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Model-independent limits on Λ_{QCD} from e^+e^- annihilation in the energy range from 14 to 46 GeV

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Abstract. Multihadronic events measured with the CELLO detector in the energy range 14 to 46 GeV have been analyzed in terms of thrust, jet masses and the asymmetry of the energy-energy correlation. The data have been compared with 2nd order QCD calculations. From a study of the general properties of fragmentation effects, model-independent limits on Λ_{QCD} and α_s have been found to be 79 MeV $<\Lambda_{\mathrm{QCD}}<628$ MeV and $0.117<\alpha_s<0.169$ (at $\sqrt{s}=35$ GeV). The dependence of these results on the renormalization scheme is discussed.

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1 Introduction

The event shape of multihadronic events produced in e^+e^- interactions at high energies has frequently been used to determine the coupling strength α_s of the strong interaction. However, it is well known that in analyses of this type the result depends significantly on the fragmentation model used to extract the value of α_s [1, 2]. Furthermore, the Monte-Carlo simulation of hadronic final states involves QCD matrix elements whose implementation always requires a phase space cut at the parton level to avoid infrared and collinear divergences, or negative values of the production cross sections for a given number of partons. The problems of this kind of approach to determine α_s have been discussed elsewhere [3].

The analysis presented here tries to avoid the difficulties outlined above. The method was first used by PLUTO [4] and Field [5] and later by JADE [6]. It is based on a direct comparison of second order QCD calculations with various event shape parameters V, sensitive to α_s, which have been extracted from the corrected data. The energy-dependence of such an observable V is assumed as a sum of a QCD term, in the form of a power series expansion in α_s, and a hadronization term $H(s)$:

$$V(s)=C_1\frac{\alpha_s(s)}{\pi}+C_2\left(\frac{\alpha_s(s)}{\pi}\right)^2+\ \dots\ +H(s) \tag{1}$$

where s is the square of centre-of-mass energy.

At present, only second-order QCD calculations exist for the event shape parameters, so that the QCD term is a second-order polynomial in α_s. The hadronization term $H(s)$ incorporates the effects of the fragmentation of quarks and gluons into observable particles as well as higher order QCD-terms neglected in the power series. The energy dependence of the QCD term follows from the energy dependence of the running coupling constant α_s which, in the $\overline{\mathrm{MS}}$ renormalization scheme, is given by:

$$\alpha_s(s,\Lambda^2_{\overline{\mathrm{MS}}})=\frac{12\pi}{(33-2N_f)\log(s/\Lambda^2_{\overline{\mathrm{MS}}})} \times\left(1-6\,\frac{(153-19N_f)\log(\log(s/\Lambda^2_{\overline{\mathrm{MS}}}))}{(33-2N_f)^2\log(s/\Lambda^2_{\overline{\mathrm{MS}}})}\right) \tag{2}$$

where N_f is the number of effective quark flavours. In our energy range $N_f=5$ and all $\Lambda_{\overline{\mathrm{MS}}}$ values given in this paper refer to 5 flavours. The relationship between $\Lambda_{\overline{\mathrm{MS}}}$ and the number of flavours has been discussed by Marciano [7].

The exact functional shape of $H(s)$ is unknown but, for some observables at least, general statements about its properties can be made which will be used in this analysis. The first is that the size of the hadronization term should decrease with energy. The second and here more important property is the sign of $H(s)$ which for some variables is known to be always positive or always negative for all reasonable fragmentation models. This allows us to use these observables to place upper and lower limits on α_s and hence on Λ_{QCD}, in a way which is largely model-independent.

2 Method

We study four quantities, which are insensitive to infrared and collinear divergences in the QCD matrix element and which have been calculated up to second order in α_s.

1. $1-T$. The thrust of an event is defined by $T=\max(\sum|p_L|/\sum|p|)$, where p_L is the longitudinal component of the particle momentum p with respect to a given event axis. The differential distribution of the thrust is divergent for $T\to 1$, but the average value of $1-T$ is insensitive to this divergence and has been calculated [8] in the $\overline{\mathrm{MS}}$ renormalization scheme as:

$$\langle 1-T\rangle=1.05\,\frac{\alpha_s}{\pi}\left(1.0+9.05\,\frac{\alpha_s}{\pi}\right). \tag{3}$$

2. M_h^2/s. The jet masses are calculated by dividing an event into two hemispheres in such a way that the sum of the two squared invariant masses is minimized. The distribution of the heavier of the two invariant masses M_h^2/s resembles that of $1-T$: The peak of the distribution shifts to lower values with increasing energy. The mean value of the heavier jet mass has been calculated by Clavelli [9] as:

$$\langle M_h^2/s\rangle=1.05\,\frac{\alpha_s}{\pi}\left(1.0+6.57\,\frac{\alpha_s}{\pi}\right). \tag{4}$$

3. $(M_h^2-M_l^2)/s$. The difference between the heavy and the light jet masses is of special interest because the second order corrections are small [9]:

$$\langle (M_h^2-M_l^2)/s\rangle=1.05\,\frac{\alpha_s}{\pi}\left(1.0+2.76\,\frac{\alpha_s}{\pi}\right). \tag{5}$$

4. A_{EEC}. The energy-energy correlation $E(\chi)$ is defined as the distribution of the angle χ between the directions of any two particles in an event, where χ is weighted by the particle energies [10]. The asymmetry of the energy-energy correlation (A_{EEC}) is the difference $E(180°-\chi)-E(\chi)$. It has been calculated to second order by several authors [10, 11]. Here we consider the χ interval between 90° and 30°, which is the angular range outside a typical jet cone so that the contribution from gluon radiation, sensitive to α_s, is expected to dominate. The integration of the angular asymmetry within the above limits yields:

$$\int_{30°}^{90°} A_{\mathrm{EEC}}\,d\chi=0.766\,\frac{\alpha_s}{\pi}\left(1.0+3.59\,\frac{\alpha_s}{\pi}\right). \tag{6}$$

As mentioned, our analysis relies on the sign of $H(s)$ for these observables. Four different fragmentation models have been studied to check the agreement on this property. We have compared two models with independent fragmentation (Ali [12] and Hoyer [13]), a parton shower model [14], and the LUND string fragmentation model. All models are used as implemented in the LUND 6.3 Monte-Carlo program [15]. All the models predict $H(s)>0$ for the observables $\langle 1-T\rangle$ and the heavy jet mass $\langle M_h^2/s\rangle$, and $H(s)<0$ for the difference of the jet masses $\langle (M_h^2-M_l^2)/s\rangle$. For $\int A_{\mathrm{EEC}}\,d\chi$ all the models also predict negative hadronization contributions, if the integration is done over a suffiently large region. Only if the lower integration limit is 40° or higher, the Hoyer model gives a small positive hadronization contribution, $H(s)\approx +0.004$. We have chosen an integration range (90° to 30°) with a sufficient margin to guarantee a negative sign of the hadronisation term for all the models.

3 Data analysis

The experiment was carried out with the CELLO detector at the e^+e^- storage ring PETRA, covering the energy range from 14 GeV to 46.8 GeV with a total integrated luminosity of about 120 pb^{-1}. A detailed description of the CELLO detector can be found in [16]. We briefly outline here only the detector components relevant to this analysis. The central tracking device consists of interleaved cyclindrical drift and proportional chambers inside a magnetic field of 1.3 tesla. Neutral particles (mainly photons) are detected by a fine grain lead/liquid argon calorimeter, sampling the energy depositions of the showers from 7 layers in depth. The total thickness of the calorimeter is 20 radiation lengths.

The cuts imposed on neutral and charged particle tracks in the selection of multihadronic events depend on the detector status and the background conditions. The cut in transverse momentum for charged particles varies from 150 MeV to 250 MeV and energy depositions of neutral showers have to exceed 400 to 600 MeV. Only tracks originating from the interaction vertex and which are contained in the fiducial volume of the central detector ($\cos\Theta<0.85$) are taken into account. Multihadronic events are selected by combinations of the following requirements:

1. Number of good charged particle tracks $\geqq 5$
2. Total energy of charged particles $>0.2\sqrt{s}$
3. Total energy of neutral particles >1 GeV
4. Total visible energy $>0.33\sqrt{s}$
5. Net charge of particle tracks <6
6. Charged particle tracks in both z-hemispheres.

Events passing conditions 1, 2, 4, 5 or 1, 3, 5, 6 are accepted as multihadrons. Additional cuts on the angular distribution $\cos\theta_{\mathrm{jet}}$ of event axes (thrust, sphericity) select a subsample of events which are well contained in the central detector.

The effects of initial state radiation, detector acceptance, detector resolution, and event selection have been corrected by an unfolding procedure [17]. The measured and the corrected distributions are connected by a convolution integral:

$$g(y)=\int A(y,x)\,f(x)\,dx,$$

when $g(y)$ is the measured distribution, $f(x)$ is the corrected distribution and $A(y,x)$ is the resolution function. Replacing the integral by a sum, we get the corrected distributions by minimizing:

$$\chi^2=\sum\frac{(\sum A_{ji}f_i-g_j)^2}{(\delta g_j)^2}$$

with the bin contents f_i as free parameters and with resolution matrices A_{ji} obtained from full Monte-Carlo simulations of the detector. From the corrected distributions the average values for our observables are then calculated. The solution of this type of integral equation is in general unstable, due to the loss of information in the detector, and generates strongly fluctuating f_i. To obtain stable solutions, additional conditions must be imposed by a regularization procedure. We have imposed the following two conditions on the f_i:

1. $f_i\geqq 0$ (histograms must not have negative entries).
2. The total curvature of the function $f(x)$ is limited by adding to the χ^2 a term proportional to the total curvature squared:

$$S=\chi^2+\tau\int(f''(x))^2\,dx.$$

The regularization constant τ is adjusted by hand to give a satisfactorily smooth shape for the resulting unfolded histogram. It turns out that the mean values, on which this analysis is based, do not depend on the value of the parameter τ.

The results of the unfolding procedure do not depend on the form of the Monte-Carlo distributions f_i which were used to calculate the resolution function. Thus any direct dependence of the corrected data on the fragmentation models used to produce the Monte-Carlo events is excluded. Only small, indirect effects remain, e.g. where different event shapes may lead to slight variations of the efficiencies or resolutions. These variations have been studied and are included in our systematic errors.

The unfolding procedure has been applied to the first three event shape variables. Owing to the definition of the energy-energy-correlations, an unfolding procedure cannot be readily applied to this variable. However, our angular resolution is good enough to make unfolding unnecessary. The effects of energy resolution, track efficiency and acceptance are corrected bin by bin using factors derived from the Monte-Carlo simulations. Also here, the small effects on the correction factors due to the various fragmentation schemes have been included in the systematic errors.

4 Results

Figure 1 shows our corrected data for $\langle 1-T\rangle$, $\langle M_h^2/s\rangle$, $\langle (M_h^2-M_l^2)/s\rangle$ and $\int A_{\mathrm{EEC}}\,d\chi$, together with the pure QCD-predictions (without hadronisation) for $\Lambda_{\overline{\mathrm{MS}}}=200$ MeV. It can be seen that the expectations for the signs of the various hadronization terms are confirmed and that the absolute size of $H(s)$ decreases with energy (except for $(M_h^2-M_l^2)/s$, where the fragmentation term is approximately energy-independent).

From the model calculations discussed above parametrisations of $H(s)$ can be derived. $H(s)$ is found to fall approximately as $C/\sqrt{s}$ (for $1-T, M_h^2/s, \int A_{\mathrm{EEC}}\,d\chi$) or to be constant (for $(M_h^2-M_l^2)/s$). A 5 parameter fit to our data (four constants C for the energy dependence $H(s)$ of the four event shape variables, and $\Lambda_{\overline{\mathrm{MS}}}$) results in $\Lambda_{\overline{\mathrm{MS}}}=375\pm 60$ MeV. The fit gives $\chi^2=32.5$ with 27 degrees of freedom. The error includes the statistical and systematic uncertainties of the data, but not the effect of the choice of the functional form of $H(s)$. Different parametrisations of $H(s)$ are found to give values of $\Lambda_{\overline{\mathrm{MS}}}$ covering a wide interval, limited only by the restrictions on the sign of $H(s)$. It is clear that this method will not result in a model-independent value for $\Lambda_{\overline{\mathrm{MS}}}$.

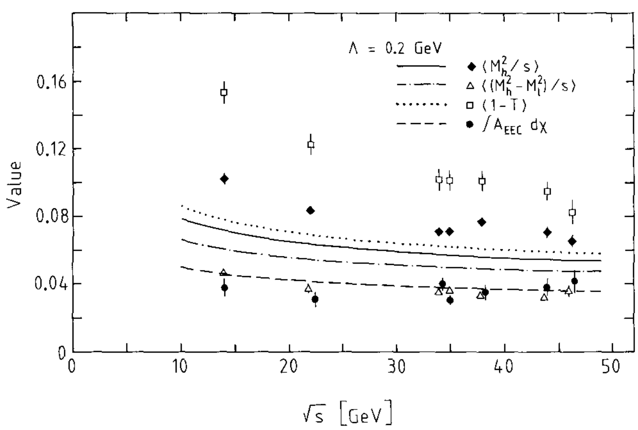

Fig. 1. Measurements of the quantites $\langle 1-T\rangle$, $\langle M_h^2/s\rangle$, $\langle (M_h^2-M_l^2)/s\rangle$ and $\int A_{\mathrm{EEC}}\,d\chi$ in the energy range 14–46.3 GeV. The data points have been corrected for detector effects and initial state radiation. The errors include statistical and systematic errors. The curves represent pure QCD predictions for the four variables using $\Lambda_{\overline{\mathrm{MS}}}=200$ MeV (this value of $\Lambda_{\overline{\mathrm{MS}}}$ is in agreement with the present experimental information [22]. Comparing the dashed curve and the data points for $\int A_{\mathrm{EEC}}\,d\chi$ one can see that for this quantity the fragmentation contributions are small

Nevertheless, conservative limits on $\Lambda_{\overline{\mathrm{MS}}}$ can be obtained, using the QCD terms alone. As $\Lambda_{\overline{\mathrm{MS}}}$ is varied, the theoretical curves in Fig. 1 move up and down, and excessive variations of $\Lambda_{\overline{\mathrm{MS}}}$ would change the sign of $H(s)$ for any given observable. The "correct" sign (see Sect. 2) of given $H(s)$ thus limits the values of $\Lambda_{\overline{\mathrm{MS}}}$. In Table 1 these limits are shown for all four variables. It is found that the most stringent limits on $\Lambda_{\overline{\mathrm{MS}}}$ result from requiring a positive sign for $H(s)$ in $\langle M_h^2/s\rangle$ (upper limit for $\Lambda_{\overline{\mathrm{MS}}}$), and a negative sign for $H(s)$ in $\int A_{\mathrm{EEC}}\,d\chi$ (lower limit for $\Lambda_{\overline{\mathrm{MS}}}$). Fitting the experimental data with $H(s)=0$ in each case gives, at the 95% confidence level:

$$\Lambda_{\overline{\mathrm{MS}}}<655\text{ MeV},\quad \alpha_s(35\text{ GeV})<0.171$$
$$\Lambda_{\overline{\mathrm{MS}}}>\ 45\text{ MeV},\quad \alpha_s(35\text{ GeV})>0.107.$$

Due to their larger fragmentation terms, $\langle 1-T\rangle$ and $\langle (M_h^2-M_l^2)/s\rangle$ give less stringent limits (Table 1). Figure 2 shows the upper and lower limits for α_s determined from M_h^2/s and $\int A_{\mathrm{EEC}}\,d\chi$, respectively, as functions of the center of mass energy. Also shown are the limits from the combined fit of the pure QCD formulae (see Sect. 2) to the data with values for Λ_{QCD} as given above.

As fragmentation effects can be assumed to decrease with energy, we have tried to improve the limits by using only the data points at energies greater than 30 GeV. However, with fewer data points there are larger statistical errors on $\Lambda_{\overline{\mathrm{MS}}}$ and the limits do not change much (see Table 1):

Table 1. Limits on $\Lambda_{\overline{\mathrm{MS}}}$ using various methods described in the text. The quoted limits correspond to the 95% confidence level. The renormalisation scale x was chosen as 1

a) Lower limits	$\langle (M_h^2-M_l^2)/s\rangle$ ($\Lambda_{\overline{\mathrm{MS}}}$ in MeV)	$\int A_{\mathrm{EEC}}\,d\chi$ ($\Lambda_{\overline{\mathrm{MS}}}$ in MeV)
$H(s)=0$, Fit over all energies	>13	>45
$H(s)=0$, $E>30$ GeV only	>10	>56
$H(s)$ polynomial	>18	>79

b) Upper limits	$\langle 1-T\rangle$ ($\Lambda_{\overline{\mathrm{MS}}}$ in MeV)	$\langle M_h^2/s\rangle$ ($\Lambda_{\overline{\mathrm{MS}}}$ in MeV)
$H(s)=0$, Fit over all energies	<1390	<655
$H(s)=0$, $E>30$GeV only	<1430	<654
$H(s)$ polynomial	<1320	<628

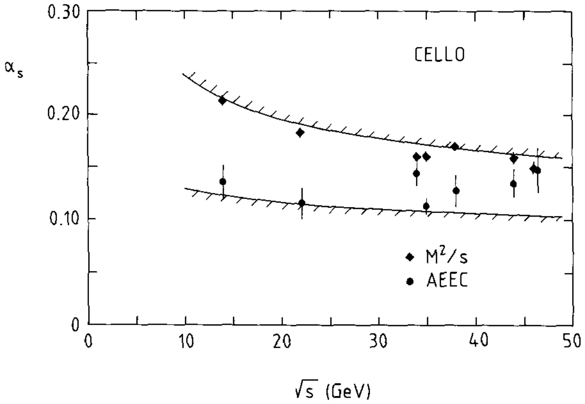

Fig. 2. Limits on α_s from $\int A_{\mathrm{EEC}}\, d\chi$ and $\langle M_h^2/s\rangle$. The data points show the values for α_s calculated from our experimental values for $\int A_{\mathrm{EEC}}\, d\chi$ and $\langle M_h^2/s\rangle$ by setting $H(s)=0$. The solid curves correspond to the 95% confidence level limits on $\Lambda_{\overline{MS}}$ which have been derived from fits of the pure QCD formulae to the data

$$\Lambda_{\overline{MS}}<654 \text{ MeV}, \quad \alpha_s(35 \text{ GeV})<0.171$$
$$\Lambda_{\overline{MS}}>56 \text{ MeV}, \quad \alpha_s(35 \text{ GeV})>0.111.$$

A further increase of the minimum energy does not lead to more stringent limits.

Finally, we use a third approach which makes use of all the data points and takes into account the hadronization term $H(s)$. We describe $H(s)$ as a third order polynomial in s, subject to the restrictions on sign and monotonous decrease already discussed, fit the formulae for $V(s)$ to the data and calculate the corresponding limits for $\Lambda_{\overline{MS}}$. This method results in our tightest limits on $\Lambda_{\overline{MS}}$ (see Table 1):

$$\Lambda_{\overline{MS}}<628 \text{ MeV}, \quad \alpha_s(35 \text{ GeV})<0.169$$
$$\Lambda_{\overline{MS}}>79 \text{ MeV}, \quad \alpha_s(35 \text{ GeV})>0.117.$$

The total cross section for e^+e^- annihilation to hadrons also gives a value for $\Lambda_{\overline{MS}}$ independent of the fragmentation model. From a combination of all the experimental results at CESR, DORIS, PEP, PETRA and TRISTAN a value of $\Lambda_{\overline{MS}}=400^{+300}_{-220}$ MeV can be obtained [18], using a second order QCD calculation. The 95% C.L. limits for $\Lambda_{\overline{MS}}$ from the total cross section data are considerably weaker than those presented here. Taking into account the uncertainties in the cross section normalisation due to higher order QED corrections one obtains $0.030<\Lambda_{\overline{MS}}<0.930$ GeV. However, the R value also yields the most probable value for $\Lambda_{\overline{MS}}$ in this interval, namely 400 MeV, which is in good agreement with the central value (375 ± 60 MeV) and consistent with the limits from the event shape analysis presented here, but is independent of assumptions concerning the hadronisation process.

5 Renormalisation scheme dependence

The calculation of the QCD-series expansion (1) requires the introduction of a specific renormalisation convention and scale. There are many possible choices of renormalisation scheme. The renormalisability of the theory ensures that the perturbative prediction for a physical observable V does not depend on the scheme, if the calculation is done to all orders. In practice, of course, a QCD calculation to all orders is not feasible and perturbative calculations for the observables investigated here are presently limited to second order. The first order coefficient (C_1 in (1)) is independent of the renormalisation scheme. But the higher order coefficients of the expansion do depend on the scheme and so will the result of Λ_{QCD} extracted by means of a low-order QCD formula from an observable V.

In calculations to second order α_s, any change of renormalisation scheme is equivalent to a change of the scale $s\rightarrow xs$ in the formula for α_s [19]. The coefficient C_2 in (1) is then transformed to

$$C_2'=C_2+0.5\,C_1(11-\tfrac{2}{3}N_f)\ln(x).$$

There are no solid theoretical guidelines for the choice of x. Stevenson [20] has suggested to choose an optimized renormalisation scale based on a "principle of minimum sensitivity" (PMS), so that the calculated value of the observable shows minimal sensitivity to the scale. This implies choosing x so that $\partial V/\partial x=0$. Note that different observables have different optimum values of x. For the above four variables, the values of x where this condition holds are almost independent of $\Lambda_{\overline{MS}}$. Using PMS we get the following limits (see also Table 2):

Table 2. Limits on $\Lambda_{\overline{MS}}$ using an optimised choice of the renormalisation scale ("PMS" scale, see text)

a) Lower limits	$\langle(M_h^2-M_l^2)/s\rangle$ ($\Lambda_{\overline{MS}}$ in MeV)	x	$\int A_{\mathrm{EEC}}\, d\chi$ ($\Lambda_{\overline{MS}}$ in MeV)	x
$H(s)=0$, Fit over all energies	>12	0.40	>40	0.33
$H(s)=0$, $E>30$ GeV only	>9	0.40	>48	0.33
$H(s)$ polynomial	>18	0.40	>65	0.33
b) Upper limits	$\langle 1-T\rangle$ ($\Lambda_{\overline{MS}}$ in MeV)	x	$\langle M_h^2/s\rangle$ ($\Lambda_{\overline{MS}}$ in MeV)	x
$H(s)=0$, Fit over all energies	<407	0.08	<354	0.15
$H(s)=0$, $E>30$ GeV only	<497	0.08	<401	0.15
$H(s)$ polynomial	<375	0.08	<340	0.15

Table 3. Examples for upper limits on $\Lambda_{\overline{MS}}$ with an arbitrary but large renormalisation scale ($x=2$)

Upper limits	$\langle 1-T \rangle$ ($\Lambda_{\overline{MS}}$ in MeV)	$\langle M_h^2/s \rangle$ ($\Lambda_{\overline{MS}}$ in MeV)
$H(s)=0$, Fit over all energies	<2170	<930
$H(s)=0$, $E>30$ GeV only	<2180	<915
$H(s)$ polynomial	<2040	<880

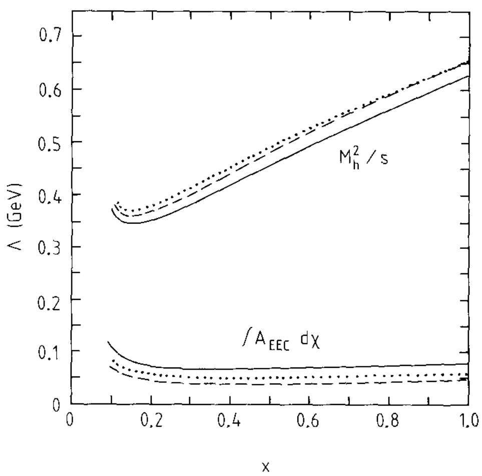

Fig. 3. Limits on $\Lambda_{\overline{MS}}$ as a function of the renormalisation parameter x. The dashed lines represent the fits over the whole energy range with $H(s)=0$. The dotted lines are for $E_{cm}>30$ GeV. The solid lines result from fits with a polynomial $H(s)$ (see text)

$$\Lambda_{\overline{MS}} < 340 \text{ MeV}, \quad x=0.15$$
$$\Lambda_{\overline{MS}} > 65 \text{ MeV}, \quad x=0.33.$$

For any given $\Lambda_{\overline{MS}}$ the PMS scale yields the maximum value for the observables given in (3–6). Consequently the value for $\Lambda_{\overline{MS}}$, obtained in a QCD fit to the measurement of an observable using PMS, will be the lowest in comparison to any other scale. It follows that PMS will lead to the tightest upper limits but to the most conservative lower limits for $\Lambda_{\overline{MS}}$. This can be verified by comparing the results for $\Lambda_{\overline{MS}}$ in Tables 1 and 2.

Without a good guideline for a choice of scale, we have varied x over a plausible range. The lower limit was chosen as $x>0.1$ ($x>0.006$ for $\langle 1-T \rangle$). Below this, the values of the observables become unphysical and no real solutions for α_s can be obtained. As to the upper limit of x, we have explored x values up to 10 with the general observation that the limits for $\Lambda_{\overline{MS}}$ increase with increasing x. For illustration, Table 3 gives our results for the upper limits for $\Lambda_{\overline{MS}}$ at the value of $x=2$. Figure 3 shows the limits on $\Lambda_{\overline{MS}}$ as a function of x up to $x=1$ for the observables M_h^2/s and $\int A_{EEC}\, d\chi$. In all observables the limits obtained from our data continue to grow with increasing x. Since there is no clear prescription as to which x to take there is also no clear upper limit on $\Lambda_{\overline{MS}}$. There is, however, a definite minimum for the lower limit on $\Lambda_{\overline{MS}}$, which is reached at the PMS scale, as discussed above.

One may ask whether experiment can help to decide about the correct scale x, given the second order QCD predictions (1). However, for each of the four observables it is possible to find a range of combinations of x values and values of $\Lambda_{\overline{MS}}$ that allow good fits to our data with $H(s)\equiv 0$. The range of possible values of x is big and we conclude that the influence of the hadronization term, the experimental errors, and the close correlation between $\Lambda_{\overline{MS}}$ and x do not allow us to make statements about experimentally preferred values for x.

There may, however, exist observables which allow a determination of optimal values for both x and $\Lambda_{\overline{MS}}$. For this purpose, the relative rates of 2, 3, and 4 jet final states in multihadronic events seem to offer promise. Preliminary evidence suggests that a rather small value of x (in the order of 0.1) is preferred [21]. Since the event shape variables studied here are directly related to the relative parton multiplicities in the final state one can use the x determined from the jet cross sections as a guide for the proper scale. We therefore consider $x=1$ a conservative choice for the determination of an upper limit of Λ_{QCD}. The lower limits, on the other hand, do not depend strongly on the scale and have a definite minimum. They can be regarded as solid experimental constraints on $\Lambda_{\overline{MS}}$. Taking into account the hadronization term $H(s)$ and the uncertainties related to the renormalisation scale dependence we conclude for the QCD scale parameter Λ_{QCD} that

$$65 \text{ MeV} < \Lambda_{QCD} < 628 \text{ MeV}$$

at the 95% C.L. (see also Table 2).

The choice of the renormalisation scale in principle also affects the limits for $\Lambda_{\overline{MS}}$ calculated from the total e^+e^- annihilation cross section (see preceeding section). The variation of $\Lambda_{\overline{MS}}$ in the x range below 1, however, is insignificant and the limits given above do not change much.

6 Conclusions

From measurements of the energy dependence of the event shape variables $1-T$, M_h^2/s, $(M_h^2-M_l^2)/s$ and $\int A_{EEC}\, d\chi$ in the energy range from 14 to 46 GeV we have derived limits for the QCD scale parameter Λ_{QCD} which are independent of fragmentation effects.

Working in the $\overline{\mathrm{MS}}$ renormalisation scheme we obtain the limits 79 MeV $< \Lambda_{\overline{\mathrm{MS}}} <$ 628 MeV at the 95% C.L. These values are consistent with similar analyses from other authors. R.D. Field [5] quotes limits of $0.02 < \Lambda_{\overline{\mathrm{MS}}} < 0.40$ GeV, while the PLUTO collaboration [4] quotes values for α_s(30 GeV) between 0.11 ± 0.01 and 0.15 ± 0.02, corresponding to $0.02 < \Lambda_{\overline{\mathrm{MS}}} < 0.55$ GeV. JADE [6] quote limits of $0.025 < \Lambda_{\overline{\mathrm{MS}}} < 0.400$ GeV at the 95% C.L., based on a single data point giving the tightest limit. All these results are for the $\overline{\mathrm{MS}}$ renormalisation scheme and do not take into account the effects of changes of the renormalisation scale x. Whereas the choice of x does not significantly affect the lower limit of Λ_{QCD} the upper limit increases with increasing x. Restricting x to the plausible range below 1 we find 65 MeV $< \Lambda_{\mathrm{QCD}} <$ 628 MeV at the 95% C.L.

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References

1. CELLO Coll. H.J. Behrend et al.: Nucl. Phys. B218 (1983) 269
2. CELLO Coll. H.J. Behrend et al.: Phys. Lett. 138B (1984) 311
3. CELLO Coll. H.J. Behrend et al.: Contribution No. 685 to the International Conference on High Energy Physics, Munich, 1988
4. PLUTO-Coll. Ch. Berger et al.: Z. Phys. C – Particles and Fields 12 (1981) 297
5. R.D. Field: Proc. Int. Symposium on Lepton and Photon Interactions at High Energies, Cornell, 1983
6. JADE Coll. A. Dieckmann, Proc. of the 22. Rencontre de Moriond, 1987, p. 337
7. W.J. Marciano: Phys. Rev. Lett. 29 (1983) 580
8. T. Chandramohan, L. Clavelli: Phys. Lett. 94B (1980) 409
9. L. Clavelli: Phys. Lett. 85B (1979) 111
10. C.L. Basham et al.: Phys. Rev. D19 (1979) 2018
11. D.G. Richards et al.: Phys. Lett. 119B (1982) 193; A. Ali, F. Barreiro: Nucl. Phys. B236 (1984) 269
12. A. Ali et al.: Phys. Lett. 93B (1979) 349
13. P. Hoyer et al.: Nucl. Phys. B161 (1979) 349
14. G. Marchesini, B.R. Webber: Nucl. Phys. B238 (1984) 1
15. T. Sjöstrand: Computer Phys. Commun. 43 (1987) 367
16. CELLO Coll. H.-J. Behrend et al.: Phys. Scr. 23 (1981) 610
17. V. Blobel: DESY 84-118 (1984); S. Provencher: Comput. Phys Commun. 27 (1982) 213; Rust & Burrus, Mathematical programming and the solution of linear equations, New York, 1972
18. CELLO Coll. G. D'Agostini et al.: contribution No. 764 to the International Conference on High Energy Physics, Munich, 1988
19. D.W. Duke, R.G. Roberts: Phys. Rep. 120 (1985) 275
20. P.M. Stevenson: Phys. Rev. D23 (1981) 2916
21. CELLO Coll. H.-J. Behrend et al.: contribution No. 540 to the International Conference on High Energy Physics, Munich, 1988
22. see, e.g., W. de Boer: Proc. of the 10th Warsaw Symposium on Elementary Particle Physics, Kazimierz, Poland 1988, p. 503, and SLAC-Pub-4428, 1988