LEPTON MASS EFFECTS IN SEMI-LEPTONIC B-MESON DECAYS

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Received 15 June 1989; revised manuscript received 5 August 1989

We calculate decay rates, decay spectra and polarization parameters for exclusive semi-leptonic (s.l.) b→c and b→u bottom meson and free bottom quark decays involving the τ-lepton. These are compared to the corresponding observables calculated for the light lepton modes. A surprising feature is a significant scalar current contribution in s.l. B→D and B→π decays which leads to an average positive longitudinal τ-polarization in the B→D mode.

Experimentally the semi-leptonic (s.l.) branching ratios of bottom meson decays are approximately 10% for the two light lepton modes e and μ each. These modes appear to be dominated (≈90%) by the two exclusive s.l. modes B→D and B→D*. Correspondingly one can expect the τ-lepton mode to be dominated by the same two exclusive channels. It is therefore worthwhile and important to theoretically analyze these two exclusive s.l. decay modes also in the τ-sector.

In this letter we calculate decay rates, decay spectra and polarization parameters for exclusive s.l. b→c and b→u bottom meson and free bottom quark decays involving the τ-lepton. These are compared to the corresponding observables calculated in the light lepton sector.

Let us begin by defining a standard set of invariant form factors by writing

$$\langle D(p_2)|V_\mu|B(p_1)\rangle$$
$$=F_+^V(p_1+p_2)_\mu+F_-^V q_\mu, \tag{1a}$$

$$\langle D^*(p_2)|A_\mu+V_\mu|B(p_1)\rangle=(F_1^A g_{\mu\alpha}+F_2^A p_{1\mu}p_{1\alpha}$$
$$+F_3^A q_\mu p_{1\alpha}+\mathrm{i}F^V\epsilon_{\mu\alpha\rho\sigma}p_1^\rho p_2^\sigma)\epsilon^*(p_2)^\alpha, \tag{1b}$$

where $q_\mu=(p_1-p_2)_\mu$ is the four-momentum transfer and $q^2=q_\mu q^\mu$.

Next we calculate helicity form factors by taking the appropriate helicity projections of the covariants in eq. (1) [1]. One has

$$H_0^D=\frac{2M_1 p}{\sqrt{q^2}}F_+^V,$$
$$H_t^D=\frac{1}{\sqrt{q^2}}[(M_1^2-M_2^2)F_+^V+q^2F_-^V], \tag{2a}$$

$$H_0^{D^*}=\frac{1}{2M_2\sqrt{q^2}}[(M_1^2-M_2^2-q^2)F_1^A+2M_1^2p^2F_2^A)],$$
$$H_t^{D^*}=\frac{M_1 p}{M_2\sqrt{q^2}}[F_1^A+\tfrac{1}{2}(M_1^2-M_2^2+q^2)F_2^A+q^2F_3^A],$$
$$H_\pm^{D^*}=F_1^A\pm M_1 pF^V. \tag{2b}$$

where p is the momentum of the D(D*) in the B rest system given by

[1] Supported by Bundesministerium für Forschung und Technologie, 05 4HH 92P/3, Bonn, RFG.

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(North-Holland Physics Publishing Division)

$$2M_1 p = (M_1^4 + M_2^4 + q^4 - 2M_1^2 M_2^2 - 2M_1^2 q^2 - 2M_2^2 q^2)^{1/2}, \quad (3)$$

and where M_1 and M_2 are the masses of the B and the D(D*), respectively.

It is instructive to analyze the partial wave structure of the quasi-two-body decays $B \to D(D^*) + W_{\text{off-shell}}$ which leads to the pseudo-threshold factors p^l ($l=0, 1, 2$) appearing in the helicity amplitudes (2). The $W_{\text{off-shell}}$ has the spin content $J^P = 0^+, 1^-$ ($0^-, 1^+$) for the vector (axial vector) current transitions. Note that the spin 0 (time) components become excited only in decays involving heavy leptons. These involve a lepton–neutrino helicity flip in the decay $W_{\text{off-shell}} \to \ell + \nu_\ell$. One has

(i) $B \to D$

V: $B(0^-) \to D(0^-) + W(1^-)$ p-wave,

$B(0^-) \to D(0^-) + W(0^+)$ s-wave, (4a)

(ii) $B \to D^*$

V: $B(0^-) \to D^*(1^-) + W(1^-)$ p-wave,

A: $B(0^-) \to D^*(1^-) + W(1^+)$ s,d-wave,

$B(0^-) \to D^*(1^-) + W(0^-)$ p-wave. (4b)

The fact that one has an s-wave scalar current excitation with its accompanying pseudo-threshold enhancement in decays involving heavy leptons in the $B \to D$ (or $B \to \pi$) case leads to interesting consequences, in particular for the semi-leptonic $B \to D$ decay involving the τ-lepton as we shall show in the following.

The double decay distribution for the decays $B(b) \to D(c)(D^*(c)) + \ell^- + \bar{\nu}_\ell$ reads [1,2]

$$\frac{d\Gamma}{dq^2 dE_\ell} = \frac{G^2}{48\pi^3} |V_{bc}|^2 \frac{q^2 - \mu^2}{M_1^2} \times [\tfrac{3}{8}(1+\cos^2\theta)\hat{H}_U + \tfrac{3}{4}\sin^2\theta\, \hat{H}_L + \tfrac{3}{4}\cos\theta\, \hat{H}_P + (\mu^2/2q^2)(\tfrac{3}{4}\sin^2\theta\, \hat{H}_U + \tfrac{3}{2}\cos^2\theta\, \hat{H}_L + \tfrac{1}{2}\hat{H}_S + 3\cos\theta\, \hat{H}_{SL})], \quad (5)$$

where E_ℓ is the lepton energy in the B rest system and μ is the lepton mass. θ is the polar angle between the D(D*) and the lepton ℓ^- in the $(\ell^- \bar{\nu}_\ell)$ CM system and is given by

$$\cos\theta = \frac{(M_1^2 - M_2^2 + q^2)(q^2 + \mu^2) - 4q^2 M_1 E_\ell}{2M_1 p (q^2 - \mu^2)}. \quad (6)$$

The hadron tensor components $\hat{H}_i$ appearing in the decay distribution (5) can be expressed in terms of the helicity amplitudes and are given by

$\hat{H}_U = |H_+|^2 + |H_-|^2$ unpolarized-transverse,

$\hat{H}_L = |H_0|^2$ longitudinal,

$\hat{H}_P = |H_+|^2 - |H_-|^2$ parity-odd,

$\hat{H}_S = 3|H_t|^2$ scalar,

$\hat{H}_{SL} = \mathrm{Re}(H_t H_0^*)$ scalar-longitudinal-interference. (7)

The E_ℓ (or $\cos\theta$) integration in eq. (5) can easily be done and results in the differential q^2-distribution

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_{U+L}}{dq^2} + \frac{d\tilde{\Gamma}_{U+L}}{dq^2} + \frac{d\tilde{\Gamma}_S}{dq^2}, \quad (8)$$

where $\Gamma_{U+L} = \Gamma_U + \Gamma_L$, and where we have defined partial helicity rates according to

$$\frac{d\Gamma_i}{dq^2} = \frac{G^2}{(2\pi)^3} |V_{bc}|^2 \frac{(q^2-\mu^2)^2 p}{12 M_1^2 q^2} \hat{H}_i, \quad i = U, L, P \quad (9a)$$

and

$$\frac{d\tilde{\Gamma}_i}{dq^2} = \frac{\mu^2}{2q^2} \frac{d\Gamma_i}{dq^2}, \quad i = U, L, S, SL. \quad (9b)$$

As the angular distribution (5) is evaluated in the $(\ell^- \bar{\nu}_\ell)$ CM system the rates $d\Gamma_i/dq^2$ and $d\tilde{\Gamma}_i/dq^2$ can be seen to correspond to the neutrino–lepton spin no-flip and flip contributions, respectively, *in that system* [1]. The flip contributions $d\tilde{\Gamma}_i/dq^2$ involve the characteristic flip factor $\mu^2/2q^2$ which vanishes when $\mu \to 0$. Since the massless antineutrino has positive helicity the flip/no-flip composition determines the longitudinal polarization P_L of the heavy lepton in the $(\ell^- \bar{\nu}_\ell)$ CM system via [#1]

$$P_L = \frac{d\tilde{\Gamma} - d\Gamma}{d\tilde{\Gamma} + d\Gamma}. \quad (10)$$

In the following we shall investigate rates and decay spectra involving the τ-lepton and τ-polarization

[#1] Note that the longitudinal polarization evaluated in the $(\ell^- \bar{\nu}_\ell)$ CM system differs from the longitudinal polarization evaluated in the B rest system as e.g. in ref. [3].

spectra for the free quark decay model (FQD) and the helicity matching model of exclusive s.l. B decays introduced in ref. [4]. For reasons of brevity the latter model will be referred to as the KS-model.

The FQD model is specified in terms of its hadron tensor components $\hat{H}_i$ which are given by [1] [#2]

$$\hat{H}_U = 8(m_1^2+m_2^2-q^2),$$
$$\hat{H}_L = 4(m_1^2+m_2^2-q^2+4m_1^2p^2/q^2),$$
$$\hat{H}_P = -16m_1 p,$$
$$\hat{H}_S = 3\hat{H}_L,$$
$$\hat{H}_{SL} = 8m_1 p(m_1^2-m_2^2)/q^2, \tag{11}$$

where m_1 and m_2 refer to the masses of the heavy and light quark.

In the KS model of exclusive s.l. $B\to D(D^*)$ and $B\to \rho(\pi)$ decays the particle helicity amplitudes are matched to the free quark decay helicity amplitudes at $q^2=0$, assuming that the spectator quark is spin-inert.

Thus one has

$$H_0^{D(D^*)} = \langle D(D^*)|J_0|B\rangle$$
$$\simeq \tfrac{1}{2}I(\langle c\downarrow|J_0|b\downarrow\rangle + (-)\langle c\uparrow|J_0|b\uparrow\rangle),$$
$$H_t^{D(D^*)} = \langle D(D^*)|J_t|B\rangle$$
$$\simeq \tfrac{1}{2}I(\langle c\downarrow|J_t|b\downarrow\rangle + (-)\langle c\uparrow|J_t|b\uparrow\rangle),$$
$$H_{-(+)}^{D^*} = \langle D^*\downarrow(\uparrow)|J_{-(+)}|B\rangle$$
$$\simeq \frac{1}{\sqrt{2}}I\langle c\downarrow(\uparrow)|J_{-(+)}|b\uparrow(\downarrow)\rangle, \tag{12}$$

where I is an overlap factor between the initial and final meson. To be definite we take $I_{bc}=0.7$ and $I_{bu}=0.33$ as in ref. [5].

Solving the matching conditions (12) to first order in q^2 one arrives at $q^2=0$ form factor values which are then continued to $q^2\geqslant 0$ by using pole-type form factors with a power behaviour given by the QCD power counting rules [6]. The KS invariant form factors are then given by

$$F_+^V(q^2) = I(1-q^2/m_{FF}^2)^{-1},$$
$$F_-^V(q^2) = -\frac{M_1-M_2}{M_1+M_2}I(1-q^2/m_{FF}^2)^{-1},$$
$$F_1^A(q^2) = (M_1+M_2)I(1-q^2/m_{FF}^2)^{-1},$$
$$F^V(q^2) = F_2^A(q^2) = -F_3^A(q^2)$$
$$= -\frac{2}{M_1+M_2}I(1-q^2/m_{FF}^2)^{-2}. \tag{13}$$

For the sake of simplicity we work only with one effective form factor pole mass m_{FF} in each of the $b\to c$ and $b\to u$ cases, for which we take $B_c^*(6.34)$ and $B_u^*(5.33)$, respectively.

In table 1 we list our results for the no-flip and flip partial rates Γ_i and $\tilde{\Gamma}_i$ appearing in the decay distribution (5) for $b\to c$ transitions in the electron and τ-case [#3]. We also list the total decay rate (for notational simplicity we use $U := \Gamma_U$, $\tilde{U} := \tilde{\Gamma}_U$ etc.)

$$\Gamma_{tot} = U+L+\tilde{U}+\tilde{L}+\tilde{S}, \tag{14}$$

and the average longitudinal polarization of the lepton

$$\langle P_L\rangle = \frac{\tilde{U}+\tilde{L}+\tilde{S}-U-L}{\tilde{U}+\tilde{L}+\tilde{S}+U+L}. \tag{15}$$

[#2] One has to remember to include the statistical factor ½ when using the rate formulas (5) and (8) for the FQD case.

[#3] Results for the s.l. modes involving the μ-lepton differ only insignificantly from the e-lepton case [1].

Table 1
Partial rates, total rates and polarization parameters for s.l. $b\to c$ transitions in the e- and τ-sectors. We take $m_b=4.73$ GeV and $m_c=1.55$ GeV. Rates are in units of $|V_{bc}|^2\ 10^{12}\ s^{-1}$.

Decay	U	L	P	$\tilde{U}$	$\tilde{L}$	$\tilde{S}$	$\tilde{S}L$	Γ	$\langle P_L\rangle$	α
$B\to D(e)$	–	8.3	–	–	–	–	–	8.3	−1	–
$B\to D^*(e)$	12.7	13.1	−6.9	–	–	–	–	25.8	−1	1.1
$b\to c(e)$	12.4	24.8	−7.6	–	–	–	–	37.2	−1	–
$B\to D(\tau)$	–	0.72	–	–	0.17	1.26	0.26	2.15	0.33	–
$B\to D^*(\tau)$	3.03	1.95	−1.39	0.64	0.43	0.46	0.25	6.50	−0.53	0.55
$b\to c(\tau)$	2.24	2.01	−1.14	0.51	0.49	1.47	0.43	6.72	−0.26	–

Table 1 also contains the value of the asymmetry parameter α which determines the polar angle decay distribution $W(\theta^*)=1+\alpha\cos^2\theta^*$ in the cascade decay $D^*\to D\pi$ ($\rho\to\pi\pi$), where θ^* is the polar angle of the D (or π) in the $D^*(\rho)$ rest system. The asymmetry parameter is given by [1]

$$\alpha=\frac{2L+2\tilde{L}-U-\tilde{U}+2\tilde{S}}{U+\tilde{U}}. \qquad (16)$$

Table 1 shows a rate reduction from the e-modes to the τ-modes which is largest for the FQD with 18%, 25% for $B\to D^*$ and smallest for $B\to D$ with 26%. In fact, the sum of the D and D* rates now exceeds the FQD rate. The strongest rate reduction occurs in the longitudinal no-flip rates which are 8.6%, 14.8% and 8.1% for $B\to D$, $B\to D^*$ and $b\to c$, respectively, of the corresponding e-rates whereas the transverse no-flip rates contributing to $B\to D^*$ and $b\to c$ are down less and occur at O(25%). That the total $B\to D$ rate reduction is comparable to the $B\to D^*$ and $b\to c$ cases can be seen to result from the fact that one has a strong scalar current contribution from $\tilde{S}$ in the τ-mode. This contribution is large because there is no pseudo-threshold factor to dampen the enhancement resulting from the time-like form factors at large q^2 as explained after (4). For the same reason the average longitudinal polarization has undergone a drastic change from the value -1 in the e-mode to a positive value of 0.33 in the τ-mode, which again results from the large scalar current contribution. In the $B\to D^*$ and $b\to c$ cases the flip rates $\tilde{U}$, $\tilde{L}$ and $\tilde{S}$ occur at only O(10%) of the total rate which explains why the average polarization is still negative in these cases. One notes also that the alignment polarization of the D* measured by the asymmetry parameter α is smaller in the τ-mode than in the e-mode. This results from the relatively strong suppression of the longitudinal contribution in the τ-mode.

In fig. 1 we show the x ($=p/p_{max}$) hadron momentum spectrum of the s.l. decays $B\to D(e,\tau)$, $B\to D^*(e,\tau)$ and $b\to c(e,\tau)$. In the e-mode the spectra rise to their highest values at $x=1$ for $B\to D$ and $b\to c$, whereas the $B\to D^*$ spectrum is softer and shows a shoulder around $x=0.6$ as also seen in the data [7]. In the τ-mode the heavy lepton kinematics forces the spectra to go to zero at $x=1$. Consequently the spectra become softer in the τ-modes. The same behaviour is observed to a lesser degree in the y ($=p_\ell/p_{\ell max}$) lepton momentum spectra, where the spectra become slightly softer going from the e-mode to the τ-mode (see fig. 2).

In fig. 3 we show the y-behaviour of the longitudi-

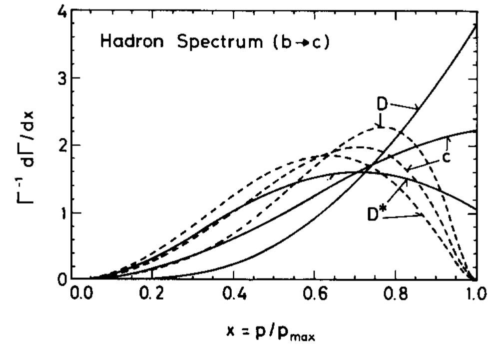

Fig. 1. x ($=p/p_{max}$) hadron momentum spectra of $b\to c(e)$ transitions (full lines) and $b\to c(\tau)$ (dashed lines).

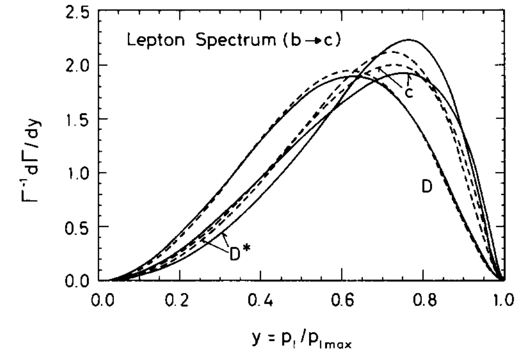

Fig. 2. y ($=p_\ell/p_{\ell max}$) lepton momentum spectra of $b\to c(e)$ transitions (full lines) and $b\to c(\tau)$ transitions (dashed lines).

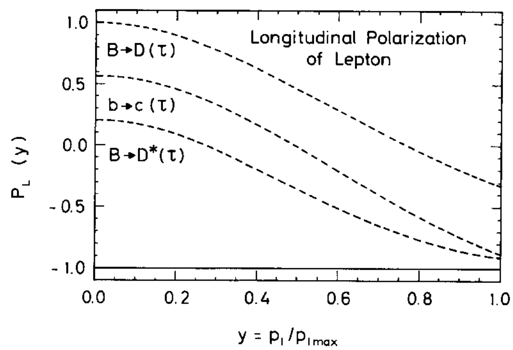

Fig. 3. y ($=p_\ell/p_{\ell max}$) lepton momentum dependence of longitudinal polarization of the e and τ in $b\to c$ transitions.

nal polarization $P_L(y)$ of the τ in the three decay modes $B\to D(\tau)$, $B\to D^*(\tau)$ and $b\to c(\tau)$ (in the e-mode the longitudinal polarization is obviously equal to -1). In all three cases the longitudinal polarization decreases uniformly from its highest (positive) value at $y=0$ to its lowest (negative) value at $y=1$. That $P_L(y=0)=1$ for $B\to D$ is due to spin kinematics as there are only spin flip contributions to $B\to D$ in this limit. The longitudinal polarization of the τ is positive for the cases $B\to D^*$ and $b\to c$ close to $y=0$ where the transverse and parity-odd no-flip contributions U and P add destructively. That $P_L(B\to D) > P_L(b\to c) > P_L(B\to D^*)$ over the whole y-range reflects the relative strength of flip and no-flip contributions as evidenced by the partial rates in table 1.

In table 2 we list our predictions for the $b\to u$ transitions. When comparing the exclusive $B\to\pi$ and $B\to\rho$ modes with the FQD modes one has to take the theoretical uncertainty in the calculation of the overlap factor I_{bu} into account which could easily deviate from its value of $I_{bu}=0.33$ used in this calculation by $\simeq 25\%$. For this reason we limit our discussion to the qualitative features of the two exclusive $b\to u$ decay modes.

The total decay rates are generally reduced going from the e- to the τ-mode. The reduction, however, is not as strong as in the corresponding $b\to c$ transitions. The τ-rates are 52%, 58% and 34% of the corresponding e-rates in the $B\to\pi$, $B\to\rho$ and $b\to u$ cases, respectively. In the τ-mode the two decay channels $B\to\pi$ and $B\to\rho$ almost saturate the FQD rate. The flip rates are generally small except again for the scalar current contribution in the $B\to\pi$ case which occurs at a level of 28% compared to the total rate. That the scalar current contribution is not as strong as in the $B\to D$ case discussed earlier is due to the fact that there is a strong cancellation of the F_+^V and F_-^V contributions to the scalar helicity amplitude H_t^π (with $F_+^V/F_-^V = -(M_1-M_2)/(M_1+M_2)\to -1$ as $M_2/M_1\to 0$) which partially compensates for the s-wave enhancement. The average longitudinal polarization is negative for all three cases $B\to\pi$, $B\to\rho$ and $b\to u$. The largest reduction from the left-handed value $P_L=-1$ in the e-mode occurs for the $B\to\pi$ case with $P_L=-0.25$ which is mainly due to the strength of the scalar current contribution S. The alignment polarization of the ρ as measured by the asymmetry parameter α is small for both the e- and the τ-modes.

The $B\to\pi$, $B\to\rho$ and $b\to u$ hadron momentum spectra are shown in fig. 4. In the e-mode the spectra rise to their highest values at $x=1$ for $B\to\pi$ and $b\to u$. The $B\to\rho$ spectrum shows a shoulder around $x=0.3$ at a lower x-value than in the corresponding $B\to D$ case. This is due to the time-like form factor effect – the ρ prefers to be produced at low momentum where q^2 is largest. In the τ-mode all spectra become slightly

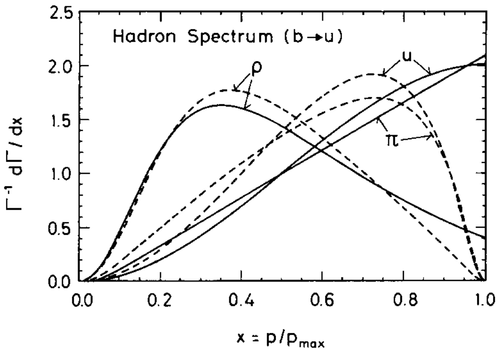

Fig. 4. x (p/p_{max}) hadron momentum spectra of $b\to u(e)$ transitions (full lines) and $b\to u(\tau)$ transitions (dashed lines).

Table 2
Partial rates, total rates and polarization parameters for s.l. $b\to u$ transitions in the e- and τ-sectors. $m_b=4.73$ GeV and $m_u=0.3$ GeV. Rates in units of $|V_{bu}|^2\, 10^{12}\, s^{-1}$. Rates are for $\overline{B}^0\to\pi^+(\rho^+)$. Rates for $B^-\to\pi^0(\rho^0)$ are down by a factor 2.

Decay	U	L	P	$\tilde{U}$	$\tilde{L}$	$\tilde{S}$	$\tilde{S}L$	Γ	$\langle P_L\rangle$	α
$B\to\pi(e)$	–	7.25	–	–	–	–	–	7.25	−1	–
$B\to\rho(e)$	21.9	11.0	−19.0	–	–	–	–	32.9	−1	0.003
$b\to u(e)$	26.1	52.2	−25.2	–	–	–	–	78.3	−1	–
$B\to\pi(\tau)$	–	2.4	–	–	0.3	1.1	0.3	3.8	−0.25	–
$B\to\rho(\tau)$	11.9	4.6	−10.1	1.4	0.6	0.8	0.4	19.2	−0.72	−0.1
$b\to u(\tau)$	9.1	9.8	−8.6	1.4	1.8	5.3	1.7	27.4	−0.38	–

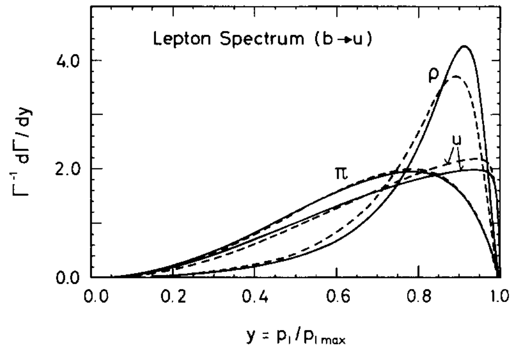

Fig. 5. y ($=p_\ell/p_{\ell max}$) lepton momentum spectra of b→u(e) transitions (full lines) and b→u(τ) transitions (dashed lines).

softer which is due to the fact that the spectra go to zero at $x=1$ in the τ-mode.

The y ($=p_\ell/p_{\ell max}$) lepton momentum spectra shown in fig. 5 remain practically unchanged for the B→π case and become slightly softer for B→ρ and slightly harder for b→u when going from the e- to the τ-mode.

Fig. 6 finally shows the y-dependence of the longitudinal polarization. As in the corresponding b→c transitions the flip contributions dominate at lower y-values whereas the no-flip contributions dominate at higher y-values leading to positive and negative values of P_L at lower and higher y-values. The dominance of the respective contributions is, however, more pronounced than in the b→c transitions with $P_L(y=0)\cong +1$ and $P_L(y=1)\cong -1$ in all three cases.

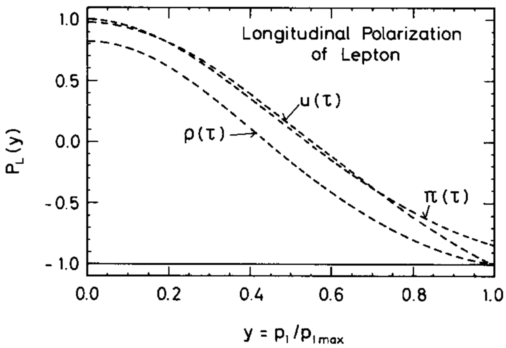

Fig. 6. y ($=p_\ell/p_{\ell max}$) lepton momentum dependence of longitudinal polarization of the e and τ in b→u transitions.

In conclusion, we have pointed out some interesting physics features in the exclusive semi-leptonic decays of the B-meson involving the heavy τ-lepton. The τ-rates occur at ≅25% and ≅50% of the e-rates for B→D(D*) and B→π(ρ), respectively. There is a significant scalar current (lepton spin-flip) contribution in the B→D and B→π cases which is strong enough to change the average longitudinal polarization from −1 to 0.33 and −0.25, respectively. It would be interesting to experimentally confirm these polarization predictions by analyzing the τ's subsequent decay distributions.

Part of this work was done while one of us (J.G.K.) was a visitor at the DESY theory group. J.G.K. would like to thank the theory group for hospitality and the DESY directorate for support. We would like to thank the referee for pointing out some numerical mistakes in the original version of this paper.

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