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Flavour-changing Yukawa coupling of the standard Higgs boson: effects of the external quark masses

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Abstract. We derive the effective off-diagonal quark-quark-Higgs boson coupling in the standard model without imposing any limitations on the quark masses. A similar vertex for the unphysical scalar present in the Feynman-'t Hooft gauge is also calculated.

Flavour-changing transitions involving the Higgs boson χ of the standard electroweak theory have been studied vigorously in recent years [1–9]. In particular, the decay $q \to q'\chi$ has been demonstrated to be observable in some quark systems for suitable Higgs boson masses [5–7]. As noticed in [7], since the Higgs boson couples to the matter fermions with the strength proportional to their masses, the usually employed approximation neglecting the external quark masses with respect to the weak scale M_W everywhere but in the couplings becomes at least questionable for the physically most interesting processes involving heavy quarks. Consequently, the authors of [7] have done the one-loop calculation of the $q \to q'\chi$ decay amplitude taking the effects of the initial and final particle masses fully into account. However, they have neither released calculational details nor discussed how the inclusion of the external masses changes the result obtained within the standard low-energy approach [1]. Since such a discussion may be of general interest, we intend to fill this gap with the present paper and to extend the usual calculation of the one-loop induced $qq'\chi$ vertex to the case of arbitrarily heavy quarks (and the Higgs boson). Our presentation includes the analytic formulae which may serve as a basis for any future applications. The dependence of the result on the external masses and on the mass of the quark exchanged in the loop is also investigated.

As is well known, the flavour-changing $qq'\chi$ vertex is absent in the tree-level standard model Lagrangian; it is generated at the one-loop approximation of the theory. For the renormalizable R_ξ gauge, the set of relevant diagrams is depicted in Figs. 1 and 2. The sum of the diagrams for the external quarks being on mass-shell (but with the Higgs boson not necessarily on-shell) can be written in the form of the following effective vertex:

$$\Gamma = -\frac{i}{(4\pi)^2}\frac{g^3}{4M_W}\sum_k \lambda_k^{ij} \cdot \bar{q}_j(p_j)[m_i F_1(m_k, m_i, m_j)O_R + m_j F_2(m_k, m_i, m_j)O_L] \cdot q_i(p_i)\chi(q), \tag{1}$$

where the summation goes over the quarks exchanged in the loop, the masses of which are called m_k. We also call m_i, p_i and m_j, p_j the masses and the momenta of the initial and final quarks, respectively. g is the $SU(2)$ coupling constant and O_L, O_R are left and right chirality projectors. λ_k^{ij} denotes the product of the mixing matrix elements. Its actual form depends on whether the external quarks are of the up- or down-type:

$$\lambda_k^{ij} = \begin{cases} V_{jk}V_{ik}^* & \text{for up-type } q_i, q_j \\ V_{kj}^* V_{ki} & \text{for down-type } q_i, q_j \end{cases} \tag{2}$$

where V represents the quark mixing matrix.

In order to calculate the form factors F_1 and F_2 we used the Feynman rules of [10]* and employed the Feynman-'t Hooft gauge ($\xi = 1$). This leaves out the question of the gauge (in)dependence of our formulae (which turned out to be essential in the case of light external quarks [3]) but guaranties their relative simplicity. For all external particles on mass-shell (a

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* The symbol χ is used in [10] to denote what we call $-\phi_3$ in this paper (the unphysical scalar) whereas the physical Higgs field is referred to as η

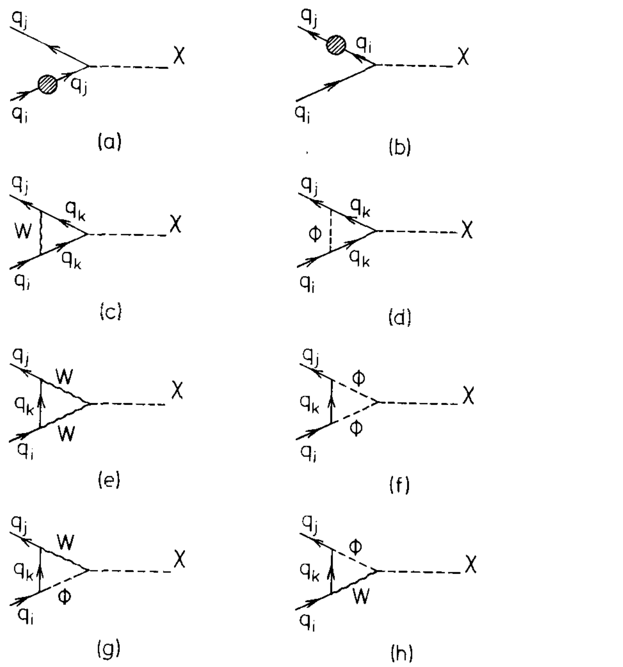


Fig. 1a–h. Diagrams contributing to the flavour-changing Yukawa coupling of the Higgs boson χ in the R_ξ gauge. Φ denotes the unphysical charged scalar present in this gauge

Fig. 2. Off-diagonal quark self-energy diagrams represented by blobs in Fig. 1

case which we will discuss in more detail), the gauge dependent terms should cancel anyway. Another assumption of practical character refers to the properties of the γ_5 matrix. Some of the diagrams of Fig. 1 (a, b, d, g, h) are UV-divergent and we calculated them with the use of dimensional regularization (of course, the intermediate step divergences cancel out in the final result), performing the Dirac algebra with the anticommuting γ_5. This prescription has been successfully used in numerous calculations of electroweak radiative corrections and—despite principal reservations—has been explicitly demonstrated to work correctly for the $qq'\chi$ vertex [9].

As usual in the case of massive external particles, it is convenient to express the results in terms of the standard one-loop functions introduced first by Passarino and Veltman [11]. Since we use the metric tensor

$$g_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1),$$

which is different from that of [11], it may be useful to display the explicit definitions relevant for the present calculation and to formulate the rules of correspondence. We have

$$\int \frac{d^n r}{(2\pi)^n} \frac{\{1, r_\mu, r_\mu r_\nu\}}{[r^2 - m_1^2 + i\varepsilon][(r+k)^2 - m_2^2 + i\varepsilon][(r+k+p)^2 - m_3^2 + i\varepsilon]} = -\frac{i}{(4\pi)^2}\begin{cases} C_0 \\ k_\mu C_{11} + p_\mu C_{12} \\ k_\mu k_\nu C_{21} + p_\mu p_\nu C_{22} + (k_\mu p_\nu + k_\nu p_\mu) C_{23} + g_{\mu\nu} C_{24} \end{cases}, \tag{3}$$

$$\int \frac{d^n r}{(2\pi)^n} \frac{\{1, r_\mu\}}{[r^2 - m_1^2 + i\varepsilon][(r+k)^2 - m_2^2 + i\varepsilon]} = \frac{i}{(4\pi)^2}\begin{cases} B_0(k^2; m_1^2, m_2^2) \\ k_\mu B_1(k^2; m_1^2, m_2^2) \end{cases}, \tag{4}$$

with the functions C in (3) depending on the same arguments

$$C_{ij} = C_{ij}(k^2, p^2, (k+p)^2; m_1^2, m_2^2, m_3^2). \tag{5}$$

In order to translate the above (and all the subsequent) formulae into the conventions of [11] one has to:

a) replace $\log 4\pi$ with $\log \pi$ in the divergent parts of functions B and C;
b) change the signs of all products and squares of momenta;
c) multiply C_{24} by -1.

Actually, since the vertex $qq'\chi$ is finite at the one-loop level due to the underlying symmetry, the rule a) is irrelevant for the present considerations.

By direct summation of the diagrams of Figs. 1 and 2 we find:

$$\begin{aligned} F_1(m_k, m_i, m_j) &= F_2(m_k, m_j, m_i) = \frac{m_k^2}{M_W^2}[B_0(q^2; m_k^2, m_k^2) \\ &+ M_W^2 C_0 + (2m_k^2 + 4M_W^2 - m_i^2 - m_j^2)(C_{11} - C_{12})] \\ &- 2m_k^2 \tilde{C}_0 - 2(m_i^2 - m_j^2 + 2M_W^2 + m_k^2)\tilde{C}_{11} \\ &+ 2(q^2 - m_j^2 + 2M_W^2 + m_k^2)\tilde{C}_{12} - \frac{M_\chi^2}{M_W^2}[m_k^2 \tilde{C}_0 \\ &+ m_k^2 \tilde{C}_{11} + (m_j^2 - m_k^2)\tilde{C}_{12}] + \frac{1}{m_i^2 - m_j^2}\left\{ \frac{m_k^2}{M_W^2} \right. \\ &\cdot [m_j^2 B_0(m_i^2; m_k^2, M_W^2) - m_i^2 B_0(m_j^2; m_k^2, M_W^2)] \\ &+ \frac{m_k^2}{M_W^2} m_j^2 [B_0(m_i^2; m_k^2, M_W^2) - B_0(m_j^2; m_k^2, M_W^2)] \\ &+ \left(2 + \frac{m_k^2}{M_W^2} + \frac{m_i^2}{M_W^2}\right) m_j^2 [B_1(m_i^2; m_k^2, M_W^2) \\ &\left. - B_1(m_j^2; m_k^2, M_W^2)] \right\}, \end{aligned} \tag{6}$$

where q is called the four-momentum of the Higgs boson. We also followed a convention according which

$$C_{ij} = C_{ij}(m_i^2, q^2, m_j^2; M_W^2, m_k^2, m_k^2), \tag{7}$$

$$\tilde{C}_{ij} = C_{ij}(m_i^2, q^2, m_j^2; m_k^2, M_W^2, M_W^2). \tag{8}$$

Let us notice that the equality $F_1(m_k, m_i, m_j) = F_2(m_k, m_j, m_i)$ follows from general considerations and was used as a check of the calculation. As another test we checked that in the limit $m_i^2 = m_j^2 = M_\chi^2 = 0$ (6) reproduces the well known low-energy expression [1]:

$$F_1 = F_2 = -\tfrac{3}{2} x_k, \tag{9}$$

where x_k is defined as $x_k = m_k^2/M_W^2$.

One of the possible strategies to obtain a numerical estimate of (6) consists of expressing the functions C_{11}, C_{12} and B_1 in terms of the scalar one-loop integrals B_0 and C_0 for which closed-form expression exist [12]. This step can be easily done with the use of the formulae from Appendices D and E of [11] (see also [13]). Since the resulting expressions are rather long we do not present them here. However, in the physically important case when one of the external quark masses, say m_j, is much smaller than the other one and than the weak boson mass M_W, they become greatly simplified and take the following form:

$$\begin{aligned}
&F_1(m_k, m_i, m_j = 0)\\
&= \frac{1}{q^2 - m_i^2}\{(2 - x_k)(q^2 - m_i^2)B_0(0; m_k^2, M_W^2)\\
&\quad + [(2 - x_k)m_i^2 + 2(x_k - 1)(2M_W^2 + m_k^2)]\\
&\quad \cdot B_0(m_i^2; m_k^2, M_W^2) + 2(-q^2 + 2M_W^2 + m_k^2)\\
&\quad \cdot B_0(q^2; M_W^2, M_W^2) + x_k(q^2 - 4M_W^2 - 2m_k^2)\\
&\quad \cdot B_0(q^2; m_k^2, m_k^2) + x_k[M_W^2(q^2 - 2m_i^2 - 2m_k^2 + 4M_W^2)\\
&\quad + m_k^2(m_i^2 - 2m_k^2)]C_0(m_i^2, q^2, 0; M_W^2, m_k^2, m_k^2)\\
&\quad + 2[M_W^2(2M_W^2 - m_k^2 - q^2) + m_k^2(m_i^2 - m_k^2)\\
&\quad + m_i^2(q^2 - m_i^2)]C_0(m_i^2, q^2, 0; m_k^2, M_W^2, M_W^2)\\
&\quad + x_k M_\chi^2[B_0(q^2; M_W^2, M_W^2) - B_0(m_i^2; m_k^2, M_W^2)\\
&\quad + (M_W^2 + m_i^2 - q^2 - m_k^2)\\
&\quad \cdot C_0(m_i^2, q^2, 0; m_k^2, M_W^2, M_W^2)]\}
\end{aligned} \tag{10}$$

(the knowledge of F_2 is immaterial, see below).

Let us consider three different choices of the external quarks q_i, q_j:

A) $q_i = b$, $q_j = s$;
B) $q_i = t$, $q_j = c$;
C) $q_i = t'$, $q_j = t$, where we assumed the existence of a fourth family of quarks and denoted its up-type member t'.

Actually, the case B) can also serve for the purpose of the estimate of the $q_i = b'$, $q_j = b$ coupling in the four family scenario, since the effect of the $m_j \neq 0$ is negligible in both instances. For each choice we compute F_1 and F_2 assuming that not only the quarks q_i, q_j but also the Higgs boson are on mass-shell. This ensures the gauge independence of the results. We investigate the quantities

$$z_{1,2} = \left| \frac{F_{1,2}(m_k, m_i, m_j) - F_{1,2}(0, m_i, m_j)}{(3/2)x_k} \right| \tag{11}$$

which take the GIM cancellation of the m_k-independent terms into account and measure directly the deviations from the low-energy approximation (9).

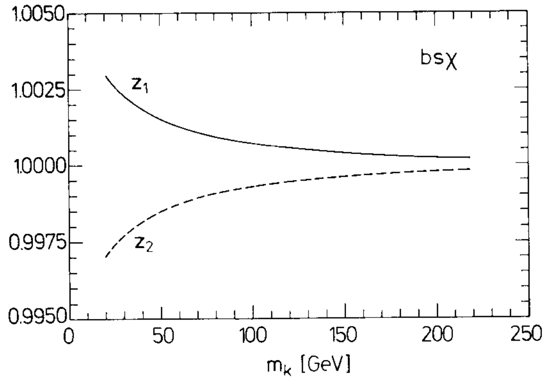


Fig. 3. Quantities z_1 (solid line) and z_2 (dashed line) for the process $b \to s\chi$ vs. the intermediate quark mass. $M_\chi = 0.2\,\text{GeV}$

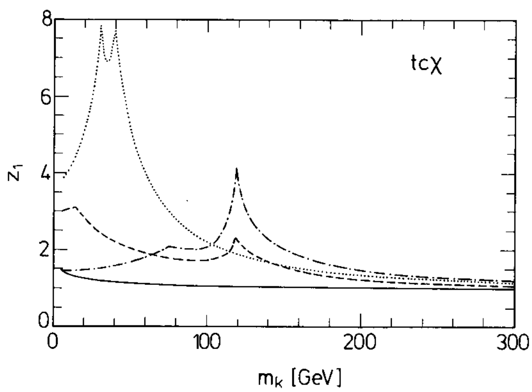


Fig. 4. The quantity z_1 for the process $t \to c\chi$ as a function of the internal quark mass m_k. Other masses are the following: $m_t = 40\,\text{GeV}$, $M_\chi = 0.2\,\text{GeV}$ (full line); $m_t = 120\,\text{GeV}$, $M_\chi = 60\,\text{GeV}$ (dotted); $m_t = 200\,\text{GeV}$, $M_\chi = 30\,\text{GeV}$ (dashed); $m_t = 200\,\text{GeV}$, $M_\chi = 60\,\text{GeV}$ (dot-dashed)

In Fig. 3 we show the dependence of z_1 and z_2 on m_k for the $bs\chi$ coupling (case A)) assuming the Higgs boson mass $M_\chi = 0.2\,\text{GeV}$, a value which is still not totally excluded [14]. As expected, the approximate result (9) works very well here and is accurate up to 0.3%. This number gets even lower for larger values of M_χ although the dependence on M_χ is mild. The form factor F_2 is slightly reduced by the external mass effects and can be safely neglected (it is multiplied in the amplitude by the small final quark mass). We have checked in a number of examples that the contribution of F_2 is immaterial also if the initial quark mass becomes large (case B)): one always finds that F_1 and F_2 have the same order of magnitude and therefore $m_i|F_1| \gg m_j|F_2|$. We show the behaviour of the dominating quantity z_1 for the $tc\chi$ vertex in Fig. 4. Again, the dependence on the intermediate quark mass m_k is depicted for several choices of other parameters: $m_t = 40\,\text{GeV}$, $M_\chi = 0.2\,\text{GeV}$; $m_t = 120\,\text{GeV}$, $M_\chi = 60\,\text{GeV}$; $m_t = 200\,\text{GeV}$, $M\chi = 30\,\text{GeV}$ and $m_t = 200\,\text{GeV}$, $M_\chi = 60\,\text{GeV}$. It is clear that the inclusion of the external masses enhances F_1. This enhancement is strongest in the regions where the cuts open in the amplitude ($m_t = M_W + m_k$, $M_\chi = 2m_k$). Unfortunately, the peak values cannot be fully trusted since in our analysis we neglect the finite particle widths. The openings of the cuts are the exact places where the width effects may become important even if, formally, they are higher order corrections (however, we would like to stress that peaks in all presented curves although sharp, are finite). Except for small regions around the maxima only insignificant modifications are expected.

Another feature of Fig. 4 worth noticing is the relatively strong enhancement of the form factor F_1 occurring for the light Higgs boson in the region of small m_k (the endpoints of the curves correspond to the value $m_k = m_b = 4.6$ GeV). We illustrate this effect also in Fig. 5, where z_1 is plotted versus M_χ in the three generation model ($m_k = 4.6$ GeV) for $m_t = 40$, 80, 120 and 200 GeV. Figure 5 shows that an enhancement of order 10 over the low energy estimate is not out of reach. The primary reason for this behaviour is the noncancellation of the terms proportional to $\log x_k$ which are absent in the massless limit [5]. For instance, for $M_\chi^2 \ll m_k^2$ and with $M_W > m_i$ one finds the following leading term in m_k:

$$F_1(m_k, m_i, m_j = 0) \sim -x_k \log x_k \cdot \frac{2M_W^2}{m_i^4}\left[2m_i^2 + (2M_W^2 - m_i^2)\log\frac{M_W^2 - m_i^2}{M_W^2}\right]$$

for

$$\frac{(m_i^2 - M_W^2)^2}{2(m_i^2 + M_W^2)} \gg m_k^2. \tag{12}$$

The presence of the logarithmic terms is irrelevant from the point of view of the observability of the $t \to c\chi$ decay. Its branching ratio remains hopelessly small (of order of at most 10^{-10}) even after their inclusion.

The above conclusion depends crucially on the

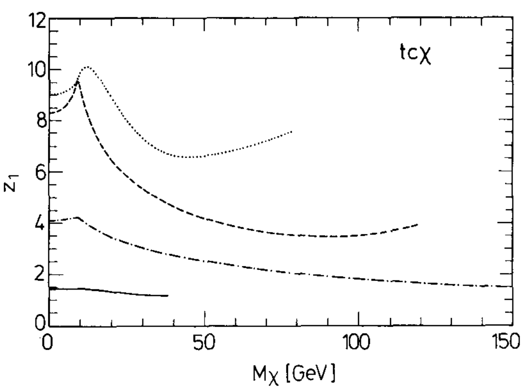
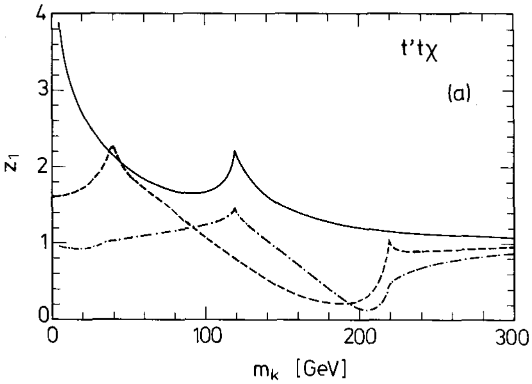


Fig. 5. z_1 vs. the Higgs boson mass for $t \to c\chi$ in the three-generation model. The top quark mass is chosen to be: $m_t = 40$ GeV (solid line); $m_t = 80$ GeV (dotted); $m_t = 120$ GeV (dashed); $m_t = 200$ GeV (dot-dashed)



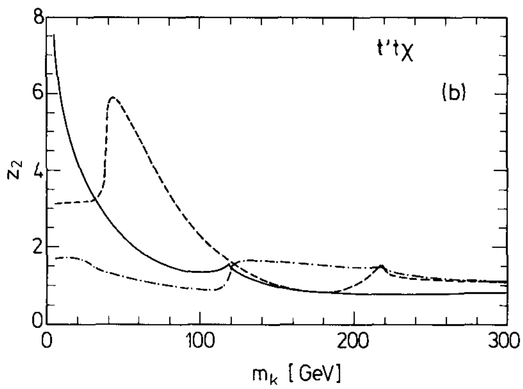


Fig. 6. Quantities **a** z_1 and **b** z_2 for the process $q_i \to q_j\chi$ vs. the intermediate quark mass. The other masses are: $m_i = 200$ GeV, $m_j = 40$ GeV, $M_\chi = 0.2$ GeV (solid); $m_i = 300$ GeV, $m_j = 120$ GeV, $M_\chi = 150$ GeV (dashed); $m_i = 300$ GeV, $m_j = 200$ GeV, $M_\chi = 60$ GeV (dot-dashed)

assumption about the number of quark families. If a fourth heavy generation existed then the decay rate for $t \to c\chi$ could be substantial [7] (let us notice here that for the internal quark mass $m_k \gtrsim 200$ GeV the low-energy expression (9) approximates F_1 remarkable well). Process involving the fourth generation external quarks would be even more important. This situation (case C)) differs from the ones considered up to now by the fact that both form factors F_1 and F_2 now become important. They (or rather quantities z_1 and z_2) are depicted in Fig. 6a and b, respectively, as functions of the internal quark mass m_k for three choices of the free parameters: $m_i = 200$ GeV, $m_j = 40$ GeV, $M_\chi = 0.2$ GeV; $m_i = 300$ GeV, $m_j = 120$ GeV, $M_\chi = 150$ GeV and $m_i = 300$ GeV, $m_j = 200$ GeV, $M_\chi = 60$ GeV. Although the curves of Fig. 6 develop complicated patterns, they do not deviate from unity by more than a factor ~ 2 unless the Higgs boson and the internal quark are both light.

In general calculations of flavour-changing processes involving heavy quarks and performed in a renormalizable R_ξ gauge, one encounters also the effective vertex $qq'\phi_3$, where ϕ_3 is the unphysical scalar partner of the Z boson. The derivation of this vertex in the Feynman-'t Hooft gauge follows that for the

Higgs boson. It is, however, simpler since the relevant diagrams form only a subset of those shown in Fig. 1 (of course, with χ replaced by ϕ_3): diagrams e and f vanish because of the absence of the $WW\phi_3$ and $\Phi\Phi\phi_3$ couplings. For the external quarks being on mass-shell the form of the effective vertex is the same as in (1) but with F_1 and F_2 given now by the following formula:

$$\begin{aligned}
&F_1(m_k, m_i, m_j)/(2iT_3) = -F_2(m_k, m_j, m_i)/(2iT_3) \\
&= \frac{m_k^2}{M_W^2}[B_0(q^2; m_k^2, m_k^2) + (M_W^2 - m_j^2)C_0 + (m_j^2 - m_i^2) \\
&\quad \cdot (C_{11} - C_{12})] - 4m_k^2\tilde{C}_0 - 2(m_i^2 + m_j^2)\tilde{C}_{11} + 2q^2\tilde{C}_{12} \\
&\quad + \frac{1}{m_i^2 - m_j^2}\left\{\frac{m_k^2}{M_W^2}[m_j^2 B_0(m_i^2; m_k^2, M_W^2)\right. \\
&\quad - m_i^2 B_0(m_j^2; m_k^2, M_W^2)] + \frac{m_k^2}{M_W^2} m_j^2 [B_0(m_i^2; m_k^2, M_W^2) \\
&\quad - B_0(m_j^2; m_k^2, M_W^2)] + \left(2 + \frac{m_k^2}{M_W^2} + \frac{m_i^2}{M_W^2}\right) \\
&\quad \left. \cdot m_j^2 [B_1(m_i^2; m_k^2, M_W^2) - B_1(m_j^2; m_k^2, M_W^2)]\right\}, \qquad (13)
\end{aligned}$$

where T_3 is called the third component of the weak isospin of the external quarks. Since the $qq'\phi_3$ coupling is gauge dependent and can only appear as a part of a larger diagram, there is no point in discussing the magnitude of the above form factors F_1 and F_2. It may be however useful to know the expressions for them in the limit $m_i^2 = m_j^2 = q^2 = 0$:

$$F_1 = -F_2 = -i2T_3 x_k \left[\frac{3x_k + 2}{(1 - x_k)^2}\log x_k + \frac{6 - x_k}{1 - x_k}\right]. \qquad (14)$$

To conclude, we have calculated and discussed the effective flavour-changing coupling to the Higgs boson, taking into account the effects of the external particle masses. It has been found that the simple low-energy expression gives a correct order of magnitude estimate of the full result provided that the mass of the quark exchanged in the loop is large. However, for the Higgs boson and the internal quark simultaneously light (as can be the case for the $t \to c\chi$ decay in the three family model) or near the mass regions where the amplitude develops a non-zero absorptive part, the inclusion of the external masses enhances the vertex substantially. The analytic formulae for the effective flavour-changing coupling to the physical Higgs boson χ as well as to the unphysical one ϕ_3 (present in the Feynman-'t Hooft gauge) have been also given.

Note added in proof

After this paper had been sent for publication, the authors of [7] released the more extensive version of their analysis (TECHNION-PH-89-9) which addresses some of the questions discussed above.

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