DETERMINATION OF α_s AND THE Z^0 MASS FROM MEASUREMENTS OF THE TOTAL HADRONIC CROSS SECTION IN e^+e^- ANNIHILATION

G. D'AGOSTINI [a], W. DE BOER [b] and G. GRINDHAMMER [b,c]

[a] *Dipartimento di Fisica, Università di Roma, "La Sapienza" and INFN, I-00185 Rome, Italy*
[b] *Max-Planck-Institut für Physik und Astrophysik, D-8000 Munich 40, FRG*
[c] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*

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We have made a fit to the measurements of the normalized cross section R for the process $e^+e^- \to$ hadrons at centre of mass energies between 7.0 and 57.0 GeV with α_s and M_Z as free parameters. At the highest TRISTAN energies, the increase in R from the tail of the Z^0 resonance allows a precise measurement of the Z^0 mass, while the lower energy data determine the value of the strong coupling constant. The result is α_s $(34^2\ \mathrm{GeV}^2) = 0.143 \pm 0.015$, if $O(\alpha_s^3)$ QCD corrections are taken into account, and $M_Z = 89.3 \pm 1.5$ for as top mass of 60 GeV and fixed $\sin^2\theta_w$. The off shell determination of the Z^0 mass depends almost linearly on the top mass through the electroweak radiative corrections; it is lowered by 1.0 GeV for $M_t = 180$ GeV.

1. Introduction

The total hadronic cross section in e^+e^- annihilation is determined by electroweak interactions and strong interactions, which can be calculated in the standard $SU(3)_C \otimes SU(2)_L \otimes U(1)$ model. Recently several efforts have been made to determine α_s from R, the total hadronic cross section normalized to the pointlike μ-pair cross section [1–6]. Such a determination from an inclusive quantity has the advantage that it is insensitive to fragmentation effects. The disadvantage is that the contribution from gluon radiation is rather small, so one has to combine the results from various experiments in order to obtain relative uncertainties in α_s below 15%. However, this requires a careful study of the systematic errors and their correlations. The first thorough study using a complete error correlation matrix has been made by the CELLO Collaboration [1]. We want to repeat this analysis for the following reasons:

– New experimental data has become available from the experiments at TRISTAN up to center of mass energies of 57.0 GeV. At these energies the tail of the Z^0 resonance is increasing R already by 30%, thus allowing for a direct measurement of the Z^0 mass. Such a mass determination has been published by the AMY Collaboration [3] and presented at various conferences [4–6]. Quoted Z^0 masses range around 89 GeV (error ± 1.3 GeV), which is slightly lower than the results from $p\bar{p}$ experiments ($M_Z = 93.1 \pm 1.0 \pm 3.1$ from UA1 and $M_Z = 91.5 \pm 1.2 \pm 1.7$ from UA2 [7]). We have repeated the fit to R after applying consistently the radiative corrections to all data.

– The Crystal Ball experiment [8] at DORIS has published a precise measurement of the continuum R at the energy of the $\Upsilon(1S)$ state. These data together with all other data in the same energy range [9] have been used in this analysis. We have not included older data between center of mass energies of 2 and 7 GeV, since at these energies the spread in the data is about one unit in R. Furthermore, such old experiments did not have access to refined Monte Carlos available nowadays and at energies below 7 GeV resonances occur, which make the uncertainties from non-perturbative QCD effects and non-trivial radiative corrections important.

– The first calculation of the $O(\alpha_s^3)$ contribution to R has been made by Gorishny et al. [10]. This contribution turns out to be larger than the second order contribution in the commonly used $\overline{\mathrm{MS}}$ scheme.

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2. Standard model formulae

Here we summarize the formulae used in fitting the hadronic cross section. The normalized cross section R is defined as the ratio

$$R \equiv \frac{\sigma[e^+e^- \to \gamma, Z^0 \to \text{hadrons}]}{\sigma[e^+e^- \to \gamma \to \mu^+\mu^-]} .$$

The $\mu^+\mu^-$ cross section is the lowest order pointlike QED cross section of massless spin $\frac{1}{2}$ particles, and is equal to $4\pi\alpha^2/3s$, where s is the square of the centre of mass energy.

The total hadronic cross section excluding QED radiative corrections is given by the sum of the following contributions:

$$\sigma^{\gamma}_{\text{had}} = \frac{4\pi\alpha^2}{3s} r_{\text{QCD}} e_e^2 \sum_{q=1}^{5} e_q^2 , \quad (1)$$

$$\sigma^{\gamma Z}_{\text{had}} = 8\pi\alpha r_{\text{QCD}} \frac{K(s-M_Z^2)}{(s-M_Z^2)^2 + s^2\Gamma_{\text{tot}}^2/M_Z^2} \times e_e v_e \sum_{q=1}^{5} e_q v_q , \quad (2)$$

$$\sigma^{Z}_{\text{had}} = 12\pi r_{\text{QCD}} \frac{K^2 s}{(s-M_Z^2)^2 + s^2\Gamma_{\text{tot}}^2/M_Z^2} \times (v_e^2 + a_e^2) \sum_{q=1}^{5} (v_q^2 + a_q^2) . \quad (3)$$

The superscripts indicate the contribution from photon exchange, Z^0 exchange and their interference and the sum is taken over five quark flavours, thus assuming the top quark is too heavy; e, v and a represent the electric charge and vector and axial vector couplings of the quarks (subscript q) and electrons (subscript e) and Γ_{tot} is the total width of the Z^0. For simplicity we have neglected small mass effects in the formulae above, but they have been taken into account in the analysis, using the formulae in ref. [1]. The factor r_{QCD} represents the effect from gluon radiation and is given in the $\overline{\text{MS}}$ scheme by [10,11]

$$r_{\text{QCD}} = 3\left[1 + \frac{\alpha_s}{\pi} + (1.986 - 0.115 n_f)\left(\frac{\alpha_s}{\pi}\right)^2 + 70.985 - 1.2 n_f - 0.005 n_f^2 - 1.679 \frac{(\sum e_q)^2}{3\sum e_q^2}\left(\frac{\alpha_s}{\pi}\right)^3\right] . \quad (4)$$

The factor 3 on the right-hand side accounts for the color of the quarks.

The energy dependence (running) of α_s is given by the third order formula [12]

$$\alpha_s(s) = \frac{4\pi}{\beta_0 \log(s/\Lambda^2)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\log[\log(s/\Lambda^2)]}{\log(s/\Lambda^2)} + \left(\frac{\beta_1}{\beta_0^2}\right)^2 \frac{1}{\log^2(s/\Lambda^2)}\left(\{\log[\log(s/\Lambda^2)] - \tfrac{1}{2}\}^2 + \frac{\beta_2\beta_0}{\beta_1^2} - \frac{5}{4}\right)\right] \quad (5)$$

with

$$\beta_0 = 11 - \tfrac{2}{3} n_f ,$$

$$\beta_1 = 2(51 - \tfrac{19}{3} n_f) ,$$

$$\beta_2 = \tfrac{2857}{2} - \tfrac{5033}{18} n_f + \tfrac{325}{54} n_f^2 .$$

The constant K can be either defined as

$$K_1 = \frac{\sqrt{2}\, G_F M_Z^2 \kappa_1}{48\pi} \quad (6)$$

$$K_2 = \frac{\alpha\kappa_2}{48\sin^2\theta_w \cos^2\theta_w} . \quad (7)$$

Here G_F is the Fermi constant, which is well known from muon decay, and $\sin^2\theta_w$ determines the electroweak mixing angle, which can be used to define the coupling constants between a pair of fermions and the Z^0 gauge boson:

$$v_f = 2I_3 - 4e_f \sin^2\theta_w , \quad (8)$$

$$a_f = 2I_3 . \quad (9)$$

Here I_3 is the third component of the weak isospin. In the definitions of K, we have explicitly included the factor κ which represents the loop corrections to the Z^0 propagator. For example, practically all data from the PEP and PETRA experiments have been corrected with the LUND Monte Carlo program [13], which uses the radiative corrections from Berends et al. [14], thus including the loop corrections for the photon propagator, but not the loop corrections for the Z^0 propagator. In this case the formulae to be fitted to the data should include this κ-factor, which can be written as follows [15,16]:

$$\kappa_1 = \frac{1 - \Delta r}{1 + \Pi_Z(s)} \quad (10)$$

or

$$\kappa_2 = \frac{1}{1+\Pi_Z(s)}, \tag{11}$$

where

$$1-\Delta r = \frac{\alpha(0)}{\alpha(M_W)} + \delta_r(M_t, M_H) \tag{12}$$

and

$$1+\Pi_Z(s) = \frac{\alpha(0)}{\alpha(M_Z)} + \delta_\Pi(M_t, M_H, s)\ . \tag{13}$$

Here Δr represents the electroweak corrections to the charged gauge boson exchange in muon decay and $\Pi_Z(s)$ represents the electroweak loop corrections to the neutral gauge boson exchange. One sees that the first term in both cases is given by the running of the QED coupling constant as a function of energy coming from the light fermion loops in the photon propagator (hence the indication of the scale in α). The remaining contributions come from box and vertex diagrams, and diagrams with Higgses and heavy quark contributions. For top quark masses below the gauge boson masses the first term is dominant in both expressions. E.g. for a top mass of 70 GeV Δr is about 7% and 6% is coming from the first term alone. However, for a top mass of 230 GeV the latter term is as large as the first term, but of opposite sign, so the total correction Δr is about zero. $\Pi_Z(s)$ shows a similar behaviour, so that the ratio in κ_1 is much less dependent on the top mass and furthermore close to 1 (see fig. 1). This is the advantage of the parametrization with K_1: one can neglect the electroweak corrections to a large extent and the results are insensitive to the unknown top mass. This was the reason why in previous fits to data on R this parametrization has been used, e.g. to determine the strong coupling constant [1]. What was considered further as an advantage compared to the K_2 parametrization was the insensitivity to the Z^0 mass at PETRA energies: the dominant term in both the numerator and denominator in eq. (3) is proportional to M_Z^6, thus largely cancelling the uncertainty in M_Z. However, at TRISTAN energies one observes the tail of the Z^0 resonance and it becomes possible to make a direct measurement of the Z^0 mass. In this case one obtains much more sensitivity with K_2, since one can measure the pure propagator effect without the compensation from the M_Z^6 factor in the numerator. However, with the K_2 parametrization the electroweak corrections (κ_2 in this case) cannot be neglected anymore, since the correction to the total hadronic cross section is of order 3% at 60 GeV, as will be discussed below. From the definitions of K_1 and K_2 one can deduce the following well known relation between $\sin^2\theta_w$ and M_Z (which allows to define the couplings through the mass, thus having only M_Z as free parameter in the electroweak sector):

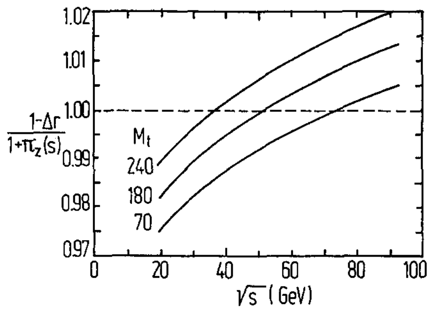

Fig. 1. Electroweak corrections in case the parametrization with the Fermi constant is used (see text).

$$\sin^2\theta_w = \frac{1}{2}\left(1-\sqrt{1-\frac{4\pi\alpha}{\sqrt{2}\,G_F M_Z^2(1-\Delta r)}}\right). \tag{14}$$

3. Analysis method

Fitting the data from different experiments is always a delicate procedure. It requires that
- all data points have been corrected to the same level and that their errors have a similar meaning;
- the correlations between the data points within the same experiment and, eventually, between different experiments must be considered.

Unfortunately, the radiative corrections have not always been applied in a consistent way. These corrections can be divided in three classes:
- Initial state radiative corrections. These corrections depend on M_Z (via the shape dependence of the propagator), as can be seen from a comparison of the two lowest curves in fig. 2, which give the correction factors to the Born cross section [defined by eqs. (1)–(3) with K_1 and $\kappa_1 = 1$] using the radiative corrections as implemented in the Lund program [13].

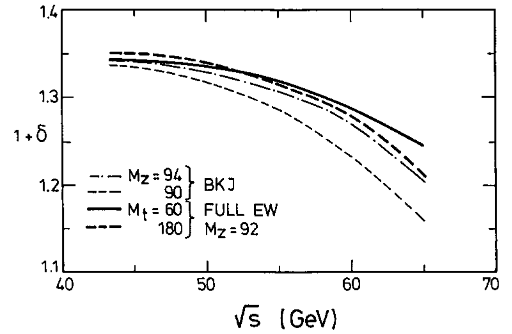

Fig. 2. Radiative correction factors including photonic corrections according to the formulae from ref. [14]. The two upper curves include in addition the electroweak corrections from ref. [16] for $M_Z=92$ GeV, a Higgs mass of 100 GeV and a top mass of 60 and 180 GeV, respectively. The two lower curves show the dependence of the radiative corrections on the M_Z mass using formulae from Berends et al. (BKJ) [14].

Some experiments have used for M_Z the default value of 94 GeV in this program, so their data had to be corrected to the fitted value of M_Z, which gives typically a 3% correction at 60 GeV. Others have used a value close to 92 GeV, which differs, however, from the best fitted value around 90 GeV (see below). So we have consistently corrected all data using the best fitted Z^0 mass. Not making these corrections would have increased the fitted mass by 0.6 GeV. We have also taken into account the dependence of the efficiency on the fitted Z^0 mass, which increases slightly for lower Z^0 masses (about 0.1% per GeV in M_Z at $\sqrt{s}=60$ GeV). We have checked that this correction is not strongly dependent on the difference in acceptance cuts between the various experiments.

Note that the radiative corrections include first order contributions only, i.e. only one photon can be radiated from the initial state. Furthermore, the maximum energy for the photon was limited to 99% of the beam energy, corresponding to a minimum center of mass of $0.1\sqrt{s}$ for light quarks, while for b-quarks the minimum center of mass energy was taken to be $2m_q+1=11$ GeV (default in Lund program). Final state radiative corrections have been neglected, since these are small, if one sums over all possible final states [17].

– Loop corrections to the photon and Z^0 propagator. These corrections are independent of the final state, but for the Z^0 propagator they depend on the unknown top and Higgs mass. Therefore we have followed the strategy to correct the data for the loop corrections to the photon propagator, but exclude the loop corrections to the Z^0 propagator. These contributions can be included in the function to be compared with the data. In this way we can easily compare the data with the standard model predictions for different top masses. It has furthermore the advantage that all of the PEP and PETRA data have been corrected to this level. So, if we exclude for TRISTAN data the loop corrections to the Z^0 propagator too, both data sets can be treated on an equal footing in the fit procedure. In fig. 2 we have also shown the radiative corrections including the loop corrections to the Z^0 propagator (upper two curves for two different top masses and $M_Z=92$ GeV). For light top masses the contribution from these loop corrections is about 3% at $\sqrt{s}=60$ GeV. Such large corrections have to be dealt with correctly. For example, ignoring them completely in the fit reduces the fitted Z^0 mass typically by more than one GeV.

– Vertex and box diagram corrections. These corrections have been taken into account for photon exchange, while for Z^0 exchange they have been found to be negligible for the total cross sections, at least in the on shell renormalization scheme [18]. This is not true for the renormalization scheme used in the program by Fujimoto et al. [19]. There vertex corrections and loop corrections are of similar size [20]. Also box diagrams can be neglected, so in the fits described hereafter we have included only the loop corrections for Z^0 exchange using the formulae from ref. [16] and after correcting all data consistently for initial state radiation (which depends on M_Z as discussed above).

It should be noted that the radiative corrections including the complete loop corrections agree well with the corrections from the program by Fujimoto et al. [19], which have been used for part of the data from TRISTAN [3,6]. We have undone these electroweak corrections and furthermore applied radiative corrections to all data using iteratively the Z^0 mass from the best fit. The original and the corrected data points [6,21] (for $M_Z=190$ GeV) have been summarized in table 1, as well as all the low energy data points [8,9] which have not been listed in ref. [1].

Correlated errors between measurements can be taken into account by defining the χ^2 via an error correlation matrix [1]:

Table 1
Experimental data R used in addition to data given in ref. [1]. The point to point systematic error (σ_{ptp}) includes also the statistical one. The second normalization error σ_{norm2} has been included, if the correlation between the data points varies as function of energy. The data R_{corr} have been corrected for initial state radiation and photonic vertex corrections using $Z^0=90$ GeV. Electroweak vertex and loop corrections have not been applied.

Experiment	$\sqrt{s}$	R	R_{corr}	σ_{ptp} (%)	σ_{norm1} (%)	σ_{norm2} (%)
CBAL	9.39	3.48		1.1	4.6	–
CLEO	10.49	3.77		1.6	6.4	–
CUSB	10.40	3.54		1.4	11.3	–
DASP2	9.50	3.73		4.3	7.5	–
DHHM	9.40	3.80		7.0	11.0	–
LENA	7.44	3.37		3.9	6.7	–
	8.91	3.42		2.9	6.7	–
	9.28	3.31		2.7	6.7	–
	9.42	3.57		7.6	6.7	–
AMY	50.00	4.50	4.55	10.7	4.1	–
	52.00	4.29	4.35	4.7	4.1	–
	55.00	4.62	4.72	5.2	4.1	–
	56.00	5.19	5.31	3.7	4.1	–
	56.50	5.32	5.45	9.0	4.1	–
	57.00	4.90	5.03	3.3	4.1	–
TOPAZ	50.00	4.08	4.11	13.0	5.0	–
	52.00	4.40	4.44	4.5	5.0	–
	55.00	4.64	4.70	5.2	5.0	–
	56.00	4.99	5.06	4.4	5.0	–
	56.50	4.97	5.05	9.1	5.0	–
	57.00	5.19	5.27	4.6	5.0	–
VENUS	50.00	4.40	4.43	11.4	2.0	8.3
	52.00	4.70	4.74	6.4	2.0	8.3
	55.00	4.24	4.30	7.1	2.0	2.2
	56.00	4.92	4.99	4.5	2.0	2.2
	56.50	4.14	4.20	11.8	2.0	2.2
	57.00	5.47	5.56	5.3	2.0	2.2

$$\chi^2 = \Delta^T V^{-1} \Delta . \qquad (15)$$

Here Δ is a column vector containing the residuals between R_i and R_{fit} and V is the $N \times N$ error correlation matrix between N measurements. The elements of V can be estimated as follows:
– the diagonal elements are given by the total variance, i.e. the quadratic sum of statistical, point-to-point and overall normalization error;
– the off diagonal elements of correlated points are given by the sum of the square of the errors common to them.

This can be easily demonstrated by studying the properties of $V_{ij} = \langle (R_i - \langle R_i \rangle)(R_j - \langle R_j \rangle) \rangle$ in presence of a covariant term. Since in practice the common uncertainty is usually given as a percent error (p) to the data points, it can been shown that the previous formula leads to off diagonal terms of the size $V_{ij,i\neq j} = (1/100^2) p^2 R_i R_j$.

In the error matrix of the R measurements, the elements connecting different experiments have been set to zero as a first approximation. A check has been performed, which shows that there is only little change in the result if one includes an overall correlation at the percent level. As in ref. [1] we have not included the uncertainty from higher order QED radiative corrections in the covariance matrix, since we believe that treating it in a probabilistic way is uncorrect. As long as the higher order radiative corrections are being applied consistently to both the hadronic cross section and the luminosity measurement, the total effect on their ratio, i.e. on R, partially cancels and the residual effect is estimated to influence the R values between 0.0 and -1.0% [22]. We will quote below the variation of the final results for such a correction.

For some experiments the separation into point-to-point and common error was not explicitly given. In these cases it was checked that the numerical values of the fitted parameters were very stable against even large variations of their splittings.

4. Results

The three parameters in the total hadronic cross section that we would like to determine are α_s, M_Z, and $\sin^2\theta_w$. At the energies considered here the Z^0 width does not play a role, and we fixed its value at 2.5 GeV. We have tried several strategies to determine the other parameters:
- The most trivial way is a three parameter fit assuming no connection between the couplings and the mass.
- Make a two parameter fit of M_Z and α_s assuming relation 14 to hold or taking for $\sin^2\theta_w$ the value obtained in neutrino–quark scattering, while taking into account the dependence on the unknown top mass [23].
- Make a one parameter fit for α_s with the G_F parametrization; in this case one is insensitive to the top mass owing to the compensation between Δr and Π_Z; the sensitivity to the Z_0 mass is weak too in this parametrization, as said above.

In table 2 we give the results of these last fits with the G_F parametrization. The values of α_s obtained from the different energy regimes are consistent and the results from PEP and PETRA data are unchanged with respect to the results from ref. [1]. Comparing the $O(\alpha_s^2)$ and $O(\alpha_s^3)$ fits in table 2, one observes a systematic reduction of the α_s values by 11–12% for the latter for all energy regimes, as expected, in contrast to variations between 6.6% and 16.5% in ref. [2].

The fit results with M_Z as an additional free parameter are presented in table 3 using data between a center of mass energy of 7 and 57 GeV. It should be noted that the main sensitivity to M_Z comes from the propagator effect in the TRISTAN energy range, while at PEP/PETRA energies the sensitivity came only through the couplings and the use of relation (14). This propagator effect, implying a direct measurement of the mass, is demonstrated in fig. 3, where we plotted the fitted M_Z mass with the couplings fixed ($\sin^2\theta_w=0.231$) as a function of the maximum center of mass energy used in the fit. Fig. 3 shows that the error rapidly decreases, if higher energies are included. If we do the opposite, i.e. keep the mass of the Z^0 in the propagator fixed and determine M_Z from the couplings only [via eq. (14)], we find that the error on M_Z is enlarged by a factor 3.1 and the energy dependence is strongly reduced. In fact the error does not change from 47 to 60.8 GeV. At 36 GeV instead, the M_Z determination from the couplings is 2.5 times better than the determination from the propagator. It must be noted that for both fits the value and error

Table 2
α_s and $\Lambda_{\overline{MS}}^{(5)}$ fitted with the G_F parametrization for $\sin^2\theta_w=0.231$ ($M_t=60$ GeV) and $M_Z=89.3$ GeV.

Data	Energy range	$O(\alpha_s^2)$	$O(\alpha_s^2)$
PEP, PETRA	14–47 GeV	$\alpha_s=0.168\pm0.025$ $\Lambda_{\overline{MS}}^{(5)}=590^{+470}_{-340}$ MeV	$\alpha_s=0.151\pm0.020$ $\Lambda_{\overline{MS}}^{(5)}=320^{+240}_{-180}$ MeV
PEP, PETRA, TRISTAN	14–57 GeV	$\alpha_s=0.170\pm0.025$ $\Lambda_{\overline{MS}}^{(5)}=620^{+460}_{-340}$ MeV	$\alpha_s=0.152\pm0.019$ $\Lambda_{\overline{MS}}^{(5)}=330^{+230}_{-180}$ MeV
CESR, DORIS, PEP, PETRA, TRISTAN	7–57 GeV	$\alpha_s=0.158\pm0.020$ $\Lambda_{\overline{MS}}^{(5)}=440^{+300}_{-230}$ MeV	$\alpha_s=0.143\pm0.015$ $\Lambda_{\overline{MS}}^{(5)}=240^{+150}_{-120}$ MeV

Table 3
M_Z and α_s values from various fits for center of mass energies between 7 and 57 GeV.

M_t (GeV)	$\sin^2\theta_w$ fixed	$\sin^2\theta_w = f(M_Z)$	$\sin^2\theta_w$ free
60	$M_Z = 89.3 \pm 1.5$ GeV $\alpha_s = 0.139 \pm 0.017$ $\sin^2\theta_w = 0.231$	$M_Z = 90.0 \pm 1.6$ GeV $\alpha_s = 0.137 \pm 0.018$ $\sin^2\theta_w = 0.244 \pm 0.014$	$M_Z = 89.4 \pm 1.9$ GeV $\alpha_s = 0.141 \pm 0.019$ $\sin^2\theta_w = 0.221^{+0.029}_{-0.022}$
120	$M_Z = 88.8 \pm 1.4$ GeV $\alpha_s = 0.139 \pm 0.017$ $\sin^2\theta_w = 0.230$	$M_Z = 89.4 \pm 1.5$ GeV $\alpha_s = 0.137 \pm 0.018$ $\sin^2\theta_w = 0.243 \pm 0.014$	$M_Z = 88.8 \pm 1.5$ GeV $\alpha_s = 0.142 \pm 0.018$ $\sin^2\theta_w = 0.221^{+0.029}_{-0.021}$
180	$M_Z = 88.3 \pm 1.4$ GeV $\alpha_s = 0.139 \pm 0.017$ $\sin^2\theta_w = 0.230$	$M_Z = 88.8 \pm 1.5$ GeV $\alpha_s = 0.137 \pm 0.018$ $\sin^2\theta_w = 0.241 \pm 0.014$	$M_Z = 88.3 \pm 1.4$ GeV $\alpha_s = 0.142 \pm 0.018$ $\sin^2\theta_w = 0.221^{+0.028}_{-0.021}$

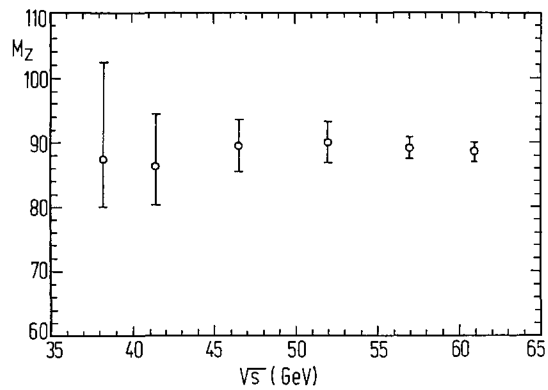

Fig. 3. The fitted M_Z mass as function of the maximum center of mass energy used in the fit.

of α_s remain absolutely stable from 36 GeV on. This is a clear proof that the QCD coupling constant is mainly determined by the data up to the PETRA energy, while the Z^0 mass can be determined in a direct way from the TRISTAN data through the propagator and not from the couplings.

The top mass dependence found for M_Z comes from the radiative corrections to the Z^0 exchange present in the κ_2 parametrization. This cannot be avoided by the use of the κ_1 parametrization, since in this case the sensitivity to the Z^0 mass is reduced (the fit would give an error on M_Z of about 4 GeV). For neutrino scattering the value of $\sin^2\theta_w$ is almost independent of M_t, which is not true for all processes [24]. It is interesting to notice that in case one could use precise $\sin^2\theta_w$ values from these other processes (they are not precise unfortunately), the M_t dependence on $\sin^2\theta_w$ would almost cancel the M_t dependence of κ_2, so M_Z would not depend on M_t anymore. Choosing for the top mass the lower edge of the presently most probable region of 60–180 GeV [25], the best estimate for the Z^0 mass is

$$M_Z = 89.3 \pm 1.5 .$$

This value decreases almost linearly to 88.3 GeV for a top mass of 180 GeV. The uncertainty on the Z^0 mass from the experimental error in $\sin^2\theta_w$ is smaller than ± 0.2 GeV. The dependence on the unknown Higgs mass is small too: changing its value from 100 to 1000 GeV increases the Z^0 mass by 0.2 GeV.

The importance of electroweak radiative corrections can be shown by making a fit where these corrections are excluded. If we fix $\sin^2\theta_w$ at $0.241 \pm 0.0030 \pm 0.0027$ (a value from deep inelastic neutrino scattering excluding electroweak corrections [26]), we obtain $M_Z = 88.1 \pm 1.2$.

Table 3 shows the result from the three parameter fit too. As can be seen the parameter most sensitive to the top mass is M_Z. The correlation coefficients between the three parameters are rather small: $\rho(\sin^2\theta_w, M_Z) = -0.29$, $\rho(\Lambda^{(5)}_{\overline{MS}}, \sin^2\theta_w) = -0.42$ and $\rho(\Lambda^{(5)}_{\overline{MS}}, M_Z) = 0.33$ independent of the top mass, and the χ^2/DF is 68/189. This excellent χ^2 may be due to an overestimate of the common normalization errors: if we calculate the χ^2 only from the diagonal elements of the matrix V^{-1}, thus ignoring the correlations, but including the complete errors, we find χ^2_D/DF is 91/89. We have also repeated the fit including the preliminary results above 57 GeV recently presented by the TRISTAN Collaborations at

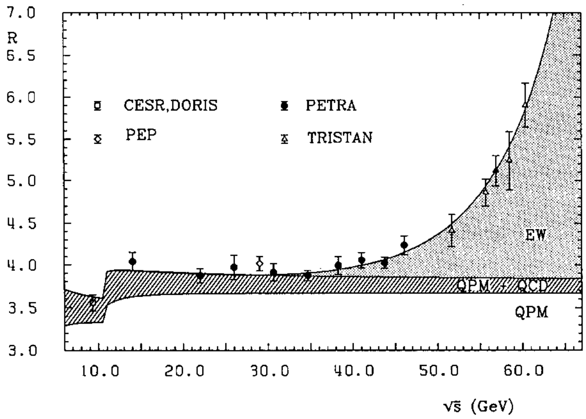

Fig. 4. Averaged experimental data on R as function of the center of mass energy, and best fit (solid line) yielding $M_Z = 89.4 \pm 1.3$, $\alpha_s = 0.142 \pm 0.018$, and $\sin^2\theta_w = 0.220^{+0.025}_{-0.020}$. The error bars include the statistical and normalization errors and their correlations.

the KEK Topical Workshop [27]. The result is in very good agreement with the data up to 57 GeV, as can be seen from fig. 4. Including this new data yields a Z^0 mass of 89.0 ± 1.0 GeV for a top mass of 60 GeV. For clarity we have averaged the data points within certain energy bins in the following way: we have fitted a constant value to the data points within a certain energy bin using the complete error correlation matrix (we have checked that this procedure exactly corresponds to a weighted average, taking correctly into account independent and correlated errors). So the error bars represent the total errors including the correlation and the data have not been renormalized.

It is interesting to express the QCD contribution (fitted with the G_F parametrization, thus being less dependent on M_Z) as f_{QCD} ($\sqrt{s} = 34$ GeV) $= R/R_{EW} = 1.056 \pm 0.008$, for its unambiguous meaning not related to a specific expansion order in α_s or renormalization scheme. An extrapolation of this value at the LEP/SLC energy yields $f_{QCD}(\sqrt{s} = 90$ GeV) $= 1.046 \pm 0.006$. A 1% reduction of all R values, which is the maximum effect expected from $O(\alpha^3)$ radiative corrections [22], would reduce α_s(34 GeV) by 11%. This reduction is below the naive expectation of about 18% due to the fact that we fit over a large energy range.

A common uncertainty of 1% between all experiments as a possible common bias to the data would increase the error on α_s without affecting its central value. However, it would increase M_Z by 0.7 GeV. A similar result is obtained if one assumes a common 1% error for the PEP/PETRA measurements and separately a common 1% error for the TRISTAN ones.

5. Conclusions

The tail of the Z^0 resonance gives a 50% increase of the multihadron cross section at $\sqrt{s} = 60$ GeV, which allows a direct measurement of M_Z. Note that this increase is due to pure Z^0 exchange, since the interference term γ/Z^0 is suppressed by the small value of the electron vector coupling. We find, for a top mass of 60 GeV,

$$M_Z = 89.3 \pm 1.5 \text{ GeV},$$

which is somewhat below the results from the on mass shell measurements at the CERN collider [7]. A top mass of 120 and 180 GeV would lower this value to 88.8 and 88.3 GeV, respectively. Including the preliminary results above 57 GeV yields $M_Z = 89.0 \pm 1.0$ GeV.

We have determined the QCD coupling constant to be

$$\alpha_s(34 \text{ GeV}) = 0.143 \pm 0.015$$

with little correlation to the Z^0 mass and practically independent from the top mass and uncertainties in $\sin^2\theta_w$. Here we have taken the third order QCD corrections into account, which lower α_s about 10% (see table 3) with respect to the second order value. The QCD series $1 + \alpha_s/\pi + ...$ has been determined from a direct fit to the data in a model independent way. Extrapolating to the Z^0 region, we find this factor to be 1.046 ± 0.006.

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