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CROSS SECTION FOR FIVE-PARTON PRODUCTION IN e^+e^- ANNIHILATION

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We describe routines written in REDUCE for the calculation of the five-parton processes $e^+e^- \to q\bar{q}3g$ and $e^+e^- \to 2q2\bar{q}g$ in lowest order QCD perturbation theory.

PROGRAM SUMMARY

Title of program: FIVE PARTON CROSS SECTION

Catalogue number: ABLH

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland (see application form in this issue)

Computer: IBM 3084 Q

Operating systems: TSO/NEWLIB

Programming language used: REDUCE [1], FORTRAN

High speed storage required: 3000 kbytes

No. of lines in combined program and test deck: 2656

Keywords: QCD, perturbation theory, jet physics, e^+e^- annihilation, multi-parton production, symbolic calculations

Nature of physical problem
Five-parton cross section in e^+e^+ annihilation is calculated.

Method of solution
The five-parton cross section in e^+e^- annihilation is calculated by REDUCE programs. FORTRAN programs that sum all contributions such that an application in Monte Carlo studies is possible are presented. Permutations of momenta of the external particles are performed numerically to keep the cross section formula short.

Running time
Calculation of the cross section: 2 hours.

Reference

[1] A.C. Hearn, REDUCE User's Manual, Version 3.2, Rand Publication CP78 (Rev. 4/85).

[1] Supported by Bundesministerium für Forschung und Technologie, 05 4HH 92P/3, Bonn, Fed. Rep. Germany. Present address: BASF AG, Abteilung ZXT, D-6700 Ludwigshafen, Fed. Rep. Germany.
[2] Supported by Bundesministerium für Forschung und Technologie, 05 4HH 92P/3 (1988), Bonn, Fed. Rep. Germany, and Studienstiftung des deutschen Volkes (1989).

LONG WRITE-UP

1. Introduction

Recently there has been much interest in multijet production in electron–positron annihilation. Experimental results for the production of up to five jets have been presented by two DESY and one KEK collaboration [1]. In QCD the production of hadron jets originates from the primordial production of quarks and gluons and their subsequent fragmentation into hadrons. The annihilation of e^+ and e^- into quarks and gluons is calculated in QCD perturbation theory. Then the number of jets is equal to the number of partons in the final state. The production of four-parton final states occurs the first time in second-order QCD. The Born cross section is proportional to α_s^2. Five-parton production is calculated in third order ($\mathcal{O}(\alpha_s^3)$). The number of diagrams for $e^+e^- \to q\bar{q}2g$ and $e^+e^- \to 2q2\bar{q}$ is still small and the final formulae for the cross sections, also calculated with the help of REDUCE, have been published [2]. In the case of five-parton production, the number of Feynman diagrams for $e^+e^- \to q\bar{q}3g$ and $e^+e^- \to 2q2\bar{q}g$ is much larger. We show the general structure of these diagrams in fig. 1. The complete list of all diagrams consists of the diagrams in fig. 1 together with the permutations of final gluon lines for $e^+e^- \to q\bar{q}3g$ and together with the permutations of quark and antiquark lines for $e^+e^- \to 2q2\bar{q}g$. Altogether we have 54 diagrams for

$$e^+e^- \to q(p_1)\bar{q}(p_2)g(p_3)g(p_4)g(p_5), \qquad (1.1)$$

and 48 diagrams for

$$e^+e^- \to q(p_1)\bar{q}(p_2)q(p_3)\bar{q}(p_4)g(p_5). \qquad (1.2)$$

The p_i stand for the momenta of the partons in the final state. The differential cross section is obtained from

$$d\sigma = \frac{e^4}{2q^6 N_s} l^{\mu\nu} \prod_{i=1}^{5} \frac{d^3p_i}{2p_{i0}(2\pi)^3} (2\pi)^4 \times \delta^{(4)}\left(p_+ + p_- - \sum_{i=1}^{5} p_i \right) H_{\mu\nu}, \qquad (1.3)$$

where $q = p_+ + p_-$, p_+ and p_- are the momenta of the incoming positron and electron, respectively. $l^{\mu\nu}$ stands for the lepton tensor which after integration over the orientation of the final parton system with respect to the incoming positron momentum can be replaced by $l_{\mu\nu} = -q^2 g_{\mu\nu}/3$.

It is the purpose of this paper to describe the routines for the calculation of H^{μ}_{μ} for the final states (1.1) and (1.2) which is needed for the differential cross section with the angular dependence with respect to the beam direction integrated out. We write $H_{\mu\nu}$ in the following form:

$$H_{\mu\nu} = (4\pi\alpha_s)^3 \sum_{k=1}^{N_f} Q_k^2 \sum_{m \geq n = 1} A(m, n)_{\mu\nu}. \qquad (1.4)$$

$A(m, n)$ stands for the product of diagram m with diagram n and of diagram n with diagram m (except for $m = n$) taken from the list of diagrams in fig. 1, summed over spins, colours and flavours of the final states (1.1) and (1.2). Therefore the sums over m and n in (1.4) run over all 54 diagrams with $m \geq n$ for (1.1) and 48 diagrams with $m \geq n$ for (1.2).

The calculation of the traces of the products of matrix elements $A(m, n)^{\mu}_{\mu}$ for every m and n was done in the Feynman gauge. To sum over the polarizations of the gluons we have taken the trace with respect to the Lorentz indices of the gluon polarization vectors. Then it is necessary to cancel the contribution of the unphysical gluon polarizations by adding ghost diagrams. In total there are 72 ghost diagrams whose products must be added in (1.4). The general structure of the ghost diagrams is shown in fig. 2.

The calculation of the various traces for summation over quark and gluon polarizations has been performed with REDUCE 3.2 [3]. The results for the contributions $A(m, n)^{\mu}_{\mu}$ come out as functions of the invariants $y_{ij} = 2p_i p_j$ ($i, j = 1, 2, \ldots, 5$) and the colour factors which are determined in a separate routine. The expressions obtained for $A(m, n)^{\mu}_{\mu}$ are very long. This is the reason why we want to publish the REDUCE program so that the matrix elements can be recalculated and then used in a Monte Carlo routine

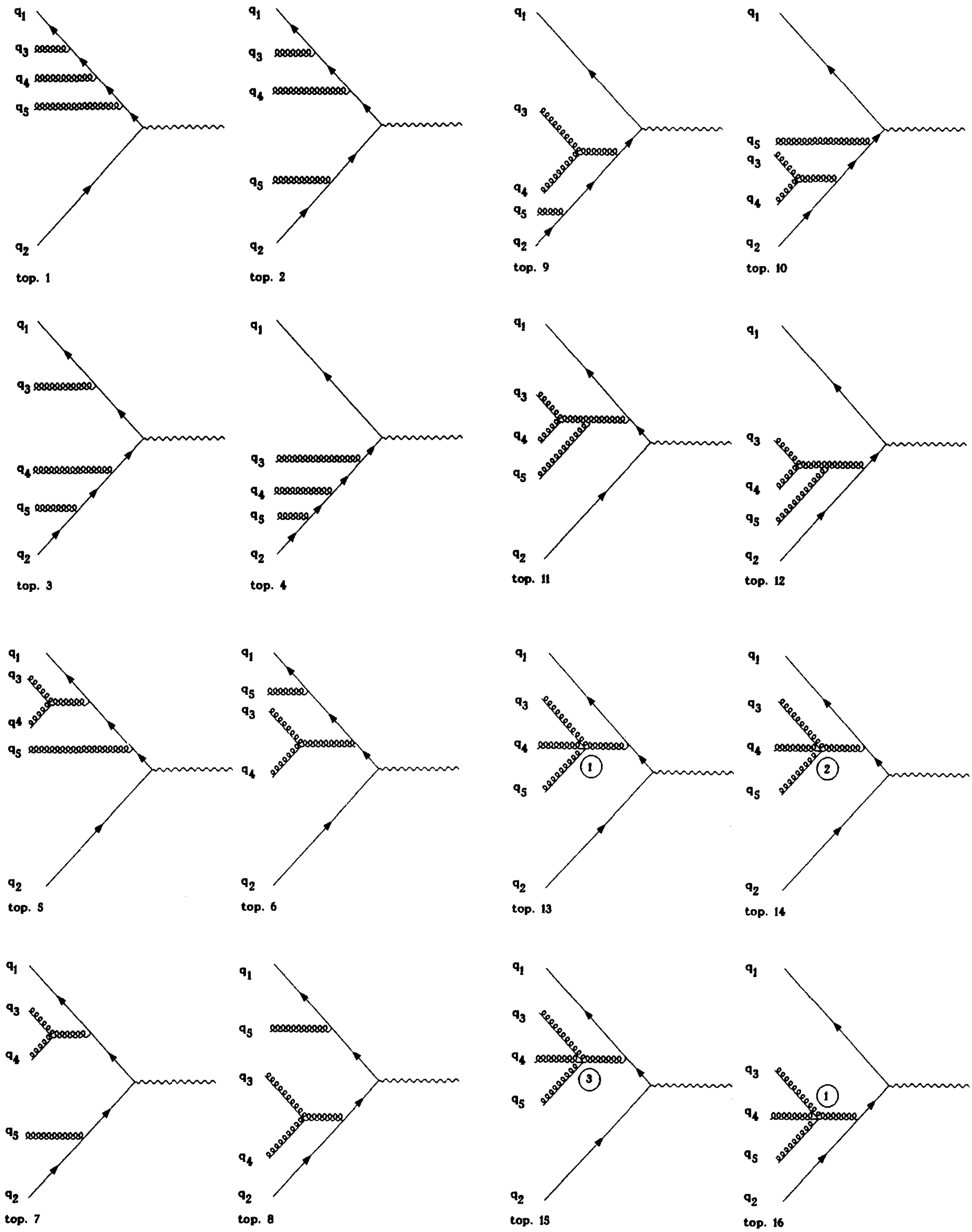

Fig. 1a. q$\bar{q}$3g-graphs.

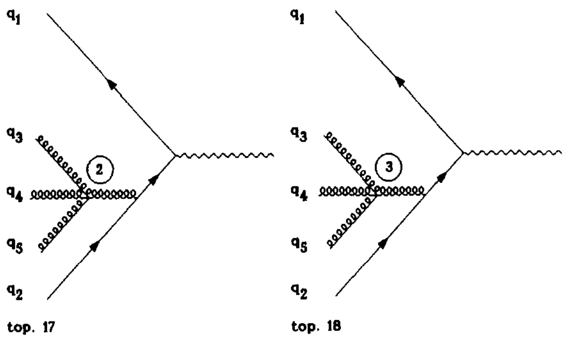

Fig. 1a (continued).

for the calculation of parton cross sections or in fragmentation models. The outline of this paper is as follows. In section 2 we describe the general setting used for our calculations. Section 3 contains the calculation of the Lorentz structure and section 4 describes the calculation of the colour factors. In section 5 we describe how these routines must be combined by FORTRAN subroutines in order to obtain the trace of the hadron tensor.

2. General description of the calculation

The hadron tensor $H_{\mu\nu}$ is given by $H_{\mu\nu} = M_\mu^* M_\nu$, where M_μ is the amplitude of the process

$$\gamma(\text{polarization vector } \mu) \to \text{final state.} \tag{2.1}$$

The amplitude M_μ is given as a sum over all possible subamplitudes M_μ^i,

$$M_\mu = \sum_i M_\mu^i, \tag{2.2}$$

each corresponding to a unique Feynman graph.

We will discuss in detail the case of the final state $q\bar{q}3g$, the calculation of the final states $2q2\bar{q}g$ and $q\bar{q}$ 2 ghosts g is then an obvious generalization.

First assume that the polarizations λ_i of the final state are specified. The quark carries the index 1, the antiquark the index 2, and the 3 gluons carry the indices 3, 4, 5. We have

$$M_\mu^i = \bar{u}(p_1, \lambda_1)\Gamma^i_{\mu\epsilon_3\epsilon_4\epsilon_5}v(p_2, \lambda_2)$$
$$\epsilon^{\epsilon_3^*}(p_3, \lambda_3)\epsilon^{\epsilon_4^*}(p_4, \lambda_4)\epsilon^{\epsilon_5^*}(p_5, \lambda_5). \tag{2.3}$$

Here $\Gamma^i_{\mu\alpha\beta\gamma}$ is a string of γ-matrices corresponding to a particular Feynman graph, possibly multiplied by three- and four-gluon vertices, and $\epsilon^\rho(p, \lambda)$ is the polarization vector of a gluon carrying momentum p and a Lorentz index ρ.

The sum over all polarizations in the final state yields the hadron tensor

$$\begin{aligned} H_{\mu\nu} &= \sum_\lambda \sum_{i,j} M_\mu^{i*}(\lambda) M_\nu^j(\lambda) \\ &= \sum_{i,j} \sum_\lambda \bar{v}(p_2, \lambda_2)\Gamma^{i^{\mathrm{T}}}_{\mu\epsilon_3'\epsilon_4'\epsilon_5'}u(p_1, \lambda_1) \\ &\quad \times \bar{u}(p_1, \lambda_1)\Gamma^j_{\mu\epsilon_3\epsilon_4\epsilon_5}v(p_2, \lambda_2) \\ &\quad \cdot \epsilon^{\epsilon_3'}(p_3, \lambda_3)\epsilon^{\epsilon_4'}(p_4, \lambda_4)\epsilon^{\epsilon_5'}(p_5, \lambda_5) \\ &\quad \cdot \epsilon^{\epsilon_3^*}(p_3, \lambda_3)\epsilon^{\epsilon_4^*}(p_4, \lambda_4)\epsilon^{\epsilon_5^*}(p_5, \lambda_5). \end{aligned} \tag{2.4}$$

We are using the relations,

$$\sum_\lambda u(p, \lambda)\bar{u}(p, \lambda) = \sum_\lambda v(p, \lambda)\bar{v}(p, \lambda) = \not{p}, \tag{2.5}$$

in the case of massless quarks, and the gluon polarization sum,

$$\sum_\lambda \epsilon^\mu(p, \lambda)\epsilon^\nu(p, \lambda) = -g^{\mu\nu}. \tag{2.6}$$

Since we have dropped some terms in the sum (2.6), we have to add ghost diagrams to ensure transversality of the external gluons, as explained elsewhere [4].

Performing the polarization sums (2.5, 2.6) in (2.4) we get

$$H_{\mu\nu} = -\sum_{i,j} \mathrm{tr}\left(\Gamma_\mu^{i^{\mathrm{T}}\epsilon_3\epsilon_4\epsilon_5}\not{p}_1\Gamma^j_{\nu\epsilon_3\epsilon_4\epsilon_5}\not{p}_2\right). \tag{2.7}$$

Therefore the trace of the hadron tensor is

$$H_\mu^\mu = -\sum_{i,j} \mathrm{tr}\left(\Gamma^{i^{\mathrm{T}}\mu\epsilon_3\epsilon_4\epsilon_5}\not{p}_1\Gamma^j_{\mu\epsilon_3\epsilon_4\epsilon_5}\not{p}_2\right). \tag{2.8}$$

We have done these trace calculations with REDUCE.

For the $q\bar{q}3g$-case, the contributing diagrams may have 18 different topologies (fig. 1a), in the $2q2\bar{q}g$-case, there are 12 different topologies (fig.

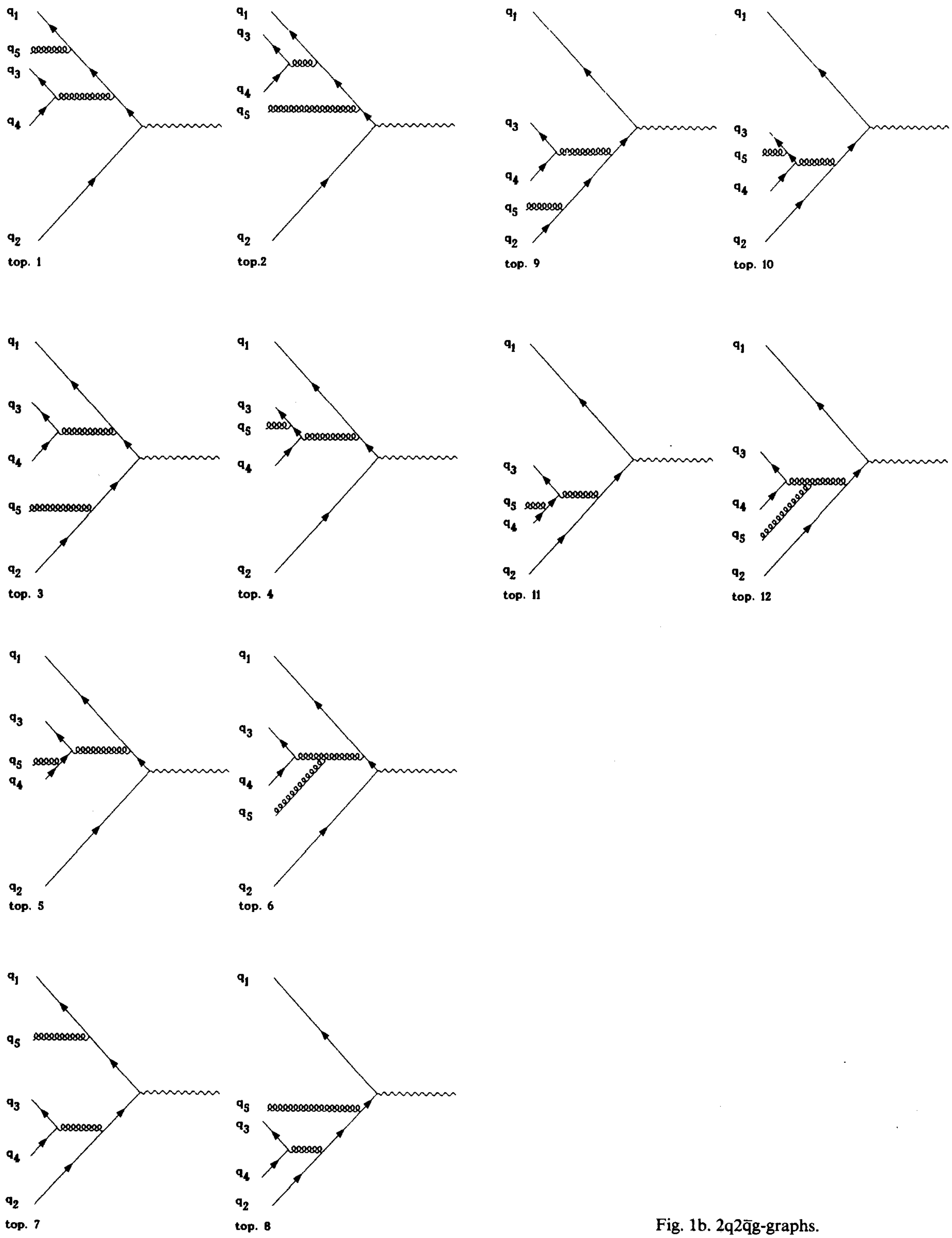

Fig. 1b. $2q2\bar{q}g$-graphs.

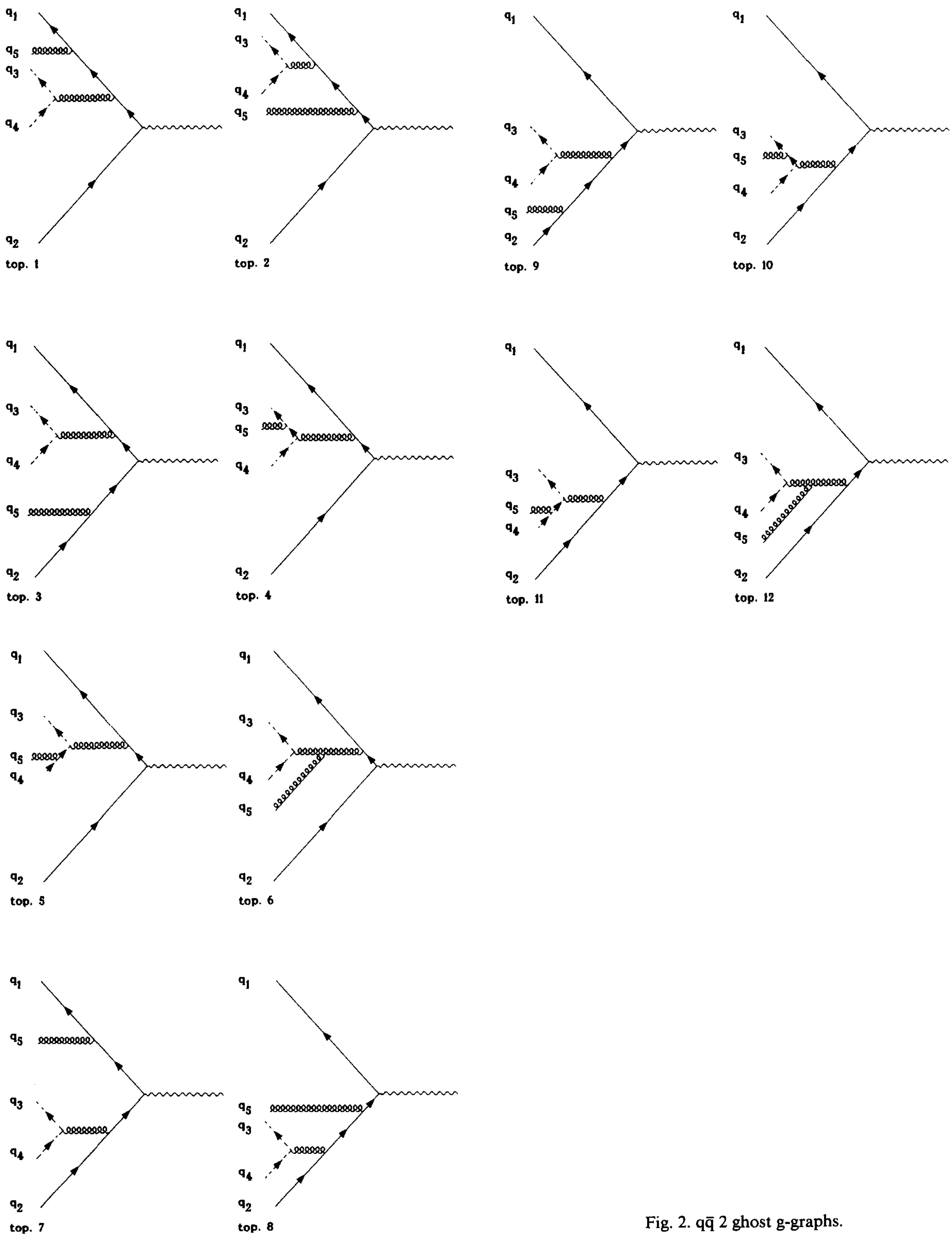

Fig. 2. $q\bar{q}$ 2 ghost g-graphs.

1b), and finally, there are 12 different topologies for the $q\bar{q}$ 2 ghost g-diagrams (fig. 2).

Now we turn back to the $q\bar{q}3g$-case. The momenta q_3, q_4 and q_5 of the external gluons have to run through all six different permutations of p_3, p_4 and p_5 in the case of the Abelian diagrams (1)–(4); we have to use the cyclic permutations of p_3, p_4 and p_5 in the case of the diagrams (5)–(12); and no permutations except the identity are allowed for the diagrams with the four-gluon vertex (13)–(18). This is due to the fact that the three- and four-gluon vertices are symmetric with respect to all legs. We have split up the four-gluon vertex into 3 parts each corresponding to a unique colour structure. In total, this yields 54 different diagrams as already mentioned in the introduction, resulting in 54^2 contributions to the hadron tensor. Now the matrix elements are real numbers, so $M^{i*}M^j = M^{j*}M^i$, therefore, we only need the $(54 \times 55)/2$ diagrams with $i \leq j$. This number can be reduced considerably if:

(a) the momentum permutation is performed numerically in the resulting FORTRAN program and
(b) obvious symmetries among the diagrams are used.

Table 1 shows the six different permutations of the gluon momenta. The permutation $\sigma = 1$ is the identity, the first three permutations are cyclic, the last three are anticyclic.

In principle, we have to sum over all physically distinct permutations of the external gluons in the left and right factors. Now it is possible to rearrange the sum such that we collect the terms M^*M which differ only in a permutation of the momenta. Each of these terms is calculated only once, the momentum permutation is performed later in a FORTRAN program.

Table 1
Momentum permutations for $q\bar{q}3g$

σ	q_3	q_4	q_5
1	p_3	p_4	p_5
2	p_4	p_5	p_3
3	p_5	p_3	p_4
4	p_3	p_5	p_4
5	p_4	p_3	p_5
6	p_5	p_4	p_3

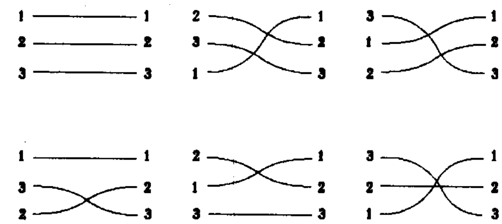

Fig. 3. Possible contractions.

Now we assume that we have chosen a topology JL on the left hand side (i.e. in M^*) and a topology JR on the right hand side (i.e. in M); JL, JR $= 1, 2, \ldots, 18$. We denote the momenta on the left side with QL_i, the momenta on the right side with QR_i. Obviously, $QL_1 = QR_1 = p_1$, $QL_2 = QR_2 = p_2$.

If we assume momentum permutations σ_L and σ_R for M^* and M, respectively, then for $i = 3, 4, 5$ we have

$$QL_i = p_{\sigma_L(i)}, \qquad QR_i = p_{\sigma_R(i)}, \tag{2.9}$$

and the same for the Lorentz indices.

There are six possibilities to contract the gluon indices as depicted in fig. 3. We will call such a correspondence a *contraction*.

If σ_L and σ_R run through all permutations that are allowed for the corresponding diagrams, then this is equivalent to a sum over all σ_R of the right diagram and all $\sigma = \sigma_L^{-1} \cdot \sigma_R$, such that only the permutions $\sigma_L = \sigma_R \cdot \sigma^{-1}$ that are appropriate for the left diagram are reached. We will call σ_R the *momentum permutation*.

The REDUCE programs described in the next section calculate the products $M_\mu^{i*} M^{j\mu}$ for all necessary contractions, but only for the momentum permutation $\sigma_R = 1$.

This means that the momentum permutations have to be performed numerically by the FORTRAN programs described later on in section 5.

3. Calculation of the Lorentz structure

We describe the REDUCE program for the $q\bar{q}3g$ case in detail, the other programs ($2q2\bar{q}g$ and

$q\bar{q}$ 2 ghost g) are quite similar. The program consists of 17 parts that will be explained now.

3.1. Parameters

Since the result is a long list of FORTRAN statements, we have to split up the result into smaller parts that can be handled by a FORTRAN compiler. There are various parameters which determine the part of the result that is calculated:
@TOPRU and @TOPRO determine the range for the right topologies;
@TOPLU and @TOPLO do the same for the left topologies;
@TOPN determines if the denominators are calculated. If @TOPN = 0, then this calculation is skipped, if @TOPN = 18, then his calculation is performed. Similar, if @TOPRO = @TOPLO = 0, @TOPRU = @TOPLU = 1, then the calculation of the numerators is *not* performed.

3.2. Open output file

We write the REDUCE output to a sequential file, this file is opened here and all other output is suppressed.

3.3. FORTRAN output

The output is specified to be in FORTRAN format (max. 19 continuation cards, etc.)

3.4. Momentum sums

$Q_{\alpha\beta\dots\epsilon}$ is a shorthand notation for $Q_\alpha + Q_\beta + \cdots + Q_\epsilon$. QDEF defines these sums, QREL defines these symbols to be vectors. Furthermore, the external momenta are set on the mass shell.

3.5. Declare some vectors

The external momenta $p_1, \dots, p_5$ are declared as vectors and set on the mass shell. The vector dimension is fixed to 4.

3.6. Lorentz scalars

The final result is expressed in terms of the Lorentz invariants $y_{ij} = 2p_i p_j$.

3.7. Indices and fermion lines

Here some Lorentz indices are defined. The fermion line is denoted by L.

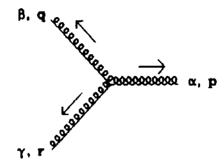

Fig. 4. The three-gluon vertex.

3.8. Squares of the inner momenta

The propagators are $1/q^2$ or $\not{q}/q^2$, where q is the momentum of the internal line. The squares of the four vectors q are defined here.

3.9. The three-gluon vertex

DGLV(α, β, γ, p, q, r) is an operator that defines the Lorentz structure of three-gluon vertex. α, β, γ are the Lorentz indices, p, q, r are the external momenta (see fig. 4).

3.10. The four-gluon vertex

The four-gluon vertex is a sum of three terms where each of them has a different colour structure. We separate the colour structures and define three different vertices 1, 2, 3. VGLV(n, α, β, γ, δ) is the vertex n with Lorentz indices α, β, γ, δ (see fig. 5).

3.11. Feynman rules

This section contains the strings $\Gamma_{\mu\alpha\beta\gamma}$ and $\Gamma^{\mathrm{T}}_{\mu\alpha\beta\gamma}$ of γ-matrices corresponding to each of the topologies TOP = 1, 2, ..., 18 for the momentum permutations $\sigma_{\mathrm{R}} = \sigma_{\mathrm{L}} = 1$. Γ and Γ^{T} are denoted by VR(TOP) and VL(TOP), respectively.

3.12. IPR

Since there are various symmetries of the diagrams, we classify them according to their symmetries. Type 1 are the Abelian diagrams, type 2 the diagrams containing one or two three-gluon vertices, and type 6 the diagrams with a four-gluon vertex.

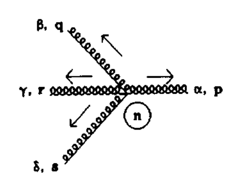

Fig. 5. The four-gluon vertex.

3.13. IM

Given a pair (TOPL, TOPR) of topologies, we need not sum over all contractions σ, but only over a subset depending on the symmetry type of TOPL and TOPR.

For example, consider a product of a type-1 diagram with a type-6 diagram. The Abelian (type-1) diagram has no internal symmetry. Therefore, in principle we should sum over all six contractions. But since the type-6 diagram contains a four-gluon vertex, all these expressions differ only in a permutation of the external momenta. Since this permutation is done in a FORTRAN program, it suffices to calculate the term for the contraction $\sigma = 1$. Therefore, in this special case, IM(1, 6) = 1.

3.14. Formula for the cross section

The contribution to the trace of the hadron tensor is calculated by means of the formula

$$\mathrm{WQ(TOPL,\ TOPR)} = -4 * \mathrm{tr}(\mathrm{VL(TOPL)} * \not{p}_1 * \mathrm{VR(TOPR)} * \not{p}_2) \quad (3.1)$$

as already indicated in eq. (2.8). The factor 4 compensates the factor 1/4 that REDUCE inserts into trace calculations, the minus sign stems from

Table 2
Parameters for the $q\bar{q}3g$-contributions

filename	@TOPLU	@TOPLO	@TOPRU	@TOPRO	@TOPN	
AA0001	1	3	1	3	0	
AA0002	1	2	4	4	0	
AA0003	3	4	4	4	0	
AA0004	1	4	5	5	0	
AA0005	1	4	6	6	0	
AA0006	1	4	7	7	0	
AA0007	1	4	8	8	0	
AA0008	1	4	9	9	0	
AA0009	1	4	10	10	0	
AA0010	1	2	11	11	0	
AA0011	3	4	11	11	0	
AA0012	1	2	12	12	0	
AA0013	3	4	12	12	0	
AA0014	1	4	13	18	0	
AA0015	5	6	5	6	0	
AA0016	5	6	7	8	0	
AA0017	7	8	7	8	0	
AA0018	5	6	9	10	0	
AA0019	7	8	9	10	0	
AA0020	9	10	9	10	0	
AA0021	5	6	11	11	0	
AA0022	7	8	11	11	0	
AA0023	9	10	11	11	0	
AA0024	11	11	11	11	0	
AA0025	5	6	12	12	0	
AA0026	7	8	12	12	0	
AA0027	9	10	12	12	0	
AA0028	11	11	12	12	0	
AA0029	12	12	12	12	0	
AA0030	5	10	13	18	0	
AA0031	11	12	13	18	0	
AA0032	13	18	13	18	0	
DA0002	1	0	1	0	18	denominators

Table 3
Parameters for the $q\bar{q}$ 2 ghost g-contributions

filename	@TOPLU	@TOPLO	@TOPRU	@TOPRO	@TOPN	
AZ0001	1	6	1	12	0	
AZ0002	7	12	1	12	0	
DG0002	1	0	1	0	12	denominators

the $(-g_{\epsilon\epsilon'})$-contractions of the polarization sum for the external gluons.

3.15. Some calculations

In this section the head of the FORTRAN subroutines is printed, the name ROUT001 has to be replaced by an appropriate name, AAxxxx for instance, where xxxx runs from 0001 to 0032 (we had to split up the whole output into 32 FORTRAN subroutines).

There are three nested loops over the different topologies on the right and left side, and over the contractions corresponding to the pair (TOPL, TOPR).

In the next step we perform the permutation of the Lorentz indices to obtain a certain contraction, furthermore, the momenta on the left side are determined from the contraction.

We calculate the traces *without* replacing the sum $Q_{\alpha\beta\ldots\epsilon}$, because then we have less terms and the calculation is therefore faster. Application of QDEF then defines these sums, the symbols $Q_{\alpha\beta\ldots\epsilon}$ are replaced, and the dot products are expressed in terms of the relativistic invariants y_{ij}.

The result is printed, and finally it is assigned to an array AS(TOPL, TOPR, V, JP). JP is a dummy index that is needed to store the results after the momentum permutations later on, JP runs from 1 to 6.

3.16. Calculate the denominators

In this section the denominators are calculated and printed.

3.17. Close output file

The output file is closed, REDUCE is left.

As mentioned above, we need 32 files containing the FORTRAN subroutines. To obtain approximately equal length programs (500–1000 lines), we chose the parameters indicated in table 2.

The denominators are calculated if @TOPLU = @TOPRU = 1, @TOPLO = @TOPRO = 0 and

Table 4
Parameters for the $2q2\bar{q}g$-contributions

filename	@TOPLU	@TOPLO	@TOPRU	@TOPRO	@TOPN	
AN0001	1	4	1	4	0	
AN0002	1	4	5	5	0	
AN0003	1	4	6	6	0	
AN0004	1	4	7	8	0	
AN0005	1	4	9	11	0	
AN0006	1	4	12	12	0	
AN0007	5	6	5	6	0	
AN0008	5	6	7	8	0	
AN0009	5	6	9	10	0	
AN0010	5	6	11	11	0	
AN0011	5	6	12	12	0	
AN0012	7	10	7	10	0	
AN0013	7	10	11	12	0	
AN0014	11	12	11	12	0	
DD0002	1	0	1	0	12	denominators

@TOPN = 18. This subroutine should be called DA0002 (ROUT001 has to be replaced, and the array AS(18, 18, 6, 6) has to be replaced by an array DS(18, 6)).

The program in the $q\bar{q}2$ ghost g-case is similar. Here, there is only one external gluon, and we have to consider only one contraction. The momentum permutation is done numerically in the FORTRAN program that calls the subroutines.

A slight complication arises in the $2q2\bar{q}g$-case. Here, depending on the contraction, we may have to calculate one or two fermion traces. This is accomplished by introducing two new arguments in the functions VR and VL: Vx(TOP, α, L). $\alpha = 1$ denotes the γ-string that is connected to the external photon, $\alpha = 2$ denotes the γ-string that is connected to the ($\alpha = 1$)-string via a gluon. L denotes the name of the corresponding fermion line. The parameters for the ghost-contributions and the four-quark terms are given in tables 3 and 4.

4. Calculation of the colour and flavour factors

Again, we will describe the $q\bar{q}3g$-case in detail. The REDUCE program that we will describe produces a FORTRAN program with a lot of assignments. For each contribution (TOPL, TOPR, σ) there is a corresponding term in the resulting FORTRAN program. The REDUCE program consists of the following 12 parts:

4.1. Open output file

The REDUCE output is written to a file TFILE.

4.2. FORTRAN output

The output should be FORTRAN-like.

4.3. The invariants of SU(N)

NC and CF are the number of colours and the eigenvalue of the Casimir-operator, respectively. We need NC = 3 (corresponding to SU(3)), therefore CF = 4/3. D1, D2, ..., D7 are some constants that appear frequently.

4.4. Colour factors

This is a complete list of the colour factors calculated by hand. We may classify the graphs according to their colour structure. There are 6 different types. Furthermore, the colour factor depends on the contraction σ.

4.5. Transposed contractions

List of the inverse contractions, VT(V) · V = 1.

4.6. The lower triangle

The GP(...) only define the upper triangle of the whole matrix, the lower triangle is obtained by exchanging the arguments for the topologies and using the transposed contractions.

4.7. TI

TI(TOP) is the colour class of the graph topology TOP.

4.8. IPR

Because of the symmetries of the graphs we do not need to calculate all contractions. IPR(TOP) specifies the symmetry of a graph.

4.9. IM

Determines the contractions σ that are needed (see the explanation of this variable in section 3. 13).

4.10. Some calculations

First, the head of the FORTRAN subroutine is written to TFILE. Then there are loops over all topologies and the relevant contractions. The colour factor CFACT is determined and printed.

4.11. Calculation of flavour factors

We define the charges of five quarks, the number of flavours is fixed to 5. S1 is the sum of the squares of the quark charges. The flavour factors for the six contractions are calculated and printed. Because of the normalization with σ_0, in this particular case the flavour factors are all 1.

4.12. Close output file

Finally, the file is closed.

The FORTRAN subroutines for the colour and flavour factors are called FA0002, FD0002 and FG0002 for the final states $q\bar{q}3g$, $2q2\bar{q}g$ and $q\bar{q}2$ ghost g, respectively.

5. FORTRAN subroutines

We describe some FORTRAN subroutines that permute momenta, create arrays containing information concerning the symmetry of the graphs, and subroutines that sum up the contributions including colour and flavour factors.

5.1. The momentum permutations

As described in previous sections, our FORTRAN subroutines only contain the results for the contributions to the trace of the hadron tensor for the momentum permutation $\sigma_R = 1$. The missing contributions are determined by a permutation of the relativistic invariants y_{ij}. For this purpose, there are two subroutines IPERM2 and IPERC2, the former for the $q\bar{q}3g$-case, the latter for the $2q2\bar{q}g$-case. The structure of these subroutines is quite simple:

An integer parameter JP (running from 1–6 or 1–4, respectively) determines the permutation. A common block IPER2 contains the invariants YAIJ of the particular event and the permuted ones, YIJ. One has to specify the permutation JP and to call the subroutine. Then the YIJ will contain the permuted invariants.

5.2. Arrays containing combinatorial information

Because of the various symmetries of the graphs we could omit the calculation of a lot of contributions. We have to pay for this simplification with the definition of arrays that contain the information that has been lost.

The $q\bar{q}3g$-case: IKOMB1

The variables IPR(TOP) and IM(TOP) have already been explained in section 3. The array IGT(τ, σ)is the group multiplication table of S_3, the symmetry group of the six permutations. We have $\tau \cdot \sigma = \text{IGT}(\tau, \sigma)$. This array is needed to calculate the momenta QL_i, if QR_i and the contraction are known.

The arrays IA1, IV1 and IP1 are needed to calculate the arrays IA, IV and IP, respectively. These arrays are handed to the calling program. We will describe how these arrays are used. Suppose that you want to calculate the following contribution: The left and right topologies are JL and JR, the contraction is JV and JP labels the permutation. We associate (JLN, JRN, JVN, JPR, JPL) to (JL, JR, JV, JP) in the following sense:

The contribution to the trace of the hadron tensor of the topologies JL, JR, the contraction JV and the permutation JP is equal to the contribution of the topologies JLN, JRN, the contraction JVN and the momentum permutation JPR, furthermore, the momenta of the external particles in the complex conjugate matrix elements is given by JPL.

The formulae for JLN, JRN, JVN, JPR, JPL in terms of JL, JR, JV, JP are

$$\begin{aligned}
&\text{JLN} = \text{JL},\\
&\text{JRN} = \text{JR} + \text{IA}(\text{JL}, \text{JR}, \text{JV}),\\
&\text{JVN} = \text{IV}(\text{JL}, \text{JR}, \text{JV}),\\
&\text{JPR} = \text{IGT}(\text{JP}, \text{IP}(\text{JL}, \text{JR}, \text{JV})),\\
&\text{JPL} = \text{IGT}(\text{JPR}, \text{JVN}).
\end{aligned} \tag{5.1}$$

These formulae and the tables IA, IV and IP may be obtained by choosing symmetry types TL, TR $\in \{1, 2, \ldots, 5\}$ and comparing the expressions with respect to the symmetries of the vertices. This is a straightforward, but tedious task.

The $2q2\bar{q}g$-case: IKOMC1

Here the graphs contain no useful internal symmetries with respect to the external legs. The only possible simplification is to do the momentum permutation by a FORTRAN program. The array IGT contains the group multiplication table of $S_2 \times S_2$ (exchange of the two quarks and the two antiquarks).

5.3. Subroutines for summing up the contributions

It is assumed that the invariants are defined in the common block INVR.

The $q\bar{q}$ 3g-case: ARUP50

In the beginning, the combinatorial arrays and the colour factors are determined by means of IKOMB1 and FA0002. Then the numerators and denominators (the Lorentz structure) are determined in all possible permutations of the outgoing momenta. The last step is the summation of all contributions. A factor of two has to be included for all off-diagonal terms. The trace of the hadron tensor is returned in double precision format.

If the three gluons are not considered to be distinguishable, then one has to multiply the trace of the hadron tensor by a factor of 1/6.

The $2q2\bar{q}$ g-case ARUD50

Here the combinatorial arrays are filled by means of IKOMC1, the colour factors are determined by FD0002. The contributions to the trace of the hadron tensor are determined for all possible permutations of the outgoing momenta. Finally, the contributions are summed up.

The $q\bar{q}$ 2 ghost g-case: ARUG50

The combinatorial arrays are determined by means of IKOMB1, the colour factors are calculated in FG0002. We have to sum over all permutations of the outgoing momenta.

A generic contribution to the hadron tensor has the form:

$$\begin{aligned}\mathrm{ANS0} = {} & \mathrm{FS(JV)} * \mathrm{GS(JL,JR,JV)} \\ & * \mathrm{AS(JL,JR,JV,JP)} \,/\, \mathrm{DS(JL,JPL)} \\ & /\, \mathrm{DS(JR,JPR)}. \end{aligned} \tag{5.2}$$

FS(...) is the flavour factor, GS(...) is the colour factor. AS(...) denotes the numerator and DS(...) is the denominator (Lorentz structure).

Now assume, for instance, that you want to obtain the trace of the hadron tensor for certain invariants y_{ij} defined in the common block INVR. If you do not want to distinguish identical particles, then you should use the following sequence in your program:

```
CALL ARUD50 ( WQ4Q )
CALL ARUP50 ( WQ3G )
CALL ARUG50 ( WQGH )                (5.3)
WQSUM = WQ4Q/4.0
        + (WQ3G - WQGH)/6.0.
```

The ghost contributions are *subtracted*.

5.4. *Some numerical examples*

Finally, in the program EX0001 we give three sets of invariants originally determined by a Monte Carlo program. The three contributions for each of these sets are given to offer the possibility of a simple check. We have calculated H^{μ}_{μ} without the factor $(4\pi\alpha_s)^2$ in eq. (1.4) and divided by σ_0.

6. Conclusions

We have presented REDUCE programs that are capable to calculate the trace of the hadron tensor for the process $\gamma \to 5$ partons. The CPU time for the calculation of the cross section is approximately 2 hours on an IBM 3084 Q. If the momentum permutations are performed numerically the length of the result can be reduced considerably (by a factor of 4). Finally we described how to use the result in Monte Carlo simulations. The REDUCE program has been written in such a way that momentum permutations are performed numerically in the FORTRAN routines. This has the consequence that the FORTRAN routines are much shorter and the time for their production is smaller approximately by a factor of 4. Explicit calculations for five-jet cross sections on the basis of these programs have been described in ref. [5].

References

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[5] N.K. Falck, D. Graudenz and G. Kramer, DESY report 88-186 and 89-027.

PROGRAM LISTINGS
2q3g

```
%=====================================================================
% 2 QUARK 3 GLUON LORENTZ STRUCTURE
%=====================================================================
COMMENT
5-JETS, FINAL STATE WITH 2 QUARKS, 3 GLUONS
CALCULATION OF THE LORENTZ STRUCTURE
FILE CREATED:      02\02\1989
LAST UPDATE:       06\03\1989
$
%=====================================================================
% ----> PARAMETERS
% --> RIGHT TOPOLOGIES
TOPRU:=@TOPRU$
TOPRO:=@TOPRO$
% --> LEFT TOPOLOGIES
TOPLU:=@TOPLU$
TOPLO:=@TOPLO$
% --> DENOMINATORS
TOPN:=@TOPN$
%=====================================================================
% ----> OPEN OUTPUT FILE
OFF ECHO$
OUT TFILE$
%=====================================================================
% ----> FORTRAN OUTPUT
ON FORT$
OFF PERIOD$
CARDNO!*:=19$
%=====================================================================
% ----> MOMENTUM SUMS
DEFINE QDEF =
BEGIN
LET
QR13=QR1+QR3,
QR15=QR1+QR5,
QR25=QR2+QR5,
QR34=QR3+QR4,
QR134=QR1+QR3+QR4,
QR234=QR2+QR3+QR4,
QR245=QR2+QR4+QR5,
QR345=QR3+QR4+QR5,
QR1345=QR1+QR3+QR4+QR5,
QR2345=QR2+QR3+QR4+QR5,
QL13=QL1+QL3,
QL15=QL1+QL5,
QL25=QL2+QL5,
QL34=QL3+QL4,
QL134=QL1+QL3+QL4,
QL234=QL2+QL3+QL4,
QL245=QL2+QL4+QL5,
QL345=QL3+QL4+QL5,
QL1345=QL1+QL3+QL4+QL5,
QL2345=QL2+QL3+QL4+QL5
$
END$
%
% ----> CLEAR MOMENTUM SUMS
DEFINE QREL =
BEGIN
CLEAR  QR1,QR2,QR3,QR4,QR5,
       QR13,QR15,QR25,QR34,
       QR134,QR234,QR245,QR345,
       QR1345,QR2345,
       QL1,QL2,QL3,QL4,QL5,
       QL13,QL15,QL25,QL34,
       QL134,QL234,QL245,QL345,
       QL1345,QL2345
$
% ----> DEFINE THEM AS VECTORS
$
VECTOR QR1,QR2,QR3,QR4,QR5,
       QR13,QR15,QR25,QR34,
       QR134,QR234,QR245,QR345,
       QR1345,QR2345,
       QL1,QL2,QL3,QL4,QL5,
       QL13,QL15,QL25,QL34,
       QL134,QL234,QL245,QL345,
       QL1345,QL2345
$
% ----> EXTERNAL MOMENTA ARE ON THE MASS SHELL
MASS QR1=0,QR2=0,QR3=0,QR4=0,QR5=0,
     QL1=0,QL2=0,QL3=0,QL4=0,QL5=0$
MSHELL QR1,QR2,QR3,QR4,QR5,
       QL1,QL2,QL3,QL4,QL5$
END$
%=====================================================================
% ----> DECLARE SOME VECTORS
VECDIM(4)$
VECTOR P1,P2,P3,P4,P5$
MASS P1=0,P2=0,P3=0,P4=0,P5=0$
MSHELL P1,P2,P3,P4,P5$
%=====================================================================
% ----> LORENTZ-SCALARS
LET P1.P2=Y12/2,
    P1.P3=Y13/2,
    P1.P4=Y14/2,
    P1.P5=Y15/2,
    P2.P3=Y23/2,
    P2.P4=Y24/2,
    P2.P5=Y25/2,
    P3.P4=Y34/2,
    P3.P5=Y35/2,
    P4.P5=Y45/2
    $
%=====================================================================
% ----> INDICES AND FERMION LINES
INDEX L$                                                % FERMION LINE
INDEX SIR,SIL,RHOR,RHOL,MU,ER3,EL3,ER4,EL4,ER5,EL5$   % INDICES
%=====================================================================
% ----> SQUARES OF THE INNER MOMENTA
OPERATOR SQUA$
FOR ALL P LET SQUA(P)=P.P$
LET SQ13  =SQUA(QL13),
    SQ15  =SQUA(QL15),
    SQ25  =SQUA(QL25),
    SQ34  =SQUA(QL34),
    SQ134 =SQUA(QL134),
    SQ234 =SQUA(QL234),
    SQ245 =SQUA(QL245),
    SQ345 =SQUA(QL345),
    SQ1345=SQUA(QL1345),
    SQ2345=SQUA(QL2345)
$
%=====================================================================
% ----> THE THREE GLUON VERTEX
OPERATOR DGLV$
FOR ALL A,B,C,P,Q,R LET
DGLV(A,B,C,P,Q,R)=-(A.B*(P-Q).C
                   +B.C*(Q-R).A
                   +C.A*(R-P).B)
$
%=====================================================================
% ----> THE FOUR GLUON VERTEX
OPERATOR VGLV$
FOR ALL A,B,C,D LET
VGLV(1,A,B,C,D)=A.B*C.D-A.C*B.D,
VGLV(2,A,B,C,D)=A.C*D.B-A.D*C.B,
VGLV(3,A,B,C.D)=A.D*B.C-A.B*D.C
$
%=====================================================================
% ----> FEYNMAN RULES
OPERATOR VR,VL$
% ----> TOP=1
LET
VR(1)=-G(L,ER3,QR13,ER4,QR134,ER5,QR1345,MU),
VL(1)=-G(L,MU,QL1345,EL5,QL134,EL4,QL13,EL3)
$
% ----> TOP=2
LET
VR(2)=-G(L,ER3,QR13,ER4,QR134,MU,-QR25,ER5),
VL(2)=-G(L,EL5,-QL25,MU,QL134,EL4,QL13,EL3)
$
% ----> TOP=3
LET
VR(3)=-G(L,ER3,QR13,MU,-QR245,ER4,-QR25,ER5),
VL(3)=-G(L,EL5,-QL25,EL4,-QL245,MU,QL13,EL3)
$
% ----> TOP=4
LET
VR(4)=-G(L,MU,-QR2345,ER3,-QR245,ER4,-QR25,ER5),
VL(4)=-G(L,EL5,-QL25,EL4,-QL245,EL3,-QL2345,MU)
$
% ----> TOP=5
LET
VR(5)=-G(L,SIR,QR134,ER5,QR1345,MU)*DGLV(SIR,ER3,ER4,-QR34,QR3,QR4),
VL(5)=-G(L,MU,QL1345,EL5,QL134,SIL)*DGLV(SIL,EL3,EL4,-QL34,QL3,QL4)
$
% ----> TOP=6
LET
VR(6)=-G(L,ER5,QR15,SIR,QR1345,MU)*DGLV(SIR,ER3,ER4,-QR34,QR3,QR4),
VL(6)=-G(L,MU,QL1345,SIL,QL15,EL5)*DGLV(SIL,EL3,EL4,-QL34,QL3,QL4)
$
% ----> TOP=7
```

```
LET
VR(7)=-G(L,SIR,QR134,MU,-QR25,ER5)*DGLV(SIR,ER3,ER4,-QR34,QR3,QR4),
VL(7)=-G(L,EL5,-QL25,MU,QL134,SIL)*DGLV(SIL,EL3,EL4,-QL34,QL3,QL4)
$
% ----> TOP=8
LET
VR(8)=-G(L,ER5,QR15,MU,-QR234,SIR)*DGLV(SIR,ER3,ER4,-QR34,QR3,QR4),
VL(8)=-G(L,SIL,-QL234,MU,QL15,EL5)*DGLV(SIL,EL3,EL4,-QL34,QL3,QL4)
$
% ----> TOP=9
LET
VR(9)=-G(L,MU,-QR2345,SIR,-QR25,ER5)*DGLV(SIR,ER3,ER4,-QR34,QR3,QR4),
VL(9)=-G(L,EL5,-QL25,SIL,-QL2345,MU)*DGLV(SIL,EL3,EL4,-QL34,QL3,QL4)
$
% ----> TOP=10
LET
VR(10)=-G'L,MU,-QR2345,ER5,-QR234,SIR)*DGLV(SIR,ER3,ER4,-QR34,QR3,QR4),
VL(10)=-G(L,SIL,-QL234,EL5,-QL2345,MU)*DGLV(SIL,EL3,EL4,-QL34,QL3,QL4)
$
% ----> TOP=11
LET
VR(11)=-G(L,SIR,QR1345,MU)*DGLV(SIR,RHOR,ER5,-QR345,QR34,QR5)
                          *DGLV(RHOR,ER3,ER4,-QR34,QR3,QR4),
VL(11)=-G(L,MU,QL1345,SIL)*DGLV(SIL,RHOL,EL5,-QL345,QL34,QL5)
                          *DGLV(RHOL,EL3,EL4,-QL34,QL3,QL4)
$
% ----> TOP=12
LET
VR(12)=-G(L,MU,-QR2345,SIR)*DGLV(SIR,RHOR,ER5,-QR345,QR34,QR5)
                           *DGLV(RHOR,ER3,ER4,-QR34,QR3,QR4),
VL(12)=-G(L,SIL,-QL2345,MU)*DGLV(SIL,RHOL,EL5,-QL345,QL34,QL5)
                           *DGLV(RHOL,EL3,EL4,-QL34,QL3,QL4)
$
% ----> TOP=13
LET
VR(13)=G(L,SIR,QR1345,MU)*VGLV(1,SIR,ER3,ER4,ER5),
VL(13)=G(L,MU,QL1345,SIL)*VGLV(1,SIL,EL3,EL4,EL5)
$
% ----> TOP=14
LET
VR(14)=G(L,SIR,QR1345,MU)*VGLV(2,SIR,ER3,ER4,ER5),
VL(14)=G(L,MU,QL1345,SIL)*VGLV(2,SIL,EL3,EL4,EL5)
$
% ----> TOP=15
LET
VR(15)=G(L,SIR,QR1345,MU)*VGLV(3,SIR,ER3,ER4,ER5),
VL(15)=G(L,MU,QL1345,SIL)*VGLV(3,SIL,EL3,EL4,EL5)
$
% ----> TOP=16
LET
VR(16)=G(L,MU,-QR2345,SIR)*VGLV(1,SIR,ER3,ER4,ER5),
VL(16)=G(L,SIL,-QL2345,MU)*VGLV(1,SIL,EL3,EL4,EL5)
$
% ----> TOP=17
LET
VR(17)=G(L,MU,-QR2345,SIR)*VGLV(2,SIR,ER3,ER4,ER5),
VL(17)=G(L,SIL,-QL2345,MU)*VGLV(2,SIL,EL3,EL4,EL5)
$
% ----> TOP=18
LET
VR(18)=G(L,MU,-QR2345,SIR)*VGLV(3,SIR,ER3,ER4,ER5),
VL(18)=G(L,SIL,-QL2345,MU)*VGLV(3,SIL,EL3,EL4,EL5)
$
%=======================================================================
% ----> 'IPR' CONTAINS THE TYPE OF A TOPOLOGY
OPERATOR IPR$
IPR(1) :=1$
IPR(2) :=1$
IPR(3) :=1$
IPR(4) :=1$
IPR(5) :=2$
IPR(6) :=2$
IPR(7) :=2$
IPR(8) :=2$
IPR(9) :=2$
IPR(10):=2$
IPR(11):=2$
IPR(12):=2$
IPR(13):=2$
IPR(13):=6$
IPR(14):=6$
IPR(15):=6$
IPR(16):=6$
IPR(17):=6$
IPR(18):=6$
%=======================================================================
% ----> 'IM' CONTAINS THE NUMBER OF CONTRACTIONS THAT ARE NECESSARY
%       (THE FORMAT IS IM(TYPEL,TYPER))
OPERATOR IM$
IM(1,1):=6$
IM(1,2):=3$
IM(1,6):=1$
IM(2,2):=2$
IM(2,6):=1$
IM(2,6):=1$
IM(6,6):=1$
%=======================================================================
% ----> FORMULA FOR THE X-SECTION
OPERATOR WQ$
%
COMMENT ----- FAKTOR -4: CONTRACTION OF THE GLUONS: (-G MU NU)**3,
                         REDUCE'S TRACE IS ONLY 1/4...
$
FOR ALL TOPL,TOPR LET
WQ(TOPL,TOPR)=-4*VL(TOPL)*G(L,QR1)*VR(TOPR)*G(L,QR2)
$
%=======================================================================
% ----> SOME  C A L C U L A T I O N S
%
OFF PERIOD$
WRITE "C ---"$
WRITE "      SUBROUTINE ROUT001(AS,JP)"$
WRITE "C ---"$
WRITE "      DOUBLE PRECISION"$
WRITE "     &Y12,Y13,Y14,Y15,Y23,Y24,Y25,Y34,Y35,Y45,"$
WRITE "     &AS(18,18,6,6)"$
WRITE "      INTEGER JP"$
WRITE "C ---"$
WRITE "      COMMON /INVR1/"$
WRITE "     &Y12,Y13,Y14,Y15,Y23,Y24,Y25,Y34,Y35,Y45"$
WRITE "C ---"$
WRITE "C     2 QUARK - 3 GLUON X-SECTIONS"$
WRITE "C##"$
WRITE "C --- TOPLU=",TOPLU,", TOPLO=",TOPLO$
WRITE "C --- TOPRU=",TOPRU,", TOPRO=",TOPRO$
WRITE "C --- TOPN =",TOPN$
WRITE "C --- ---------------------------------------------------------"$
%
FOR TOPR:=TOPRU:TOPRO DO BEGIN
%
% ----> ENSURE THAT TOPL<=TOPR:
TOPLMAX:=MIN(TOPLO,TOPR)$
FOR TOPL:=TOPLU:TOPLMAX DO BEGIN
%
VO:=IM(IPR(TOPL),IPR(TOPR))$
FOR V :=1:VO DO BEGIN
%
% ----> CALCULATE THE TRACE USING THE VARIABLES Q... WITHOUT
%       REPLACING THEM BY THE P'S
%
% ----> MOMENTUM PERMUTATION LEFT SIDE
QREL$
IF V=1 THEN LET QL3=QR3,QL4=QR4,QL5=QR5,EL3=ER3,EL4=ER4,EL5=ER5 ELSE
IF V=2 THEN LET QL3=QR4,QL4=QR5,QL5=QR3,EL3=ER4,EL4=ER5,EL5=ER3 ELSE
IF V=3 THEN LET QL3=QR5,QL4=QR3,QL5=QR4,EL3=ER5,EL4=ER3,EL5=ER4 ELSE
IF V=4 THEN LET QL3=QR3,QL4=QR5,QL5=QR4,EL3=ER3,EL4=ER5,EL5=ER4 ELSE
IF V=5 THEN LET QL3=QR4,QL4=QR3,QL5=QR5,EL3=ER4,EL4=ER3,EL5=ER5 ELSE
IF V=6 THEN LET QL3=QR5,QL4=QR4,QL5=QR3,EL3=ER5,EL4=ER4,EL5=ER3
$
QR1:=P1$
QR2:=P2$
QR3:=P3$
QR4:=P4$
QR5:=P5$
% ----
QL1:=P1$
QL2:=P2$
% ---
WQ1:=WQ(TOPL,TOPR)$
QDEF$
OFF PERIOD$
WRITE "C --- TOPL=",TOPL,", TOPR=",TOPR,", V=",V$
%
% ----> PRINT THE X-SECTION
ON PERIOD$
WRITE WQ1$
OFF PERIOD$
WRITE "      AS(",TOPL,",",TOPR,",",V,",JP)=ANS"$
WRITE "C --- ---------------------------------------------------------"$
END$
END$
END$
```

```
%=========================================================================
% ----> CALCULATE THE DENOMINATORS
OPERATOR NE$
LET
NE(1)=SQ13*SQ134*SQ1345,
NE(2)=SQ13*SQ25*SQ134,
NE(3)=SQ13*SQ25*SQ245,
NE(4)=SQ25*SQ245*SQ2345,
NE(5)=SQ34*SQ134*SQ1345,
NE(6)=SQ15*SQ34*SQ1345,
NE(7)=SQ25*SQ34*SQ134,
NE(8)=SQ15*SQ34*SQ234,
NE(9)=SQ25*SQ34*SQ2345,
NE(10)=SQ34*SQ234*SQ2345,
NE(11)=SQ34*SQ345*SQ1345,
NE(12)=SQ34*SQ345*SQ2345,
NE(13)=SQ345*SQ1345,
NE(14)=SQ345*SQ1345,
NE(15)=SQ345*SQ1345,
NE(16)=SQ345*SQ2345,
NE(17)=SQ345*SQ2345,
NE(18)=SQ345*SQ2345
$
QREL$
QDEF$
FOR TOP:=1:TOPN DO BEGIN
OFF PERIOD$
WRITE "C --- TOP=",TOP$
ON PERIOD$
%
QL1:=P1$
QL2:=P2$
QL3:=P3$
QL4:=P4$
QL5:=P5$
WRITE NE(TOP)$
OFF PERIOD$
WRITE "      DS(",TOP,",JP)=ANS"$
WRITE "C --- ---------------------------------------------------------"$
END$
%=========================================================================
WRITE "      RETURN"$
WRITE "      END"$
WRITE "C --- END OF TEXT"$
SHUT TFILE$
BYE$
%=========================================================================
```

ARUP50

```
C   07/11/88 903170901  MEMBER NAME  ARUP50   (*JBT3G)       FORTRAN
C -----------------------------------------------------------------------
C
C
C     ==> ARUP50 <==
C
C --- 2 QUARK 3 GLUON FINAL STATE
C
C --- SUBROUTINE CALLS THE CALCULATION FOR ONE EVENT
C
C     FILE CREATED:     08\02\1989
C     LAST UPDATE:      06\03\1989
C
C -----------------------------------------------------------------------
C
      SUBROUTINE ARUP50(WQUER)
C
      DOUBLE PRECISION
     &WQUER,ANS0,ANS1,
     &YD12,YD13,YD14,YD15,YD23,YD24,YD25,YD34,YD35,YD45,
     &YE12,YE13,YE14,YE15,YE23,YE24,YE25,YE34,YE35,YE45,
     &YF12,YF13,YF14,YF15,YF23,YF24,YF25,YF34,YF35,YF45,
     &AS(18,18,6,6),DS(18,6),FS(6),GS(18,18,6)
C
      COMMON /KOMB1/
     &IM(18),IGT(6,6),
     &IA(18,18,6),IV(18,18,6),IP(18,18,6)
C
      COMMON /IPER2/
     &YD12,YD13,YD14,YD15,YD23,YD24,YD25,YD34,YD35,YD45,
     &YE12,YE13,YE14,YE15,YE23,YE24,YE25,YE34,YE35,YE45
C
      COMMON /INVR1/
     &YF12,YF13,YF14,YF15,YF23,YF24,YF25,YF34,YF35,YF45
C
      COMMON /INVR/
     &GG,B11,B22,B33,B44,B55,B12,B13,B14,B15,B23,B24,B25,B34,B35,B45,
     &YB12,YB13,YB14,YB15,YB23,YB24,YB25,YB34,YB35,YB45
C
C --- COMBINATORIAL ARRAYS
C
      CALL IKOMB1
C
C --- COLOUR AND FLAVOUR FACTORS
C
      CALL FA0002(FS,GS)
C
C --- STORE SINGLE PRECISION INVARIANTS IN DOUBLE PRECISION VARIABLE
C
      YD12=YB12
      YD13=YB13
      YD14=YB14
      YD15=YB15
      YD23=YB23
      YD24=YB24
      YD25=YB25
      YD34=YB34
      YD35=YB35
      YD45=YB45
C
C --- DETERMINE DENOMINATORS AND NUMERATORS IN ALL POSSIBLE
C     DISTINCT PERMUTATIONS OF THE OUTGOING MOMENTA
C
      DO 100 JP=1,6
C
C ---  MOMENTUM PERMUTATION
C
       CALL IPERM2(JP)
C
       YF12=YE12
       YF13=YE13
       YF14=YE14
       YF15=YE15
       YF23=YE23
       YF24=YE24
       YF25=YE25
       YF34=YE34
       YF35=YE35
       YF45=YE45
C
C ---  DENOMINATORS
C
       CALL DA0002(DS,JP)
```

```
C --- NUMERATORS
C
      CALL AA0001(AS,JP)
      CALL AA0002(AS,JP)
      CALL AA0003(AS,JP)
      CALL AA0004(AS,JP)
      CALL AA0005(AS,JP)
      CALL AA0006(AS,JP)
      CALL AA0007(AS,JP)
      CALL AA0008(AS,JP)
      CALL AA0009(AS,JP)
      CALL AA0010(AS,JP)
      CALL AA0011(AS,JP)
      CALL AA0012(AS,JP)
      CALL AA0013(AS,JP)
      CALL AA0014(AS,JP)
      CALL AA0015(AS,JP)
      CALL AA0016(AS,JP)
      CALL AA0017(AS,JP)
      CALL AA0018(AS,JP)
      CALL AA0019(AS,JP)
      CALL AA0020(AS,JP)
      CALL AA0021(AS,JP)
      CALL AA0022(AS,JP)
      CALL AA0023(AS,JP)
      CALL AA0024(AS,JP)
      CALL AA0025(AS,JP)
      CALL AA0026(AS,JP)
      CALL AA0027(AS,JP)
      CALL AA0028(AS,JP)
      CALL AA0029(AS,JP)
      CALL AA0030(AS,JP)
      CALL AA0031(AS,JP)
      CALL AA0032(AS,JP)
C
100   CONTINUE
C
C --- SUM UP ALL CONTRIBUTIONS
C
      ANS1=0.0
      DO 200 JR=1,18
       JMR=IM(JR)
       DO 201 JL=1,JR
        JML=IM(JL)
        DO 202 JP=1,JMR
         DO 203 JV=1,JML
          JLN=JL
          JRN=JR+IA(JL,JR,JV)
          JVN=IV(JL,JR,JV)
          JPR=IGT(JP,IP(JL,JR,JV))
          JPL=IGT(JPR,JVN)
          ANSO=FS(JVN)*GS(JLN,JRN,JVN)
     &         *AS(JLN,JRN,JVN,JPR)/DS(JL,JPL)/DS(JR,JPR)
          IF (JR.NE.JL) THEN
           ANSO=2.0*ANSO
          ENDIF
          ANS1=ANS1+ANSO
203      CONTINUE
202     CONTINUE
201    CONTINUE
200   CONTINUE
C
      WQUER=ANS1
C
      RETURN
      END
```

TEST RUN PROGRAM EX0001

```
C
      DOUBLE PRECISION WQ4Q, WQ3G, WQGH, WQSUM
      COMMON/INVR/GG,B11,B22,B33,B44,B55,B12,B13,B14,B15,B23,B24,B25,
     $B34,B35,B45,Y12,Y13,Y14,Y15,Y23,Y24,Y25,Y34,Y35,Y45
C ---------------------------------------------------------------------
C
C     ==> EX0001 <==
C
C     SOME NUMERICAL EXAMPLES
C
C     SOURCE CREATED:   06\03\1989
C     LAST UPDATE:      06\03\1989
C
C ---------------------------------------------------------------------
C
      WRITE(6,9999)
      Y12 = 0.259435E+03
      Y13 = 0.620182E+02
      Y14 = 0.989886E+02
      Y15 = 0.214198E+03
      Y23 = 0.321739E+02
      Y24 = 0.183717E+03
      Y25 = 0.627708E+02
      Y34 = 0.819113E+02
      Y35 = 0.786124E+02
      Y45 = 0.151171E+03
      SY=Y12+Y13+Y14+Y15+Y23+Y24+Y25+Y34+Y35+Y45
      WRITE(6,100) SY
      CALL ARUD50(WQ4Q)
      CALL ARUP50(WQ3G)
      CALL ARUG50(WQGH)
      WQSUM = WQ4Q/4.0 + (WQ3G - WQGH)/6.0
      WRITE(6,1500) WQ4Q
      WRITE(6,1501) WQ3G
      WRITE(6,1502) WQGH
      WRITE(6,1503) WQSUM
C
C ---------------------------------------------------------------------
C
      WRITE(6,9999)
      Y12 = 0.482830E+02
      Y13 = 0.367569E+02
      Y14 = 0.917490E+02
      Y15 = 0.917818E+02
      Y23 = 0.119737E+03
      Y24 = 0.130760E+03
      Y25 = 0.766626E+02
      Y34 = 0.358296E+03
      Y35 = 0.239548E+03
      Y45 = 0.314220E+02
      SY=Y12+Y13+Y14+Y15+Y23+Y24+Y25+Y34+Y35+Y45
      WRITE(6,100) SY
      CALL ARUD50(WQ4Q)
      CALL ARUP50(WQ3G)
      CALL ARUG50(WQGH)
      WQSUM = WQ4Q/4.0 + (WQ3G - WQGH)/6.0
      WRITE(6,1500) WQ4Q
      WRITE(6,1501) WQ3G
      WRITE(6,1502) WQGH
      WRITE(6,1503) WQSUM
C
C ---------------------------------------------------------------------
C
      WRITE(6,9999)
      Y12 = 0.161919E+03
```

```
      Y13 = 0.460678E+02
      Y14 = 0.208431E+03
      Y15 = 0.106025E+03
      Y23 = 0.169903E+03
      Y24 = 0.105832E+03
      Y25 = 0.407410E+02
      Y34 = 0.187181E+03
      Y35 = 0.607370E+02
      Y45 = 0.138161E+03
      SY=Y12+Y13+Y14+Y15+Y23+Y24+Y25+Y34+Y35+Y45
      WRITE(6,100) SY
      CALL ARUD50(WQ4Q)
      CALL ARUP50(WQ3G)
      CALL ARUG50(WQGH)
      WQSUM = WQ4Q/4.0 + (WQ3G - WQGH)/6.0
      WRITE(6,1500) WQ4Q
      WRITE(6,1501) WQ3G
      WRITE(6,1502) WQGH
      WRITE(6,1503) WQSUM
      WRITE(6,9999)
C
C ---------------------------------------------------------------------
C
 100  FORMAT(2X,'SUM OF INVARIANTS:             ',E16.8)
9999  FORMAT(2X,'---------------------------------------------')
1500  FORMAT(2X,'4Q1G CONTRIBUTION TO H_MU_NU: ',D16.8)
1501  FORMAT(2X,'2Q3G CONTRIBUTION TO H_MU_NU: ',D16.8)
1502  FORMAT(2X,'  GH CONTRIBUTION TO H_MU_NU: ',D16.8)
1503  FORMAT(2X,' SUM INCL. COMB. FACTORS:     ',D16.8)
C
C ---------------------------------------------------------------------
C
      STOP
      END
```

TEST RUN OUTPUT

```
---------------------------------------------
SUM OF INVARIANTS:               0.12249954E+04
4Q1G CONTRIBUTION TO H_MU_NU:   -0.38740376D+01
2Q3G CONTRIBUTION TO H_MU_NU:   -0.44361243D+02
  GH CONTRIBUTION TO H_MU_NU:   -0.24467497D+00
 SUM INCL. COMB. FACTORS:       -0.83212707D+01
---------------------------------------------
SUM OF INVARIANTS:               0.12249954E+04
4Q1G CONTRIBUTION TO H_MU_NU:   -0.14602477D+02
2Q3G CONTRIBUTION TO H_MU_NU:   -0.26149353D+02
  GH CONTRIBUTION TO H_MU_NU:   -0.38528598D+00
 SUM INCL. COMB. FACTORS:       -0.79446304D+01
---------------------------------------------
SUM OF INVARIANTS:               0.12249968E+04
4Q1G CONTRIBUTION TO H_MU_NU:   -0.20096614D+01
2Q3G CONTRIBUTION TO H_MU_NU:   -0.28258509D+02
  GH CONTRIBUTION TO H_MU_NU:   -0.26731649D+00
 SUM INCL. COMB. FACTORS:       -0.51676141D+01
---------------------------------------------
```