

## Exclusive semileptonic decays of $B$ and $D$ mesons

James M. Cline and William F. Palmer

*Department of Physics, The Ohio State University, Columbus, Ohio 43210*

G. Kramer

*II. Institut für Theoretische Physik der Universität Hamburg, D-2000 Hamburg 50, West Germany*

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Exclusive semileptonic decays of  $D$  and  $B$  mesons to one and two pseudoscalar mesons are studied with respect to form-factor dependence, finite-width effects in intermediate resonant states, and non-resonant background terms calculated from chiral Lagrangians. Branching ratios, electron decay spectra, and asymmetry parameters for angular distributions are given for Cabibbo-Kobayashi-Maskawa-favored and -unfavored transitions, and compared to experimental data.

### I. INTRODUCTION

The study of semileptonic decays of bottom mesons is of particular interest for determining the Cabibbo-Kobayashi-Maskawa (CKM) mixing parameters  $V_{cb}$  and  $V_{ub}$ . To extract these parameters from the measured lepton spectrum or other observables of  $B$ -meson decays one needs theoretical input in the form of weak-current matrix elements between the initial  $B$  meson and the possible final hadron states. Weak decays of  $D$  and  $B$  mesons are clearly nonperturbative processes and cannot be calculated reliably from the QCD Lagrangian. Therefore one relies on phenomenological models. Originally the free-quark-decay model<sup>1</sup> (with and without QCD corrections<sup>2</sup>) was the favored tool for discussing the semileptonic decays of heavy quarks. It is thought to give a reasonable description of the absolute decay rates and perhaps also of the semileptonic branching ratios. But, as has been emphasized many times, and in much detail recently by Isgur *et al.*,<sup>3</sup> it is not able to predict the shape of the lepton spectrum, in particular the shape and normalization of the  $B \rightarrow X e \bar{\nu}_e$  end-point region, which is the interesting region for determining  $V_{ub}$ . This is the region of low recoil masses  $m_X$ , which is controlled by a set of discrete states:  $D$ ,  $D^*$ , etc., in the case of  $B$  decays via  $b \rightarrow c$  transitions or  $\pi$ ,  $\rho$ ,  $\eta$ ,  $\eta'$ , etc., for  $b \rightarrow u$  transitions and similarly in the case of semileptonic  $D$  decays. Thus the study of exclusive semileptonic decays, of which the lepton spectrum is built, is a more realistic endeavor than the free-quark-decay approach, if the spectra are to be understood, especially in the end-point region.

In general we limit the discussion to the most simple and low-lying states and do not aim at the description of the complete lepton spectrum. In some cases, however, it seems that these low-lying states almost exhaust the semileptonic decay rate.  $D \rightarrow K, K^*$  is such a case, so that a realistic description of the whole lepton spectrum is possible.

In order to get better insight into the validity of the phenomenological models, it is advisable to test them in cases where the weak mixing angles are known, for exam-

ple,  $D$  decays with  $c \rightarrow s$  transitions and  $B$  decays with  $b \rightarrow c$ . With the understanding obtained from these cases one might try to model the  $b \rightarrow u$  transitions in  $B$  decay like  $B \rightarrow \pi, \pi\pi, \pi\rho, \eta, \eta'$ , etc., which provides information on  $V_{ub}$ . Therefore we shall consider all semileptonic transitions of  $D$  and  $B$  mesons to one and to two pseudoscalar mesons on the same footing.

This field is presently under much experimental<sup>4</sup> and theoretical investigation. On the experimental side the semileptonic decay modes  $D \rightarrow K$  (Refs. 4 and 5),  $D \rightarrow K^*$  (Ref. 4), and  $B \rightarrow D^*$  (Refs. 4 and 6) have been clearly identified. The theoretical side of the field is much older. Rough estimates for exclusive decay rates were given already in (Ref. 1). Later work employed current algebra,<sup>7</sup> flavor-SU(4) symmetry, and various quark-model approaches to calculate individual form factors in transitions to final pseudoscalar, vector, or higher spin states.<sup>8</sup> In Ref. 9 effective chiral Lagrangians including a Wess-Zumino term have been used to predict weak matrix elements, especially for final states with two pseudoscalars. The chiral Lagrangian yields constraints on the weak matrix elements at very low energies, which are fully determined by the pion decay constant. This approach is particularly suited to calculate  $K_{e4}$  decays, which occur almost at threshold, and less suitable for the semileptonic decays of the heavier mesons, such as  $D$  and  $B$ , which are characterized by a large energy release. In the latter case the low-energy chiral approximation had to be improved by pole enhancements describing final-state interactions in intermediate states. These pole enhancement factors, however, were quite *ad hoc* and left no room to introduce sufficient structure into the weak transition matrix elements to vector-resonance intermediate states.

It is one of the purposes of this paper to remedy this drawback. We shall write down weak transition amplitudes to two pseudoscalars which have the same structure concerning transitions to intermediate vector-meson states as some of the models in Ref. 8 and have in addition the constraints arising from low-energy theorems codified in the chiral Lagrangian.<sup>9</sup> The low-energy constraints lead to additional terms, which we consider as a

reasonable model for a nonresonant background underneath the dominant resonant contribution. Unfortunately these contributions cannot be taken too seriously as they come from the lowest-order tree diagrams in an effective chiral Lagrangian. Because the  $D$  and  $B$  mesons are heavier than the chiral-symmetry-breaking scale  $\Lambda_{\text{CSB}} \sim 1$  GeV, there would be significant (and incalculable) contributions from higher-derivative interactions, loop diagrams, and higher-order symmetry-breaking terms. Therefore the current-algebra terms must be regarded as an educated guess rather than a quantitative estimate of the nonresonant contributions. It would certainly be wrong to omit them, even though their exact form is not predicted. Of course, such nonresonant terms will be relatively more important the larger the width of the intermediate resonance state is. Therefore they are more relevant for  $D \rightarrow \rho$  than for  $D \rightarrow K^*$  transitions. We expect them to be negligible for  $B \rightarrow D^*$ , but important for  $B \rightarrow \rho$  transitions. This approach for semileptonic transitions of  $B$ 's and  $D$ 's to two pseudoscalar final states is in the same spirit as an extension of the chiral-Lagrangian framework to mainly electromagnetic transitions by two of us.<sup>10</sup> In this work we constructed transition amplitudes for  $\eta, \eta' \rightarrow \pi^+ \pi^- \gamma$ ,  $K_{e4}$ ,  $\omega \rightarrow \pi^+ \pi^- \pi^0$ , and  $\gamma \rightarrow \pi^+ \pi^- \pi^0$  that respect the low-energy theorems but also have realistic vector-meson pole continuation in the intermediate state.

In Sec. II we present the weak vector- and axial-vector-current matrix elements for  $D \rightarrow K$  and  $D \rightarrow K \pi$  with realistic intermediate pole structure in the case of  $D \rightarrow K \pi$ . From this the structure functions of the hadron tensor are derived, which determine the partial semileptonic width, the lepton spectrum, and the differential distribution of the lepton-neutrino invariant mass. Section III is reserved for the numerical results. We give results for  $D, B$  going to one pseudoscalar, just for completeness, and  $D, B$  going to two pseudoscalar mesons. For the latter case we consider contact terms based on the chiral Lagrangian representing nonresonant background and compare with the cases of no contact term and with the zero-width approximation. We discuss how the results depend on form factors and try to determine these form factors by comparing the helicity structure with recently measured parameters of the decay angular distribution of the vector-meson decay products. Section IV contains a summary and the conclusions.

## II. SEMILEPTONIC DECAY SPECTRUM AND PARTIAL WIDTH

Following Wilson<sup>8</sup> and Chao *et al.*,<sup>9</sup> we write the total semileptonic decay rate as

$$\Gamma = \frac{2G_F^2 f(\theta_C)}{m_1 (2\pi)^2} \int \frac{d^3 k}{2k_0} \frac{d^3 k'}{2k'_0} H^{\mu\nu} l_{\mu\nu} \quad (1)$$

$$\Gamma = \frac{G_F^2 f(\theta_C) m_1^5}{8} \int dx dt d\mu^2 \{ t H_1(\mu^2, t) + \frac{1}{2} [x(1-\mu^2-x) - t(1-x)] H_2(\mu^2, t) + \frac{1}{2} t(2x-1-t+\mu^2) H_3(\mu^2, t) \}, \quad (9)$$

with the lepton tensor

$$l_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - k' \cdot k g_{\mu\nu} + i \epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta \quad (2)$$

and the hadron tensor

$$H_{\mu\nu} = \sum_X \prod_{i=2}^n \int \frac{d^3 p_i}{2p_{i0} (2\pi)^{3(n-1)}} \delta^{(4)} \left[ p_1 - q - \sum_{i=2}^n p_i \right] \times \langle D(p_1) | J_\mu^\dagger(q) | X(p_2, p_3, \dots) \rangle \times \langle X(p_2, p_3, \dots) | J_\nu(q) | D(p_1) \rangle. \quad (3)$$

The momenta correspond to a general process

$$p_1 \rightarrow p_2 + p_3 + \dots + k + k', \quad (4)$$

where  $p_i$  are momenta of hadron decay products,  $k$  is the electron momentum,  $k'$  the neutrino momentum,  $q = k + k'$  the momentum transfer to the lepton system, and  $m_1$  the decaying meson mass. We shall integrate over the final hadronic momenta for single-pseudoscalar and two-pseudoscalar particle states, and vector-meson states.  $H_{\mu\nu}$  is a dimensionless function of  $p_1$ , the momentum of the decaying meson, and  $q$ . We have neglected the lepton mass.

Instead of the variables  $k$  and  $k'$ , describing the lepton system, we introduce new variables

$$x = \frac{2k_0}{m_1}, \quad t = \frac{q^2}{m_1^2}, \quad \mu^2 = \frac{(p_1 - q)^2}{m_1^2}, \quad (5)$$

where  $k_0$  is the energy in the rest frame of particle 1. The width  $\Gamma$  is then given by

$$\Gamma = \frac{G_F^2 f(\theta_C) m_1^3}{8} \int dx \int dt \int d\mu^2 H^{\mu\nu} l_{\mu\nu} \quad (6)$$

with the integration boundaries

$$\mu_{\min}^2 \leq \mu^2 \leq 1-x, \quad 0 \leq t \leq x(1-\mu^2-x)/(1-x), \quad (7)$$

where  $\mu_{\min}^2, m_1^2$  is the smallest mass appearing in the intermediate state.

The hadron tensor, in its most general form, has the decomposition

$$H_{\mu\nu} m_1^2 = -g_{\mu\nu} H_1 m_1^2 + p_{1\mu} p_{1\nu} H_2 + i \epsilon_{\mu\nu\alpha\beta} p_1^\alpha q^\beta H_3 + q_\mu q_\nu H_4 + (p_{1\mu} q_\nu + q_\mu p_{1\nu}) H_5 + i(p_{1\mu} q_\nu - q_\mu p_{1\nu}) H_6, \quad (8)$$

where the  $H_i$  are real functions of  $\mu^2$  and  $t$ . Only the first three structure functions contribute when we neglect the lepton mass,  $q^\mu l_{\mu\nu} = l_{\mu\nu} q^\nu = 0$ . Substituting this decomposition into the width formula, we obtain

where  $f(\theta_C)$  stands for the appropriate weak-mixing matrix element.

Note the electron momentum dependence arises only from the factors of  $x$  in the integrand and in the integration boundaries. Suppressing the curly-bracketed integrand in Eq. (9) these limits are

$$\begin{aligned} \Gamma &= \frac{G_F^2 f(\theta_C) m_1^3}{8} \int_{\mu_{\min}^2}^{\mu_{\max}^2} d\mu^2 \int_0^{(1-\mu)^2} dt \int_{x_-}^{x_+} dx \{ \\ &= \frac{G_F^2 f(\theta_C) m_1^3}{8} \int_{\mu_{\min}^2}^{\mu_{\max}^2} d\mu^2 \int_0^{1-\mu^2} dx \int_0^{x[1-\mu^2/(1-x)]} dt \{ \end{aligned} \quad (10)$$

where

$$x_{\pm} = \frac{1}{2}(1 - \mu^2 + t) \pm \frac{1}{2}\sqrt{(1 - \mu^2 + t)^2 - 4t} . \quad (11)$$

The expression (8) yields the most general form of the lepton spectrum  $d\Gamma/dx$ . From the definition of  $H_{\mu\nu}$  in Eq. (3) it is obvious that  $r_{\mu}^* H^{\mu\nu} r_{\nu} \geq 0$  for any  $r_{\mu}$ . This condition leads to the following positivity constraints (Wilson, Ref. 8):

$$\Delta^2 H_2 + t H_1 \equiv \Delta^2 H_0 \geq 0, \quad H_1 \pm \Delta H_3 \equiv \Delta^2 H_{\pm} \geq 0, \quad (12)$$

$$\Delta = \frac{1}{2}\sqrt{(1 + \mu^2 - t)^2 - 4\mu^2} = p/m_1, \quad (13)$$

where  $p$  is the momentum  $2m_1 p = [\lambda(tm_1^2, \mu^2 m_1^2, m_1^2)]^{1/2}$  of the intermediate state  $X(p_2, p_3, \dots)$  in the  $D$ -meson rest system and where  $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ . In terms of the positive functions  $H_+$ ,  $H_-$ , and  $H_0$  the total width can be written

$$\begin{aligned} \Gamma &= \frac{G_F^2 f(\theta_C) m_1^5}{32} \\ &\times \int dx dt d\mu^2 [t H_+(\mu^2, t)(x_+ - x)^2 \\ &\quad + t H_-(\mu^2, t)(x - x_-)^2 \\ &\quad + 2H_0(\mu^2, t)(x_+ - x)(x - x_-)] . \end{aligned} \quad (14)$$

Instead of the variable  $x$  we can introduce into Eq. (14) the angle  $\theta$ , which is the polar angle between the momentum of the intermediate state  $X$  and the lepton  $l^-$  in the  $(l^- \bar{\nu}_l)$  c.m. system. The relation between  $x$  and  $\cos\theta$  is

$$2m_1 p \cos\theta = m_1^2 - m_X^2 + q^2 - 4m_1 k_0, \quad (15)$$

or, if written in terms of  $\mu^2$ ,  $t$ , and  $x$ ,

$$2\Delta \cos\theta = 1 - \mu^2 + t - 2x . \quad (16)$$

Then we get, for  $\Gamma$ ,

$$\begin{aligned} \Gamma &= \frac{G_F^2 f(\theta_C) m_1^5}{32} \\ &\times \int dx dt d\mu^2 [t \Delta^2 H_+(\mu^2, t)(1 + \cos\theta)^2 \\ &\quad + t \Delta^2 H_-(\mu^2, t)(1 - \cos\theta)^2 \\ &\quad + 2\Delta^2 H_0(\mu^2, t)(1 - \cos^2\theta)] . \end{aligned} \quad (17)$$

From Eq. (17) it is clear that  $H_+$ ,  $H_-$ , and  $H_0$  must be the helicity projections of  $H_{\mu\nu}$ , where the helicity  $\lambda = \pm, 0$  is the helicity label of the current.<sup>11</sup> It is clear that each of the terms in Eq. (17) must separately be positive. The problem of finding the lepton spectrum or any other differential distribution is now reduced to finding the three structure functions  $H_1$ ,  $H_2$ ,  $H_3$  or their equivalents  $H_+$ ,  $H_-$ ,  $H_0$ .

In the following we evaluate  $H_1$ ,  $H_2$ , and  $H_3$  for final hadronic states of one and two pseudoscalar mesons and vector-meson resonances. The  $VA$  interference term is contained in  $H_3$  and does not contribute to the total decay rate but does influence the electron spectrum shape. The effect in  $D$  decay is to shift the spectrum to lower electron momentum and in  $B$  decay to higher electron momentum.

For the processes with one pseudoscalar in the final state, let us consider the example

$$D^+ \rightarrow \bar{K}^0 + e^+ + \nu$$

or

$$(D^0 \rightarrow \pi^+ + e^- + \bar{\nu}) = \sqrt{2}(D^+ \rightarrow \pi^0 + e^+ + \nu) .$$

With the notation  $p_1 \rightarrow p_2 + k + k'$ , the transverse part of the current is

$$J_{\mu} = (p_1 + p_2)_{\mu} \frac{m_{F^*}^2}{m_{F^*}^2 - q^2} I \equiv (p_1 + p_2)_{\mu} G_V(q^2) I, \quad (18)$$

where  $G_V$  is a form factor describing the  $q^2$  dependence of the coupling of the vector current to the  $c\bar{s}$  hadronic state:

$$G_V(q^2) = \frac{m_{c\bar{s}}^2}{m_{c\bar{s}}^2 - q^2} \quad (19)$$

and  $m_{c\bar{s}}^2$  is the mass of the nearest vector meson with the hadron current quantum numbers. Following Wirbel, Stech, and Bauer<sup>12</sup> we introduce a factor  $I$  which in the quark model can be understood as the overlap integral between the initial and final meson wave functions. In the chiral  $SU(N)$  limit,  $I = 1$ . Dominguez and Paver have estimated corresponding overlap factors for  $c\bar{s}$ ,  $c\bar{d}$ , and  $b\bar{u}$  transitions<sup>13</sup> using the QCD sum-rule technique. With suitable changes in the mass in  $G_V$  and the overlap integral the same expression, Eq. (18), describes

$$B^+ \rightarrow \bar{D}^0 + e^+ + \nu \quad \text{or} \quad B^0 \rightarrow \pi^+ + e^- + \bar{\nu} .$$

The corresponding hadronic structure functions are

$$H_1 = 0, \quad H_3 = 0, \quad H_2 = \frac{m_1^2}{2\pi^3} |G_V I|^2 \delta(p_2^2 - m_2^2) . \quad (20)$$

[The expression  $\delta(p_2^2 - m_2^2) = (1/m_1^2) \delta(\mu^2 - m_2^2/m_1^2)$  appears because the hadronic state is a single-particle state.] The total exclusive decay rate is given by

$$\begin{aligned}\Gamma &= \frac{G_F^2 f(\theta_C) m_1^5}{32\pi^3} \int_0^{1-m_2^2/m_1^2} dx \int_0^{x[1-m_2^2/m_1^2/(1-x)]} dt \left[ x \left( 1 - \frac{m_1^2}{m_1^2} - x \right) - t(1-x) \right] |G_V I|^2 \\ &= \frac{G_F^2 f(\theta) m_1^5}{192\pi^3} \int_0^{t_-} dt [(t-t_+)(t-t_-)]^{3/2} |G_V I|^2,\end{aligned}\quad (21)$$

where

$$t_{\pm} = \frac{q_{\pm}^2}{m_1^2} = \left[ \frac{m_1 \pm m_2}{m_1} \right]^2.$$

For the hadronic structure function corresponding to two pseudoscalars we consider, to be specific, the process

$$D^+ \rightarrow K^- + \pi^+ + e^+ + \nu, \quad p_1 \rightarrow p_2 + p_3 + k + k'.$$

The current-algebra threshold result in the limit of SU(4) symmetry is

$$J_{\mu} = \frac{2\sqrt{2}}{\pi^2 F_{\pi}^3} \epsilon_{\mu\nu\alpha\beta} p_3^{\nu} p_2^{\alpha} p_1^{\beta} G_V(q^2) + \frac{2\sqrt{2}i}{3F_{\pi}} (2p_3 + p_1 - p_2)^{\nu} \left[ g_{\mu\nu} G_V(q^2) - \frac{q_{\mu} q_{\nu}}{q^2 - m_F^2} \right], \quad (22)$$

where the  $q^2$  pole behavior corresponds to the graph of Fig. 1. The  $q_{\mu} q_{\nu}$  terms which we shall now drop, have negligible contribution in the limit of vanishing lepton mass, where the lepton current is conserved. We now add resonance behavior in the two-body pseudoscalar channels, following the procedure of Chao *et al.*<sup>9</sup> (where the parameters  $3\alpha f_V$  and  $3\alpha f_A$  defined below were taken to be unity) as further developed in Ref. 10. The vector current is given by

$$J_{\mu} = \frac{2\sqrt{2}}{\pi^2 F_{\pi}^3} \epsilon_{\mu\nu\alpha\beta} p_3^{\nu} p_2^{\alpha} p_1^{\beta} \frac{1}{2} \left[ \frac{m_{D^*}^2}{m_{D^*}^2 - s_{13}} + (1 - 3\alpha f_V) + 3\alpha f_V \frac{m_{K^*}^2 - im_{K^*}\Gamma}{m_{K^*}^2 - s_{23} - im_{K^*}\Gamma} \right] G_V I \quad (23)$$

and the axial vector by

$$\begin{aligned}J_{\mu} &= \frac{2\sqrt{2}i}{3F_{\pi}} \left\{ (1 - 3\alpha f_A)(p_3 - p_2)_{\mu} \right. \\ &\quad + 3\alpha f_A \frac{m_{K^*}^2 - im_{K^*}\Gamma}{m_{K^*}^2 - s_{23} - im_{K^*}\Gamma} \left[ (p_3 - p_2)_{\mu} + (p_3 + p_2)_{\mu} \frac{m_2^2 - m_3^2}{m_{K^*}^2} \right. \\ &\quad \left. \left. + \frac{F_2^A(0)}{F_1^A(0)} p_{1\mu} \left[ p_1(p_3 - p_2) + p_1(p_3 + p_2) \frac{m_2^2 - m_3^2}{m_{K^*}^2} \right] \right] \right\} \\ &\quad + \frac{m_{D^*}^2}{m_{D^*}^2 - s_{13}} (p_1 + p_3)_{\mu} \left. \right\} G_A I. \quad (24)\end{aligned}$$

(In Ref. 9 the spin projection for the axial-vector part of the intermediate vector state was incomplete and the effect propagated into our subsequent formula for  $H_i$ . The effect is small on the numerical results of that paper, which did not account for the  $F_2^A$  form factor and used a much stronger vector form factor than is now believed correct.) In Eqs. (23) and (24) the introduction of the parameter  $\alpha$  decouples the pole enhancement from the low-energy behavior. Without it, relations between different coupling constants evolve which are in disagreement with experiment. (For more discussion on this point see Ref. 10.)  $f_V$  and  $f_A$  parametrize the possibility that this decoupling is different for vector and axial-vector currents.  $I$  stands for overlap integrals, which are as

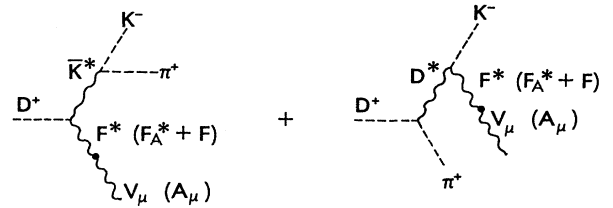


FIG. 1. Resonance-enhanced diagrams for  $D \rightarrow K\pi e\nu$ .  $K^*$  and  $D^*$  dominate the  $K\pi$  and  $D\pi$  channels, respectively. The vector- and axial-vector-current channels are enhanced by the vector, axial-vector, and pseudoscalar hadrons with the appropriate flavor.

sumed to be equal for the vector and axial-vector current. In the chiral-symmetry limit Eqs. (23) and (24) reduce to Eq. (22) when  $s_{13}$  and  $s_{23}$  are evaluated at soft threshold  $s_{13}=s_{23}=0$  and  $m_2=m_3=0$  for any values of the parameter  $\alpha f_V$  and  $\alpha f_A$ . We shall treat the slowly varying distant pole contribution in the 13 channel by evaluating these terms at current-algebra threshold  $s_{13}=0$ . Otherwise it would have been necessary to modify the pole term in the 13 channel in the same way as the 23 channel with threshold behavior and pole enhancement decoupled. This way this term becomes part of the contact term. These contact terms along with terms proportional to  $(1-3\alpha f_{V,A})$  describe small constant terms required by current algebra. Even far from the chiral-symmetry limit they may represent nonresonant background underneath the resonant terms or the accumulated effect of higher resonances, as discussed below. We notice that the terms proportional to  $F_2^A$  are irrelevant at the threshold point. Therefore they do not lead to a modification of the contact term.

In narrow width approximation, for  $D \rightarrow K^* e \nu$  where only the  $K^*$  intermediate state is important, we can relate our form factor to  $D \rightarrow K^*$  transition factors, as they appear in the literature:<sup>8</sup>

$$\langle K^*(p) | J_\mu | D(p_1) \rangle = e_v^* (F_1^A g_\mu^\nu + F_2^A p_{1\mu} p_1^\nu + i F^V \epsilon_{\mu\nu\rho\sigma} p_1^\rho p^\sigma). \quad (25)$$

The correspondence between the form factors is

$$3\alpha f_V I G_V = \sqrt{2} g_{K^*0K-\pi} + F^V(0) G_V \frac{\pi^2 F_\pi^3}{m_{K^*}^2} \equiv C_V, \quad (26)$$

$$3\alpha f_A I G_A = \sqrt{2} g_{K^*0K-\pi} + F_1^A(0) G_A \frac{3F_\pi}{4m_{K^*}^2} \equiv C_A. \quad (27)$$

An early discussion of the behavior of  $F^V(0)$ ,  $F_1^A(0)$ , and  $F_2^A(0)$  in the quark model was given by Ali *et al.*<sup>14</sup> Körner and Schuler<sup>11</sup> have taken this up and match the spin properties of the mesonic transitions to the free quark decay transitions at  $q^2=0$ , obtaining the relations

$$\begin{aligned} F_1^A(0) &= (m_1 + m_X) I, \\ F_2^A(0) &= \frac{-2}{m_1 + m_X} I, \\ F^V(0) &= \frac{2}{m_1 + m_X} I, \end{aligned} \quad (28)$$

where  $m_1$  is the decaying meson mass and  $m_X$  is the vector-meson hadronic final-state mass. [We have taken the sign of  $F^V$  here for  $D \rightarrow K^*$  semileptonic decay; for  $B \rightarrow K^*$  semileptonic decay,  $F^V(0)$  changes sign.] The effect is seen in the  $F^V(0)F_1^A(0)$  interference term which for  $D$  ( $B$ ) decay is positive (negative) at low electron momentum and negative (positive) at high electron momentum in  $d\Gamma/dE_e$ .

Overlap factors have been estimated by several authors. Körner and Schuler<sup>11</sup> take a common factor  $I=0.7$  for the  $b \rightarrow c$  transition form factor, following Wirbel, Stech, and Bauer,<sup>12</sup> who use an infinite-momentum-frame quark model to estimate  $I=0.71, 0.65,$  and  $0.69$  for overlap factors multiplying  $F^V, F_1^A,$  and  $F_2^A$ , respectively, when the average transverse quark momentum is  $0.400$  GeV. In a refined analysis of overlap factors for semileptonic  $B$  and  $D$  decays, Bauer and Wirbel<sup>8</sup> find possible variations of overlap factors for  $F_V, F_1^A,$  and  $F_2^A$  in the ranges  $0.92-0.35, 0.85-0.33,$  and  $1.12-(-0.04)$  respectively for  $B$  decays to  $D^*$  mesons and  $1.65-0.63, 1.14-0.44,$  and  $1.89-(-0.10)$  for  $D$  mesons to  $K^*$  mesons, using monopole form factors. This range can be regarded as some measure of the theoretical uncertainty in estimating overlaps for the underlying dynamics. Of course, one has to keep in mind that these results for the overlap factors in the framework of the infinite-momentum-frame quark model<sup>12</sup> or the quark potential model<sup>3,8</sup> come from model calculations and are purely phenomenological. They only give us indications of the values of these parameters. The same holds for the Körner-Schuler relations, Eqs. (28). All these form factors should be determined experimentally. If this is done we shall have better confidence in the validity of the various quark-model results referred to above.

As mentioned in the Introduction, perturbative QCD calculations for inclusive decays would seem to be an attractive escape from the uncertainties involved with the nonperturbative physics of exclusive processes. Recently, however, Isgur, Scora, Grinstein, and Wise<sup>3</sup> have criticized the usual free quark model concerning the shape and end-point spectrum of heavy-quark inclusive semileptonic decay and argued the need for detailed nonperturbative analysis of dominant exclusive channels, especially concerning the end-point spectrum. They are thus in harmony with earlier literature studying exclusive channels using predictions of nonperturbative, phenomenological models. They point out the wide range of predictions of these models, consistent with the range of overlap integrals mentioned above. It is not the purpose of this paper to add more predictions in this respect. Instead we take the attitude of first fixing  $F^V, F_1^A,$  and  $F_2^A$  according to Eq. (28) and then studying the consequences of changes of these parameters in the vicinity of their fiducial values. We treat the overlap parameter  $I$  similarly. As we mentioned in the Introduction our next objective is to study backgrounds beneath these states and the accuracy of the narrow-width approximation, a task for which our formalism is uniquely suited. Though we will present results for both full two-body kinematics and the narrow-width approximation, the formulas for  $H_1, H_2,$  and  $H_3$  shall be now written down in the narrow-width approximation because they are too lengthy in the general case.

They shall be written in terms of the parameters

$$\gamma_1 = \frac{1}{2} \frac{m_2^2 - m_3^2}{m_{K^*}^2}, \quad (29)$$

$$\gamma_2 = \frac{F_2^A}{4F_1^A s_{23}} \lambda^{1/2}(s_{23}, m_1^2, q^2) \lambda^{1/2}(s_{23}, m_2^2, m_3^2), \quad (30)$$

$$C = \frac{\lambda^{1/2}(s_{23}, m_2^2, m_3^2)}{4(2\pi)^5 s_{23}}. \quad (31)$$

In the narrow-width approximation  $s_{23}$  is evaluated at the resonance mass:

$$s_{23} = \mu^2 m_1^2 = m_{K^*}^2. \quad (32)$$

Then we get

$$H_1 = \lambda(s_{23}, m_2^2, m_3^2) \frac{C}{3s_{23}} \frac{m_{K^*}^3}{\Gamma_{K^*}} \delta(s_{23} - m_{K^*}^2) \left[ \frac{|C_V|^2 \lambda(s_{23}, m_1^2, q^2)}{8\pi^3 F_\pi^6} + \frac{8\pi}{9F_\pi^2} |C_A|^2 \right], \quad (33)$$

$$H_2 = \frac{C}{3s_{23}^2} \frac{m_{K^*}^3}{\Gamma_{K^*}} \delta(s_{23} - m_{K^*}^2) m_1^2 \left[ -s_{23} |C_V|^2 \frac{2}{2\pi^3 F_\pi^6} \lambda(s_{23}, m_2^2, m_3^2) q^2 \right. \\ \left. + |C_A|^2 \frac{8\pi}{9F_\pi^2} \left[ \lambda(s_{23}, m_2^2, m_3^2) + 12s_{23}^2 (\gamma_1^2 + \frac{1}{3}\gamma_2^2) + 3(m_2^2 - m_3^2)^2 \right. \right. \\ \left. \left. - 12s_{23}(m_2^2 - m_3^2)\gamma_1 - 4s_{23}\gamma_2(s_{23} - m_1^2 + q^2) \frac{\lambda^{1/2}(s_{23}, m_2^2, m_3^2)}{\lambda^{1/2}(s_{23}, m_1^2, q^2)} \right] \right], \quad (34)$$

$$H_3 = C_V C_A \frac{4}{3\pi F_\pi^4} \frac{\lambda(s_{23}, m_2^2, m_3^2)}{3s_{23}} \frac{m_{K^*}^3}{\Gamma_{K^*}} \delta(s_{23} - m_{K^*}^2) C m_1^2. \quad (35)$$

These equations for  $H_1$ ,  $H_2$ , and  $H_3$  agree with the equations for these structure functions in Ref. 9, if  $3af_{V,A} = 1$  and  $\gamma_1 = \gamma_2 = 0$ .  $\gamma_2$  parametrizes the contribution of the new form factor  $F_2^A$ , which is irrelevant for the low-energy behavior.  $\gamma_1$  appears because we have built in the correct propagator for the intermediate vector state. We see that all terms proportional to  $m_2^2 - m_3^2$  cancel. From these structure functions we obtain the helicity structure functions introduced in Eq. (12):

$$\Delta^2 H_\pm = \lambda(s_{23}, m_2^2, m_3^2) \frac{C}{3s_{23}} \frac{m_{K^*}^3}{\Gamma_{K^*}} \delta(s_{23} - m_{K^*}^2) \frac{\pi}{2} \\ \times \left[ \frac{\pm C_V \lambda^{1/2}(s_{23}, m_1^2, q^2)}{2\pi^2 F_\pi^3} + \frac{4C_A}{3F_\pi} \right]^2, \quad (36)$$

$$\Delta^2 H_0 = \lambda(s_{23}, m_2^2, m_3^2) \frac{C}{12m_1^2 s_{23}^2} \frac{m_{K^*}^3}{\Gamma_{K^*}} \delta(s_{23} - m_{K^*}^2) \frac{\pi}{2} \\ \times \frac{16}{9F_\pi^2} |C_A|^2 \left[ (m_1^2 - s_{23} - q^2) \right. \\ \left. + \lambda(q^2, s_{23}, m_1^2) \frac{F_2^A}{2F_1^A} \right]^2. \quad (37)$$

We may now substitute  $H_\pm$  and  $H_0$  into Eq. (14) and perform the  $\mu^2$  integration. Then the first two terms correspond to the transversely polarized  $K^*$  and the last term corresponds to the longitudinally polarized  $K^*$ . This equation agrees with the corresponding equation in Ref. 11 if we substitute the relations (26) and (27) and use the well-known formula for the  $K^*$  width.

### III. NUMERICAL RESULTS

In presenting our results we shall take as input the weak mixing matrix elements when they are reliably known from lifetime measurements. Our objects of study are overlap integrals, weak mixing angles for  $b \rightarrow u$  transitions, and the effects of current-algebra contact terms and the narrow-width approximation on extracting these parameters.

$D \rightarrow K e \nu$ . For  $D$  decays to strange states we take  $|V_{cs}| = 0.974$ . Then from the decays  $D^0 \rightarrow K^- + e^+ + \nu$  and  $D^+ \rightarrow \bar{K}^0 + e^+ + \nu$ , using the monopole form factor with mass  $m_{cs} = 2.114$  GeV we find

$$\Gamma(D \rightarrow K e \nu) = |V_{cs} I|^2 \times 10.27 \times 10^{-14} \text{ GeV}. \quad (38)$$

Taking the measured values from the E691 experiment at Fermilab,<sup>5,15</sup>

$$\Gamma(D^0 \rightarrow K^- + e^+ + \nu) = (6.0 \pm 1.2) \times 10^{-14} \text{ GeV}, \\ \Gamma(D^+ \rightarrow \bar{K}^0 + e^+ + \nu) = (6.3 \pm 1.3) \times 10^{-14} \text{ GeV}, \quad (39)$$

or from the Mark III experiment at SLAC (Ref. 15),

$$\Gamma(D^0 \rightarrow K^- + e^+ + \nu) = (6.5 \pm 1.1) \times 10^{-14} \text{ GeV}, \\ \Gamma(D^+ \rightarrow \bar{K}^0 + e^+ + \nu) = (4.5 \pm 1.3) \times 10^{-14} \text{ GeV}, \quad (40)$$

we determine the overlap integrals from these data. The result is

$$\text{E691, } D^0: I = 0.78 \pm 0.08; \quad D^+: I = 0.80 \pm 0.07, \quad (41)$$

$$\text{Mark III, } D^0: I = 0.82 \pm 0.06; \quad D^+: I = 0.68 \pm 0.09.$$

The average value is  $I = 0.77 \pm 0.04$ . This is in good

agreement with the theoretical values that Wirbel, Stech, and Bauer obtain in their infinite-momentum-frame approach,  $I=0.76\pm 0.06$  (Ref. 12), and Dominguez and Paver get from QCD sum rules,  $I=0.75\pm 0.05$  (Ref. 13). We consider therefore that the value  $I=0.7$  is a good choice for overlap integrals, which we will use as a universal value for  $D$  and  $B$  transitions. We also remark that in the E691 experiment the form-factor mass has been determined from the data with the result  $m_{\bar{c}s}=2.1\pm 0.3$  GeV, in agreement with the assumed value above.

$D \rightarrow \pi e \nu$ ,  $\eta e \nu$ ,  $\eta' e \nu$ . For  $D$  decays to  $\pi$ ,  $\eta$ , and  $\eta'$ , using an  $\eta, \eta'$  mixing angle of  $-11^\circ$ , we find the relative couplings

$$\frac{D^+}{\sqrt{2}}(\pi^0 - 0.71\eta - 0.70\eta') + D^0\pi^+$$

and the rates

$$\begin{aligned} \Gamma(D^+ \rightarrow \pi^0 e^+ \nu) &= |V_{cd}I|^2 \times 1.005 \times 10^{-13} \text{ GeV}, \\ \Gamma(D^+ \rightarrow \eta e^+ \nu) &= |V_{cd}I|^2 \times 0.231 \times 10^{-13} \text{ GeV}, \\ \Gamma(D^+ \rightarrow \eta' e^+ \nu) &= |V_{cd}I|^2 \times 0.053 \times 10^{-13} \text{ GeV}, \end{aligned} \quad (42)$$

where we use the form-factor mass  $m_{\bar{c}\bar{d}}=2.01$  GeV. If we use the same overlap integral  $|I|\approx 0.7$  found in  $D \rightarrow K e \nu$  and  $|V_{cd}|=0.222$  we find

$$\begin{aligned} \Gamma(D^+ \rightarrow \pi^0 e^+ \nu) &= \Gamma(D^0 \rightarrow \pi^- e^+ \nu) / 2 \\ &= 0.243 \times 10^{-14} \text{ GeV}, \\ \Gamma(D^+ \rightarrow \eta e^+ \nu) &= 0.558 \times 10^{-15} \text{ GeV}, \\ \Gamma(D^+ \rightarrow \eta' e^+ \nu) &= 1.28 \times 10^{-16} \text{ GeV}. \end{aligned} \quad (43)$$

The one-pion transition has been observed,<sup>15</sup>

$$\Gamma(D^0 \rightarrow \pi^- e^+ \nu) = (0.62 \pm 0.36) \times 10^{-14} \text{ GeV}, \quad (44)$$

and corresponds to an overlap integral  $I=0.79\pm 0.20$

Thus a reasonably clear, consensus picture of semileptonic decays of the  $D$  meson to a single pseudoscalar emerges when these results are compared to the related work of Dominguez and Paver,<sup>13</sup> and the overlap integral analysis of Wirbel, Stech, and Bauer.<sup>8</sup> We shall see whether this consensus survives when  $B$  decays to a single pseudoscalar are studied below.

$D \rightarrow K \pi e \nu$  ( $D \rightarrow K^* e \nu$ ). Before that we turn to the decay of  $D$  to two-body resonance states  $K^*$  and  $\rho$ . Our formalism allows us to study the resonance cases: (1) two-body kinematics with contact terms, (2) two-body kinematics without contact terms, and (3) the narrow-width approximation (no contact terms). We shall also present results for the decay rate to longitudinal and transversely polarized  $K^*$ 's.

We first consider  $D^+ \rightarrow (K^- \pi^+ e^+ \nu + \bar{K}^0 \pi^0 e^+ \nu)$  and ( $D^+ \rightarrow \bar{K}^{*0} e^+ \nu$ ) and to be specific use  $I=0.7$ ,  $|V_{cs}|=0.974$ , the form-factor mass  $m_{\bar{c}s}=2.114$  GeV, and the form-factor parametrization of Körner and Schuler, as given in Eq. (28) of the preceding section. The results are

$$\Gamma = 4.42 \times 10^{-14} \text{ GeV}$$

(two-body kinematics with contact terms),

$$\Gamma = 4.15 \times 10^{-14} \text{ GeV}$$

(two-body kinematics without contact terms),

$$\Gamma = 4.59 \times 10^{-14} \text{ GeV} \quad (45)$$

(narrow-width approximation without contact terms).

The effect of the narrow-width approximation is to increase the rate by 11% over the corresponding calculation using full two-body kinematics with no contact term. The effect of the contact terms is to increase the rate by 7% over the corresponding two-body calculation without contacts. The electron decay spectra for each case is shown in Fig. 2. It is clear that we are dealing here with small effects but it is interesting to note that the contact term, equivalent to a constant nonresonant background, is in rough agreement with that data as reported by Morrison.<sup>15</sup> In the E691 experiment the nonresonant background in the  $D^0$  and in the  $D^+$  transition is of the order of 10%, with, however, rather large experimental errors. The Mark III experiment claims to have found rather large nonresonant contributions. The ratio of nonresonant to resonant branching ratio is here roughly 1:2.

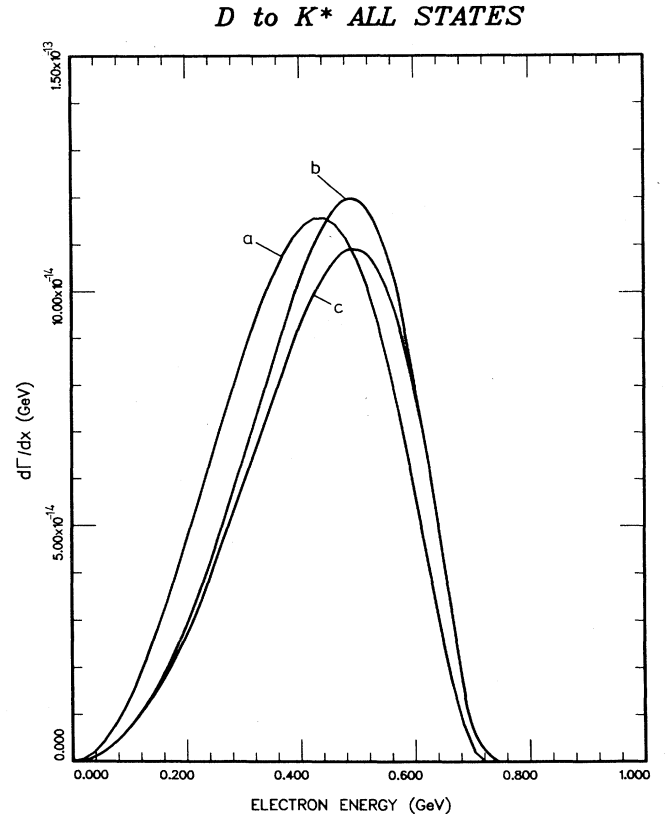


FIG. 2. Electron spectrum for  $D^+$  semileptonic decay to  $\pi K$ . (a) narrow-width approximation. (b) two-body kinematics *with* contact terms. (c) two-body kinematics *without* contact terms.

As seen above in our model the nonresonant terms increase the partial width by 7%.

Having determined the effects of contact terms and the narrow-width approximation in  $D \rightarrow K\pi$  semileptonic decay we now study the variation of the decay rate and spectrum with respect to the form factors  $F^V$ ,  $F_1^A$ , and  $F_2^A$ . We will vary the ratio  $F_2^A/F_1^A \equiv -2R/(m_D + m_{K^*})^2$  from  $R=2$  to  $-2$  and allow  $F^V$  to take on the values 0,  $F^V$ , and  $2F^V$ , where  $F^V$  is given by Eq. (28). By adjusting  $F_1^A$  the rate can be made to fit the experimental value. The results will be presented as curves with a common (unit) area, to illustrate how the electron spectrum shape changes. Figure 3 shows how the spectral shapes change as  $F_2^A/F_1^A$  varies from  $-5$  to  $+5$ . The *shape* change is not dramatic, although the decay rate itself (if not adjusted by scaling  $F_1^A$ ) changes by a factor of 3.3, as shown in Table I, if  $R$  is varied only between  $-2$  and 2.

Similarly, we can vary  $F^V$  for fixed  $F_2^A$  and  $F_1^A$ . The results, normalized to constant area, are shown in Fig. 4 and Table I. (The change in the absolute rate in this case is also slight because the vector contribution is such a small fraction of the total rate.) The shape of the spectrum changes to some extent although the vector contribution is small. If  $F^V$  is decreased to zero the maximum of the spectrum shifts nearer to the end point.

#### D to K\* ALL STATES

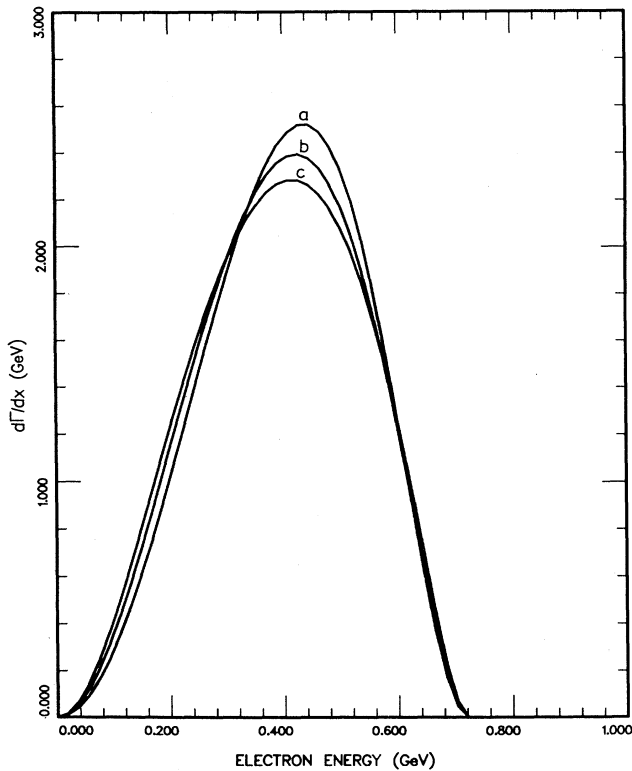


FIG. 3. Electron spectrum for  $D \rightarrow K^* e \nu$  for various values of  $R$ , where  $F_2^A/F_1^A = -2R/(m_D + m_{K^*})^2$ . The curves have been adjusted to have a common area. (a)  $R=1$ , (b)  $R=5$ , (c)  $R=-5$ .

TABLE I. Decay rate for  $D^+ \rightarrow \bar{K}^{0*} \nu$  for various values of  $R$ , where  $F_2^A/F_1^A = -2R/(m_D + m_{K^*})^2$ , and various values of  $F^V=0$ ,  $F^V=F^V$  [Eq. (28)], and  $F^V=2F^V$  [Eq. (28)]. The last column is the asymmetry parameter which appears in the angular distribution  $1 + \alpha \cos^2 \theta$  of the  $K$  momentum relative to the  $K^*$  in the center-of-mass system of the leptons. The last row is the decay rate for transversely polarized  $K^*$  mesons.

$R$	$\Gamma(F^V=0)$ ( $10^{-14}$ GeV)	$\Gamma(F^V)$ ( $10^{-14}$ GeV)	$\Gamma(2F^V)$ ( $10^{-14}$ GeV)	$\alpha(F^V)$
2.0	3.30	3.40	3.69	0.183
1.0	4.49	4.59	4.88	1.30
0.5	5.29	5.39	5.68	2.05
0	6.23	6.32	6.62	2.93
-0.5	7.30	7.39	7.69	3.93
-1.0	8.50	8.60	8.89	5.06
-2.0	11.3	11.4	11.7	7.70
$\Gamma_T$	2.04	2.13	2.43	

Also shown in Table I is the transverse decay rate and the corresponding value of the asymmetry parameter

$$\alpha = \frac{2\Gamma_L}{\Gamma_T} - 1 \quad (46)$$

which appears in the angular distribution  $1 + \alpha \cos^2 \theta$  of the  $K$  momentum relative to the  $K^*$  in the center-of-mass system of the leptons. This parameter is sensitive to the

#### D to K\* ALL STATES

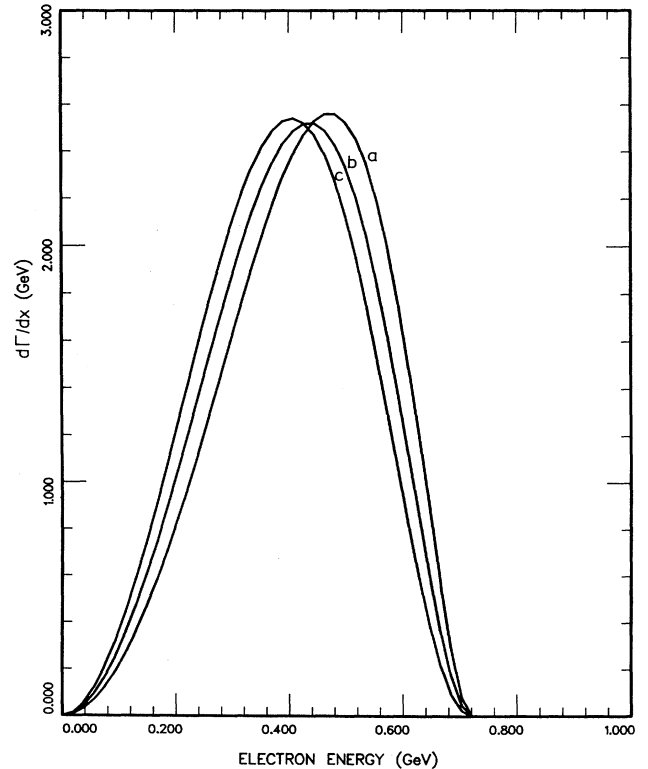


FIG. 4. Electron spectrum for  $D \rightarrow K^* e \nu$  for various values of  $F^V$ . The curves have been adjusted to a common area. (a)  $F^V=0$ , (b)  $F^V=F^V$  [Eq. (28)], (c)  $F^V=2F^V$  [Eq. (28)].



$F_2^A/F_1^A$  ratio, as shown in Table I. It has been evaluated at the central value of  $F^V$ .

The experimental data with which we can compare come again from the Mark III Collaboration and the E691 experiment. Since the Mark III data show an appreciable nonresonant background in contrast with our model, we compare only to the E691 (Ref. 15) data, which are consistent with our results with respect to the nonresonant contribution.<sup>16</sup> These results are  $\Gamma(D^0 \rightarrow K^{*-} e^+ \nu) = (2.62 \pm 0.56) \times 10^{-14}$  GeV, the nonresonant background is  $(0.31 \pm 0.22) \times 10^{-14}$  GeV. The partial width  $\Gamma(D^+ \rightarrow K^{0*} e^+ \nu) = (2.83 \pm 0.54) \times 10^{-14}$  GeV, with nonresonant background  $(0.32 \pm 0.25) \times 10^{-14}$  GeV. So the  $D^+$  and the  $D^0$  decays have comparable rates. The average, including the 10% background, is equal to  $(3.07 \pm 0.32) \times 10^{-14}$  GeV. Another data set is the recently measured  $K^*$ -decay angular distribution. The result quoted is  $\Gamma_L/\Gamma_T = [2.4 + 1.7(-0.9) \pm 0.2]$ . This means that the  $K^*$  is strongly longitudinally polarized. Because of the large error the parameter  $R$  could still lie between  $-2.0$  and  $1.0$ , as can be seen from Table I. If we assume that  $\Gamma_L/\Gamma_T$  has its central value 2.4, then  $R = -0.4$  and from the rate we can determine the overlap factor  $I$  with the result that  $I = 0.42 \pm 0.02$  [we took the result from  $\Gamma(FV)$  in Table I] instead of  $I = 0.7$ , which was assumed to generate the numbers in Table I. With better data for  $\alpha$  and good measurements of the electron spectrum for the  $K^*$  channel it will eventually be possible to pin down the three form factors  $F^V$ ,  $F_1^A$ , and  $F_2^A$ .

$D \rightarrow \pi\pi e\nu$  ( $D \rightarrow \rho e\nu$ ). As in the  $K\pi$  case, we study  $D \rightarrow \pi\pi e\nu$  ( $D^- \rightarrow \rho^0 e\nu$ ) with  $\rho$  pole in the  $\pi\pi$  channel to examine effects of the contact terms and the narrow-width approximation. To be specific and for comparisons with  $B \rightarrow \pi\pi e\nu$  ( $B \rightarrow \rho e\nu$ ) we present our results in terms of the overlap integrals  $I = 0.33$  and  $I = 0.7$ , discussed in the  $B \rightarrow \pi\pi$  section, and still using  $V_{cd} = 0.222$ . The results are, for  $I = 0.33$ ,

$$\Gamma = 3.98 \times 10^{-16} \text{ GeV}$$

(two-body kinematics, no contact term),

$$\Gamma = 4.64 \times 10^{-16} \text{ GeV}$$

(narrow-width approximation, no contact term),

(47)

$$\Gamma = 6.13 \times 10^{-16} \text{ GeV}$$

(two-body kinematics with contact terms);

and for  $I = 0.7$ ,

$$\Gamma = 3.98 \times 10^{-16} \text{ GeV}$$

(two-body kinematics, no contact term),

$$\Gamma = 20.9 \times 10^{-16} \text{ GeV}$$

(narrow-width approximation, no contact terms),

(48)

$$\Gamma = 19.5 \times 10^{-16} \text{ GeV}$$

(two-body kinematics with contact terms).

First we discuss the  $I = 0.33$  calculation, where the

contact terms are relatively more important because the resonance term is more sharply damped by the overlap factor. As before, the narrow-width approximation overestimates the resonance rate (no contact terms), now by 17%, but by far the larger effect is that of the contact terms, which enhance the rate by about 54%, when  $I = 0.33$ . The contact terms are *relatively* more important here because they contribute to the  $\pi\pi$  amplitude roughly the *same* as in  $D \rightarrow K\pi e\nu$  whereas the resonance contribution is *less* because it is of order  $I(m/\Gamma)$ , which is 1.68 for  $\rho$  and 12.25 for  $K^*$ . If contact terms are present at the magnitude suggested by chiral SU(4)—admittedly a badly broken symmetry but perhaps useful for estimating rough magnitudes—then they will clearly have significant effects on the analysis of  $D \rightarrow \rho e\nu$ . As in the  $K\pi$  case they may be evidenced by large backgrounds beneath the  $\rho$  resonance. For their effect on the electron spectrum, see Fig. 5. When the larger overlap,  $I = 0.7$ , is taken, the effects are less dramatic, as expected.

Such direct couplings of the current to three pseudoscalars (here  $D\pi\pi$ ) have been discussed by Braaten *et al.*<sup>17</sup> and by two of us<sup>10</sup> in connection with  $\rho^- \rightarrow \pi^- \gamma$ ,  $\omega \rightarrow 3\pi$ ,  $\omega \rightarrow \pi^0 \gamma$ , and  $\omega \rightarrow \pi^0 \mu^+ \mu^-$ . The contact terms also have an interpretation in the framework of generalized vector dominance where they model the sum of higher mass resonance terms where masses are large com-

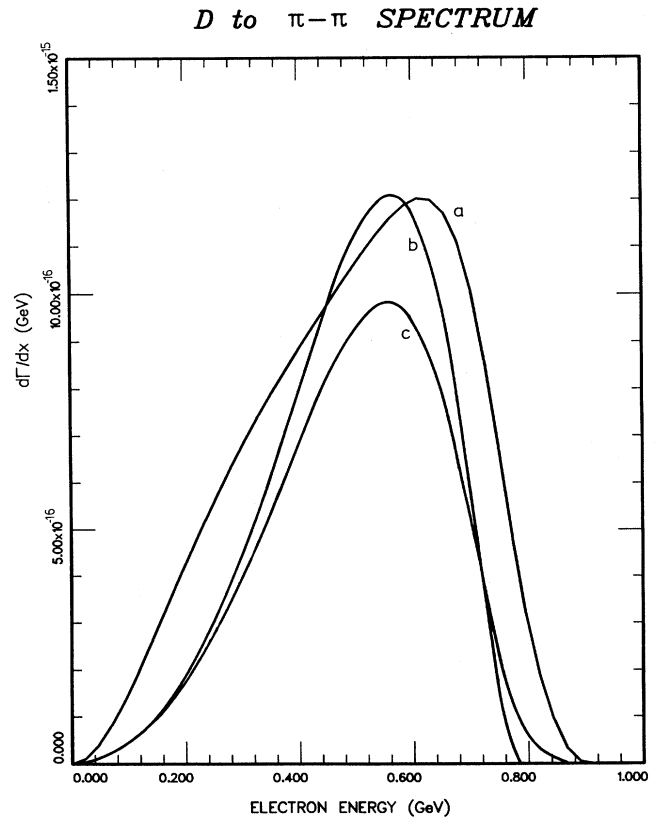


FIG. 5. Electron spectrum for  $D^- \rightarrow \pi^+ \pi^- e \nu$  ( $D^- \rightarrow \rho^0 e \nu$ ): (a) two-body kinematics *with* contact terms, (b) narrow-width approximation, (c) two-body kinematics *without* contact terms.

pared to the  $\pi\pi$  energy. (For an application of such higher resonances to vector-meson decays in the framework of the dual model see Dominguez.<sup>18</sup>)

$B \rightarrow D e \nu$ . We find

$$\Gamma(B^- \rightarrow \bar{D}^0 e \nu) = |V_{cb} I|^2 \times 1.21 \times 10^{-11} \text{ GeV}, \quad (49)$$

where we have used a form-factor mass  $m_{b\bar{c}} = 6.340 \text{ GeV}$ . Using  $|V_{cb}| = 0.045$  we find the estimate<sup>12</sup>  $I \sim 0.7$  corresponds to an exclusive rate  $\Gamma(B^- \rightarrow \bar{D}^0 e \nu) = 1.20 \times 10^{-14} \text{ GeV}$ . We shall continue to use  $I = 0.7$  below to present our results.

$B^- \rightarrow D \pi e \nu$  ( $B^- \rightarrow D^0 e \nu$ ). We treat  $B^- \rightarrow \bar{D}^0 e \nu$  only in the narrow-width approximation, where the rate we find is

$$\Gamma(B^- \rightarrow \bar{D}^0 e \nu) = |V_{cb} I|^2 \times 3.61 \times 10^{-11} \text{ GeV}. \quad (50)$$

Again we test for variation with  $F^V$  and  $F_2^A/F_1^A$  by varying  $R$  in  $F_2^A/F_1^A = -2R/(m_{D^*} + m_B)^2$  between 5 and  $-5$  and allowing  $F^V$  to take on the values 0,  $F^V$ , and  $2F^V$ , where  $F^V$  is given in Eq. (28). The results, along with the transverse decay rate  $\Gamma_T$  and the asymmetry parameter  $\alpha$ , are given in Table II and presented in Figs. 6 and 7. Table II is based on  $|V_{cb}| = 0.045$  and  $I = 0.7$ . Figures 6 and 7 are normalized to unit area to show how the shape

### B to D\* ALL STATES

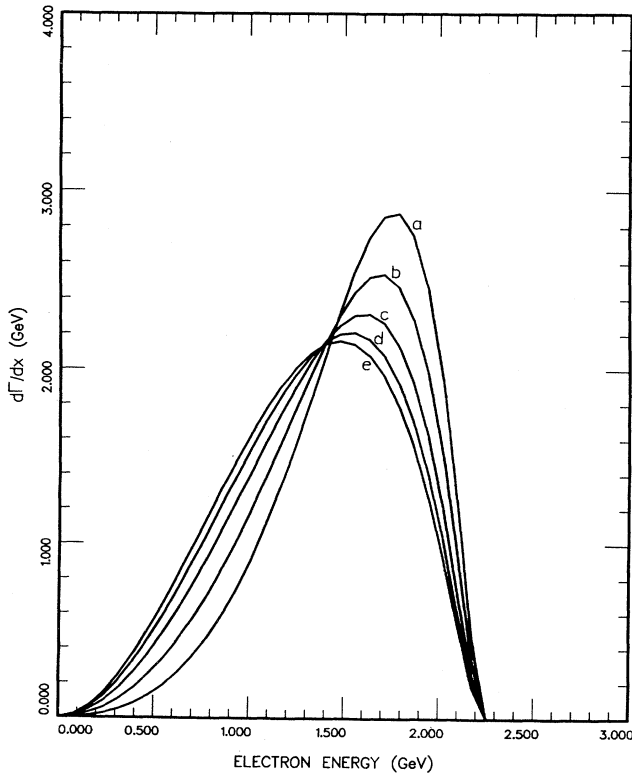


FIG. 6. Electron spectrum for  $B \rightarrow D^* e \nu$  for various values of  $R$ , where  $F_2^A/F_1^A = -2R/(m_{D^*} + m_B)^2$ . The curves have been adjusted to have a common area. (a)  $R=2$ , (b)  $R=1$ , (c)  $R=0$ , (d)  $R=-1$ , (e)  $R=-2$ .

TABLE II. Decay rate for  $B^- \rightarrow D^{0*} e \nu$  for various values of  $R$ , where  $F_2^A/F_1^A = -2R/(m_B + m_{D^*})^2$  and various values of  $F^V$  ( $F^V=0$ ,  $F^V=F^V$  [Eq. (28)] and  $F^V=2F^V$  [Eq. (28)]). The last column is the asymmetry parameter which appears in the angular distribution  $1 + \alpha \cos^2 \theta$  of the  $D$  momentum relative to the  $D^*$  in the center-of-mass system of the leptons. The last row is the decay rate for transversely polarized  $D^*$  mesons.

Ratio $R$	$\Gamma(F^V=0)$ ( $10^{-14} \text{ GeV}$ )	$\Gamma(F^V)$ ( $10^{-14} \text{ GeV}$ )	$\Gamma(2F^V)$ ( $10^{-14} \text{ GeV}$ )	$\alpha(F^V)$
5.0	3.87	3.99	4.33	1.77
2.0	2.29	2.41	2.75	-0.120
1.0	3.47	3.58	3.93	1.28
0.5	4.38	4.49	4.84	2.38
0	5.50	5.61	5.96	3.71
-0.5	6.83	6.95	7.29	5.31
-1.0	8.38	8.49	8.84	7.16
-2.0	12.1	12.2	12.5	11.6
-5.0	28.4	28.5	28.9	31.1
$\Gamma_T$	1.56	1.67	2.02	

changes with variations in  $F^V$  and  $F_2^A/F_1^A$ . Increasing  $F^V$  and the ratio  $R$  shifts the decay spectrum to higher electron energy. It is again notable how sensitive the  $\alpha$  asymmetry parameter is to  $F_2^A/F_1^A$ . This experimental measurement yields very useful constraints on the determination of the form factors. The experimental situation is as follows. The  $B$  lifetime is  $(1.18 \pm 0.14) \times 10^{-12} \text{ sec}$

### B to D\* ALL STATES

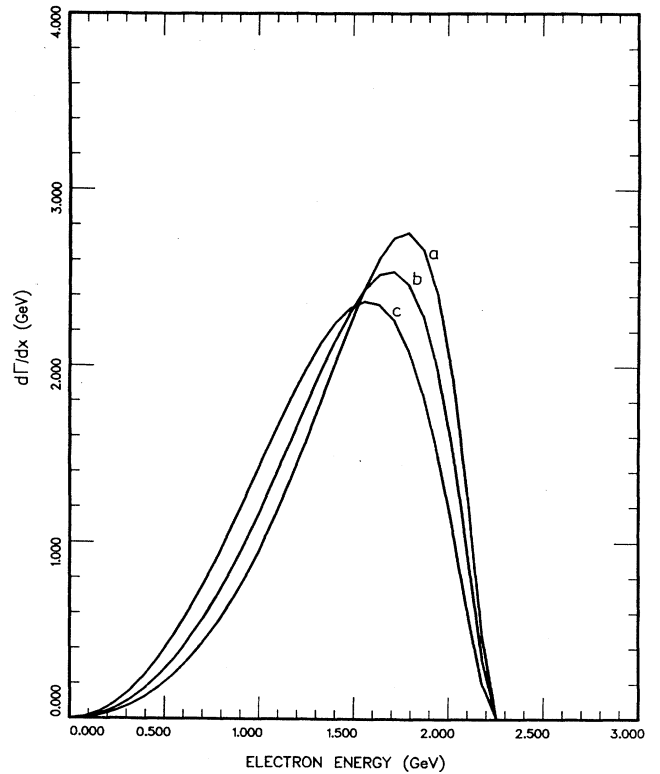


FIG. 7. Electron spectrum for  $B \rightarrow D^* e \nu$  for various values of  $F^V$ . The curves have been adjusted to have a common area. (a)  $F^V=2F^V$  [Eq. (28)], (b)  $F^V=F^V$  [Eq. (28)], (c)  $F^V=0$ .

(world average).<sup>4</sup> The semileptonic branching ratio to  $D^*$  has been measured by the Argus Collaboration at DESY with the result  $B(B^0 \rightarrow D^* e \nu) = (7.0 \pm 1.2 \pm 1.9)\%$ .<sup>4</sup> The world average of the total semileptonic branching ratio is  $(11.8 \pm 0.7)\%$ .<sup>4</sup> It is reasonable to assume that the  $D$  and the  $D^*$  channel dominate the semileptonic decay of the  $B$ 's. Then the branching fraction to  $De\nu$  is  $(4.8 \pm 2.4)\%$  and  $D$  and  $D^*$  are 41% and 59% of the total semileptonic decay. Furthermore the  $\alpha$  parameter has been measured as  $\alpha = 0.3 \pm 0.6 \pm 0.3$  (Ref. 4) and  $\alpha = 0.7 \pm 0.9$  (Ref. 19). The experimental width  $B \rightarrow D^* e \nu = (3.90 \pm 1.25) \times 10^{-14}$  GeV can be compared with our results in Table II. The  $\alpha$  parameter comes out right if  $R \sim 1.5$  and  $I$  is increased to  $I = 0.8$ . [We note that also the transition to single  $D$ 's needs a larger overlap than 0.7 since experimentally  $\Gamma(B \rightarrow De\nu) = (2.68 \pm 1.34) \times 10^{-14}$  GeV, which is more than a factor of 2 larger than the theoretical value with  $I = 0.7$ .] We remark that Körner and Schuler<sup>8</sup> have obtained somewhat different results since they assumed dipole form factors for  $F^V$  and  $F_2^A$ . Since the masses for such dipole form factors are even less known than for the monopole form factors, we preferred to work with monopoles only.

$B \rightarrow \pi e \nu$ . We find

$$\begin{aligned} \Gamma(B^- \rightarrow \pi^0 e^- \nu) &= \Gamma(B^0 \rightarrow \pi^+ e \nu) / 2 \\ &= |V_{ub} I|^2 \times 2.19 \times 10^{-11} \text{ GeV} \end{aligned} \quad (51)$$

using the form-factor masses

$$m_{b\bar{u}} = 5.324 \text{ GeV}$$

in  $G_V$ . If we test for sensitivity to  $G_V$  by using a dipole instead of a monopole form factor we find the result

$$\begin{aligned} \Gamma(B^- \rightarrow \pi^0 e^- \nu) &= \Gamma(B^0 \rightarrow \pi^+ e \nu) / 2 \\ &= |V_{ub} I|^2 \times 8.52 \times 10^{-11} \text{ GeV} \end{aligned} \quad (52)$$

with a spectrum very sharply peaked at the end point. The two spectra are shown in Fig. 8. The sensitivity of the spectrum to the dipole mass is illustrated by showing also the result for a dipole mass of 5.916 GeV. Of course we have no theoretical reason to assume a dipole form factor at this place with the mass value given above. A better way would be to assume generalized vector-dominance form factors, where the smallest intermediate mass is the mass we have used above.<sup>18</sup>

We can also express these results as branching ratios, using  $1.18 \times 10^{-12}$  sec for the  $B$  lifetime as reported by Schröder,<sup>4</sup>

$$B(B^0 \rightarrow \pi^+ e \nu) = \begin{cases} 79 |V_{ub} I|^2, & \text{monopole form factor,} \\ 305 |V_{ub} I|^2, & \text{dipole form factor.} \end{cases} \quad (53)$$

These may be compared with recent CLEO results<sup>20</sup>

$$B(B \rightarrow \pi^+ e \nu) < 2.5 \times 10^{-4} \quad (54)$$

and yield the limits

$$\begin{aligned} |V_{ub} I| &< 1.78 \times 10^{-3}, & \text{monopole form factor,} \\ |V_{ub} I| &< 0.905 \times 10^{-3}, & \text{dipole form factor.} \end{aligned} \quad (55)$$

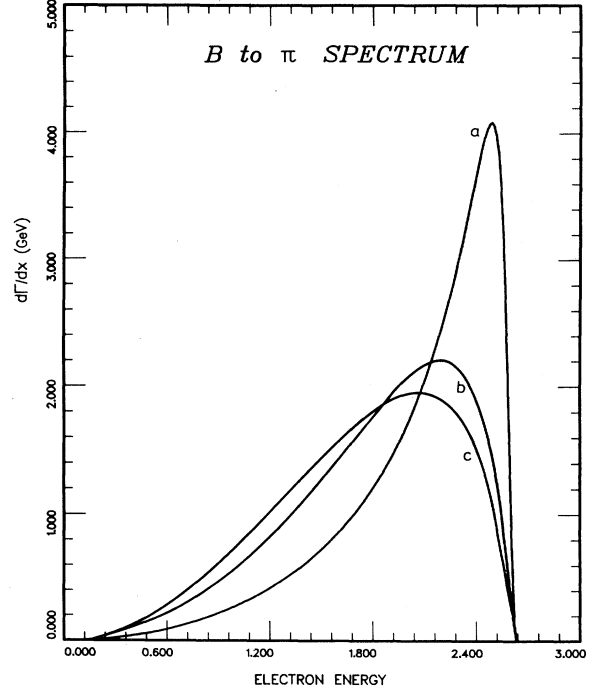


FIG. 8. Electron spectrum for  $B \rightarrow \pi e \nu$  for various vector form factors. The curves have been adjusted to have a common area (a) dipole form factor with mass  $m_{b\bar{u}} = 5.324$  GeV, (b) monopole form factor with same mass as in (a), (c) dipole form factor with mass  $m_{b\bar{u}} = 5.916$  GeV.

$B \rightarrow \pi \pi e \nu$  ( $B^- \rightarrow \rho^0 e^- \nu$ ). For the narrow-width approximation we find

$$\begin{aligned} \Gamma &= |V_{ub} I|^2 \times 1.29 \times 10^{-10} \text{ GeV, monopole form factor,} \\ \Gamma &= |V_{ub} I|^2 \times 1.00 \times 10^{-10} \text{ GeV, dipole form factor.} \end{aligned} \quad (56)$$

While the widths are quite similar, the dipole form factor shifts the spectrum sharply to higher electron momentum, as shown in Fig. 9. These results can also be expressed as branching ratios using the lifetime quoted above:

$$B(B^- \rightarrow \rho^0 e^- \nu) = \begin{cases} 232 |V_{ub} I|^2, & \text{monopole form factor,} \\ 179 |V_{ub} I|^2, & \text{dipole form factor.} \end{cases} \quad (57)$$

These may be compared to recent CLEO results<sup>20</sup>

$$B(B^- \rightarrow \rho^0 e \nu) < 3.0 \times 10^{-4} \quad (58)$$

and yield the limits

$$\begin{aligned} |V_{ub} I| &< 1.14 \times 10^{-3}, & \text{monopole form factor,} \\ |V_{ub} I| &< 1.29 \times 10^{-3}, & \text{dipole form factor.} \end{aligned} \quad (59)$$

As we expect  $|I| > 0.25$ , this would imply that  $|V_{ub}/V_{cb}| < 0.1$ .

To investigate effects of two-body kinematics and contact terms we use the overlap factor  $I = 0.33$  and express our results in terms of  $|V_{ub}|^2$ . They are, for  $I = 0.33$  and a monopole form factor,

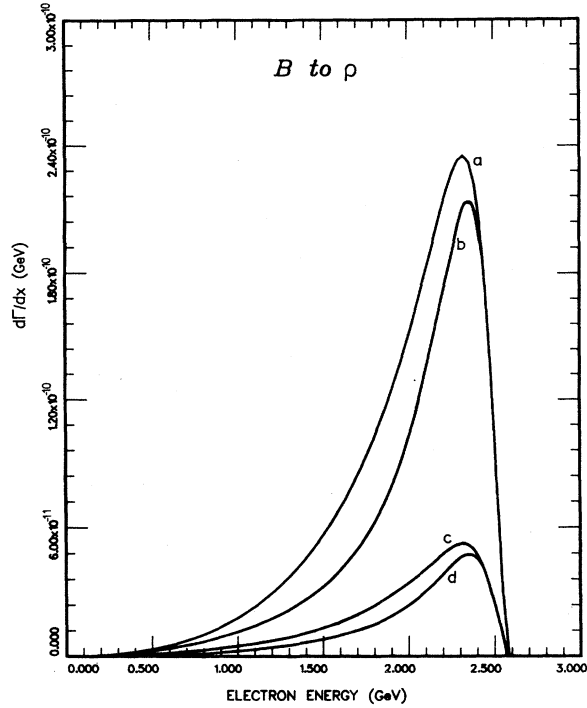


FIG. 9. Electron spectrum for  $B \rightarrow \rho^0 e \nu$  for overlap integrals  $I=0.7$  and  $0.33$  and monopole and dipole form factors: (a)  $I=0.7$ , monopole form factor; (b)  $I=0.7$ , dipole form factor; (c)  $I=0.33$ , monopole form factor; (d)  $I=0.33$ , dipole form factor.

$$\Gamma = |V_{ub}|^2 \times 1.29 \times 10^{-11} \text{ GeV}$$

(two-body kinematics, no contact term),

$$\Gamma = |V_{ub}|^2 \times 1.41 \times 10^{-11} \text{ GeV}$$

(narrow-width approximation, no contact terms),

$$\Gamma = |V_{ub}|^2 \times 62.0 \times 10^{-11} \text{ GeV}$$

(two-body kinematics with contact terms),

and, for  $I=0.33$  and a dipole form factor,

$$\Gamma = |V_{ub}|^2 \times 1.01 \times 10^{-11} \text{ GeV}$$

(two-body kinematics, no contact terms),

$$\Gamma = |V_{ub}|^2 \times 1.09 \times 10^{-11} \text{ GeV}$$

(narrow-width approximation, no contact terms),

$$\Gamma = |V_{ub}|^2 \times 103 \times 10^{-11} \text{ GeV}$$

(two-body kinematics with contact terms).

These results can also be expressed as branching ratios using the lifetime quoted above:

$$B(B^- \rightarrow \rho^0 e^- \nu) = 1112 |V_{ub}|^2,$$

(two-body kinematics with contact terms)

for a monopole form factor and  $I=0.33$ ,

$$B(B^- \rightarrow \rho^0 e^- \nu) = 1859 |V_{ub}|^2,$$

(two-body kinematics with contact terms)

for a dipole form factor and  $I=0.33$ . Comparing to the recent CLEO result,<sup>20</sup> Eq. (58), we find the limits

$$|V_{ub}| < 5.19 \times 10^{-4}$$

(two-body kinematics with contact terms)

for a monopole form factor and  $I=0.33$ ,

$$|V_{ub}| < 4.02 \times 10^{-4}$$

(two-body kinematics with contact terms)

for a dipole form factor and  $I=0.33$ .

Striking here is the dramatic effect of the contact terms, which boost the rate by two orders of magnitude. Further analysis reveals the effect to be primarily the vector contact term; putting it to zero results only in a doubling of the rate. The presence of the contact terms has been suggested by the extrapolation of a current-algebra result to very high energy, admittedly a crude estimate, but in the absence of a detailed dynamical theory for these current-algebra “backgrounds,” they cannot be ruled out. The moral seems to be that extraction of  $V_{bu}$  from these rates through their contributions to the electron end-point spectrum seems to be a very difficult task that will require careful study of high-statistics data such as those which might be available from a  $B$  factory to see whether or not such nonresonant background is present.

#### IV. SUMMARY AND CONCLUSIONS

In this work we have incorporated the “low-energy” results of chiral symmetry into resonance decay models for semileptonic decays of  $D$  and  $B$  mesons to two pseudoscalar mesons. The resonance models for  $D \rightarrow K^*$ ,  $D \rightarrow \rho$ ,  $B \rightarrow D^*$ , and  $B \rightarrow \rho$  were considered in the zero-width approximation and with finite-width effects with and without the contact terms originating from the low-energy effective Lagrangian. The weak transition matrix elements between  $D$  and  $B$  mesons and the intermediate vector mesons have sufficient structure so that there are no unjustified constraints on the three independent helicity structure functions  $H_+$ ,  $H_-$ , and  $H_0$  as was the case in many of the older works. The dependence of the lepton spectrum on the form factors  $F^V$  and  $F_2^A$  and the variation of the asymmetry parameter  $\alpha$  as a function of  $F_2^A/F_1^A$  were studied in detail. The results for  $\alpha$  were compared to experimental data in the cases  $D \rightarrow K^*$  and  $B \rightarrow D^*$ .

The finite-width effect for  $D \rightarrow K^* \rightarrow K\pi$  is approximately 11%. The background described by contact terms increases the width by 7%, in reasonable agreement with the data from the E691 experiment at Fermilab. For  $D \rightarrow \rho \rightarrow \pi\pi$  the finite-width effect and the background contributions are much larger. Taking the same overlap factor  $I$  as for  $D \rightarrow K^*$ , the finite-width effect causes the partial semileptonic width to decrease by 16% and the contact terms increase the width by 9%. This is clearly connected to the larger width of the  $\rho$  as com-

pared to the  $K^*$  width. Clearly there are no such effects for  $B \rightarrow D^* \rightarrow D\pi$  because of the extremely small total width of the  $D^*$ . On the other hand, for  $B \rightarrow \rho \rightarrow \pi\pi$  these effects are large again. The finite-width effect is only 9%, but the contribution of the contact terms is huge. This is partly due to the assumed small overlap factor in the resonance transition and partly due the large energy difference between initial and final states. Although we expect that these large background terms as described by the low-energy contact terms will be modified by higher-derivative terms and higher-order loop corrections, our result indicates that the  $B \rightarrow \rho \rightarrow \pi\pi$  transition is much less suitable for determining  $V_{ub}$  from experimental data to be obtained in the future. To get information on  $V_{ub}$  the single-pion final state seems much more suitable although it also depends on one unknown form factor which, however, is under much better theoretical control than the background terms in the  $\pi\pi$  final state.

Accurate experimental data on the partial semileptonic widths, the lepton spectrum and the asymmetry parameter  $\alpha$  are needed to determine the universal overlap factor  $I$  and the relative magnitude of the vector-meson form factors  $F_V$ ,  $F_1^A$ , and  $F_2^A$ . The parameter  $\alpha$  has been measured for  $D \rightarrow K^*$  and  $B \rightarrow D^*$ , although with large errors. From these data it seems that the ratio  $F_2^A/F_1^A$  for both decays is not in agreement with  $-2/(m_p + m_v)^2$  ad-

vocated in the work of Körner and Schuler.<sup>8</sup>

*Note added in proof.* After submission of this paper it was brought to our attention that the CLEO results<sup>20</sup> in Eqs. (54) and (58) are based on electron energies between 2.3 and 2.7 GeV. If we include in our branching ratios only electron energies above 2.3 GeV the results of Eq. (53) are reduced by factors 7.19 (for the monopole form factor) and 2.52 (for the dipole form factor), respectively. Similarly the branching ratios in Eq. (57) are reduced by factors 4.04 and 3.18. This increases the upper limits in Eq. (55) to  $4.77 \times 10^{-3}$  (monopole form factor) and to  $1.44 \times 10^{-3}$  (dipole form factor) and those in Eq. (59) to  $2.29 \times 10^{-3}$  (monopole form factor) and to  $2.30 \times 10^{-3}$  (dipole form factor), respectively. The limits following from two-body kinematics with contact terms are  $3.11 \times 10^{-3}$  and  $1.92 \times 10^{-3}$  instead of  $5.19 \times 10^{-4}$  and  $4.02 \times 10^{-4}$ .

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