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Cancellation of spectator interactions in the Drell–Yan and deep inelastic processes

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Abstract. We present an explicit calculation of the one-gluon soft interactions involving one spectator quark in the Drell–Yan and in the deep inelastic processes. We verify that these interactions are suppressed in the high energy limit. The gluon has a wavelength ranging from infinity to a value that is much smaller than the wavelength of the high energy photon. This includes a subregion, ranging from the meson size to values much bigger than the meson size, that has not been explicitly considered in the previous calculations.

1 Introduction

The factorization conjecture in the Drell–Yan process states that the cross section at high energy can be calculated using perturbative QCD for the hard process of quark–antiquark annihilation (active quarks), given the structure functions of the hadrons as measured in the process of deep inelastic scattering. If this conjecture is correct, then the interactions involving the quarks that do not participate in the hard process of annihilation (spectator quarks) should be suppressed at high energy.

Explicit computations of interactions with spectator quarks have considered different regions of the gluon momenta. It was found some time ago that if the gluon momenta are in the "Glauber" region then the spectator interactions are not suppressed [1]. Subsequently contributions from outside the "Glauber" region were included and the desired suppression was verified. This was done to two-gluon order in [2] in the regions of "soft", collinear and "Glauber" gluons, but not for "very-soft" gluons. "Very soft" gluons were included in [3] to one gluon order and more recently this was extended to two-gluon order [4]; both results show that the spectator interactions cancel in the "very soft" region too.

In the present paper it is taken into account, to one-gluon order, the contributions of gluon momenta not only from the "soft" and "very soft" regions but also from the region that lies in between. It is easy to see explicitly which region is this. Indeed, if we denote the quark mass by m, the transverse gluon momentum by $\mathbf{K}$ and the 4-momentum of the virtual photon by Q, it is evident that there is a gap between the "soft", $m < |\mathbf{K}| \ll \sqrt{Q^2}$, and the "very soft" regions, $|\mathbf{K}| \ll m$.

As in the previous calculations we use the framework of the Sachrajda–Yankielowicz scalar-field model, where the hadrons are described by scalar fields ϕ and the "quarks" by scalar-fields χ. The hadrons interact with the quarks through terms $\lambda\phi\chi^\dagger\chi$ and this interaction is used only to the lowest order in the coupling constant λ. We apply this model to the process

$$\text{quark} + \text{meson} \to \text{lepton pair} + \text{anything}. \tag{1}$$

In this simple case the Drell–Yan cross section is given by [2]

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9Q^4} x F_{\mathrm{DY}}(x), \tag{2}$$

where $Q^2 = xs$ is the invariant mass squared of the lepton pair and s is the incident energy squared.

In the Drell–Yan model $F_{\mathrm{DY}} = F_{\mathrm{DI}}$, where F_{DI} is the deep inelastic structure function. Thus, in the Drell–Yan model the factorization is trivially obeyed. When we consider the one gluon corrections, we will still have factorization *if*

$$F_{\mathrm{DY}}^{(1)} = K^{(1)} F_{\mathrm{DI}}^{(0)} + F_{\mathrm{DI}}^{(1)}, \tag{3}$$

where $K^{(1)}$ is the one-gluon contribution to the on-shell active-quark/active-quark annihilation process and $F_{\mathrm{DI}}^{(1)}$ is the lowest order contribution to the deep inelastic structure function.

In the next section we study the one-gluon corrections to F_{DY}. We consider all the relevant graphs which describe the active-quark/active-quark and active-quark/spectator interactions. The technique used in [5] to derive the eikonal result from the Sachrajda–Yankielowicz model is generalized to include the region $m < |k_\mu| \ll \sqrt{Q^2}$, where k_μ are the components of the gluon momenta. We find that

$$F_{\mathrm{DY}}^{(1)} = \bar{K}^{(1)} F_{\mathrm{DI}}^{(0)} + \mathscr{A}, \tag{4}$$

where $\bar{K}^{(1)}$ represents the contribution to $K^{(1)}$ from the region $|k_\mu| \ll \sqrt{Q^2}$ and $\mathscr{A}$ denotes the on-shell active-quark/spectator interactions. We show that $\mathscr{A}$ is suppressed in the high energy limit. In the Appendix it is verified that the same is true for $\bar{K}^{(1)}$ (contributions to $K^{(1)}$ come only from hard gluons). Thus, if the expression (3) is valid then $F_{\mathrm{DI}}^{(1)}$ should be suppressed in the high energy limit too. In the Sect. 3 we verify that this is indeed the case.

2 Quark-meson scattering

The relevant graphs that contribute to the quark-meson scattering cross section are shown in Fig. 1. The graphs involving the quark self-energy (which represents the renormalization of the quark line) and the ones involving the interaction between active and spectator quarks inside the meson (these interactions are already described in the context of the Sachrajda–Yankielowicz model) were not included.

We work in the Feynman gauge and parametrize the momenta of the incoming meson and quark using the light cone metric as follows:

$$P = (1, M^2, \mathbf{0}), \tag{5}$$

$$P_q = (m^2, 1, \mathbf{0}). \tag{6}$$

Thus the meson and the quark have respectively mass M and m. We set $M < 2m$ so that the meson is stable against spontaneous decay. The scale of energy is fixed in such a way that $s \equiv (P + P_q)^2 = 1 + \mathcal{O}(m^2)$ ($m^2 \ll 1$). The spectator coming from the meson has momentum.

$$P_s = \left(1 - x, \frac{m^2 + \mathbf{P}^2}{1 - x}, \mathbf{P}\right), \tag{7}$$

and the gluon momentum is given by

$$k = (\alpha, \beta, \mathbf{K}). \tag{8}$$

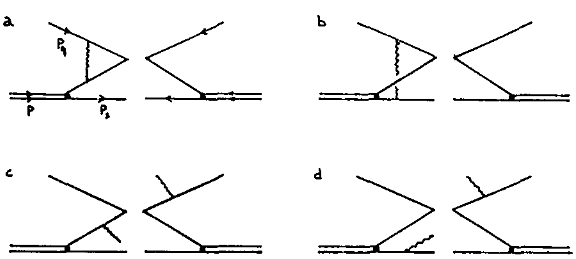

Fig. 1a–d. Graphs contributing to $F_{\mathrm{DY}}^{(1)}$. Gluons are denoted by wavy lines. The right-hand half of the graphs represents a complex conjugate amplitude

The vectors, $\mathbf{P}$ etc, are two-dimensional and transverse to the direction of propagation of the incident quark and meson.

Using the expressions (5) to (8) we can write the spectator and the active quark propagators respectively as

$$D_s(k) = \left[(1-x)\left(\beta + \frac{m^2 + \mathbf{P}^2}{1-x}\alpha - \frac{2\mathbf{P}\cdot\mathbf{K}}{1-x} + \frac{\alpha\beta - \mathbf{K}^2}{1-x} + i\varepsilon\right)\right]^{-1} \tag{9}$$

and

$$D_a(k) = \frac{-1}{x}\left[\beta + \left(\frac{M^2}{x} - \frac{m^2 + \mathbf{P}^2}{x(1-x)}\right)\alpha - M^2 + \frac{m^2 + \mathbf{P}^2}{x(1-x)} - \frac{2\mathbf{P}\cdot\mathbf{K}}{x} - \frac{\alpha\beta - \mathbf{K}^2}{x} - i\varepsilon\right]^{-1}, \tag{10}$$

and the propagator of the incoming quark as

$$D_q(k) = (m^2\beta + \alpha + \alpha\beta - \mathbf{K}^2 + i\varepsilon)^{-1}. \tag{11}$$

In terms of D_s, D_a and D_q, the graphs in Fig. 1a, b, c and d gives contributions that are proportional to $D_a(0)D_a(k)D_q(k)$, $D_s(k)D_a(0)D_q(k)$, $D_a(0)D_a(k)D_q^*(-k)$ and $D_s(k)D_a(0)D_q^*(-k)$ respectively.

In what follows we consider

$$|\alpha|, |\beta|, |\mathbf{K}| \ll 1. \tag{12}$$

This covers the region dealt with by Lindsay et al. [2], the region considered by Landshoff and Stirling [3] and also the region in between.

Performing the transformation [2]

$$\left.\begin{aligned} &\beta \to \beta + \frac{2\mathbf{P}\cdot\mathbf{K}}{1-x} - \alpha\frac{\mathbf{P}^2}{(1-x)^2} + \frac{\mathbf{K}^2}{1-x} \\ &\mathbf{K} \to \mathbf{K} + \alpha\frac{\mathbf{P}}{1-x} \end{aligned}\right\}, \tag{13}$$

the propagators (9), (10) and (11) change to

$$D'_s(k) = \left[(1-x)\left(\beta + \frac{m^2}{(1-x)^2}\alpha + i\varepsilon\right)\right]^{-1}, \tag{14}$$

$$D'_a(k) = \left[-x\left(\beta + \frac{m^2}{(1-x)^2}\alpha + A(\mathbf{K}) - i\varepsilon\right)\right]^{-1}, \tag{15}$$

and

$$D'_q(k) = \left(m^2\beta + \alpha - \mathbf{K}^2 + \frac{2m^2}{1-x}\mathbf{P}\cdot\mathbf{K} + i\varepsilon\right)^{-1}, \tag{16}$$

where

$$A(\mathbf{K}) = \frac{(\mathbf{P} + \mathbf{K})^2 + m^2 - x(1-x)M^2}{x(1-x)}. \tag{17}$$

We have neglected $\alpha\beta$ compared with β and $\mathcal{O}(m^2)\alpha$ compared with $\mathcal{O}(m^2)$. The gluon propagator does not change under the transformation (13). In other words, (13) is a *Lorentz transformation.*

The other approximation that we are allowed to make is to neglect α and β in the expressions for the quark-gluon vertices as well as α, β and $\mathcal{O}(m^2)$ in the quark–photon vertices. Adding the complex conjugate of the graphs in Fig. 1 and using the expressions (14–16) we obtain

$$F_{DY}^{(1)} = 2\,\mathrm{Re}\left\{\frac{\lambda^2}{(2\pi)^2}\frac{\alpha_s C_F}{x(1-x)}\int\frac{d^2\mathbf{P}}{A(\mathbf{0})}[I^V(x,\mathbf{P}^2)+I^R(x,\mathbf{P}^2)]\right\}, \tag{18}$$

where

$$I^V = -2i\int d\alpha d\beta d^2\mathbf{K}\left[\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha+i\varepsilon}+\frac{1}{A(\mathbf{0})}\right]\cdot\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha+A(\mathbf{K})-i\varepsilon}\cdot\frac{1}{m^2\beta+\alpha-\mathbf{K}^2+\frac{2m^2}{1-x}\mathbf{P}\cdot\mathbf{K}+i\varepsilon}\,\frac{1}{\alpha\beta-\mathbf{K}^2+i\varepsilon} \tag{19}$$

and

$$I^R = 4\pi\int d\alpha d\beta d^2\mathbf{K}\left[\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha+i\varepsilon}+\frac{1}{A(\mathbf{0})}\right]\cdot\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha+A(\mathbf{K})-i\varepsilon}\cdot\frac{1}{m^2\beta+\alpha+\mathbf{K}^2+\frac{2m^2}{1-x}\mathbf{P}\cdot\mathbf{K}+i\varepsilon}\,\delta(\alpha\beta-\mathbf{K}^2)\theta(\alpha+\beta) \tag{20}$$

are the contributions from the graphs with virtual and real gluon corrections respectively.

Using the identity

$$\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha+i\varepsilon}=\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha-i\varepsilon}-2\pi i\delta\left(\beta+\frac{m^2}{(1-x)^2}\alpha\right), \tag{21}$$

we can write (19) and (20) as

$$I^V = -2i\int d\alpha d\beta d^2\mathbf{K}\cdot\left[\frac{F(k)}{A(\mathbf{0})}\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha-i\varepsilon}-\frac{2\pi i}{A(\mathbf{K})}\delta\left(\beta+\frac{m^2}{(1-x)^2}\alpha\right)\right]\cdot\frac{1}{m^2\beta+\alpha-\mathbf{K}^2+\frac{2m^2}{1-x}\mathbf{P}\cdot\mathbf{K}+i\varepsilon}\,\frac{1}{\alpha\beta-\mathbf{K}^2+i\varepsilon} \tag{22}$$

and

$$I^R = 4\pi\int d\alpha d\beta d^2\mathbf{K}\frac{F(k)}{A(\mathbf{0})}\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha-i\varepsilon}\cdot\frac{1}{m^2\beta+\alpha+\mathbf{K}^2+\frac{2m^2}{1-x}\mathbf{P}\cdot\mathbf{K}+i\varepsilon}\cdot\delta(\alpha\beta-\mathbf{K}^2)\theta(\alpha+\beta) \tag{23}$$

where

$$F(k)=\frac{\beta+\frac{m^2}{(1-x)^2}\alpha+A(\mathbf{0})}{\beta+\frac{m^2}{(1-x)^2}\alpha+A(\mathbf{K})}. \tag{24}$$

The second term in (21) does not contribute to (20) because its δ-function is inconsistent with the on-shell condition for the real gluon.

If we could replace F by 1 in (22) and in (23), then we would have the standard eikonal result obtained in [5]. In this case the first term in (22) and the expression (23) would be the contribution to the on-shell active-quark/active-quark interaction.

We now show that even when $|\mathbf{K}| \gg m$ it is possible to replace F by 1 in the integrands of expressions (21) and (22). In order to do that, let us choose λ, μ, and τ such that

$$\lambda \ll m \ll \mu \ll 1, \tag{25}$$

and

$$|\alpha|, |\beta| < \tau \ll 1. \tag{26}$$

We consider two regions: $|\mathbf{K}| < \lambda$ and $\lambda < |\mathbf{K}| < \mu$. In the first region there is no problem and we can approximate F by 1. This is the "very soft" region where it is already known that the eikonal result is valid.

Suppose now that $|\mathbf{K}|$ is in the second region. Let us first consider the expression (23). The δ-function implies that $\beta = \mathbf{K}^2/\alpha$ and therefore $\beta > \lambda^2/\tau$. If λ, μ, and τ can be chosen in such a way that

$$\frac{\lambda^2}{\tau} \gg \mu^2, \tag{27}$$

then we would be again allowed to replace F by 1. To see that this is indeed possible, let us take $\mu = m^x$, $\tau = m^y$ and $\lambda = m^z$ and combine (25) with (27). This gives the following inequalities:

$$x + \frac{y}{2} > z > 1 > x. \tag{28}$$

For $\mu = \tau$ we can have for instance $x = y = 3/4$ and $z = 17/16$. We can choose $y > x$ as well ("Glauber region"). For example: $x = 3/4$, $y = 3/2$ and $z = 5/4$.

In the case of the first term in the expression (22),

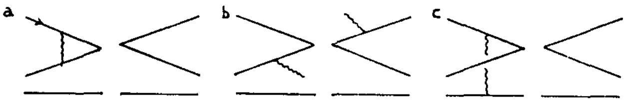

Fig. 2a–c. Graphs **a** and **b** represent the contribution to $\bar{K}^{(1)}$. The contribution to $\mathcal{A}$ is represented in **c**. The external lines represent on-shell eikonalized quarks. Wavy lines denote "soft gluons". The right-hand half of the graphs represents a complex conjugate amplitude

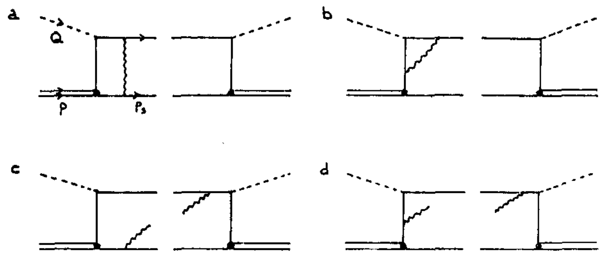

Fig. 3a–d. Graphs contributing to $F_{\mathrm{DI}}^{(1)}$. In these graphs the wavy line denotes gluons and the dotted line denotes the high energy photon. The right-hand half of the graphs represents a complex conjugate amplitude

we can evaluate the β-integral by closing the contour in the lower half-plane where there are two poles. One is from the incoming quark propagator which gives a contribution that is suppressed by the factor m^2. The other is from the gluon propagator and is in the region $|\beta| > \lambda^2/\tau$ where again F can be replaced by 1.

With $F(k)=1$, both the frist term in (22) and (23) are *factorized* in the sense that the $\mathbf{P}$-integral and the k-integral in (18) do not interfere with each other. The factorized $\mathbf{P}$-integral is equal to the lowest order correction to the deep inelastic structure function ($F_{\mathrm{DI}}^{(0)}$). The k-integral is represented diagrammaticaly by the Fig. 2a and b and is equal to the factor $\bar{K}^{(1)}$ defined in (4). In the Appendix we show that this integral vanishes. This is consistent with the fact that $K^{(1)}$ is an ultraviolet-dominated function [6].

The second term in (22) describes the active-quark/spectator interaction and is represented diagrammaticaly by Fig. 2c. Substituting this term in (18) we get the expression for the second term in (4) which is given by

$$\mathcal{A} = 2\,\mathrm{Re}\left\{ \frac{\lambda^2}{\pi}\frac{\alpha_s C_F}{x(1-x)}\int\frac{d^2\mathbf{P}}{A(\mathbf{0})}\int\frac{d^2\mathbf{K}}{A(\mathbf{K})}\int d\alpha \cdot \frac{1}{\alpha-\mathbf{K}^2+\dfrac{2m^2}{1-x}\mathbf{P}\cdot\mathbf{K}+i\varepsilon}\,\frac{1}{\dfrac{m^2}{(1-x)^2}\alpha^2+\mathbf{K}^2}\right\}, \tag{29}$$

where we have used the δ-function in the second term of (22) to integrate over β. Making the transformation $\alpha \to \alpha + \mathbf{K}^2$, neglecting $2m^2\mathbf{K}^2\alpha/(1-x)^2$ compared with $\mathbf{K}^2$ and performing the result α-integral in the upper half-plane we obtain

$$\mathcal{A} = 2\lambda^2\frac{\alpha_s C_F m^3}{x(1-x)^3}\int\frac{d^2\mathbf{P}}{A(\mathbf{0})}\int\frac{d^2\mathbf{K}}{A(\mathbf{K})}\frac{2\mathbf{P}\cdot\mathbf{K}}{|\mathbf{K}|^3}. \tag{30}$$

Outside the infrared region the $\mathbf{K}$-integral is strongly suppressed by a factor of order m^4 (the $\mathbf{P}$-integral is of order m). In the infrared region, $|\mathbf{K}| \ll m$, one can neglect $\mathbf{K}$ in $A(\mathbf{K})$ and the resulting angular integral vanishes.

3 Deep inelastic structure function

The deep inelastic structure function is represented to one-gluon order by the graphs in Fig. 3. We use the same parametrization for P, P_s and $\mathbf{K}$ as given in expressions (5), (7) and (8). For the incoming photon we use

$$Q = (-x + m^2 + \mathbf{P}^2, 1, \mathbf{0}) \tag{31}$$

or

$$Q' = (-x + m^2 + \mathbf{P}^2 + \alpha, 1 + \beta, \mathbf{K}), \tag{32}$$

in the case of virtual or real gluon graphs respectively. Neglecting $\mathcal{O}(m^2)$, α and β compared with x, we get the standard relation: $Q^2 \simeq Q'^2 \simeq -x$.

With this notation, the formal difference between the deep inelastic structure function and the quark meson process is that instead of the expression (11) we now have

$$\tilde{D}_q(k) = -[\alpha + \beta(m^2+\mathbf{P}^2) + 2\mathbf{P}\cdot\mathbf{K} + \alpha\beta - \mathbf{K}^2 - i\varepsilon]^{-1}, \tag{33}$$

for the propagator of the outgoing active quark.

We now perform the transformation (13). Under this transformation $\tilde{D}_q$ is not altered. The extra terms in the transformed β are suppressed by the factor $(m^2 + \mathbf{P}^2)$ in (33). In the case of the quark-meson process we had to keep the extra term $(2m^2/(1-x))\mathbf{P}\cdot\mathbf{K}$, but in (33) such a contribution can be neglected when compared with $2\mathbf{P}\cdot\mathbf{K}$. We can again neglect α and β in the expressions for the vertices in Fig. 3. Adding the complex conjugate of the graphs, the resulting expression for $F_{\mathrm{DI}}^{(1)}$ can be written as

$$F_{\mathrm{DI}}^{(1)} = 2\,\mathrm{Re}\left\{\frac{\lambda^2}{(2\pi)^2}\frac{\alpha_s C_F}{x(1-x)}\int\frac{d^2\mathbf{P}}{A(\mathbf{0})} \cdot [J^V(x,\mathbf{P}^2) + J^R(x,\mathbf{P}^2)]\right\}, \tag{34}$$

where J^V and J^R denote the contributions from the graphs with a virtual and a real gluon respectively. The expressions for J^V and J^R are obtained from (22) and (23) replacing the incoming quark propagator by (33) and are given by

$$J^V = 2i\int d\alpha\, d\beta\, d^2\mathbf{K}\left[\frac{F(k)}{A(\mathbf{0})}\,\frac{1}{\beta + \dfrac{m^2}{(1-x)^2}\alpha - i\varepsilon}\right.$$

$$-\frac{1}{A(\mathbf{K})}2\pi i\delta\left(\beta+\frac{m^2}{(1-x)^2}\alpha\right)\Bigg]$$

$$\cdot\frac{1}{\alpha+(m^2+\mathbf{P}^2)\beta+2\mathbf{P}\cdot\mathbf{K}-\mathbf{K}^2-i\varepsilon}\,\frac{1}{\alpha\beta-\mathbf{K}^2+i\varepsilon} \tag{35}$$

and

$$J^R=-4\pi\int d\alpha d\beta d^2\mathbf{K}\frac{F(k)}{A(\mathbf{0})}\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha-i\varepsilon}$$

$$\cdot\frac{1}{\alpha+(m^2+\mathbf{P}^2)\beta+2\mathbf{P}\cdot\mathbf{K}+\mathbf{K}^2-i\varepsilon}$$

$$\cdot\delta(\alpha\beta-\mathbf{K}^2)\theta(\alpha+\beta). \tag{36}$$

Integrating the first term in (35) by closing the contour in the β-integral in the lower half-plane, the *only* contribution we get is from the gluon propagator. The resulting expression would cancel exactly with J^R if the sign of $\mathbf{K}^2$ in the quark propagator was the same in both J^V and J^R. However, since for this contribution we have $\mathbf{K}^2=\alpha\beta\ll\alpha$, this difference can be neglected and therefore the resulting contributions cancel each other.

Using the δ-function to perform the β-integral in the second term of (35) and inserting the result in (34) gives

$$F^{(1)}_{\mathrm{DI}}=-2\,\mathrm{Re}\left[\frac{\lambda^2}{\pi}\frac{\alpha_s C_F}{x(1-x)}\int\frac{d^2\mathbf{P}}{A(\mathbf{0})}\int\frac{d^2\mathbf{K}}{A(\mathbf{K})}\int d\alpha\right.$$

$$\left.\cdot\frac{1}{\alpha-\mathbf{K}^2+2\mathbf{P}\cdot\mathbf{K}-i\varepsilon}\,\frac{1}{\frac{m^2}{(1-x)^2}\alpha^2+\mathbf{K}^2}\right]. \tag{37}$$

Making the transformation $\alpha\to\alpha+\mathbf{K}^2$ and performing the α-integral by closing the contour in the lower half-plane we obtain

$$F^{(1)}_{\mathrm{DI}}=-2\lambda^2\frac{\alpha_s C_F m}{x(1-x)^2}\int\frac{d^2\mathbf{P}}{A(\mathbf{0})}\int\frac{d^2\mathbf{K}}{A(\mathbf{K})}\frac{2\mathbf{P}\cdot\mathbf{K}}{|\mathbf{K}|^3}. \tag{38}$$

This expression is not exactly equal to the result obtained for $\mathscr{A}$ in (30), as it would be expected if equation (3) were exactly obeyed. Nevertheless, one can see that $F^{(1)}_{\mathrm{DI}}$ is suppressed too. As in (30), the angular integral vanishes in the infrared region. Outside the infrared region the result is suppressed by a factor $\mathcal{O}(m^2)$.

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Appendix

In this appendix we verify the suppression of $\bar{K}^{(1)}$. The contributions to $\bar{K}^{(1)}$ are represented in Fig. 2a and b, and the respective analytic expressions are obtained from the first term in (22) and (23) by replacing $F(k)$ by 1. The sum of these two expressions is proportional to

$$i\int d\alpha d\beta d^2\mathbf{K}\left[\frac{1}{m^2\beta+\alpha-\mathbf{K}^2+\frac{2m^2}{1-x}\mathbf{P}\cdot\mathbf{K}+i\varepsilon}\,\frac{1}{\alpha\beta-\mathbf{K}^2+i\varepsilon}\right.$$

$$+\frac{1}{m^2\beta+\alpha+\mathbf{K}^2+\frac{2m^2}{1-x}\mathbf{P}\cdot\mathbf{K}+i\varepsilon}$$

$$\left.\cdot2\pi i\delta(\alpha\beta-\mathbf{K}^2)\theta(\alpha+\beta)\right]\frac{1}{\beta+\frac{m^2}{(1-x)^2}\alpha-i\varepsilon}. \tag{A1}$$

In the first term we can perform the α-integral by closing the contour in the lower half-plane. The contribution from the gluon propagator cancels with the second term. As in the case of the deep inelastic structure function, the first and second terms in (A1) have a different sign for $\mathbf{K}^2$ in the incoming quark propagator, but this difference is neglegible since for this contribution $\mathbf{K}^2=\alpha\beta\ll\alpha$. There is also a pole from the incoming quark propagator which gives

$$\frac{-2\pi}{m^2}\int d\beta d^2\mathbf{K}\frac{1}{\beta+\frac{m^2}{(1-x)^2}\mathbf{K}^2+\frac{2m^4}{(1-x)^3}\mathbf{P}\cdot\mathbf{K}-i\varepsilon}$$

$$\cdot\frac{1}{\left(\beta+\frac{\mathbf{P}\cdot\mathbf{K}}{1-x}+i\frac{|\mathbf{K}|}{m}\right)\left(\beta+\frac{\mathbf{P}\cdot\mathbf{K}}{1-x}-i\frac{|\mathbf{K}|}{m}\right)}$$

$$=-2\pi^2\int\frac{d^2\mathbf{K}}{|\mathbf{K}|}\frac{1}{\frac{m}{1-x}\mathbf{P}\cdot\mathbf{K}-i|\mathbf{K}|}. \tag{A2}$$

where we have performed the β-integral by closing the contour in the lower half plane. After taking the real part of this expression, the resulting angular integral vanishes.

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