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Electroweak baryon number violation at finite temperature

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Abstract. We consider baryon and lepton number violating processes in the electroweak theory induced by gauge and Higgs fields passing the sphaleron solution at finite temperature. We show that for temperatures larger than 19 GeV the anomalous baryon and lepton number violating processes are suppressed by the Boltzmann factor $\exp(-\beta E_{sp})$, where E_{sp} is the sphaleron energy, rather than by the instanton tunneling factor $\exp(-8\pi^2/g^2)$. We caculate the rate of baryon and lepton number violating processes at finite temperature and determine the freezing temperature of the anomalous processes in the early universe as a function of the Higgs mass. We compare the freezing temperature with the critical temperature of the electroweak phase transition infered from the one-loop finite-temperature effective potential. We obtain a critical Higgs mass of the order of 100 GeV, slightly depending on the top mass and the magnitude of the pre-exponential factor in the rate of the B non-conservation, above which the anomalous processes are certainly in equilibrium after the electroweak phase transition. Assuming that the temperature-dependence of the sphaleron energy is given by that found from the one-loop finite-temperature effective potential, this critical Higgs mass is lowered to a value of the order of 50 GeV.

1 Introduction

Baryon and lepton number are not conserved at quantum level in the standard electroweak theory [1]. In fact, if gauge and Higgs fields fluctuate over the barrier between topological inequivalent vacua [2, 3] which are characterized by different winding numbers then due to the anomaly [4, 5] of the baryon and lepton number currents (unless otherwise stated we take the limit $\theta_w = 0$)

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = -f_g \frac{g^2}{16\pi^2} \operatorname{tr}(F_{\mu\nu} F^{*\mu\nu}), \tag{1.1}$$

where f_g is the number of generations, $F^{(*)}_{\mu\nu}$ is the (dual) $SU(2)$ field strength, there will be baryon and lepton number violating processes according to the selection rule

$$B(t_2) - B(t_1) = L(t_2) - L(t_1) = -f_g Q(t_1, t_2). \tag{1.2}$$

$B(L)$ denotes the number of baryons (leptons) minus the number of anti-baryons (anti-leptons), and $Q(t_1, t_2)$ denotes the winding number difference between two spacelike hypersurfaces at equal time (t_1 and t_2)

$$Q(t_1, t_2) = \frac{g^2}{16\pi^2} \int_{t_1}^{t_2} dt \int d^3x \operatorname{tr}(F_{\mu\nu} F^{*\mu\nu}). \tag{1.3}$$

From (1.2) it is clear that $B-L$ is conserved whereas $B+L$ is violated.

At zero temperature $B+L$ violating processes are associated with instantons which describe tunneling between vacuas and are therefore exponentially suppressed by the euclidean action $S_E = 8\pi^2/g^2$ [1]. This makes the effects unobservably small. .

However at high temperatures the situation seems to be different. A key role in the calculation of $B+L$ violating processes at high temperatures plays a new static, but unstable solution of the fundamental $SU(2)$ Higgs theory [6–10], called sphaleron [10], which correspond to a saddle point configuration of the potential between two topological distinct vacua [9, 10]. It has been argued by Kuzmin et al. [11] that thermal fluctuations cause classical transitions from one vacuum to another passing via the sphaleron. The effects of the sphaleron should be suppressed by the Boltzmann factor $\exp(-\beta E_{sp})$, where $E_{sp} = \mathcal{O}(m_w/\alpha_w)$, rather than by the factor $\exp(-8\pi^2/g^2)$. Arnold and McLerran [12] and one of us (A.R.) [13] have estimated the prefactors before the Boltzmann factor and have confirmed the observation of Kuzmin et al.

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[11] that the rate of $B+L$ violating processes exceeds the Hubble expansion rate of the universe at temperature above $\mathcal{O}(200)$ GeV.

An important question is at which temperature the transition between instanton dominated and sphaleron dominated baryon and lepton number violating processes appears [14]. A preliminary investigation of this question can be found in [15]. We derive a formula which can be used at all temperatures and which includes both quantum tunneling and classical thermal transitions.

Another important question is the dependence of the freezing temperature of the anomalous processes on the Higgs mass. This has to be compared to the critical temperature of the electroweak phase transition. It is possible to define a critical Higgs mass m_{crit} in that way that for a Higgs mass larger (smaller) than m_{crit} the anomalous processes are in (out of) equilibrium after the electroweak phase transition. The value of m_{crit} is important for the following reason: if $m_H < m_{\text{crit}}$ then all the baryon asymmetry of the universe (BAU) generated by the $B-L$ conserving decays of leptoquarks in grand unified theories has been washed out to the freezing moment of the anomalous electroweak processes [16]. In the opposite case a part of the BAU proportional to the ratio of the mass square of the heaviest lepton to the freezing temperature survives [16]. In addition, it was pointed out by Shaposhinkov [17] that the BAU can be produced already in the standard electroweak theory if the phase transition is of first order. This requires a non-trivial degeneracy of the high temperature ground state with respect to the Chern-Simons number. The BAU generated in this way survives to the present time only if the Higgs mass satisfies the upper bound $m_H < m_{\text{crit}}$. A strikingly low value of about 45 GeV has been given for m_{crit} [18]. A Higgs boson in this mass range should be observed relatively soon at LEP, however. Therefore it seems to be very important to point out clearly, to what extend this estimate of m_{crit} could be trusted.

The bound on m_{crit} [18] has been obtained on the basis of various assumptions. The starting point is a high temperature approximation for the one-loop temperature-dependent W-mass which is obtained by the replacement of $m_w = (1/2)gv$ by the effective mass $m_w(T) = (1/2)g(T)v(T)$ where the temperature-dependent expectation value $v(T)$ follows from the one-loop finite-temperature effective potential. The temperature-dependent barrier height between neighbouring topologically inequivalent vacua is obtained from the sphaleron energy by replacing m_w and α_w by their temperature dependent running values [11–13, 18]. This is the crucial assumption which is not justified in general. For the $1+1$ dimensional Abelian Higgs model it has been shown [19] that this procedure does not reproduce the correct coefficient of the leading one-loop term proportional to T. This could be a consequence of the severe infrared problems of this model and one could argue that this result is irrelevant for the $3+1$ dimensional theory. In this case the leading temperature contribution would indeed arise if the one-loop free energy of the sphaleron is evaluated as a weak-field expansion in powers of the external field. This temperature-dependent term is directly related to the quadratically divergent contribution. However, the appearence of an unstable mode for fluctuations around the sphaleron indicates that the weak-field expansion presumably breaks down before the sphaleron is reached. To clarify this problem a direct calculation of the one-loop free energy for a sphaleron background is urgently needed. For the special case of spherical symmetric fluctuations of the gauge and Higgs fields such a calculation has been carried out [20]. In this case only a weak temperature dependence is found, but this does not finally settle this question since there is also no quadratic divergence in this approximation. In order to determine the influence of the temperature-dependence we calculate the rate of the anomalous processes with the zero temperature W-mass as well as with the temperature-dependent W-mass found from the effective potential.

Apart from the temperature-dependence of the barrier height a substantial uncertainty arises from the fact that not much is known on the order of the electroweak phase transition and the critical temperature. This information should eventually become available from numerical lattice studies. Results available so far [21, 22] do not indicate a first order transition but it is presumably too early to draw definite conclusions. In particular, one should not base estimates of limits on the Higgs mass too strongly on the value of the critical temperature found from the one-loop finite-temperature effective potential.

In order to get the critical Higgs mass one needs the freezing temperature of the anomalous processes and the critical temperature of the electroweak phase transition as a function of the Higgs mass. We get an upper bound $m_{\text{crit}} = 97(113)$ GeV for a top mass $m_t = 44$ GeV ($m_t \simeq m_w$), with an uncertainty of a few percent due to an unknown constant in the pre-exponential factor in the rate of the anomalous processes, if we work with the zero temperature W-mass and assume that the one-loop finite-temperature effecitve potential correctly gives the critical temperature. If we use the temperature-dependence for the W-mass found from the one-loop effective potential this upper bound is lowered to $m_{\text{crit}} = 47(48)$ GeV, in accordance with [18].

The paper is organized as follows: In Sect. 2 we review the spherical symmetric ansatz in the fundamental $SU(2)$ Higgs model [20, 23, 24]. The sphaleron as well as the unstable eigenmode of the second functional derivative of the static Hamiltonian are spherically symmetric. We represent a simple variational ansatz for the sphaleron radial functions with the help of which many analytical results can be obtained. We consider massless left-handed fermions in a spherical symmetric background field which passes the sphaleron adiabatically. We show that there exists a normalizable solution of the Dirac equation where the energy of the solution crosses zero as the background field passes the sphaleron. This level crossing gives a physical interpretation of the anomaly (1.2) [25–28] and demonstrates the importance of the sphaleron for the anomalous processes. In Sect. 3 we specialize the path passing the sphaleron. We consider the energy functional along the unstable eigenmode in the sphaleron background found in [20]. In this way we obtain a one-dimensional potential barrier which has the form of a double well. This has to be overcome by quantum or thermal fluctuations. We calculate the winding number difference between configurations which sit on the minima of the potential and the energy of the normalizable solution of the Dirac equation in the background field. In Sect. 4 we calculate the one-dimensional transition rate over the potential barrier found in Sect. 3. It is observed that for temperatures below $T_0 = \Omega/2\pi$, where Ω is the magnitude of the negative eigenvalue of the unstable mode, the transition is dominated by quantum tunneling whereas for $T > T_0$ thermal transitions dominate, in accordance with the general considerations in [29]. The transition region is very narrow. In Sect. 5 we generalize the results to field theory. We take into account the normalization of the zero modes in the sphaleron background which can be calculated analytically with the help of the variational ansatz for the sphaleron radial functions from Sect. 2. We show where the effective temperature-dependence of the sphaleron energy arises in our formalism and point out that it is not strictly proven that it is the same temperature-dependence which follows from the one-loop finite-temperature effective potential. In Sect. 6 we calculate the rate of anomalous B and L violating processes for the case of a $B - L = 0$ universe. We determine the freezing temperature of the anomalous processes as a function of λ/g^2, where λ is the Higgs self coupling and g is the $SU(2)$ gauge coupling. We compare the freezing temperature with the critical temperature obtained from the one-loop finite-temperature effective potential and get, as already quoted, $m_{\text{crit}} = 97(113)$ GeV, if we use the zero temperature W-mass. This represents actually a quite save *upper bound* on the critical Higgs mass. Using the temperature-dependence found from the one-loop finite-temperature effective potential we establish the result of [18], $m_{\text{crit}} = 47(48)$ GeV. Section 7 contains the conclusions.

2 Spherical symmetric ansatz

We consider the fundamental $SU(2)$ Higgs theory, given by the Lagrangian density

$$\mathscr{L} = -\tfrac{1}{2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda\left(\Phi^\dagger \Phi - \frac{v^2}{2}\right)^2, \tag{2.1}$$

where

$$F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu], \tag{2.2a}$$

$$W_\mu = (\tau^a/2) W^a_\mu, \tag{2.2b}$$

$$D_\mu = \partial_\mu - ig W_\mu. \tag{2.2c}$$

The general spherical symmetric ansatz in the temporal gauge ($W_0 = 0$) is [20, 23, 24]

$$\mathbf{W}(\mathbf{x}, t) = \frac{1}{2g}\left[\frac{1 - f_A(r,t)}{r}(\mathbf{n} \times \tau) + \frac{f_B(r,t)}{r}(\tau - (\mathbf{n}\tau)\mathbf{n}) + f_C(r,t)(\mathbf{n}\,\tau)\mathbf{n}\right], \tag{2.3a}$$

$$\Phi(\mathbf{x}, t) = \frac{v}{\sqrt{2}}[H(r,t) + K(r,t) i(\mathbf{n}\tau)]\begin{pmatrix}0\\1\end{pmatrix}, \tag{2.3b}$$

where $\mathbf{n} = \mathbf{x}/r$.

The field equations for H, K, and f_i, $i = A, B, C$, can be found in [20, 23, 24]. They follow either directly by inserting the ansatz (2.3) in the general field equations obtained from (2.1), or by variation of the effective Lagrangian, which can be found by inserting (2.3) in (2.1),

$$\begin{aligned} L = \frac{4\pi}{g^2}\int_0^\infty dr \Bigg\{ & \dot f_A^2 + \dot f_B^2 + \frac{r^2}{2}\dot f_C^2 + 2m_w^2 r^2[\dot H^2 + \dot K^2] \\ & - \left[(f'_A + f_C f_B)^2 + (f'_B - f_C f_A)^2 + \frac{(f_A^2 + f_B^2 - 1)^2}{2r^2}\right] \\ & - 2m_w^2 r^2\Bigg[(H' + \tfrac{1}{2}f_C K)^2 + (K' - \tfrac{1}{2}f_C H)^2 \\ & + \frac{1}{2r^2}(Hf_A + Kf_B - H)^2 + \frac{1}{2r^2}(Kf_A - Hf_B + K)^2\Bigg] \\ & - \frac{(m_w m_H)^2}{2} r^2 (H^2 + K^2 - 1)^2 \Bigg\}, \end{aligned} \tag{2.4}$$

where $m_w = (1/2)gv$ and $m_H = \sqrt{2\lambda}v$ denote the classical W-boson and Higgs boson masses, respectively. A dot denotes the derivative with respect to t and a prime denotes the derivative with respect to r.

The sphaleron [6–10] corresponds to

$$\begin{aligned} f_A(r,t) &= f_A^{\text{sp}}(r) = 1 - 2f(r), \\ K(r,t) &= K^{\text{sp}}(r) = h(r), \\ f_B^{\text{sp}} &= f_C^{\text{sp}} = H^{\text{sp}} \equiv 0, \end{aligned} \tag{2.5}$$

where the functions $f(r)$ and $h(r)$ have to be determined numerically. A simple variational ansatz for f and h,

$$f(\xi) = \frac{\xi^2}{\xi^2 + a^2}, \tag{2.6a}$$

$$h(\xi) = \frac{\xi}{\sqrt{\xi^2 + b^2}}, \tag{2.6b}$$

where $\xi = m_w r$, gives for the sphaleron energy

$$E_{sp}(a,b)=\left[\frac{3}{4a}+\frac{b}{8}+a\left(1+\frac{b}{a}\right)^{-2}+\frac{\lambda}{g^2}b^3\right]\pi\frac{m_w}{\alpha_w}. \tag{2.7}$$

The sphaleron represents a minimum of the energy functional for fields restricted to the ansatz (2.5). The values of the variational parameters a and b which minimize the energy (2.7) are given in Fig. 1. In Fig. 2 we have plotted the corresponding sphaleron energies from (2.7), for comparison the numerical results, E_{sp}^{num} [10, 24], and E_{sp}^{int} which has been obtained by interpolating the numerical results. The latter values for the sphaleron energies will be used later on for the calculation of the rate of the anomalous processes. As can be seen, the trial functions (2.6) give reasonable approximations to the sphaleron radial functions, and the sphaleron energies agree with the numerical results within a few percents. The advantage of our ansatz is that we have a closed expression with the help of which we can do some analytical calculations (see below).

Now let us consider gauge and Higgs fields which pass the sphaleron [20]

$$\begin{aligned}
f_A(r,t)&=f_A^{sp}(r)+r\phi_A(r,t),\\
f_B(r,t)&=r\phi_B(r,t),\\
f_C(r,t)&=\sqrt{2}\phi_C(r,t),\\
H(r,t)&=\frac{1}{\sqrt{2}m_w}\phi_H(r,t),\\
K(r,t)&=K^{sp}(r)+\frac{1}{\sqrt{2}m_w}\phi_K(r,t),
\end{aligned} \tag{2.8}$$

where $\phi_i, i=A,\ldots,K$ obey the boundary conditions $r\phi_i\to 0$ for $r\to 0$ and $\phi_i\to 0$ for $r\to\infty$. The normalizations of ϕ_i are chosen such that the kinetic Hamiltonian for the modes passing the sphaleron is given by

$$H_{kin}=\frac{4\pi}{g^2}\int_0^\infty r^2dr\sum_{i=A}^{K}\dot{\phi}_i^2. \tag{2.9}$$

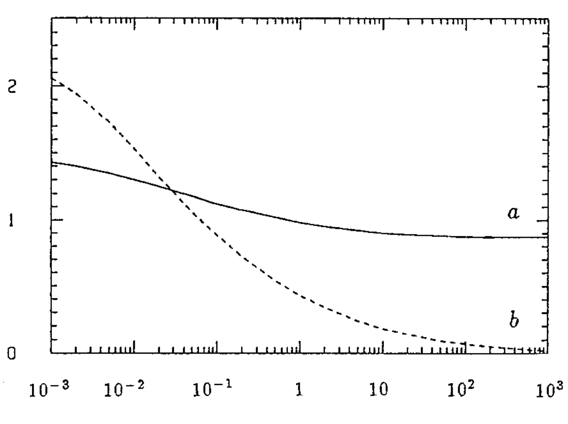

Fig. 1. Sphaleron variational parameters

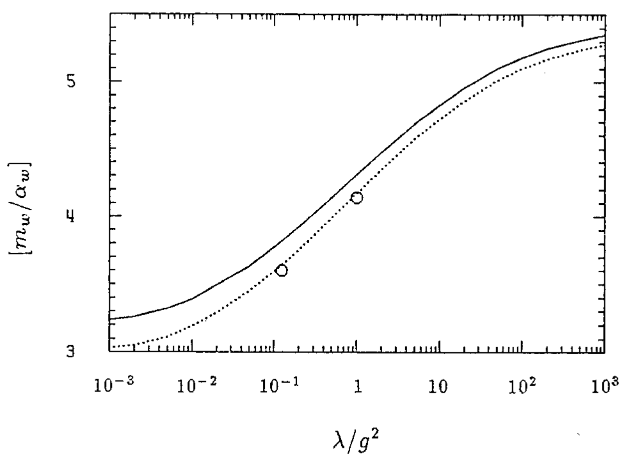

Fig. 2. Sphaleron energy. The solid line corresponds to the variational result (2.7), the circles are the numerical results [10, 24], and the dots represent the interpolation of the numerical results, E_{sp}^{int}

Consider the Dirac equation for a massless doublett in the background (2.3a) and (2.8),

$$i\sigma^\mu(\partial_\mu-igW_\mu)\psi_L=0, \tag{2.10}$$

where $\sigma^\mu=(\mathbf{1},\boldsymbol{\sigma})$ acts in spinor space. It has been shown [30, 31] that (2.10) has a normalizable zero energy solution in the background of the sphaleron (2.5), i.e. for $\phi_i\equiv 0$. It has the form

$$\psi_{La\alpha}^{(0)}=\varepsilon_{a\alpha}u(r), \tag{2.11}$$

where $\alpha=1,2$ and $a=1,2$, are spinor and weak isospin indices, respectively, $u(r)$ is given by

$$u(r)=N\exp\left\{-2\int_0^r dr'\frac{f(r')}{r'}\right\}, \tag{2.12}$$

where N is some normalization constant. With the help of the trial ansatz (2.6a) we obtain

$$u(r)=N\frac{\rho^2}{r^2+\rho^2}, \tag{2.13}$$

where $\rho=a/m_w$.

Now assume that the background fields which pass over the sphaleron are adiabatically changing in time. We take the ansatz

$$\psi_L(\mathbf{x},t)\propto\exp(-iE(t)\cdot t) \tag{2.14}$$

and obtain from (2.10)

$$i\boldsymbol{\sigma}(\partial-ig\mathbf{W}(\mathbf{x},t))\psi_L=E(t)\psi_L. \tag{2.15}$$

From first order perturbation theory it follows that there exists a normalizable solution of (2.15) with energy

$$E_f(t)=\frac{\langle\psi_L^{(0)}|g\boldsymbol{\sigma}\delta\mathbf{W}|\psi_L^{(0)}\rangle}{\langle\psi_L^{(0)}|\psi_L^{(0)}\rangle}=-\frac{1}{\sqrt{2}}\frac{\int_0^\infty r^2dr\phi_C(r,t)u^2(r)}{\int_0^\infty r^2dru^2(r)}, \tag{2.16}$$

where $\delta\mathbf{W}=\mathbf{W}-\mathbf{W}^{\mathrm{sp}}$. The fermionic energy crosses zero as $\phi_C(r,t)$ crosses the sphaleron ($\phi_C\equiv 0$). The physical interpretation of (2.16) is clear. If $E_f(t)$ crosses zero from below (above) fermions (anti-fermions) are created by the time-dependent background field.

In general this level crossing is expected to occur if the winding number difference (1.3), which is given in terms of our ansatz by [23]

$$Q(t_1,t_2)=\frac{1}{2\pi}\int_0^\infty dr\{f_A f'_B-f'_A f_B-f_C(f_A^2+f_B^2-1)\}\Big|_{t_1}^{t_2}, \tag{2.17}$$

is nonvanishing [25–28].

3 A one-dimensional potential barrier

We want to calculate the rate at which the gauge and Higgs fields pass the sphaleron. In this section we construct gauge and Higgs field configuration which describe the creation and decay of the sphaleron in the vicinity of the sphaleron. Thereby we obtain a one-dimensional potential barrier which has to be overcome by quantum or thermal fluctuations.

The linearized field equations for the ansatz (2.8) possess an unstable eigenmode in the channels B, C and H [20]. There is no unstable mode in the sphaleron channels A and K. This is already clear from the fact that the sphaleron is a minimum of the energy functional for fields restricted to the ansatz (2.5). The creation and decay of the sphaleron in the vicinity of the sphaleron therefore happens in the directions B, C, and H. For this reason we argue that the following ansatz describes the decay of the sphaleron at early times:

$$\begin{aligned} f_A(r,t)&=f_A^{\mathrm{sp}}(r),\\ f_B(r,t)&=c(t)f_B^{(-)}(r),\\ f_C(r,t)&=c(t)f_C^{(-)}(r),\\ H(r,t)&=c(t)H^{(-)}(r),\\ K(r,t)&=K^{\mathrm{sp}}(r), \end{aligned} \tag{3.1}$$

where the $(-)$ indicates the unstable modes, which are normalized according to [20]

$$\int_0^\infty dr\left\{f_B^{(-)2}+\frac{r^2}{2}f_C^{(-)2}+2m_w^2r^2H^{(-)2}\right\}=m_w^{-1}. \tag{3.2}$$

With the help of this ansatz we obtain from (2.4) the following effective Hamiltonian for the variable $c(t)$:

$$H=\frac{M}{2}\dot{c}^2+V(c), \tag{3.3}$$

where the effective potential is given by

$$V(c)=-\tfrac{1}{2}\Omega^2Mc^2+\tfrac{1}{4}\Lambda Mc^4+E_{\mathrm{sp}}. \tag{3.4}$$

With the help of (3.2) we find for the effective "mass"

$$M=\frac{8\pi}{g^2}\int_0^\infty dr\left\{f_B^{(-)2}+\frac{r^2}{2}f_C^{(-)2}+2m_w^2r^2H^{(-)2}\right\}$$

$$=\frac{8\pi}{g^2m_w}. \tag{3.5}$$

The parameters Ω and Λ in the effective potential are related to the following integrals

$$\begin{aligned} -\Omega^2=m_w\int_0^\infty dr\Bigg\{&2f_A^{\mathrm{sp}\prime}f_B^{(-)}f_C^{(-)}\\ &+(f_B^{(-)\prime}-f_C^{(-)}f_A^{\mathrm{sp}})^2+\frac{f_B^{(-)2}}{r^2}(f_A^{\mathrm{sp}2}-1)\\ &+2m_w^2r^2\Bigg[(H^{(-)\prime}+\tfrac{1}{2}f_C^{(-)}K^{\mathrm{sp}})^2\\ &+\frac{1}{2r^2}(H^{(-)}f_A^{\mathrm{sp}}+K^{\mathrm{sp}}f_B^{(-)}-H^{(-)})^2\\ &-K^{\mathrm{sp}\prime}f_C^{(-)}H^{(-)}-\frac{f_B^{(-)}H^{(-)}K^{\mathrm{sp}}}{r^2}(f_A^{\mathrm{sp}}+1)\Bigg]\\ &+(m_wm_H)^2r^2(K^{\mathrm{sp}2}-1)H^{(-)2}\Bigg\}, \end{aligned} \tag{3.6}$$

$$\begin{aligned} \Lambda=2m_w\int_0^\infty dr\Bigg\{&f_B^{(-)2}f_C^{(-)2}+\frac{1}{2r^2}f_B^{(-)4}\\ &+2m_w^2r^2\left[\tfrac{1}{4}f_C^{(-)2}H^{(-)2}+\frac{1}{2r^2}f_B^{(-)2}H^{(-)2}\right]\\ &+\tfrac{1}{2}m_w^2m_H^2r^2H^{(-)4}\Bigg\}. \end{aligned} \tag{3.7}$$

Note that $-\Omega^2$ is just the negative eigenvalue of the unstable eigenmode at the sphaleron. Ω^2 and Λ are strictly positive. The effective potential $V(c)$ has the shape of a double well.

Considering fields of the form (3.1) we obtained the potential well which has to be overcome by thermal or quantum fluctuations. The minima occur at

$$c_\pm=\pm\frac{\Omega}{\sqrt{\Lambda}}, \tag{3.8}$$

where the potential has the value

$$V(c_\pm)=-\frac{1}{4}\frac{\Omega^4}{\Lambda}M+E_{\mathrm{sp}}. \tag{3.9}$$

We have numerically calculated the integrals in (3.6) and (3.7) for the case $m_H=m_w$, that is $\lambda/g^2=1/8$, using the unstable eigenmode of [20]. We obtain

$$\omega_-^2\equiv-\Omega^2=-2.3m_w^2, \tag{3.10}$$

in accordance with [20], and

$$\Lambda=2.8m_w^2. \tag{3.11}$$

The corresponding effective potential is plotted in Fig. 3. For $\lambda/g^2=1/8$ we get for (3.8) and (3.9)

$$c_\pm=\pm 0.9, \tag{3.12}$$

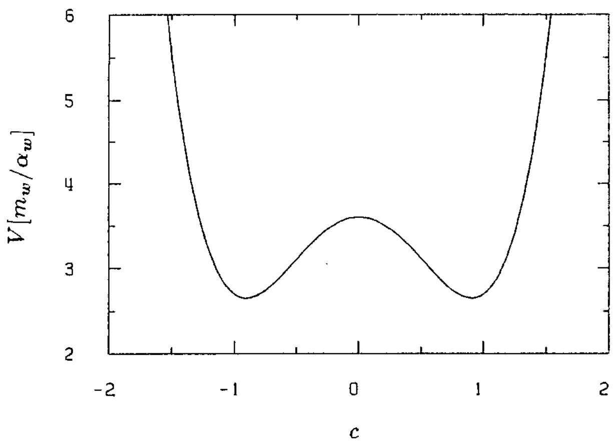

Fig. 3. The one-dimensional effective potential

$$V(c_\pm) = -0.9\frac{m_w}{\alpha_w} + E_{sp} = 2.7\frac{m_w}{\alpha_w}, \tag{3.13}$$

where we have used $E_{sp} = 3.6\, m_w/\alpha_w$ [24].

Now consider the winding number difference (2.17). If our ansatz (3.1) is inserted we obtain

$$Q(t_1, t_2) = q_1[c(t_2) - c(t_1)] - q_2[c^3(t_2) - c^3(t_1)], \tag{3.14}$$

where

$$q_1 = \frac{1}{2\pi}\int_0^\infty dr\{f_A^{sp} f_B^{(-)\prime} - f_A^{sp\prime} f_B^{(-)} - f_C^{(-)}(f_A^{sp2} - 1)\}, \tag{3.15}$$

$$q_2 = \frac{1}{2\pi}\int_0^\infty dr f_C^{(-)} f_B^{(-)2}. \tag{3.16}$$

We have also numerically evaluated q_1 and q_2 for $\lambda/g^2 = 1/8$ and find

$$q_1 = 0.4, \tag{3.17a}$$

$$q_2 = 0.1. \tag{3.17b}$$

If c changes from c_- to c_+ we get for the winding number difference

$$Q(-,+) = 2q_1 c_+ - 12 q_2 c_+^3 = 0.6. \tag{3.18}$$

That is, we obtain a winding number difference not too much different from one, in spite of the fact that we are far away from the vacuum (see (3.13)).

The fermion energy (2.16) is given for our ansatz (3.1) by

$$E_f(t) = -\varepsilon c(t), \tag{3.19}$$

where

$$\varepsilon = \frac{1}{2}\frac{\int_0^\infty r^2 dr f_C^{(-)}(r) u^2(r)}{\int_0^\infty r^2 dr u^2(r)}. \tag{3.20}$$

For $\lambda/g^2 = 1/8$ this yields

$$\varepsilon = 0.1\, m_w. \tag{3.21}$$

Note that the minus sign in (3.19) nicely fits onto the minus sign in the anomaly (1.2). If c changes from a negative to a positive value, the winding number difference will be positive (at least for $|c| \leqq c_+$), and $E_f(t)$ will change from a positive to negative value, indicating that the fermion number is decreasing, in accordance with (1.2).

In conclusion, we have found a path in configuration space which passes the sphaleron thereby creating or destroying fermions according to the selection rule (1.2). With the help of this path the potential barrier in "c-space" or, equivalently, in winding number space, has been found explicitely. The effective potential will be used in the following for the calculation of the transition rate over the barrier between topological distinct vacua.

4 One-dimensional transition rate

In this section we consider the one-dimensional problem of the transition over the barrier in c-space. The general framework has been set up by Affleck [29]. We consider both quantum tunneling and thermal transitions. We assume that the system is prepared at an initial time to sit in one of the potential wells. We note in passing that the degeneracy of the minima of $V(c)$ is lifted in the presence of fermions, due to $E_f \sim c$. The one-dimensional equilibrium transition rate is given by the Boltzmann average of the probability current over a set of quantum states [29]

$$\Gamma_1 = Z_0^{-1}\int_0^\infty dE \rho(E) T(E) \exp(-\beta E), \tag{4.1}$$

$$Z_0 = \sum_{n=0}^\infty \exp[-(n+\tfrac{1}{2})\beta\omega_0] = [2\sinh(\tfrac{1}{2}\beta\omega_0)]^{-1}, \tag{4.2}$$

where $\omega_0/2$ denotes the ground-state energy. We take $\omega_0 = m_w$ in the following. The incident flux per unit energy, $\rho(E)$ is set equal to the classical value $1/2\pi$.

For $E < E_{sp}$ the transmission coefficient $T(E)$ can be inferred from the WKB linear turning-point formula

$$T(E)_{WKB} = \exp\left\{-2\int_{c_1}^{c_2} dc'[2M(V(c') - E)]^{1/2}\right\}, \tag{4.3}$$

where c_1 and c_2 are the classical turning points at energy E.

For $E \geqq E_{sp}$ the linear turning-point formula is invalid but since the transmission occurs very close to the top of the well one can use the transmission coefficient for a parabolic barrier

$$T(E)_{par} = \{1 + \exp[-2\pi(E - E_{sp})/\Omega]\}^{-1}. \tag{4.4}$$

Note that this is exact for $\Lambda = 0$ at all energies.

Figure 4 shows the transmission coefficient for the parabolic potential, $T(E)_{par}$, and the WKB transmission coefficient for our potential, $T(E)_{WKB}$, as a

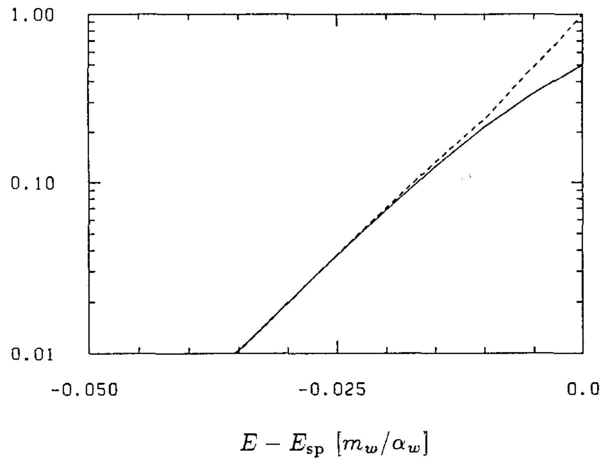

Fig. 4. Transmission coefficients. The solid line give the transmission coefficient for the parabolic potential (4.4), the dashed line corresponds to the WKB result (4.3)

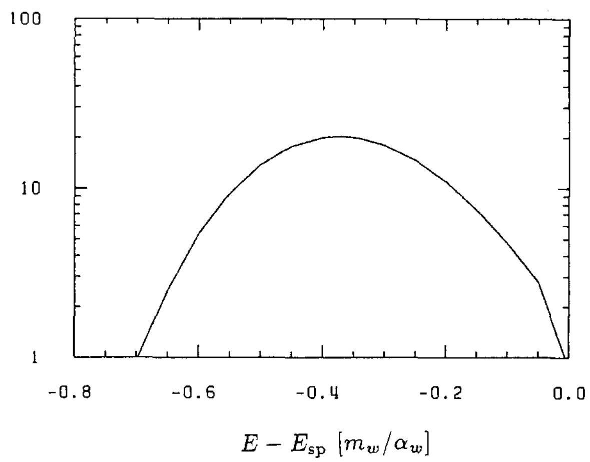

Fig. 5. Energy spectrum $d\Gamma_1/dE$ at temperature $T = 0.22\, m_w$

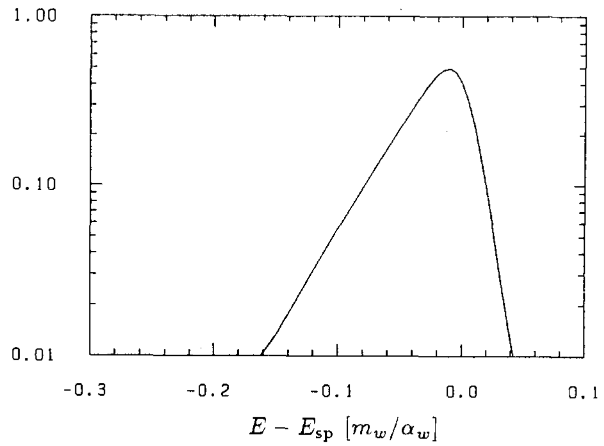

Fig. 6. Energy spectrum $d\Gamma_1/dE$ at temperature $T = 0.30\, m_w$

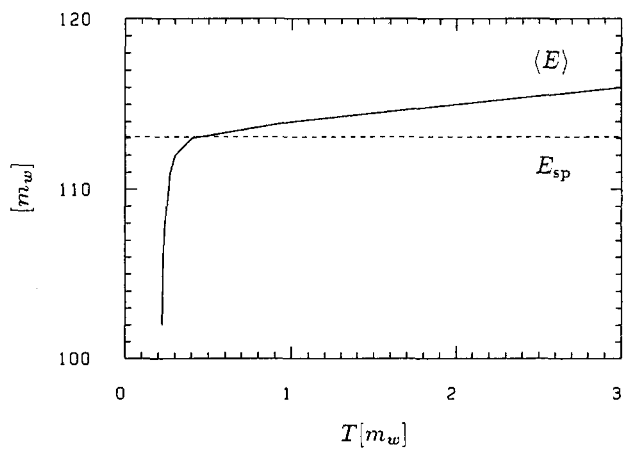

Fig. 7. Mean energy of transition

function of $\delta = E - E_{sp}$. As can be seen, both expressions cross at about $\delta = -0.03\, m_w/\alpha_w$. In the following we take $T(E)_{par}$ for δ larger than this value and $T(E)_{WKB}$ in the opposite case. We have checked our numerical integrations in the *WKB* case by putting Λ equal to zero. We find perfect agreement between the transmission coefficient obtained in this way and the transmission coefficient of the parabolic potential.

In Figs. 5 and 6 we have plotted the energy spectra

$$\frac{d\Gamma_1}{dE}_{|(E - E_{sp})} \tag{4.5}$$

for different temperatures. Here we observe the following behavior which was argued to be true on general grounds by Affleck [29]: For temperatures below $T_0 \equiv \Omega/2\pi = 0.24\, m_w$ the transition is dominated by quantum tunneling, whereas above this temperature the transition is dominated by classical thermal transitions. In Fig. 5 we see that, at a temperature $T = 0.22\, m_w < T_0$, a broad maximum exists in the energy spectrum in an energy range strictly below the sphaleron energy. This means that the typical transition is a quantum tunneling. On the other hand, the energy spectrum at temperature $T = 0.3\, m_w > T_0$, (see Fig. 6) is sharply peaked at the sphaleron energy. Here classical thermal transitions begin to dominate. This behavior can be clearly seen in Fig. 7 where we have plotted the mean energy

$$\langle E \rangle = \frac{\int_0^\infty dE E \rho(E) T(E) \exp(-\beta E)}{\int_0^\infty dE \rho(E) T(E) \exp(-\beta E)}, \tag{4.6}$$

as a function of temperature. We observe a narrow transition from quantum tunneling to thermal transitions at the temperature T_0. Note that there exists a smooth transition region between both cases. There appears to be no discontinous change as has been found in [15] by the use of an ad hoc ansatz for the gauge and Higgs fields which pass the sphaleron. The order of magnitude of T_0, however, agrees with the value found by Aoyama et al. [15].

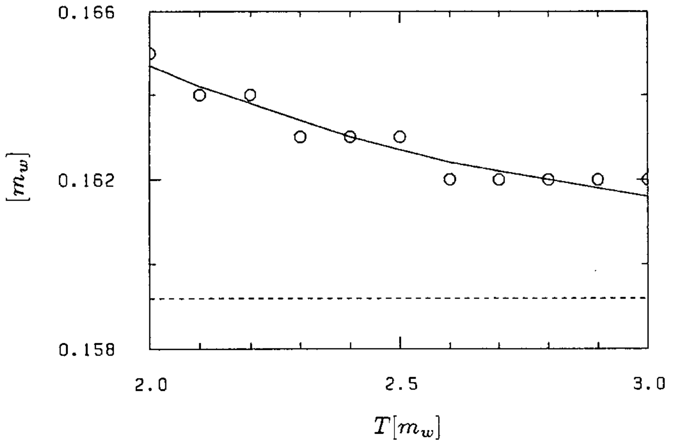

Fig. 8. The one-dimensional prefactor. The circles represent the numerically integrated values of $\Gamma_1 \exp(\beta E_{sp})$, the solid line gives the corresponding expression computed from (4.7), and the dashed line corresponds to the classical result (4.8), $\omega_0/2\pi$

Since for $T > T_0$ the integral is dominated by $E \geqq E_{sp}$ (see Fig. 7), where the transmission coefficient for the parabolic potential is valid, we can to a very good approximation evaluate Γ_1 [29],

$$\Gamma_1 \simeq \Gamma_1^A \equiv Z_0^{-1} \int_{-\infty}^{\infty} dE (2\pi)^{-1} \cdot \{1 + \exp[-2\pi(E - E_{sp})/\Omega]\}^{-1} \exp(-\beta E) = Z_0^{-1} \Omega [4\pi \sin(\tfrac{1}{2}\beta\Omega)]^{-1} \exp(-\beta E_{sp}). \tag{4.7}$$

For large temperatures this gives the classical transition rate

$$\Gamma_1 \to \Gamma_1^{cl} = \frac{\omega_0}{2\pi} \exp(-\beta E_{sp}). \tag{4.8}$$

In Fig. 8 we have plotted the $\Gamma_1 \exp(\beta E_{sp})$, which has been numerically calculated, the corresponding rate from (4.7) and the classical rate for a temperature range above $2 m_w$. We see that (4.7) gives an excellent approximation to Γ_1 for these temperatures and that the classical transition rate is approached from above. In the following, however, we use unless otherwise stated the more general numerically integrated rate.

We note in passing that Γ_1^A can be written as [29]

$$\Gamma_1^A = \frac{\Omega\beta}{\pi} \operatorname{Im} F, \tag{4.9}$$

where F is the free energy of the saddle-point of V. This form was the starting point of previous calculations [12, 18]. Here we have justified the use of this formula for temperatures larger than $T_0 = \Omega/2\pi = 19$ GeV.

5 Transition rate in field theory

Now we come to the calculation of the actual transition rate in field theory. Since there is only one unstable eigenmode in the background of the sphaleron [20], the field theoretic transition rate is, in the Gaussian, i.e. one-loop, approximation, given by [29]

$$\Gamma = (\mathcal{N}\mathcal{V}) \frac{\prod_i 2\sinh(\tfrac{1}{2}\beta\omega_0^i)}{\prod_i' 2\sinh(\tfrac{1}{2}\beta\omega^i)} \Gamma_1, \tag{5.1}$$

where

$$(\mathcal{N}\mathcal{V}) = (\mathcal{N}\mathcal{V})_{tr} (\mathcal{N}\mathcal{V})_{rot} \tag{5.2}$$

denotes the normalized volume factors for physical zero modes in the sphaleron background related to the translation and rotation invariance. The ω^i are the eigenfrequencies of the stable modes in the sphaleron background, and the ω_0^i are the eigenfrequencies in the vacuum. A prime means that the zero modes have to be omitted. The normalized volumes of the zero modes can be calculated according to [20]

$$(\mathcal{N}\mathcal{V})_{tr} = V \left[\frac{m_w \xi_{tr}}{2\pi\beta\alpha_w} \right]^{3/2}, \tag{5.3}$$

$$(\mathcal{N}\mathcal{V})_{rot} = 8\pi^2 \left[\frac{\xi_{rot}}{2\pi\beta m_w \alpha_w} \right]^{3/2}, \tag{5.4}$$

where V denotes the physical volume of the system. ξ_{tr} and ξ_{rot} are related to the following integrals of the sphaleron radial functions (2.5)

$$\xi_{tr} = \frac{4}{3m_w} \int_0^{\infty} dr \left\{ 4f'^2 + \frac{8}{r^2} f^2 (1-f)^2 + m_w^2 r^2 h'^2 + 2m_w^2 h^2 (1-f)^2 \right\}, \tag{5.5}$$

$$\xi_{rot} = \frac{8m_w}{3} \int_0^{\infty} dr \{ r^2 f'^2 + 4f^2(1-f)^2 + m_w^2 r^2 h^2 (1-f)^2 \}. \tag{5.6}$$

Using our trial ansatz (2.6) we can evaluate these expressions analytically and find

$$\xi_{tr} = \frac{2\pi}{3} \left[\frac{3}{2a} + \frac{b}{8} + \frac{a}{(1+b/a)^2} \right], \tag{5.7}$$

$$\xi_{\rm rot}=\frac{2\pi a}{3}+\frac{4a^4\pi}{3}\left[\frac{3}{2}\frac{a}{a^2-b^2}-\frac{a^3-b^3}{(a^2-b^2)^2}\right]. \qquad (5.8)$$

From Fig. 1 we get $\xi_{\rm tr}=3.8$ and $\xi_{\rm rot}=4.4$ for $\lambda/g^2=1/8$, in comparison to the values obtained numerically [20], $\xi_{\rm tr}=3.6$ and $\xi_{\rm rot}=3.5$.

The calculation of the products over the transverse modes in (5.1) lies beyond the scope of the present paper. We simply extract a factor by counting the number of zero modes, which equals to six, three from translations and three from rotations, and get for the transition rate per unit volume

$$\frac{\Gamma}{V}=\frac{(\xi_{\rm tr}\xi_{\rm rot})^{3/2}}{\pi\alpha_{\rm w}^3\beta^3}\kappa[2\sinh(\tfrac{1}{2}\beta\omega_0)]^6\Gamma_1, \qquad (5.9)$$

where κ is a constant of order one. This procedure corresponds to the dimensional arguments in [12, 13, 18]. It is amusing to note that it leads to the exact formula for high temperatures in the $1+1$ dimensional $O(3)$ sigma model [32]. κ can also mimick possible damping effects in the plasma [12, 13] which reduce the rate. Note that (5.1) can be written for $T>T_0$ as (see 4.7)

$$\Gamma=Z_0^{-1}\Omega[4\pi\sin(\tfrac{1}{2}\beta\Omega)]^{-1}(\mathcal{N}\mathcal{V})\cdot\exp\left[-\beta\left\{E_{\rm sp}+\sum_i{}'\tfrac{1}{2}\omega^i-\sum_i\tfrac{1}{2}\omega_0^i\right\}\right]\cdot\exp\left[\sum_i\ln\frac{1-e^{-\beta\omega_0^i}}{1-e^{-\beta\omega^i}}\right]. \qquad (5.10)$$

This shows that the infinite product in (5.1) gives the infinite zero temperature contribution to the vacuum energy and the sphaleron energy (first exponent), and the $T\neq 0$ contribution to the free energy of the sphaleron (second exponent). The zero temperature infinities can be absorbed into the sphaleron energy. It is important to note that by estimating (5.1) in the form (5.9), i.e. by replacing

$$\frac{\prod_i 2\sinh(\tfrac{1}{2}\beta\omega_0^i)}{\prod_i{}'2\sinh(\tfrac{1}{2}\beta\omega^i)}\exp\{-\beta E_{\rm sp}\}\to\kappa[2\sinh(\tfrac{1}{2}\beta\omega_0)]^{N_0}\cdot\exp\{-\beta E_{\rm sp}\}, \qquad (5.11)$$

where N_0 denotes the number of zero modes, we have already taken into account a part of the one-loop finite-temperature correction to the transition rate per unit volume. In the $1+1$ dimensional $O(3)$ sigma model the procedure (5.11) gives just the right answer (for $T\gg\omega_0/2$), apart from a constant of order one which can be absorbed into κ [32]. In this model the finite-temperature contribution just leads, in addition to (5.11), to the appearance of the temperature-dependent running coupling constants in the final expression [32]. Since the dependence of the coupling constant on T is weak in the range of temperatures we are considering this has a negligible effect on the rate of the anomalous processes. In previous calculations [12, 13, 18] in addition to (5.11) the classical W-boson mass has been replaced by the temperature-dependent W-boson mass $m_{\rm w}(T)=(1/2)gv(T)$, where $v(T)$ has been read off from the one-loop finite-temperature effective potential of the theory. In our formalism the temperature contribution to the sphaleron energy comes from the second exponent in (5.10), which formally is the same as the one-loop finite-temperature contribution to the classical potential. Indeed, the replacement of v by $v(T)$, found from the effective potential, gives the leading temperature effect on the sphaleron energy, if we take the frequencies ω_i to be the eigenfrequencies in a *space-* and *time-independent* back-ground field. But the sphaleron is *space-dependent* and therefore this assumption is not justified in general (see for example [19]). In particular, there will be also corrections to the kinetic terms not only to the potential. For large Higgs masses these corrections are presumably inessential [33] but for low Higgs masses of the order of the Coleman–Weinberg mass [34] they cannot be neglected [35]. What we want to stress is that the temperature-dependence of the sphaleron energy can be expressed through a temperature-dependence of the expectation value but that this temperature-dependence *need not* be of the same functional form than that found from the one-loop finite-temperature effective potential.

6 Rate of $B+L$ violating processes

In the case of a baryon and lepton number symmetric universe, $B=L$, the rate of the transitions over the sphaleron and the rate of the baryon and lepton number violating processes are related by [12, 13, 18]

$$\Gamma_B=f_g\tfrac{13}{2}\beta^3\frac{\Gamma}{V}. \qquad (6.1)$$

This has to be compared with the Hubble expansion rate of the universe

$$H=1.66\sqrt{g_*}\frac{T^2}{m_{\rm pl}}, \qquad (6.2)$$

where g_* denotes the effective relativistic degrees of freedom and $m_{\rm pl}=1.2\cdot 10^{19}$ GeV denotes the Planck mass. We take the number of generations, f_g, to be three. In this case $g_*=\mathcal{O}(100)$.

We want to determine quantitatively the influence of the assumed sphaleron temperature-dependence on the value of the critical Higgs mass. For this end we first calculate the rate of the anomalous processes using the zero temperature W-mass. This results in a larger value for the critical Higgs mass han the one obtained in [18], because the anomalous processes are more suppressed in this case due to a larger exponent in the Boltzmann factor.

Figure 9 shows the ratio Γ_B/H for a range of temperatures in the case $\lambda/g^2=1/8$ and $\kappa=1$. The

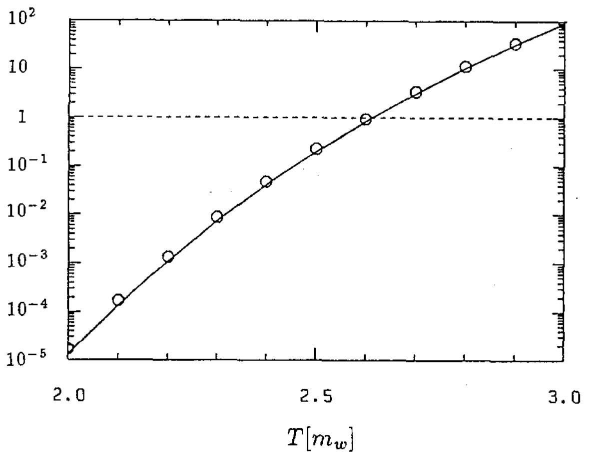

Fig. 9. Γ_B/H for $\lambda/g^2=1/8$. The circles represent the numerical results, the solid line corresponds to the use of (4.7)

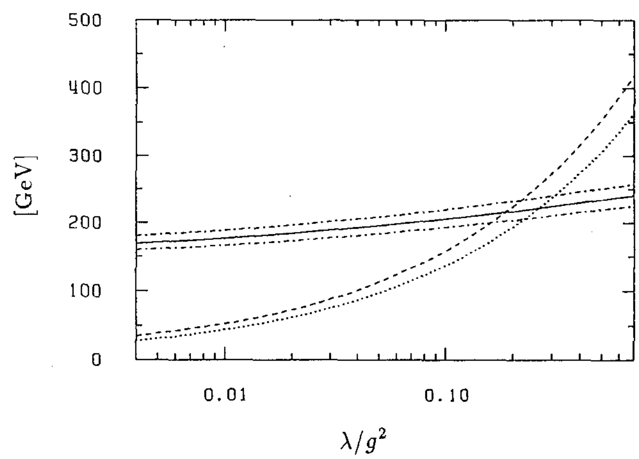

Fig. 10. Freezing temperature T_* and critical temperature T_{c_1}. The solid line gives T_* for $\kappa=1$, the upper (lower) dashed-dotted line gives T_* for $\kappa=0.1(10)$. The dashed (dotted) line represents T_{c_1} for $m_t=44\,\text{GeV}$ ($m_t=0.97\,m_w$)

circles represent the values which have been found by the numerical integration of the one-dimensional transition rate. We used the numerical values quoted in the previous sections for the effective potential $V(c)$ and the zero mode normalizations. For comparison we have plotted also the values of Γ_B/H which can be obtained from (4.7), (5.9) together with the variational results from Fig. 1, Fig. 2, (5.7), and (5.8) (solid line). Here we used $E_{\text{sp}}^{\text{int}}=3.64$ from Fig. 2, $a=1.11$, and $b=0.82$ from Fig. 1. We obtain nice agreement with the numerical results. This is to be expected because we are in a temperature range large compared to T_0. In this range we can work with formula (4.7) for the one-dimensional transition rate (see also Fig. 8). We see that the rate of the baryon number violating processes exceeds the Hubble expansion rate for temperatures larger than $T_*=2.61\,m_w=209\,\text{GeV}$. If we vary κ from 0.1 to 10, T_* varies only moderately from 224 GeV to 196 GeV.

We have repeated the calculations for different values of λ/g^2, i.e. for different Higgs masses. We used the high temperature formula (4.7) for the one-dimensional transition rate, the results from the trial ansatz for the zero mode factors, and $E_{\text{sp}}^{\text{int}}$ from Fig. 2 for the sphaleron energies. Ω has been set equal to the values in the case $\lambda/g^2=1/8$. We assume that the major effect comes from the different sphaleron energies. Furthermore it is expected that Ω does not vary very much with λ/g^2 in the range we are considering because the sphaleron radial functions do not change very much in that range (see Fig. 1). Anyway, for $T\gg\Omega/2$ the actual value of Ω is not important, because it drops out in the one-dimensional transition rate (4.7). What we found can be seen in Fig. 10, which gives the freezing temperatures, determined as

$$(\Gamma_B/H)_{|T_*}=1, \tag{6.3}$$

as a function of λ/g^2. As can be seen, the freezing temperature does not depend too much on κ.

Of course, the calculations are valid only below the the critical temperature, where the $SU(2)$ symmetry is spontaneously broken, because the sphaleron ceases to be a solution in the unbroken phase. As already mentioned in Sect. 1 not much is known about the critical temperature of the electroweak phase transition. Nevertheless, in order to get the critical Higgs mass we need also the critical temperature as a function of λ/g^2. In the following we assume that the one-loop finite-temperature effective potential correctly gives the critical temperature. For the Higgs self coupling we concentrate on the region

$$\frac{g^4}{64\pi^2}\ll\lambda\ll g^2. \tag{6.4}$$

In this region one has to take into account one-loop effects from gauge bosons and the top quark. The one-loop finite-temperature effective potential reads [18, 36]

$$V(\phi,T)=V(\phi,0)+V^T,$$

$$V(\phi,0)=-(\lambda/2+B)v^2\phi^2+\frac{\lambda}{4}\phi^4+B\phi^4\ln\frac{\phi^2}{v^2}, \tag{6.5}$$

where $(1/2)\phi^2=\Phi^\dagger\Phi$. V^T behaves for large temperatures or near $\phi=0$ as

$$V^T=-g_*(T)\frac{\pi^2}{90}T^4+\frac{1}{32}g^2 \cdot\left[2+\frac{1}{\cos^2\theta_w}+2\left(\frac{m_t}{m_w}\right)^2\right]T^2\phi^2 -\frac{g^3}{32\pi}\left[2+\frac{1}{\cos^3\theta_w}\right]T\phi^3+\frac{\Delta\lambda(T)}{4}\phi^4, \tag{6.6}$$

where

$$\Delta\lambda(T) = \tfrac{3}{16}\alpha_w^2\left[2\ln\frac{T^2\xi_B}{m_w^2} + \frac{1}{\cos^4\theta_w}\ln\frac{T^2\xi_B\cos^2\theta_w}{m_w^2} - 4\left(\frac{m_t}{m_w}\right)^4\ln\frac{T^2\xi_F}{m_t^2}\right], \tag{6.7}$$

$$\ln\xi_B = 3.91, \tag{6.8}$$

$$\ln\xi_F = 1.34. \tag{6.9}$$

m_t denotes the mass of the top quark which is larger than 44 GeV experimentally [37]. The constant B is given by

$$B = \tfrac{3}{64}\alpha_w^2\left[2 + \frac{1}{\cos^4\theta_w} - 4\left(\frac{m_t}{m_w}\right)^4\right]. \tag{6.10}$$

The Higgs mass found from the zero temperature effective potential is

$$m_H^2 = (12B + 2\lambda)v^2. \tag{6.11}$$

The electroweak phase transition practically coincides with the moment of the absolute instability of the phase with $\phi = 0$ [36] because tunneling transitions are strongly suppressed. The corresponding critical temperature T_{c_1} can be found by looking at which temperature $d^2V/d\phi^2$ at $\phi = 0$ vanishes. One finds [18, 36]

$$T_{c_1}^2 = \frac{(B + \lambda/2)v^2}{\frac{1}{32}g^2\left[2 + \frac{1}{\cos^2\theta_w} + 2\left(\frac{m_t}{m_w}\right)^2\right]}, \tag{6.12}$$

We have plotted the critical temperature as a function of λ/g^2 in Fig. 10 for $m_t = 44$ GeV ($m_t = 0.97\,m_w$). As can be seen, for λ/g^2 larger than

$$(\lambda/g^2)_{\text{crit}} = 0.19(0.26), \tag{6.13}$$

corresponding to a Higgs mass (6.11)

$$m_{\text{crit}} = 97(113)\,\text{GeV}, \tag{6.14}$$

the freezing temperature of the anomalous processes is smaller than the critical temperature. That means that for $m_H > 97(113)$ GeV the anomalous processes are in equilibrium after the electroweak phase transition. Here we took $\kappa = 1$. For $\kappa = 0.1$ the corresponding critical masses are $m_{\text{crit}} = 102(121)$ GeV, whereas for $\kappa = 10$ we obtain $m_{\text{crit}} = 86(101)$ GeV. Since at temperatures above the phase transition the anomalous processes are presumably unsuppressed [11, 12] the actual freezing temperature of the anomalous processes is given by

$$T_{\text{fr}} = \min\{T_{c_1}, T_*\}. \tag{6.15}$$

Working with the zero temperature sphaleron energy gives the most efficient suppression of the anomalous processes. That means that (6.14) actually gives an upper bound on m_{crit}. It is reasonable to assume that finite-temperature effects lead to a decrease of the sphaleron energy. These finite-temperature effects can be represented by replacing m_w, Ω and E_{sp} by

$$m_w(T) = \frac{F(T)}{v}m_w,$$

$$\Omega(T) = \frac{F(T)}{v}\Omega,$$

$$E_{\text{sp}}(T) = \frac{F(T)}{v}E_{\text{sp}}. \tag{6.16}$$

As argued in the previous section $F(T)$ *need not* coincide with $v(T)$, the minimum of the one-loop finite-temperature effective potential (6.5). Following [11–13, 18] let us now assume that setting $F(T) = v(T)$ gives the right order of magnitude for the temperature-dependent sphaleron energy. This is reasonable since in the range of Higgs masses in which we are working, (6.4), we are far away from the Coleman–Weinbrg mass, $m_{CW}^2 = 8Bv^2$, [34]. Since we are interested in the crossing point of T_{c_1} and T_*, we can take for $v(T)$ the value at $T = T_{c_1}$, which follows from (6.5) and (6.6)

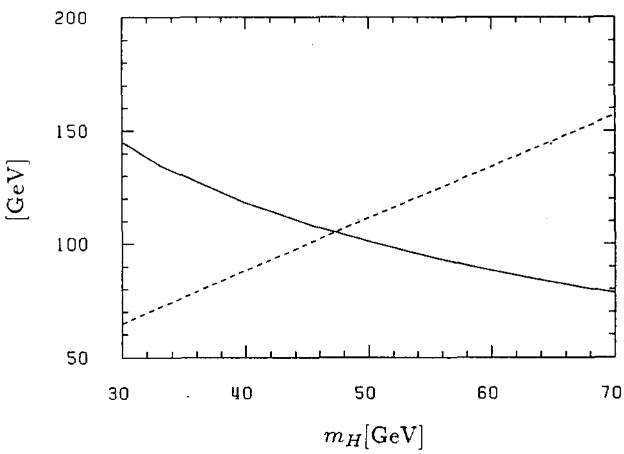

Fig. 11. Determination of m_{crit} in the approach [18] for $m_t = 44$ GeV. The solid line represents T_*, the dashed line T_{c_1}

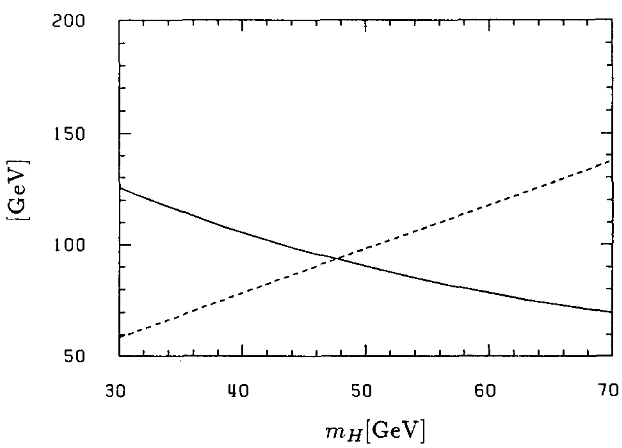

Fig. 12. Determination of m_{crit} in the approach of [18] for $m_t = 0.97\,m_w$. The solid line represents T_*, the dashed line T_{c_1}

$$v(T_{c_1}) = \frac{3T_{c_1}}{32\pi\lambda_{\text{eff}}} g^3 \left[2 + \frac{1}{\cos^3 \theta_w} \right], \tag{6.17}$$

where $\lambda_{\text{eff}} = \lambda + \Delta\lambda(T)$. What we found can be seen in Fig. 11 and Fig. 12. T_* has been calculated according to (6.3), but in Γ_B we used (6.16) at the temperature T_{c_1}. From the crossing points with T_{c_1} we infer

$$m_{\text{crit}} = 47(48)\,\text{GeV} \tag{6.18}$$

for $m_t = 44\,\text{GeV}$ ($m_t = 0.97\,m_w$) and $\kappa = 1$, in accordance with [18], where a value of $m_{\text{crit}} = 45\,\text{GeV}$ was obtained. In this case m_{crit} is very insensitive to the top mass.

7 Conclusions

We discussed baryon and lepton number violating processes in the electroweak theory induced by gauge and Higgs fields passing the sphaleron. We reduced the complicated problem to a one-dimensional problem by considering the energy functional along a path in configuration space which passes the sphaleron along the unstable eigenmode found in [20]. In this way we got a better understanding of the shape of the potential between topological inequivalent vacua near the sphaleron. It turned out that a parabolic approximation, requiring knowledge of Ω only, is already adequate, because there is no substantial tunneling down to $T_0 = 19\,\text{GeV}$ where the transition rate is totally negligible. This confirms previous calculations [12, 13, 15]. We showed that the gauge fields along the path create and destroy fermions according to the anomaly (1.2).

Our main result is the freezing temperature T_* of the anomalous processes as a function of λ/g^2 (Fig. 10). This was obtained by working with the zero temperature W-mass and therefore gives an upper bound on the actual freezing temperature. By calculating T_* it was also assumed that we are in the broken phase of the theory. T_* contains a number of uncertainties: (i) the magnitude of the pre-exponential factor κ, which however changes the freezing temperature only by a few percent (see Fig. 10); (ii) further finite temperature effects which could eventually lower T_* [18]; (iii) the magnitude of Ω, which was set equal to the value at $\lambda/g^2 = 1/8$ [20]; (iv) corrections from $\theta_w \neq 0$. Points (i) and (ii) are of course intimately related. A complete one-loop calculation of the free energy of the sphaleron could resolve these uncertainties. Point (iii) requires the calculation of the unstable mode for different values of λ/g^2. But it should have little influence on the rate because Ω drops out for $T \gg \Omega/2$. Corrections from $\theta_w \neq 0$ could be taken into account perturbatively [10]. The sphaleron energy does not change significantly for $\theta_w \neq 0$ [10], so we expect no dramatic change from that.

We determined a critical Higgs mass m_{crit} which is defined in that way that for a Higgs mass larger than m_{crit} the anomalous processes are in equilibrium after the electroweak phase transition. In the context of the generation of the BAU within the electroweak standard model [17] m_{crit} represents an upper bound on the Higgs mass. To turn $T_*(\lambda/g^2)$ into m_{crit} one needs to know the critical temperature of the electroweak phase transition. This brings the next uncertainty because the transition temperature is not known exactly. If we take for granted the results for the critical temperature from the one-loop finite-temperature effective potential, we obtain

$$m_{\text{crit}} = \mathcal{O}(100)\,\text{GeV}, \tag{7.1}$$

if we work with the zero temperature W-mass Equation (7.1) actually represents the most conservative *upper bound* on the critical mass. Inclusion of finite-temperature effects on the sphaleron energy lead to a decrease of the critical mass. If we assume that the temperature-dependence of the sphaleron is dictated by the one-loop finite-temperature effective potential, which is reasonable in the range of relevant Higgs masses, we get the low value

$$m_{\text{crit}} = \mathcal{O}(50)\,\text{GeV}, \tag{7.2}$$

in accordance with [18]. A calculation of the exact one-loop free energy of the sphaleron and an accurate determination of the critical temperature as a function of the Higgs mass by lattice calculations are urgently needed in order to strengthen the bound.

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