NON-PERTURBATIVE STUDY OF THE YUKAWA COUPLING IN AN SU(2)⊗SU(2) MODEL WITH QUENCHED NAIVE LATTICE FERMIONS ☆

W. BOCK [a], A.K. DE [a], K. JANSEN [b], J. JERSÁK [a,b] and T. NEUHAUS [c]

[a] *Institut für Theoretische Physik E, RWTH Aachen, D-5100 Aachen, FRG*

[b] *HLRZ c/o KFA Jülich, P.O. Box 1913, D-5170 Jülich, FRG*

[c] *Fakultät für Theoretische Physik, Universität Bielefeld, D-4800 Bielefeld, FRG*

Received 14 July 1989; revised manuscript received 24 August 1989

We initiate a non-perturbative study of Yukawa coupling in an $SU(2)_L \otimes SU(2)_R$ invariant scalar–fermion lattice model. As a first step we calculate fermion masses in the broken phase using naive fermions in the quenched approximation. At small Yukawa coupling, taking into account appreciable finite-size effects, our results are consistent with perturbation theory. At large bare Yukawa coupling we find the fermion mass and the renormalized Yukawa coupling growing with increasing scalar correlation length, indicating a non-perturbative behaviour.

1. Introduction

Non-perturbative investigations of fermion mass generation through the Yukawa coupling to scalar fields have recently stimulated a lot of interest. In simple lattice models with one-component scalar fields, first calculations of the masses in the quenched [1,2] and unquenched [3,4] simulations have already been performed. Simultaneously the influence of the Yukawa coupling on the phase diagram of the one-component [5] and two-component [6] scalar field models with fermions has been studied. Some effects of the Yukawa coupling have recently been analytically investigated also in lattice models with scalars and fermions coupled to U(1) and SU(2) gauge fields [7].

The most important long-term motivations of these investigations are: (i) the determination of a possible upper bound on the mass of heavy quarks [8], in analogy to the recent numerical estimate of the upper bound on the Higgs boson mass in the lattice Φ^4 model with O(4) symmetry [9–12], (ii) a study of the influence of the strong Yukawa coupling on the scalar sector and on the mentioned upper bound on the Higgs boson mass, and, (iii) a search for new critical points suitable for the construction of a – possibly non-trivial – continuum limit of the lattice formulation of the electroweak theory.

We plan on a systematic investigation of a lattice regularized model with the "chiral" $SU(2)_L \otimes SU(2)_R$ symmetry consisting of a four-component scalar field and a Yukawa coupled fermion field doublet using essentially the Wilson fermion approach where the scalar field appears in the Wilson mass term (Wilson–Yukawa term) [13–15]. This model has a promise to be physically realistic in the sense that if the doublers can be made sufficiently heavy it becomes a special case of the SU(2) sector of the electroweak theory with the weak gauge interaction turned off. The removal of the doublers is a non-perturbative issue which will be investigated in a subsequent work [16]. Its complexity requires to understand first the effects of the strong bare Yukawa coupling on the fermion masses.

Therefore in this letter we study, for the moment in the quenched approximation, the effects of the strong Yukawa coupling with naive fermions. We concentrate on the determination of the fermion masses in the broken phase and give a tentative estimate of the flow lines of constant renormalized

☆ Supported by the Deutsches Bundesministerium für Forschung und Technologie and by the Deutsche Forschungsgemeinschaft.

0370-2693/89/$ 03.50

Yukawa coupling. Similar to the findings of ref. [2] in the Z(2) case we find a region of large bare Yukawa couplings where the fermion mass increases while the expectation value of the scalar field in the broken phase approaches zero.

2. The model

The model under consideration in this letter is defined by the following action on the euclidean lattice:

$$S=-\kappa\sum_x\sum_{\mu=1}^{4}\tfrac{1}{2}\mathrm{Tr}\{\Phi_x^\dagger\Phi_{x+\mu}+\text{h.c.}\}$$
$$+\tfrac{1}{2}\sum_x\sum_{\mu=1}^{4}\bar\Psi_x\gamma_\mu\{\Psi_{x+\mu}-\Psi_{x-\mu}\}$$
$$+y\sum_x\bar\Psi_x\{\Phi_x P_R+\Phi_x^\dagger P_L\}\,\Psi_x. \qquad (1)$$

In the above, the scalar field is radially frozen (the bare quartic self-coupling is infinite) and Φ_x is a 2×2 SU(2) matrix, the fermion fields Ψ_x and $\bar\Psi_x$ are SU(2) doublets, κ is the hopping parameter for the scalar field and y is the Yukawa coupling. The operators P_L and P_R are the left- and right-hand projectors. The action is invariant under the global chiral $SU(2)_L\otimes SU(2)_R$ transformations

$$\Psi\to(V_LP_L+V_RP_R)\Psi,\quad \bar\Psi\to\bar\Psi(V_L^\dagger P_R+V_R^\dagger P_L),$$
$$\Phi\to V_L\Phi V_R^\dagger, \qquad (2)$$

where $V_L\in SU(2)_L$ and $V_R\in SU(2)_R$.

The model is a simplified version of a possible lattice formulation of the electroweak theory suggested by Smit [14] and Swift [15]. Here we restrict ourselves to only the SU(2) sector of the Smit–Swift model and neglect the weak gauge interaction. We leave out also the Wilson–Yukawa term so that we have naive fermions giving rise to 16 degenerate species in the continuum.

In the quenched approximation the phase diagram of the model (1) has, for each value of y, a symmetric ($\kappa<\kappa_c$) and a broken ($\kappa>\kappa_c$) phase, where the value of κ_c is independent of y. It is an open question, whether the model possesses more phase transition lines.

For $y=0$ the model is the O(4) symmetric Φ^4 theory at the bare quartic coupling $\lambda=\infty$, which has been investigated extensively [10,11,17,18]. From these studies it is known that $\kappa_c=0.3045(7)$. In the broken phase of this model the behaviour of the scalar mass m_σ and the field expectation value $\langle\Phi\rangle$ has been found to be consistent with the scaling laws if $m_\sigma<0.8$ ($\kappa_c<\kappa<0.33$). The scalar wave function renormalization constant Z_Φ is very close to unity ($Z_\Phi\approx0.97$) and nearly independent of κ. The renormalized quartic coupling appears to be a logarithmically decreasing function of $(\kappa-\kappa_c)$ in the range in which m_σ has the value between 0.25 and 1 in the lattice units.

For small values of the Yukawa coupling y, we expect the fermion mass and the renormalized Yukawa coupling going to zero with κ approaching κ_c at fixed value of y, in accordance with the perturbation theory. The condensate $\langle\bar\Psi\Psi\rangle$ should be proportional to y and the tree level relation $m_F=y\langle\Phi\rangle$ should hold. On the contrary, for large values of y, a strong y expansion of the unquenched model in refs. [2,19] shows that the fermion mass increases when κ approaches κ_c. The renormalized Yukawa coupling should also behave correspondingly. From the strong y expansion the condensate $\langle\bar\Psi\Psi\rangle$ is expected to be proportional to $1/y$.

3. Techniques and measurements

At an equilibrated scalar field configuration, obtained by a standard Metropolis Monte Carlo algorithm on an L^3T lattice with periodic boundary conditions, we perform conjugate gradient inversions of the fermion matrix for several source points $x_0=(\boldsymbol{x}_0, t_0)$ to get the zero spatial momentum fermion propagators

$$P^{ab}_{\alpha\beta}(\tau)=(1/L^3)\Sigma_{\boldsymbol{x}}\langle(\bar\Psi_x)^a_\alpha(\Psi_{x_0})^b_\beta\rangle\ .$$

Here $\alpha,\beta=1,2$ are the $SU(2)_{L,R}$ indices; $a,b=1,2,3,4$ are the Dirac matrices; $x=(\boldsymbol{x},t)$ and $\tau=|t-t_0|$. Choosing the so-called chiral representation of the Dirac matrices, we find the transformation properties of the propagators P^{ab} (suppressing the indices α, β) to be $P^{13}\to V_LP^{13}V_L^\dagger$, $P^{31}\to V_RP^{31}V_R^\dagger$, $P^{11}\to V_LP^{11}V_R^\dagger$, $P^{33}\to V_RP^{33}V_L^\dagger$, etc. Because of the drift of the scalar field magnetization in the broken phase (no spontaneous symmetry breaking occurs for a finite size), it is important to construct the

$SU(2)_L \otimes SU(2)_R$ invariant propagators $G^{13} = \mathrm{Tr}_{SU(2)_L} P^{13}$, $G^{31} = \mathrm{Tr}_{SU(2)_R} P^{31}$. For each pair (κ, y) we obtain the propagator data typically from 32 configurations, separated from each other by 1500 Monte Carlo iterations. We use mostly the $8^3 \cdot 16$ lattice, but also $6^3 \cdot 12$, $10^3 \cdot 16$ and 16^4 lattices to get information about the finite size effects. Fermion masses m_F (defined by $m_F = \sinh E_F$, where E_F is the lowest lying energy) are extracted, using the CERN Minuit fit program, from the exponential fall-off of the averaged propagators.

For $\langle \bar{\Psi}\Psi \rangle$ measurement, we apply the common technique using a gaussian noise. Because of the non-invariance of $\langle \bar{\Psi}\Psi \rangle$ we do an inversion of the fermion matrix for scalar configurations rotated always so that the scalar field magnetization points in one predetermined direction. The same technique has been used in refs. [10,11] to assure that $\langle \Phi \rangle$ does not vanish because of the vacuum drift during a simulation on a finite lattice.

We block the data for the correlation functions into blocks of 8 configurations and determine for each block the fermion mass. The error of the average correlation function for each block needed for the Minuit fit is obtained using the jackknife method. Regarding the fermion mass independent for each block and for each of G^{13} and G^{31} we calculate the average and also estimate the error of the mass by statistical error analysis. The errors for the chiral condensate $\langle \bar{\Psi}\Psi \rangle$ and for the scalar field expectation value $\langle \Phi \rangle$ are obtained similarly.

4. Fermion mass m_F and condensate $\langle \bar{\Psi}\Psi \rangle$

In fig. 1 we show our results for the fermion mass m_F as a function of the scalar hopping parameter κ ($\kappa = 0.31$–0.4) for different fixed values of the Yukawa coupling y ($y = 0.6$–1.5). The behaviour of m_F at small and large y clearly shows two different regions. One observes a decrease of m_F as $\kappa \searrow \kappa_c$ for $y \leqslant 1.3$ whereas for $y \geqslant 1.4$ one observes an increase of m_F as $\kappa \searrow \kappa_c$. At a value of y between 1.3 and 1.4 the fermion mass appears to be independent of κ. We call it the "cross-over point" y^*_m. However, it is not excluded by our data that at y^*_m a real phase transition takes place.

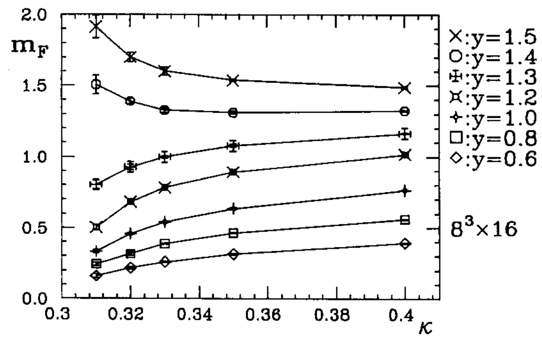

Fig. 1. The fermion mass m_F as function of κ for different values of the bare Yukawa coupling y. Here and in the following figures the straight lines connecting the data points are to guide the eye.

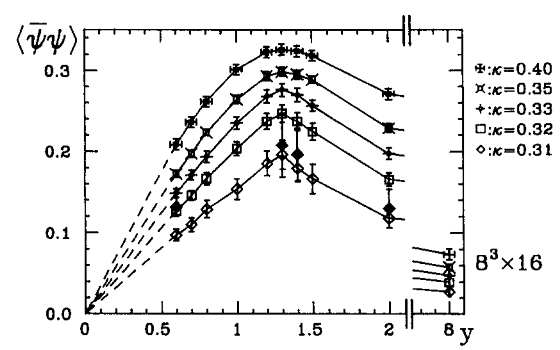

Fig. 2. The fermion condensate $\langle \bar{\Psi}\Psi \rangle$ as a function of y for different values of κ. The filled diamonds correspond to the data at $\kappa = 0.31$ on a $6^3 \cdot 12$ lattice.

Our conjugate gradient converges everywhere but around y^*_m we observe a substantial rise in the number of iterations per inversion of the fermion matrix. For example, for $\kappa = 0.32$ this number at $y = 1.3$ is 5–10 times larger than its values at $y = 0.6$ and $y = 2.0$.

Close to κ_c, for $\kappa \leqslant 0.32$ where the scalar correlation length $m_\sigma^{-1} \geqslant 1.4$, we observe large finite-size effects for the fermion mass at small values of y. Increasing the lattice size from $6^3 \cdot 12$ to $8^3 \cdot 16$ and $10^3 \cdot 16$ at $\kappa = 0.32$ and $y = 0.6$ the fermion mass decreases from 0.246(11) to 0.217(9) and 0.204(2), respectively.

In fig. 2 we show our results for the condensate $\langle \bar{\Psi}\Psi \rangle$ as a function of y for various values of κ. $\langle \bar{\Psi}\Psi \rangle$ has a maximum also around y^*_m. $\langle \bar{\Psi}\Psi \rangle$ is seen to be proportional to y for small values of y as suggested from the perturbation theory, and although not shown in the figure, we find it also to be proportional to $1/y$ for large y, as expected from the strong y expansion.

5. Renormalized Yukawa coupling

In this section we concentrate on the ratio $y_R = m_F\sqrt{Z_\Phi}/\langle\Phi\rangle$. This ratio is a possible definition of the renormalized Yukawa coupling y_R. The deviation of Z_Φ from unity [11,17,18] is insignificant on the precision level of the present investigation, so we set $Z_\Phi = 1$ in y_R.

In the weak y region, characterized by $y < y^*_m$, the perturbative triviality of the non-asymptotically free Yukawa coupling suggests that y_R should go to zero as the cut-off is raised to infinity. In fig. 3 y_R is plotted as a function of κ at $y = 0.6$ on various lattices. For small y and m_F an appreciable lattice size dependence is seen. Actually we find on small lattices qualitatively different behaviour from that expected on an infinite lattice. Our present data at $y = 0.6$ is insufficient for a reliable extrapolation to the infinite lattice but indicates at least that y_R decreases slowly with decreasing κ for $\kappa \leqslant 0.33$ and is around 0.68 at $\kappa = 0.33$. Judging from the trend of the data in fig. 3 as we increase the lattice size from $6^3 \cdot 12$ to 16^4, a farther decrease of y_R with increasing cut-off seems possible if even larger lattices could be used. We make similar observations at other small y values. We notice an analogy to the finite-size effects for the ratio $m_\sigma/\langle\Phi\rangle$ in the O(4) symmetric Φ^4 theory [11].

On the other hand, at y values larger than or around y^*_m, m_F is sufficiently large and therefore we do not see substantial finite-size effects. Data for y_R on various lattices are consistent within error bars. In fig. 4

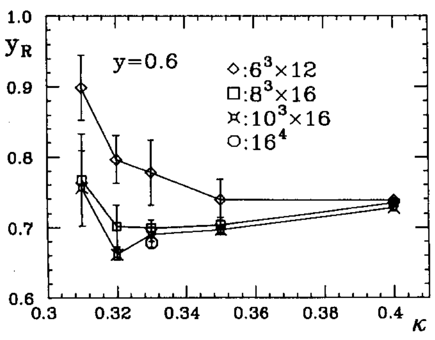

Fig. 3. y_R for $y = 0.6$ as a function of κ on different lattices.

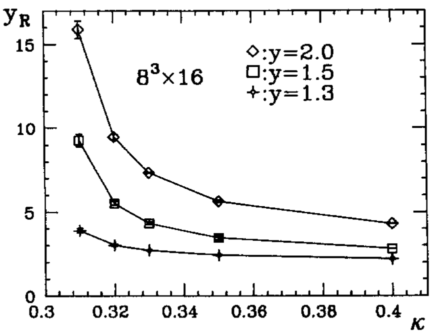

Fig. 4. y_R for $y = 1.3$, 1.5 and 2.0 as a function of κ.

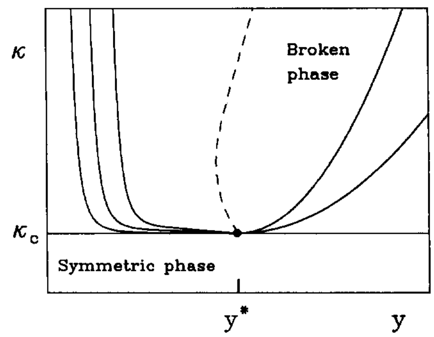

Fig. 5. Schematic plot of the lines of constant y_R in the (κ, y) phase diagram. The dashed line vaguely indicates the position of the crossover in the broken phase.

y_R is plotted against κ for $y = 1.3$, 1.5, 2.0 on an $8^3 \cdot 16$ lattice. There is a clear increase of y_R as $\kappa \searrow \kappa_c$. We see similar behaviour at all values of $y > y^*_m$ we have investigated. In the quenched calculation with naive fermions there is certainly a different behaviour of y_R for $y > y^*_m$ than that expected from perturbation theory.

In fig. 5 we show a schematic diagram of possible lines of constant y_R in the (κ, y) space. The diagram shows a different flow structure for $y < y^*$ and $y > y^*$. In fig. 5 the lines drawn for $y < y^*$ are motivated by the predictions of the perturbation theory and are similar to the constant λ_R lines in the Φ^4 theory [18]. The trend of the lines for $y > y^*$ is motivated by the data in fig. 4. From our data, however, we cannot conclude that $y^* = y^*_m$. The issue whether the conjectured point $(\kappa = \kappa_c, y = Y^*)$ in fig. 5 is a non-trivial fixed point where the fermion mass and the scalar mass could possibly be tuned independently deserves certainly more attention in future calculations [4].

6. Conclusions

In spite of appreciable finite size effects, our results for the fermion mass and the renormalized Yukawa coupling seem to be in accordance with the perturbation theory in the small Yukawa coupling region. In our quenched simulation with naive fermions there is a clear indication, however, of a non-perturbative Yukawa coupling region where the fermion mass increases as we approach the critical point. This might have an important implication for the question of decoupling of fermions whose masses are generated by Yukawa coupling. Though we expect the quenched approximation to be qualitatively valid for large Yukawa couplings, we intend to investigate the existence of a non-perturbative Yukawa coupling region in an unquenched simulation. We think that the unquenched numerical calculations in the one-component model performed until now [3] did not find such a region because the large parameter space has not yet been sufficiently explored. When we include also the Wilson–Yukawa term [16], a very interesting issue naturally will be whether we can make use of the observed non-perturbative behaviour of the fermion mass for strong bare Yukawa coupling in decoupling the unwanted doublers.

Acknowledgement

We thank J. Smit for various helpful suggestions. We have also benefited from discussions with M. Göckeler and C.B. Lang. The continuous support by H.A. Kastrup is gratefully acknowledged. The numerical simulations have been performed on the Cray X-MP/48 at HLRZ Jülich.

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