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Charged majorons in e^+e^- and e^-e^- collisions

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Abstract. The experimental limits on charged majorons, i.e. charged scalars which carry lepton number, are discussed in a model independent way. Possible signals in e^+e^- and e^-e^- collisions are then estimated. It is stressed that the limits on majorons which conserve the different lepton flavors separately are presently very poor. These limits can however be improved by studying e^+e^- and e^-e^- collisions.

0 Introduction

The only scalar particle which appears in the standard model is the Higgs boson. Nearly all extensions of the standard model increase the number of scalar particles, sometimes drastically. In this paper the phenomenology of a specific set of scalar particles will be considered, namely of the majorons. In the present context a majoron is defined as a scalar particle which can have a Yukawa coupling to $\overline{f_1^c} f_2$, where f_1, f_2 are leptons which are present in the standard model. Several models contain this kind of majorons, e.g. left-right symmetric models [1] and the so called triplet majoron model [2]. In these models the majorons play a variety of roles. They can e.g. be used to break lepton number L, or $B-L$ spontaneously and thereby generate Majorana masses for the neutrinos. In such cases there appears a massless electrically neutral Goldstone boson which could lead to interesting phenomenology.

Majorons can also carry electric charge. An interesting feature of the doubly charged Majoron is that it can appear as an s-channel resonance in e^-e^- collisions, while the standard model predicts only less spectacular t-channel physics for this process. At the moment there haven't been experiments with high energy e^-e^- collisions for a long time. For good reasons the experiments have concentrated at e^+e^- collisions. It is also impossible to change a storage ring for e^+e^- slightly to make it into a storage ring for e^-e^-, since it is impossible to store two colliding e^- beams in the same magnetic field. For linear colliders as they might appear in the coming decade the situation is different. In principle it is possible to reverse the polarity of a linear collider, so that one could use a pair of linear colliders for both e^+e^- and e^-e^- studies.

The phenomenology of majorons has been studied several times in the context of specific models [3–5]. Here it is the purpose to study the phenomenology of the charged majorons as model independent as possible. This means e.g. that one cannot assume that the neutral components of the majorons develop a vacuum expectation value (vev). By assuming that the vev's vanish one can avoid many limits which arise from neutrino masses [6] or lepton number violations [7, 8]. In this case the spectrum doesn't contain a Goldstone boson to which many limits are tied [9–12], and also relations between the masses of the majorons with different charges, as they exist in many models, do not exist anymore.

When one adds a set of majorons to the standard model, without giving their neutral components a vev, the result is obviously a rather ugly model which does not solve any theoretical or experimental problem. However the question of the existence of majorons can only be answered by the experiment.

The contents of this paper is as follows. In the first section the quantumnumbers and the interactions of the majorons are discussed. Sect. 2 deals with the existing limits on doubly charged majorons, while the singly charged majorons are covered by section three. These limits are then used to estimate the possible signals in e^+e^- and e^-e^- collisions in Sects. 4 and 5. A few concluding remarks make up Sect. 6.

1 Spectrum and interactions

Given the fermion content of the standard model the number of scalars that can have Yukawa interactions with the known fermions is rather limited. When we restrict ourselves to scalars which are color singlets, then the only possibilities are [3,4]: (i) an $SU(2)$ doublet with hypercharge $Y=+1$. This is nothing but the ordinary Higgs boson. (ii) an $SU(2)$ singlet, $Y=-4$ the a^{--}. (iii) an $SU(2)$ singlet, $Y=-2$, the b^-. (iv) an $SU(2)$ triplet, $Y=-2$, the c^i, containing a neutral component, a component with charge -1 and a component with charge -2. One can make total lepton number still a symmetry of the Yukawa couplings when one assigns $L=+2$ to the majorons a^{--}, b^-, c^i.

Apart from the usual gauge interactions the majorons have the following Yukawa couplings to leptons:

$$\begin{aligned}\mathcal{L}_{Yuk} = {} & \tfrac{1}{2} Y^a_{ee} a^* e_R^T C e_R + Y^a_{e\mu} a^* e_R^T C \mu_R + \tfrac{1}{2} Y^a_{\mu\mu} \mu_R^T C \mu_R \\ & + Y^b_{e\mu} b^* \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L^T \varepsilon C \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \\ & + \tfrac{1}{2} Y^c_{ee} c_i^* \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L^T \varepsilon \tau^i C \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \\ & + Y^c_{e\mu} c_i^* \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L^T \varepsilon \tau^i C \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \\ & + \tfrac{1}{2} Y^c_{\mu\mu} c_i^* \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L^T \varepsilon \tau^i C \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L + c.c \end{aligned} \tag{1.1}$$

where C is the charge conjugation matrix and ε the two by two antisymmetric tensor in isospin space. Only the couplings of the first two generations have been shown. Note that the couplings of the b, the singly charged singlet, are antisymmetric with respect to lepton flavour. By restricting the Yukawa interactions and assigning L_e, L_μ and L_τ to the scalars properly, it is even possible to conserve the different lepton flavours separately.

The most general $SU(2) \times U(1)$ invariant potential one can write down for these fields (taking only one copy of each) contains four mass terms, three threepoint interactions and fifteen independent fourpoint interactions. It turns out that due to the $SU(2) \times U(1)$ symmetry total lepton number is violated by the three point couplings and by nothing else. Therefore one can avoid all limits coming from lepton number violating processes by assuming these three point couplings to be absent or small. One thereby also eliminates the coupling $c^i \phi^T \varepsilon \tau^i \phi$ which would cause the neutral component of c^i to develop a vev as soon as the $SU(2) \times U(1)$ symmetry is broken by the Higgs field.

When the Higgs field develops its vev, the two singlet majorons will both mix with the components of the triplet majoron which have the same charge. The scalar potential has enough free parameters, even when lepton number conservation is imposed, to allow any value of the masses and mixing angles. The physical states are:

$$\begin{aligned} a_1^{--} &= \cos\alpha a^{--} + \sin\alpha c^{--} \\ a_2^{--} &= -\sin\alpha a^{--} + \cos\alpha c^{--} \\ b_1^- &= \cos\beta b^- + \sin\beta c^- \\ b_2^- &= -\sin\beta b^- + \cos\beta c^- \end{aligned} \tag{1.2}$$

where a^{--} and b^- are the $SU(2)$ singlet majorons and

$$\begin{aligned} c^- &= c^3 \\ c^{--} &= \frac{c^1 - ic^2}{\sqrt{2}}. \end{aligned} \tag{1.3}$$

One can allways choose the angles such that $m_{a_1} < m_{a_2}$ and $m_{b_1} < m_{b_2}$.

Since the majorons can have an $SU(2)$ invariant massterm and also get a contribution to their mass from the vev of the Higgs field, their couplings to the Higgs field are not proportional to their mass matrix. There also exist off diagonal couplings like $Ha_1^{--}a_2^{++}$ and $Hb_1^- b_2^+$, which are however proportional to $\sin 2\alpha$ resp. $\sin 2\beta$ and therefore suppressed when the mixing angles are small.

Both the singly charged b^- and the singly charged component of the triplet carry hypercharge $Y=-2$ and third component of weak isospin $T^3=0$. Therefore the rotation to their masseigenstates will not introduce offdiagonal Z-couplings. This is different for the doubly charged components. Here one mixes two fields with different hyercharge and will in general create a nonzero offdiagonal coupling $Za_1^{--}a_2^{++}$.

The mixing of the charged scalars also gives rise to a source of CP-violation. When one considers a process which involves only a limited number of Yukawa couplings, one can frequently get rid of the CP violating phases by redefining the scalar fields. When one ties different scalar fields into mass eigenstates the freedom to redefine the phases of the fields is restricted. This leads e.g. to an electric dipole moment for the electron.

2 Limits on the doubly charged majoron

It has been realized by a large number of authors that the best limits on virtual doubly charged majorons come from the process $\mu^- \to e^- e^- e^+$. First of all there exists a very stringent upper limit on this branching ratio [13]: $\mathrm{BR}(\mu^- \to e^- e^- e^+) < 1.0 \cdot 10^{-13}$ and secondly the doubly charged majoron contributes at tree level to this process. When the Yukawa couplings are given by

$$\mathcal{L} = a^* [e^T C (f_{e\mu} + i g_{e\mu} \gamma_5) \mu + \tfrac{1}{2} e^T C (f_{ee} + i g_{ee} \gamma_5) e] \tag{2.1}$$

where the coupling constants f and g are combinations of the Yukawa couplings in (1.1) and the mixing angles from (1.2), then the rate is given by

$$\Gamma(\mu \to e^- e^- e^+) = \frac{m_\mu^5}{192 \pi m_a^4} \alpha_{\mathrm{eff}}^2 \tag{2.2}$$

where

$$\alpha_{\mathrm{eff}}^2 = \frac{|f_{e\mu}|^2 + |g_{e\mu}|^2}{4\pi} \cdot \frac{|f_{ee}|^2 + |g_{ee}|^2}{4\pi}. \tag{2.3}$$

Together with the upperlimit on the branching ratio this implies

$$m_a/\alpha_{\mathrm{eff}}^{1/2} > 900\,\mathrm{TeV}. \tag{2.4}$$

It has to be realized however that this lower limit can be circumvented by constructing a model in which the contributions from two majorons interfere destructively.

Other limits on $m_a/\alpha_{\mathrm{eff}}^{1/2}$, where α_{eff} is an appropriate combination of Yukawa coupling constants, will now be discussed briefly.

The charged majorons also contribute to $\mu\to e\gamma$ [14]. Defining the Yukawa couplings analogous to (2.1) we find for the effective $\mu e\gamma$ coupling due to the exchange of a virtual doubly charged majoron

$$\begin{aligned}\Gamma^\mu = \frac{e\sigma^{\mu\nu}q_\nu}{16\pi^2 m_a^2}\sum_{l=e,\mu,\tau} & [(m_l(f_{\mu l}f_{el}^* - g_{\mu l}g_{el}^*) \\ & -\tfrac{1}{6}m_\mu(f_{\mu l}f_{el}^* + g_{\mu l}g_{el}^*)) + (m_l(-f_{\mu l}g_{el}^* + g_{\mu l}f_{el}^*) \\ & +\tfrac{1}{6}m_\mu(f_{\mu l}g_{el}^* + g_{\mu l}f_{el}^*))\gamma_5].\end{aligned} \tag{2.5}$$

At first sight this expression could be enhanced by the presence of a heavy lepton, like the τ, but the coefficients of m_l are such that they vanish if one considers only one doubly charged majoron since a single majoron couples exclusively left- or right-handed. When one couplies more than one majoron the contribution of the heavy lepton is suppressed by $\sin 2\alpha$. The typical size of this coupling is therefore given by the contributions which are proportional to the muon mass. This leads to a decay rate

$$\Gamma(\mu\to e\gamma) = \frac{\alpha\alpha_{\mathrm{eff}}^2 m_\mu^5}{1152\pi^2 m_a^4} \tag{2.6}$$

where this time

$$(4\pi\alpha_{\mathrm{eff}})^2 = \sum_l (|f_{\mu l}f_{el}^* + g_{\mu l}g_{el}^*|^2 + |f_{\mu l}g_{el}^* + g_{\mu l}f_{el}^*|^2). \tag{2.7}$$

Using the experimental upper bound [15] $\mathrm{BR}(\mu\to e\gamma) < 4.9\cdot 10^{-11}$ one obtains the limit

$$m_a\alpha_{\mathrm{eff}}^{-1/2} > 27\,\mathrm{TeV}. \tag{2.8}$$

The doubly charged majorons contribute to rare τ decays just like they contribute to rare μ decays. The experimental upper bounds [16] on rare τ decays are not that tight, so when one translates them into limits on virtual doubly charged majorons one finds the rather poor limits.

$$\begin{aligned} m_a/\alpha_{\mathrm{eff}}^{1/2} &> 4.8\,\mathrm{TeV} \quad \text{from } \tau^-\to\mu^-\mu^-\mu^+ \\ &> 4.5\,\mathrm{TeV} \quad \text{from } \tau^-\to e^-e^-e^+ \\ &> 5.5\,\mathrm{TeV} \quad \text{from } \tau^-\to\mu^-e^-e^+ \\ &> 5.5\,\mathrm{TeV} \quad \text{from } \tau^-\to\mu^-\mu^+e^-. \end{aligned} \tag{2.9}$$

Of course α_{eff} is in each of these limits a different combination of Yukawa couplings, but unless something special happens they are of the same order of magnitude.

The limits which follow from the precise knowledge one has of the electromagnetic formfactors of the electron and the muon are not very tight. They are nevertheless of interest since they are the best limits one can obtain when for some reason not only the total lepton number is conserved, but also each lepton flavor separately. The contribution of one doubly charged majoron to the anomalous magnetic and electric dipole moments of the electron is given by

$$\begin{aligned}\Gamma^\mu(q) = \frac{e\sigma^{\mu\nu}q_\nu}{16\pi^2 m_a^2}\sum_l \Bigg[& \frac{-2m_e}{3}(|f_{el}|^2 + |g_{el}|^2) \\ & - m_l\left(\ln\frac{m_a^2}{m_l^2} - \frac{5}{2}\right)(|f_{el}|^2 + |g_{el}|^2) \\ & + m_l\left(\ln\frac{m_a^2}{m_l^2} - \frac{5}{2}\right)(f_{el}g_{el}^* - g_{el}f_{el}^*)\gamma_5\Bigg].\end{aligned} \tag{2.10}$$

The second term on the rhs gives a contribution to the anomalous magnetic moment of the electron which is enhanced by a factor m_l and a logarithm. Its coefficient is not of definite sign however and vanishes when one considers only one doubly charged majoron. In general this term will be suppressed by a mixing angle. The third term on the rhs contributes to the electric dipole moment of the electron. For this term not to vanish, several requirements have to be met. Consider for example the case in which one has one singlet and one triplet majoron. To start with the contribution is proportional to $\mathrm{Im}\, Y_{el}^{a*} Y_{el}^c$, i.e. the Yukawa couplings have to have a nontrivial relative phase. The coefficient is also proportional to $\sin 2\alpha$ and a nonzero electric dipole moment requires nontrival mixing. Finally the contribution of the heavier mass eigenstate exactly cancels the contribution from the lighter mass eigenstate, when both states are degenerate in mass. This gives a final suppression with a factor proportional to $m_{a_2}^2 - m_{a_1}^2$. Therefore the only unavoidable contribution comes from the first term. Comparing this with the uncurtainty in the value of the electrons magnetic moment, we find

$$m_a/\alpha_{\mathrm{eff}}^{1/2} > 30\,\mathrm{GeV}. \tag{2.11}$$

Where this time

$$4\pi\alpha_{\mathrm{eff}} = \sum_l (|f_{el}|^2 + |g_{el}|^2). \tag{2.12}$$

Repeating the same for the muon one finds

$$m_a/\alpha_{\mathrm{eff}}^{1/2} > 340\,\mathrm{GeV}. \tag{2.13}$$

These bounds are obviously very weak, but the best bounds available when all leptons flavors are separately conserved.

Recently the upper limit on muonium–antimuonium conversion has been improved drastically [17]:

$$|\langle \overline{M}|\mathscr{H}|M\rangle|^2 < 9.3\cdot 10^{-13}\,\mathrm{eV} \tag{2.14}$$

This still leads to a rather poor limit on charged majorons however

$$m_a/\alpha_{\text{eff}}^{1/2} = \left|\frac{4\pi|\psi(0)|^2}{\langle \overline{M}|\mathscr{H}|M\rangle}\right|^{1/2} > 470\,\text{GeV}. \qquad (2.15)$$

3 Limits on the singly charged majoron

In many models featuring majorons there exist relations between the masses of singly and doubly charged majorons. Using these mass relations together with the strong bounds on the masses of the doubly charged majorons, one also obtains strong bounds on the singly charged majorons. This is obviously a model dependent procedure. The strongest, model independent bounds on singly charged majorons arises when one considers the corrections to weak interaction parameters due to tree-level exchange of majorons.

The weak mixing angle $\sin\theta_W$ and the neutral vector boson mass can both be determined from the elastic cross sections for $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering. When one adds the singly charged majorons, two things change. The charged majorons also contribute to these scattering processes and secondly processes like $\nu_\mu e \to \nu_e e$ now also take place, which cannot be distinguished experimentally from elastic scattering. The latter effect only adds incoherently to the standard model, while the former is enhanced due to interference, therefore we ignore the latter from now on. The standard model effective action for $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering is:

$$\mathscr{L}_{\text{eff}} = \frac{e^2}{16 s_\theta^2 c_\theta^2 m_Z^2}\,\bar{\nu}_\mu\gamma^\lambda(1-\gamma_5)\nu_\mu \bar{e}\gamma_\lambda(v_\theta-\gamma_5)e \qquad (3.1)$$

where $s_\theta^2 = \sin^2\theta_W$, $c_\theta^2 = 1 - s_\theta^2$, $v_\theta = 1 - 4s_\theta^2$. When the majoron b has the following coupling

$$\mathscr{L} = b^*\left(f_{\mu\nu_e}\mu^T C\frac{1-\gamma_5}{2}\nu_e + f_{e\nu_\mu} e^T C\frac{1-\gamma_5}{2}\nu_\mu\right) \qquad (3.2)$$

it contributes the following to the effective action:

$$\delta\mathscr{L} = \frac{|f_{e\nu_\mu}|^2}{8m_b^2}\,\bar{\nu}_\mu\gamma^\lambda(1-\gamma_5)\nu_\mu\bar{e}\gamma_\lambda(1-\gamma_5)e. \qquad (3.3)$$

When one determines the weak mixing angle from the ratio $\sigma(\bar{\nu}_\mu e)/\sigma(\nu_\mu e)$ one gets

$$\delta\sin^2\theta_W = \sin^2\theta_{LE} - \sin^2\theta_W = \frac{-2s_\theta^4 c_\theta^2 m_Z^2}{e^2}\frac{|f_{e\nu_\mu}|^2}{m_b^2} \qquad (3.4)$$

where θ_W is the parameter appearing in the action and θ_{LE} is the mixing angle as determined from the low energy data without corrections for the presence of the majoron.

From the total cross-section one can analogously determine the shift in the neutral vector boson mass

$$\frac{\delta m_Z^2}{m_Z^2} = \frac{-2s_\theta^4 m_Z^2}{e^2}\frac{|f_{e\nu_\mu}|^2}{m_b^2}. \qquad (3.5)$$

The standard model effective action for μ decay

$$\mathscr{L}_{\text{eff}} = \frac{-e^2}{8m_W^2 s_\theta^2}\,\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\mu\bar{e}\gamma^\lambda(1-\gamma_5)\nu_e \qquad (3.6)$$

also gets a contribution

$$\delta\mathscr{L}_{\text{eff}} = \frac{f_{\mu\nu_e}f_{e\nu_\mu}^*}{8m_b^2}\,\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\mu\bar{e}\gamma^\lambda(1-\gamma_5)\nu_e \qquad (3.7)$$

due to majoron exchange. This causes a shift in the charged vectorboson mass

$$\frac{\delta m_W^2}{m_W^2} = \frac{m_W^2 s_\theta^2}{e^2}\,\frac{2|f_{e\nu_\mu}|^2 + \text{Re}\,f_{\mu\nu_e}f_{e\nu_\mu}^*}{m_b^2}. \qquad (3.8)$$

When one constructs the ρ parameter, $\rho = m_W^2/m_Z^2\cos^2\theta_W$ from these low energy measurements, one finds

$$\frac{\delta\rho}{\rho} = \frac{\delta m_W^2}{m_W^2}, \qquad (3.9)$$

What can one learn from this? The shift in the ρ parameter due to other causes than standard model radiative corrections is known to be less than about 2% [18].

$$\frac{\delta\rho}{\rho} = \frac{3m_W^2 s_\theta^2\alpha_{\text{eff}}^2}{\alpha m_b^2} < 2\% \qquad (3.10)$$

and therefore $m_b\alpha_{\text{eff}}^{-1/2} > 5.5\,\text{TeV}$. At LEP I one will be able to measure the Z mass with an accuracy of 100 MeV. This will raise the limit to $m_b\alpha_{\text{eff}}^{-1/2} > 7.3\,\text{TeV}$. In case one will be able to determine the W mass with the same accuracy at LEP II, the limit again improves $m_b\alpha_{\text{eff}}^{-1/2} > 16\,\text{TeV}$.

Note that through a judicious choice of phases for the majoron couplings the values of $\delta\rho$ and δm_W can be reduced, but that this is not the case for δm_Z. It is also interesting to observe that these limits do not involve processes which violate lepton flavor.

To finish this section a few other limits on singly charged majorons will be briefly discussed. The presence of a singly charged majoron gives rise to a $\mu e\gamma$ vertex

$$\Gamma^\mu(q) = \frac{-e\sigma^{\mu\nu}q_\nu m_\mu}{96\pi^2 m_b^2}(1+\gamma_5)\sum_l (f_{\mu l}f_{el}^*). \qquad (3.11)$$

Together with the experimental bound on $\text{BR}(\mu\to e\gamma)$ this leads to

$$m_b\alpha_{\text{eff}}^{-1/2} > 8.4\,\text{TeV} \qquad (3.12)$$

but there are many ways to circumvent this limit. There could e.g. be destructive interference with the contribution of the doubly charged majorons.

It is amusing to note that the, in itself amazing, limit on the process $\mu\to e\nu_e\bar{\nu}_\mu$ [19], i.e. the μ decay to the "wrong" kind of neutrinos, leads to a nontrivial bound on charged majorons. The rate for this process is given by

$$\Gamma(\mu \to e\nu_e\bar{\nu}_\mu) = \frac{m_\mu^5}{384\pi^3 m_b^4} |f_{\mu\nu_\mu}|^2 |f_{e\nu_e}|^2 \tag{3.13}$$

which leads to the lower limit on the mass off

$$m_b \alpha_{\text{eff}}^{-1/2} > 1.3\,\text{TeV}. \tag{3.14}$$

Finally the limits from the anomalous magnetic moments of the electron and the muon are even weaker

$$m_b \alpha_{\text{eff}}^{-1/2} > 10\,\text{GeV from the electron} \tag{3.15}$$

$$m_b \alpha_{\text{eff}}^{-1/2} > 122\,\text{GeV from the muon.} \tag{3.16}$$

The singly charged majorons do not contribute to the electric dipole moments of leptons.

4 Signals in e^+e^- collisions

Since the majorons that can have Yukawa couplings to the ordinary leptons are necessarily gauge non-singlets, they can be easily pairproduced in e^+e^- collisions. When one assumes that there is exactly one majoron with each set of quantum numbers one finds for the pair production crosssection

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \lambda^{3/2}(1, m_1^2/s, m_2^2/s)}{8s} \sin^2\theta \cdot \left(\left| Q + \frac{Q_Z(1 - 4\sin^2\theta_W)}{4\sin^2\theta_W \cos^2\theta_W} \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2 + \left| \frac{Q_Z}{4\sin^2\theta_W \cos^2\theta_W} \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2 \right)$$

where Q and Q_Z for each process can be found in Table 1. Under most circumstances this proceeds with ordinary electroweak strength. The only exception is the case when the mixing angle of the doubly charged majorons has precisely the right value to make the coupling of the lightest doubly charged majoron to the Z-boson vanish, i.e. $\sin^2\alpha = 2\sin^2\theta_W$. In this case the crosssection for pairproduction of this scalar does not feel the Z resonance although it cannot be made to vanish completely. It is therefore quite save to assume that the lower limit for the mass of a charged majoron is about the same as the search limit for any other charged particle: a little below the beam energy of the highest energy e^+e^- machine available.

Table 1. This table contains the coefficients which appear in the expression for the pair-production cross-section of scalar particles

Particles	Q	Q_Z
$a_1\bar{a}_1$	-2	$2s_\theta^2 - s_\alpha^2$
$a_1\bar{a}_2$	0	$-s_\alpha c_\alpha$
$a_2\bar{a}_1$	0	$-s_\alpha c_\alpha$
$a_2\bar{a}_2$	-2	$2s_\theta^2 - c_\alpha^2$
$b_1\bar{b}_1$	-1	s_θ^2
$b_2\bar{b}_2$	-1	s_θ^2

An obvious possibility to look at virtual majorons is $e^+e^- \to \mu^+e^-$, which can be mediated by a doubly charged majoron in the u channel. Its total crosssection is given by

$$\sigma = \frac{\pi\alpha_{\text{eff}}^2}{s} \left(\frac{s + 2m_a^2}{s + m_a^2} - \frac{2m_a^2}{s} \ln \frac{s + m_a^2}{m_a^2} \right) \tag{4.2}$$

where this time

$$(4\pi\alpha_{\text{eff}})^2 = (|f_{ee}|^2 + |g_{ee}|^2)(|f_{e\mu}|^2 + |g_{e\mu}|^2). \tag{4.3}$$

The stringent lower limit on the majoron mass (2.4), which arises from the crossed process $\mu \to eee$, leads however to an upper limit on the cross-section for this process of about

$$\sigma(e^+e^- \to \mu^+e^-) < 5 \cdot 10^{-12} \frac{s}{m_Z^2}\,\text{pbarn}. \tag{4.4}$$

It is therefore rather unlikely to see the charged majoron this way.

Another possibility is to look at the forward–backward asymmetry in $e^+e^- \to \mu^+\mu^-$ collisions. As an example, consider the differential cross-section for this process on top of the Z resonance

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{1024 c_\theta^4 s_\theta^4} \frac{1}{\Gamma_Z^2} ((1 + v_\theta^2)^2 (1 + \cos^2\theta) + 8 v_\theta^2 \cos\theta) - \frac{\alpha\alpha_{\text{eff}}}{8} \frac{1}{m_a^2} (1 + \cos\theta)^2 \tag{4.5}$$

where $4\pi\alpha_{\text{eff}} = (|f_{e\mu}|^2 + |g_{e\mu}|^2)$ and m_a is the mass of the doubly charged majoron. This leads to a forward–backward asymmetry

$$A_{F-B} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)} = 3v_\theta^2 - \frac{27}{8} \frac{\alpha_{\text{eff}}}{m_a^2} \frac{\Gamma_Z^2}{\alpha}. \tag{4.6}$$

When one measures this asymmetry with a 1% accuracy, one establishes the following bound

$$m_a \alpha_{\text{eff}}^{-1/2} > 650\,\text{GeV}.$$

This is very meagre compared to the bound (2.4) obtained from the decay $\mu^- \to e^-e^-e^+$. However the process considered here does not involve lepton flavor violation. In case all lepton flavors are separately conserved, even in the presence of majorons, this is the strongest bound one can obtain.

5 e^-e^- scattering

When one considers e^-e^- scattering it is natural to contemplate the doubly charged majorons, since they are by construction the only scalar particles which can produce an s-channel resonance. The bound on the Yukawa couplings of the doubly charged majorons are however that tight that it is very unlikely that one can see majorons this way. As an example the cross-section for $e^-e^- \to \mu^-\mu^-$ on top of the majoron resonance is roughly $\sigma_{TOP} = \pi\alpha_{eff}^2/(8\Gamma_a^2)$. When the majoron decays through its Yukawa couplings $\Gamma_a \approx 1/4 m_a \alpha_{eff}$ and therefore $\sigma_{TOP} \approx 2\pi/m_a^2$. This means that for $m_a \approx 100$ GeV the cross-section on top of the resonance is $2.5 \cdot 10^5$ pbarn, a sizeable number. It has to be realized however that due to the bound (2.4) on the Yukawa coupling, the width Γ_a has to be smaller than 1 keV. To keep the smeared cross-section at the level of a few pbarn, the beam energy spread has to be smaller than 10 MeV. This might be hard. However, due to the poor limits on the lepton flavor conserving majorons, it is still not excluded to see a large signal here.

One could also look at the process $e^-e^- \to b^-W^-$. The total cross-section for this process is given by

$$\sigma(e^-e^- \to b^-W^-) = \frac{2\pi\alpha\alpha_{eff}}{s_\theta^2} \frac{\lambda^{1/2}(1, m_b^2/s, m_W^2/s)}{s} \cdot \left(\frac{(s-m_W^2)^2 + m_b^4}{(s-m_W^2-m_b^2)\lambda^{1/2}} \ln \frac{s-m_W^2-m_b^2+\lambda^{1/2}}{s-m_W^2-m_b^2-\lambda^{1/2}} - 1 \right) \tag{5.1}$$

where $\lambda = \lambda(s, m_b^2, m_W^2)$ and $\alpha_{eff} = |f_{e\nu_e}|^2/4\pi$. When one almost saturates the bound (3.12) by assuming $\alpha_{eff} \approx (m_b/10\,\text{TeV})^2$ one obtains for center of mass energies of 300 GeV and 1 TeV the numbers of Tables 2 and 3. Apparently it is still possible to have a sizeable effect in this channel.

It is also possible to use e^-e^- to obtain drastically improved bounds on virtual doubly charged majorons when all lepton flavours are separately conserved, by studying the angular distribution of Møller scattering. The differential cross-section for elastic e^-e^- scattering is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} f(\theta) \tag{5.2}$$

where a symmetry factor $\frac{1}{2}$ is included. When one includes the contribution of a doubly charged majoron $f(\theta)$ can be expressed as

$$f(\theta) = f_0(\theta) - \frac{2s\alpha_{eff}}{m_a^2\alpha} f_1(\theta) \tag{5.3}$$

where the standard model value is given by

$$f_0(\theta) = \frac{9 + 6\cos^2\theta + \cos^4\theta}{(1-\cos^2)^2} + 8(v^2+a^2) \cdot \frac{1 + 2m_Z^2/s - \cos^2\theta}{[(1+2m_Z^2/s)^2 - \cos^2\theta][1-\cos^2\theta]} + (v^2+a^2)^2 \cdot \frac{9(1+2m_Z^2/s)^2 + (6 + 12m_Z^2/s + 4m_Z^4/s^2)\cos^2\theta + \cos^4\theta}{[(1+2m_Z^2/s)^2 - \cos^2\theta]^2} + 4v^2a^2 \frac{7(1+2m_Z^2/s) + \cos^2\theta(-4 - 4m_Z^2/s + 4m_Z^4/s^2) - \cos^4\theta}{[(1+2m_Z^2/s)^2 - \cos^2\theta]^2} \tag{5.4}$$

and the contribution of the majoron by

$$f_1(\theta) = \frac{1}{1-\cos^2\theta} + \frac{(v^2+a^2)(1+2m_Z^2/s)}{(1+2m_Z^2/s)^2 - \cos^2\theta}. \tag{5.5}$$

In these expressions $v = -v_\theta/4s_\theta c_\theta$, $a = 1/4s_\theta c_\theta$ and $4\pi\alpha_{eff} = |f_{ee}|^2 + |g_{ee}|^2$. For simplicity terms which are proportional to $av\,\mathrm{Re}\, f_{ee}g_{ee}^*$ are ignored in this expression. To get some feeling for the sensitivity of the angular distribution we consider the ratio

$$R = \frac{d\sigma/d\Omega(\theta = 45°)}{d\sigma/d\Omega(\theta = 90°)} = R_0 + \delta R \tag{5.6}$$

where

$$R_0 = \frac{f_0(45°)}{f_0(90°)} \tag{5.7}$$

and

$$\frac{\delta R}{R_0} = \frac{2s\alpha_{eff}}{m_a^2\alpha}\left(\frac{f_1(90°)}{f_0(90°)} - \frac{f_1(45°)}{f_0(45°)}\right). \tag{5.8}$$

Table 2. The crosssection for b^-W^- production in e^-e^- collisions for center off mass energy $\sqrt{s} = 300$ GeV

m_b(GeV)	$\sigma <$ (pbarn)
60	0.14
80	0.23
100	0.34
120	0.47
140	0.62
160	0.81
180	1.00
200	1.10

Table 3. The same as Table 2, but for $\sqrt{s} = 1$ TeV

m_b(GeV)	$\sigma <$ (pbarn)
100	0.07
200	0.24
300	0.50
400	0.89
500	1.47
600	2.38
700	3.89
800	6.43
900	7.80

Assuming that one can verify experimentally that $\delta R/R_0 < 10^{-3}$ one obtains the following bounds

$$\begin{aligned} m_a \alpha_{\rm eff}^{-1/2} &> 4\,\mathrm{TeV}\,(\sqrt{s} = 10\,\mathrm{GeV}) \\ &> 22\,\mathrm{TeV}\,(\sqrt{s} = 50\,\mathrm{GeV}) \\ &> 50\,\mathrm{TeV}\,(\sqrt{s} = m_Z). \end{aligned} \tag{5.9}$$

It is clear that already the bound which could be obtained at $\sqrt{s} = 10\,\mathrm{GeV}$ means a dramatic improvement over the strongest presently existing bound on doubly charged majorons which conserve lepton flavor, i.e. the bound (2.13) arising from the anomalous magnetic moment of the muon. The second best limit on any kind of doubly charged majoron, the one from $\mu \to e\gamma$ (2.8) is already surpassed for energies of about 50 GeV. To reach the limit from $\mu \to eee$ one has to increase $\sqrt{s}$ beyond 1 TeV however.

6 Conclusions

In this paper the experimental limits on various types of charged majorons have been considered.

It has been found that the limits on doubly charged majorons which have lepton flavour off-diagonal couplings are very tight. They have to be very heavy or couple very weakly. Even if one finetunes the couplings such that the limit (2.4) from $\mu \to eee$ is circumvented, one still has the limit from $\mu \to e\gamma$

$$m_a/\alpha_{\rm eff}^{1/2} > 27\,\mathrm{TeV}. \tag{6.1}$$

This is not the case for doubly charged majorons which conserve all lepton numbers separately. The best limit on virtual majorons of this kind comes from the anomalous magnetic moment of the muon and is two orders of magnitude weaker than (6.1) and can still be circumvented by finetuning rather easily. The limits on this kind of majorons can be drastically improved by detailed studies of e^+e^- and e^-e^- collisions.

The best limits on the singly charged majorons, which do not involve model dependent relations between the masses of the different majorons, arise when one considers the corrections to the weak interaction parameters

$$m_b/\alpha_{\rm eff}^{1/2} > 7.3\,\mathrm{TeV}. \tag{6.2}$$

with the possibility to be improved by a factor of two in the near future. Therefore this limit does not weaken when one considers majorons which couple flavour diagonally.

The only way to get limits on their mass which are independent of the Yukawa couplings is by actually producing them onshell. This can ofcourse be done through pair-production in e^+e^- collisions, but also, for the singly charged majoron, by producing it in association with a W^- in e^-e^- collisions. The doubly charged majoron can only be visible as an s-channel resonance in e^-e^- collisions when it does not couple flavor off-diagonal, but can in that case produce a large signal.

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