\frac{\mathrm{i}\tilde{g}\hat{\hat{\pi}}^{\mathrm{T}}}{a-1}+\frac{\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1}(a-1)}{4\pi}\right)$$
$$\left.+\frac{eA_1a^2}{4\pi(a-1)}\right], \tag{20}$$

$$G_-=\left(-\mathrm{i}g\hat{\pi}^{\mathrm{T}}+\frac{\mathrm{i}g\partial_1 g^{-1}}{4\pi}\right), \tag{21}$$

$$G_+=\frac{a-2}{a}\left(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\frac{\mathrm{i}g^{-1}\partial_1 g}{4\pi}\right)$$
$$+\left(\frac{2\mathrm{i}\hat{\hat{\pi}}^{\mathrm{T}}\tilde{g}}{a}-\frac{2(a-1)\mathrm{i}\tilde{g}^{-1}\partial_1\tilde{g}}{4\pi a}\right). \tag{22}$$

Thus, after a redefinition of the charge, the equivalence of the abelian chiral theory with the vector theory plus free fermions is preserved. The physical spectrum of the abelian theory consists then of a massless state and a massive state of mass $m^2=e^2a^2/4\pi(a-1)$. The gauge and free currents couple only to the massive and massless states respectively. As in the vector theory, the screening phenomenon also takes place in this theory, and only charge-zero states are observed in the spectrum.

One must explain the origin of the anomalous dimension at low distances found in the theory in its variant formulation. This behaviour appears paradoxical from the point of view of the above equivalence with the vector gauge theory plus a free fermion theory. On the other hand, it seems likely that one could be able to identify the asymptotic fermionic states found in the gauge-variant formulation with the free fermions of the theory discussed above.

Before proceeding with the analysis of the fermion propagator, let us show how one can reinterpret the two poles of the gauge boson propagator in the unitary gauge of the abelian theory. In the unitary gauge the additional constraints

$$(\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1})=(\omega_3), \tag{23}$$

$$\left(-\mathrm{i}\tilde{g}\hat{\hat{\pi}}^{\mathrm{T}}-\frac{e(a-1)A_0}{4\pi}-\frac{eA_1}{4\pi}\right)=(\omega_4) \tag{24}$$

must be imposed on the theory.The constraints are now of second class and, following the Dirac procedure, the relation $\omega_i=0$ should be imposed at the operator level. Then, A_- and A_+ may be rewritten in terms of the physical currents $J_\pm$ and $G_\pm$ as

$$e\frac{(A_0-A_1)}{4\pi}=\frac{J_-}{\sqrt{a-1}}, \tag{25}$$

$$e\frac{(A_0+A_1)}{4\pi}=\frac{(a-2)J_+}{a\sqrt{(a-1)}}-\frac{2G_+}{a}. \tag{26}$$

Thus, the correlation function of the vector field in the unitary gauge must show the existence of both the massive state of the equivalent vector gauge theory and of a massless, asymptotic, state. This is, in fact, what was obtained in ref. [1]. Let us discuss this is more detail. In momentum space, the correlation function of the gauge bosons in the unitary gauge reads [1]

$$\left\langle \mathrm{T}\frac{e^2}{4\pi^2}A^\mu(x)A^\nu(0)\right\rangle(k)=\frac{ie^2}{(k^2-m^2)4\pi^2}$$
$$\times\left(-g^{\mu\nu}+\frac{1}{a-1}\left\{\left[k^\mu k^\nu\left(\frac{4\pi}{e^2}-\frac{2}{k^2}\right)\right]\right.\right.$$
$$\left.\left.+\frac{\epsilon^{\mu\alpha}k_\alpha k^\nu}{k^2}+\frac{\epsilon^{\nu\alpha}k_\alpha k^\mu}{k^2}\right\}\right), \tag{27}$$

where we have included the factor $e^2/4\pi^2$ for future purposes and $m^2=e^2a^2/4\pi(a-1)$. From now on we will define $x^\mu_\pm=x_\nu(g^{\nu\mu}\pm\epsilon^{\nu\mu})$. The correlation functions of A^μ_+ and A^μ_- are

$$\left\langle \mathrm{T}\frac{e^2}{4\pi^2}A^\mu_-(x)A^\nu_-(0)\right\rangle(k)$$
$$=\frac{\mathrm{i}k^\mu_-k^\nu_-}{\pi(k^2-m^2)(a-1)}, \tag{28}$$

$$\left\langle \mathrm{T}\frac{e^2}{4\pi^2}A^\mu_+(x)A^\nu_+(0)\right\rangle(k)$$
$$=\frac{\mathrm{i}4k^\mu_+k^\nu_+}{\pi a^2k^2}+\frac{\mathrm{i}(a-2)^2k^\mu_+k^\nu_+}{\pi(a-1)a^2(k^2-m^2)}. \tag{29}$$

Eqs. (28), (29) are consistent with the operator relations given in eqs. (25), (26). In fact, after being multiplied by constant factors the correlation function of A_- is nothing but the correlation function of the right-handed gauge current in the vector theory [17], while the correlation function of A_+ is a sum of correlations of a free left-handed current and a left-handed gauge current in the vector theory. The factor that appears in front of each term is in complete agreement with eqs. (25), (26).

Let us return to the original fermionic version of the theory. The partition function of the model reads [5]

$$\mathcal{Z}=\int \mathcal{D}A_\mu\,\mathcal{D}\alpha\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\exp(\mathrm{i}S)\,, \tag{30}$$

and the action S of the model is given by

$$S=\int \mathrm{d}^2x\left(-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}+\mathrm{i}\bar{\psi}\not{D}_{\mathrm{L}}\psi+\frac{(a-1)(\partial_\mu\alpha)^2}{8\pi}\right.$$
$$\left.-e\frac{A_\mu[g^{\mu\nu}(a-1)+\epsilon^{\mu\nu}]\partial_\nu\alpha}{4\pi}\right)\,, \tag{31}$$

where

$$\not{D}_{\mathrm{L(R)}}=\not{\partial}-\mathrm{i}e\not{A}\frac{(1-\gamma_5)}{2}\quad\left(\not{\partial}-\mathrm{i}e\not{A}\frac{(1+\gamma_5)}{2}\right)$$

and $\gamma^\mu\gamma_5=\epsilon^{\mu\nu}\gamma_\nu$. The field α is related to $\tilde{g}$ by $\tilde{g}=\mathrm{e}^{\mathrm{i}\alpha}$. In this formulation, the gauge variance of the Wess–Zumino action is exactly canceled by the jacobian of the fermionic measure $\mathcal{D}\psi$ [18] under gauge transformations. As we have already said, the first proof of the existence of asymptotic fermion states in the theory was given by computing the correlations of the physical, composite operator [8]

$$\psi_{\mathrm{L}}^{\mathrm{phys.}}=\hat{g}\psi_{\mathrm{L}}\,, \tag{32}$$

which is neutral under gauge transformations. In the abelian chiral theory, one can define the new physical currents

$$J_\mu^5=\epsilon_{\mu\nu}J^\nu,\quad G_\mu^5=\epsilon_{\mu\nu}G^\nu\,, \tag{33}$$

and the charges

$$Q_{\mathrm{f}}=\int G_0\,\mathrm{d}x^1,\quad Q_{\mathrm{f}}^5=\int G_0^5\,\mathrm{d}x^1\,. \tag{34}$$

Here J_μ^5 corresponds to the gauge-invariant chiral current in the vector theory and Q_{f} and Q_{f}^5 generate global vector and chiral transformations in the free fermion sector. It follows from the hamiltonian and the current algebra that

$$\partial^\mu J_\mu^5=\left(\frac{ea}{2\sqrt{a-1}}\right)\frac{\epsilon^{\mu\nu}F_{\mu\nu}}{2\pi}\,,$$
$$\partial^\mu J_\mu=\partial^\mu G_\mu=\partial^\mu G_\mu^5=0\,, \tag{35}$$

in complete agreement with the equivalence proved above. The theory is invariant under the global transformation generated by Q_{f} and Q_{f}^5. From the expression of $G_\pm$, eqs. (21), (22), we see that g transforms as $g\to\exp[\mathrm{i}(a-2)\beta_{\mathrm{L}}/a-\mathrm{i}\beta_{\mathrm{R}}]g$ under this global $\mathrm{U}(1)_{\mathrm{L}}\otimes\mathrm{U}(1)_{\mathrm{R}}$ symmetry, while $\tilde{g}\to\exp(\mathrm{i}2\beta_{\mathrm{L}}/a)\tilde{g}$. From the bosonization procedure [6], it follows that the transformation properties of ψ are given by $\psi_{\mathrm{L}}\to\exp[\mathrm{i}(a-2)\beta_{\mathrm{L}}/a]\psi_{\mathrm{L}}$ and $\psi_{\mathrm{R}}\to\exp(\mathrm{i}\beta_{\mathrm{R}})\psi_{\mathrm{R}}$. This global symmetry is part of a restricted local symmetry of the theory, which is obtained by promoting the global phases β_{L} and β_{R} to local phases $\beta_{\mathrm{L}}(x_-)$ and $\beta_{\mathrm{R}}(x_+)$. This is a generalization, for all values of $a>1$, of the restricted local symmetry found in the $a=2$ case [10]. This restricted local symmetry reflects the presence of free massless left-handed and right-handed fermions in the physical spectrum of the theory. Unlike the right-handed free fermion, which is identified with ψ_{R}, the left-handed free fermion operator cannot be identified with any field appearing explicitly in the lagrangian density. However, its expression may be obtained from a correct identification of the physical operators of the theory, in a way similar to the analysis made in ref. [6].

It also follows from the bosonization procedure [6] that $\psi_{\mathrm{L}}^{\mathrm{phys.}}$ carriers the quantum numbers of a left-handed fermion in the free sector and $(2/\sqrt{a-1})$ units of chiral charge in the interacting sector. The transformation properties of $\psi_{\mathrm{L}}^{\mathrm{phys.}}$ suggest to us the following picture: The composite field $\psi_{\mathrm{L}}^{\mathrm{phys.}}(x)$ is given by a local product of a physical, left-handed free fermion and a bosonic, physical operator which has no dependence on the free fermion fields. The correlation function of the composite fermion is given as a product of the correlation function of the free left fermion and the correlation function of the bosonic operator in the equivalent vector gauge theory. The scaling behaviour at small distances of the bosonic operator appears as an anomalous dimension in the gauge-variant formulation of the theory. Furthermore, the quantum numbers of $\psi_{\mathrm{L}}^{\mathrm{phys.}}$, together with the operator relations, eqs. (25), (26), explain the commutation relations

$$\left[\frac{eA_1(x)}{2\pi}, \psi_L^{\text{phys.}}(y)\right]_t = 0 ,$$

$$\left[\frac{eA_0(x)}{2\pi}, \psi_L^{\text{phys.}}(y)\right]_t$$

$$= -\frac{2i}{(a-1)} \psi_L^{\text{phys.}}(x)\delta(x^1 - y^1) , \qquad (36)$$

found in the unitary gauge of the abelian theory [10].

From now on we will concentrate on the $a=2$ case. We will follow another strategy to analyse the behaviour of the propagator of the composite fermion. We will fermionize the Wess–Zumino term. This was already done a few years ago in ref. [19] and also more recently in ref. [10]. The lagrangian density of the equivalent fermionic theory reads

$$\mathscr{L} = i\bar{\psi}\not{D}_L\psi + i\bar{\psi}_{WZ}\not{D}_R\psi_{WZ} , \qquad (37)$$

where we have called ψ_{WZ} the fermion that appears from the fermionization of the Wess–Zumino field. Both fermion theories are gauge invariant at the classical level. At the quantum level one must introduce regulators for both interacting fermion theories. Consistent regulators for both sectors can be shown to preserve the gauge invariance at the quantum level of the whole theory [#3]. The jacobian under chiral gauge transformation of the left-handed fields ψ_L is exactly canceled with the one that comes from the right-handed fields ψ_R^{WZ}. The theory acquires then a left–right gauge symmetry at the quantum level. The gauge-invariant effective action of the gauge fields coincides with the one obtained in the Schwinger model [11], while the left-handed field ψ_L^{WZ} and the right-handed field ψ_R propagate as free massles fermions. In other words, the theory is equivalent to a vector gauge theory plus a free Dirac fermion as we have shown above.

For studying the correlation function of the composite field $\psi_L^{\text{phys.}}$ in the framework of the fermionic theory we must follow a correct fermionization procedure that allows the identification of bosonic operators in terms of fermionic ones. The operator corresponding to $\tilde{g}$ in the fermionic theory is given by

$$\tilde{g} = K\bar{\psi}_R^{WZ}\psi_L^{WZ} , \qquad (38)$$

where K is a renormalization mass that depends on the normal-ordering prescription [6]. The correlation function may be obtained from the usual path integral expression. Due to the local left–right symmetry of the theory, one must regularize the local product in eq. (32) in a gauge-invariant way. It is now easy to see that the correlation function of the composite, gauge-invariant fermionic field factorizes in a part corresponding to the left-handed free fermion ψ_L^{WZ} and a part that depends on the interacting fermion fields as follows:

$$\langle \mathrm{T}\, \psi_L^{\text{phys.}}(x)\bar{\psi}_L^{\text{phys.}}(0)\rangle$$

$$= (\langle \mathrm{T}\, \bar{\chi}_R(x)\chi_L(x)\, \bar{\chi}_L(0)\chi_R(0)\rangle_{SM})$$

$$\times \langle \mathrm{T}\, \psi_L^{WZ}(x)\bar{\psi}_L^{WZ}(0)\rangle , \qquad (39)$$

where $\langle O\rangle_{SM}$ means that the correlation is the same as the one in the Schwinger model and we have omitted a normalization constant. The last term in eq. (39) is the propagator of the free left fermions, while the first factor was calculated in ref. [17]. The final result is then, apart from a renormalization constant,

$$\langle \mathrm{T}\, \psi_L^{\text{phys.}}(x)\bar{\psi}_L^{\text{phys.}}(0)\rangle$$

$$= (\exp[i4\pi\Delta_F(x^2, m^2)])S_L(x, 0) , \qquad (40)$$

which coincides with the result obtained in ref. [8]. The correlation function of the composite field $\psi_L^{\text{phys.}}$ corresponds to a sum of correlation functions in which one free massless left fermion and any number of free massive bosons (0, 1, 2, ...) are exchanged between the points 0 and x. Thus, in the $a=2$ case of the chiral theory, the anomalous dimension of the operator $\psi_L^{\text{phys.}}$ is a consequence of its compositeness. In fact, the anomalous dimension observed in the gauge-variant formulation is given by the well-known scaling behaviour of the operator $\bar{\chi}_R\chi_L$ in the equivalent vector gauge theory. The asymptotic state is a left-handed free fermion, as suggested by our previous discussions.

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[#3] Using the point splitting method, the bosonic sector in eq. (31) ($a=2$) is equivalent to the fermionized WZ theory, only if the regularization $\bar{\psi}_R^{WZ}(x-\epsilon/2)\gamma^\mu[\exp(ie\int_{x-\epsilon/2}^{x+\epsilon/2} A_+ dx)] \times \psi_R^{WZ}(x+\epsilon/2)$ for the right-handed current in the ψ^{WZ} theory is used.

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