FOURTH GENERATION EFFECTS IN B PHYSICS

David LONDON [1]
Deutsches Elektronen Synchrotron - DESY, D-2000 Hamburg 52, FRG

Received 29 August 1989; revised manuscript received 4 November 1989

Effects of a fourth generation in B_s^0–$\overline{B}_s^0$ mixing (x_s) and CP violation in the B system are discussed. Although four-generation models can accomodate practically any value of x_s, most of the four-generation parameter space predicts x_s to be in the standard model range. At most ~1% of the parameter space predicts small B_s^0–B_s^0 mixing ($x_s < 2$). The effects are much more striking in CP violating hadronic decay asymmetries. The CP asymmetry in $B_d \to \Psi K_S$ is found to be negative in about half of the four-generation parameter space, and ~40% of the space predicts an asymmetry $|A(B_s \to \Psi\phi)| > 0.2$.

It is well known that the measurement of B_s^0–B_s^0 mixing will be an important test of the three-generation standard model (SM) [1] [#1]. In the SM, B_q^0–$\overline{B}_q^0$ mixing is dominated by t-quark exchange [3] (fig. 1), so that

$$\frac{x_s}{x_d} = \left(\frac{\eta_{B_s} M_{B_s} \tau_{B_s} f_{B_s}^2 B_{B_s}}{\eta_{B_d} M_{B_d} \tau_{B_d} f_{B_d}^2 B_{B_d}}\right) \left|\frac{V_{ts}^* V_{tb}}{V_{td}^* V_{tb}}\right|^2 , \tag{1}$$

where the mixing parameter $x_q = \Delta M_{B_q}/\Gamma_{B_q}$. In eq. (1), η_{B_q} are QCD corrections, $f_{B_q}^2 B_{B_q}$ represents our lack of knowledge of the hadronic matrix element $\langle B_q^0 | [\bar{q}\gamma^\mu(1-\gamma_5)b]^2 | \overline{B}_q^0 \rangle$, and the V_{ij} are the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements. Using the Wolfenstein parametrization of the CKM matrix [4],

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\rho\lambda^3 \exp(-i\delta) \\ -\lambda[1+A^2\lambda^4\rho\exp(i\delta)] & 1-\frac{1}{2}\lambda^2 - A^2\rho\lambda^6\exp(i\delta) & A\lambda^2 \\ A\lambda^3[1-\rho\exp(i\delta)] & -A\lambda^2[1+\lambda^2\rho\exp(i\delta)] & 1 \end{pmatrix} , \tag{2}$$

where $\lambda = 0.22$, $A \simeq 1.00$, and $\rho \leqslant 0.9$ [5], one would naively expect $x_s \sim x_d/\lambda^2 \sim 14$. In fact, taking all uncertainties into account, and assuming that $\eta_{B_s} M_{B_s} \tau_{B_s} f_{B_s}^2 B_{B_s} \simeq \eta_{B_d} M_{B_d} \tau_{B_d} f_{B_d}^2 B_{B_d}$, the SM prediction for x_s becomes

$$2 \leqslant x_s \leqslant 35 . \tag{3}$$

($SU(3)_F$-breaking effects generally act in favour of increasing x_s – for instance, lattice calculations [6] indicate that $f_{B_s}^2 B_{B_s}/f_{B_d}^2 B_{B_d} \sim 1.5$.) Therefore a measurement of $x_s < 2$ would be a clear indication of physics beyond the standard model.

[1] Address after October 1, 1989: Nuclear Physics Laboratory, Université de Montréal, Montréal, Quebec, Canada H3C 3J7.
[#1] For an overview of mixing in heavy quark systems with three and four generations, see ref. [2].

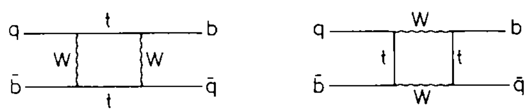

Fig. 1. Standard model contribution to B_q^0–$\overline{B}_q^0$ mixing.

0370-2693/90/$ 03.50

How can one obtain values of $x_s < 2$? Apart from certain extremely fine-tuned left–right symmetric models [7], the only possibility is to add a fourth generation [8]. In this case, the expression for B_q^0–$\bar{B}_q^0$ mixing is altered. In addition to the SM contribution, one must also consider diagrams in which one or both of the t-quarks in fig. 1 are replaced by t′-quarks. Ignoring again $SU(3)_{fl}$-breaking effects, and assuming all QCD corrections to be equal, one has

$$\frac{x_s}{x_d} = \frac{|E(y_t, y_t)(V_{ts}^* V_{tb})^2 + 2E(y_t, y_{t'})(V_{ts}^* V_{tb})(V_{t's}^* V_{t'b}) + E(y_{t'}, y_{t'})(V_{t's}^* V_{t'b})^2|}{|E(y_t, y_t)(V_{td}^* V_{tb})^2 + 2E(y_t, y_{t'})(V_{td}^* V_{tb})(V_{t'd}^* V_{t'b}) + E(y_{t'}, y_{t'})(V_{t'd}^* V_{t'b})^2|}, \tag{4}$$

where $y_i = m_i^2/M_W^2$, and

$$E(y_i, y_j) = y_i y_j \left[\left(\frac{1}{4} + \frac{3}{2}\frac{1}{(1-y_j)} - \frac{3}{4}\frac{1}{(1-y_j)^2} \right) \frac{\ln y_j}{y_j - y_i} + (y_i \leftrightarrow y_j) - \frac{3}{4}\frac{1}{(1-y_i)(1-y_j)} \right]. \tag{5}$$

Here it is possible to obtain small x_s. First of all, in the three-generation SM, CKM matrix unitarity constraints gave $V_{td} \sim \lambda V_{ts}$, which led automatically to large x_s. With four generations, these constraints are relaxed, so that the t-quark contribution to B_s^0–$\bar{B}_s^0$ mixing need not be sizeable. Secondly, one can have cancellations among the terms in the numerator of eq. (4). It is clear, therefore, that small values of x_s are possible in fourth-generation models.

One might wonder, however, how likely this is. That is, what fraction of the allowed parameter space actually predicts small x_s? In a recent paper [9], it was claimed by Hewett and Rizzo that this fraction is quite large. Using $x_s/x_d < 1$ to be their definition of small x_s, they found that, depending on the t- and t′-quark masses, between 20% and 45% of the parameter space gave $x_s < x_d$! This is an extremely surprising result. First of all, one would expect the cancellations needed in eq. (4) to give $x_s < x_d$ to be rather delicate, and therefore less probable. Secondly, the experimental limit on the CKM matrix element $|V_{cs}|$ is rather weak: $|V_{cs}| > 0.66$ (90% CL) [10]. It is only unitarity which restricts it to the range $0.9733 \leqslant |V_{cs}| \leqslant 0.9754$ in three generations [10]. With four generations, this unitarity constraint no longer applies. In fact, large values of V_{ts} (and $V_{t's}$) are allowed. Therefore, one would guess that four-generation models favour *larger* values of x_s than the SM, not smaller values. For these reasons it seems worthwhile to repeat this analysis, and it is this work which is the main point of this paper.

I will use the Botella–Chau [11] parametrization of the four-generation CKM matrix shown in table 1. This parametrization is particularly convenient because the allowed ranges of the angles are easily obtainable from experimental information about the V_{ij}. That is, $s_x \simeq \lambda$ (from V_{us}), $s_z \lesssim \lambda^3 (V_{ub})$, $s_y \simeq \lambda^2 (V_{cb})$, and $s_w \lesssim \lambda^2$ (unitarity). Since, in four-generation models, V_{cs} can be as small as 0.66, s_v can be as large as 0.72. s_u and the phases ϕ_i are free. The t and t′ masses are taken in the ranges

$$78\ \text{GeV} \leqslant m_t \leqslant 200\ \text{GeV}, \quad m_t \leqslant m_{t'} \leqslant 400\ \text{GeV}. \tag{6}$$

The lower m_t limit comes from the latest CDF measurements [12]; the upper bound comes from considering

Table 1
The four-generation CKM matrix.

$c_w c_x c_z$	$c_w s_x c_z$	$c_w s_z \exp(-i\phi_1)$	$s_w \exp(-i\phi_2)$
$-s_v s_w c_x c_z \exp[i(\phi_2-\phi_3)] - c_v s_x c_y - c_v c_x s_y s_z \exp(i\phi_1)$	$-s_v s_w s_x c_z \exp[i(\phi_2-\phi_3)] + c_v c_x c_y - c_v s_x s_y s_z \exp(i\phi_1)$	$-s_v s_w s_z \exp[i(\phi_2-\phi_3-\phi_1)] - c_v s_y c_z$	$s_v c_w \exp(-i\phi_3)$
$-s_u c_v s_w c_x c_z \exp(i\phi_2) + s_u s_v s_x c_y \exp(i\phi_3) + s_u s_v c_x s_y s_z \exp[i(\phi_3+\phi_1)] + c_u s_x s_y - c_u c_x c_y s_z \exp(i\phi_1)$	$-s_u c_v s_w s_x c_z \exp(i\phi_2) - s_u s_v c_x c_y \exp(i\phi_3) + s_u s_v s_x s_y s_z \exp[i(\phi_3+\phi_1)] - c_u c_x s_y - c_u s_x c_y s_z \exp(i\phi_1)$	$s_u c_v s_w s_z \exp[i(\phi_2-\phi_1)] - s_u s_v s_y c_z \exp(i\phi_3) + c_u c_y c_z$	$s_u c_v c_w$
$-c_u c_v s_w c_x c_z \exp(i\phi_2) + c_u s_v s_x c_y \exp(i\phi_3) + c_u s_v c_x s_y s_z \exp[i(\phi_3+\phi_1)] - s_u s_x s_y + s_u c_x c_y s_z \exp(i\phi_1)$	$-c_u c_v s_w s_x c_z \exp(i\phi_2) - c_u s_v c_x c_y \exp(i\phi_3) + c_u s_v s_x s_y s_z \exp[i(\phi_3+\phi_1)] + s_u c_x s_y + s_u s_x c_y s_z \exp(i\phi_1)$	$c_u c_v s_w s_z \exp[i(\phi_2-\phi_1)] - c_u s_v s_y c_z \exp(i\phi_3) - s_u c_y c_z$	$c_u c_v c_w$

radiative corrections to the ρ parameter [13]. The upper bound on $m_{t'}$ comes from perturbative unitarity [14]. Above this value perturbation theory breaks down and the simple box diagram approximation is no longer valid. Note that the upper bound of 200 GeV from the ρ parameter applies also to the t′–b′ mass difference. Thus, unless the t′ and b′ quarks are fairly degenerate (which seems somewhat unlikely, based on our experience with t and b quarks), the bound of 200 GeV applies to the t′ as well.

The above angles, phases and masses must also satisfy constraints from ΔM_K, ϵ and x_d. The theoretical expressions are

$$\Delta M_K = \frac{G_F^2 M_K M_W^2 f_K^2}{6\pi^2} B_K \left| \sum_{i,j=c,t,t'} \eta_{ij}^K E(y_i, y_j) \mathrm{Re}(V_{id}^* V_{is} V_{jd}^* V_{js}) \right| , \tag{7}$$

$$\epsilon = \exp(i\pi/4) \frac{G_F^2 M_K M_W^2 f_K^2}{12\sqrt{2}\pi^2 \Delta M_K} B_K \left| \sum_{i,j=c,t,t'} \eta_{ij}^K E(y_i, y_j) \mathrm{Im}(V_{id}^* V_{is} V_{jd}^* V_{js}) \right| . \tag{8}$$

$$x_{B_q} = \frac{G_F^2 M_{B_q} M_W^2 \tau_{B_q}}{6\pi^2} f_{B_q}^2 B_{B_q} \left| \sum_{i,j=t,t'} \eta_{ij}^{B_q} E(y_i, y_j) (V_{iq}^* V_{ib} V_{jq}^* V_{jb}) \right| . \tag{9}$$

The experimental values are [#2]

$$\Delta M_K = (3.521 \pm 0.014) \times 10^{-12}\ \mathrm{MeV}, \quad |\epsilon| = (2.28 \pm 0.02) \times 10^{-3}, \quad x_d = 0.70 \pm 0.13. \tag{10}$$

In the above equations, the η's are QCD corrections. For the kaon system, they are taken to be [#3] $\eta_{cc}^K \simeq 0.7$, $\eta_{tt}^K \simeq 0.6$, $\eta_{ct}^K \simeq 0.5$, $\eta_{ct'}^K \simeq 0.5$, $\eta_{tt'}^K \simeq 0.5$, $\eta_{t't'}^K \simeq 0.6$. In the B system, $\eta_{tt}^B \simeq \eta_{tt'}^B \simeq \eta_{t't'}^B$ is assumed and a value $\eta_{tt}^B \simeq 0.85$ [3] is used, although it must be noted that a recent calculation [17] obtains a lower value, $\eta_{tt}^B \simeq 0.63$. (The difference is basically due to the scale at which the QCD corrections are evaluated.) In eqs. (7), (8), the bag parameter B_K denotes our ignorance of the hadronic matrix element $\langle K^0 | [\bar{d}\gamma^\mu (1-\gamma_5) s]^2 | \bar{K}^0 \rangle$. Reasonable ranges for the hadronic uncertainties are

$$\tfrac{1}{3} \leqslant B_K \leqslant 1, \quad (100\ \mathrm{MeV})^2 \leqslant f_{B_d}^2 B_{B_d} \leqslant (200\ \mathrm{MeV})^2. \tag{11}$$

In the case of ΔM_K, long-distance effects are important, so that considering only the short-distance contributions (eq. (7)) is not a good approximation. However, it is reasonable to demand that the short-distance contributions do not exceed the experimental value.

In principle, there are other experimental data which could further constrain the four-generation parameter space. For example, the NA31 group has measured ϵ'/ϵ to be $(33.3 \pm 1.1) \times 10^{-1}$ [18]. Theoretically, however, the hadronic uncertainties are enormous, making it very difficult to relate the experimental value to the CKM matrix elements. In fact, realistically, the only information which is useful is the sign of ϵ'/ϵ. But this only has the effect of cutting the allowed parameter space in half; the fraction of this space which predicts any particular range for x_s is unchanged. Furthermore, preliminary results from Fermilab [19] give $\epsilon'/\epsilon = (-0.5 \pm 1.5) \times 10^{-3}$, in mild conflict with the above measurement. For these reasons, ϵ'/ϵ is not included in the analysis. Other processes, such as D^0–$\bar{D}^0$ mixing (given a value for the b′ mass), $K \to \pi\nu\bar{\nu}$, etc., constrain the parameter space very little (if at all), and are likewise not included.

One problem is that the criteria for deciding which points in the parameter space satisfy experimental constraints are somewhat arbitrary. For example, should one require that the central values of $|\epsilon|$ and x_d be reproduced exactly, or should one take a 90% CL (or 3σ) region? Similarly, one must deal with the fact that some of the V_{ij} have only 90% CL limits, while others have been measured.

One possible prescription is to use only 90% CL limits. The angles and phases (s_k, ϕ_l, $k = x, y, z, u, v, w$; $l = 1, 2, 3$) are required to reproduce the V_{ij} within their 90% CL ranges. m_t and $m_{t'}$ are varied randomly within their

[#2] For ΔM_K, $|\epsilon|$, see ref. [10]; for x_d, see ref. [15].

[#3] Standard model QCD corrections in the kaon system have been calculated in ref. [16].

allowed ranges. This set of points (s_k, ϕ_l, m_t, $m_{t'}$) is then considered to satisfy experiment if there exist values of B_K and $f_{B_d}^2 B_{B_d}$ in their allowed ranges such that the calculated value of ΔM_K is less than the experimental value, and such that the calculated $|\epsilon|$ and x_d fall within their 90% CL ranges. (In this paper, this will be referred to as the "90% CL prescription".) Using this set of points, the value of the ratio x_s/x_d is then determined (eq. (4)).

Another possibility is to assign a statistical weight to each set of points [20]. In this case, $SU(3)_f$-breaking is ignored, and it is assumed that all values of B_K and $f_{B_d}^2 B_{B_d}$ in the ranges of eq. (11) are equally likely. Each set of points (s_k, ϕ_l, m_t, $m_{t'}$, B_K, $f_{B_d}^2 B_{B_d}$) is required to satisfy the constraint from ΔM_K. $|\epsilon|$ and x_d are calculated and this set of points is then assigned a weight $\exp(-\chi^2/2)$, where

$$\chi^2 = \left(\frac{|\epsilon|_{\rm calc} - |\epsilon|_{\rm exp}}{\Delta|\epsilon|}\right)^2 + \left(\frac{(x_d)_{\rm calc} - (x_d)_{\rm exp}}{\Delta x_d}\right)^2. \tag{12}$$

(This will be called the "χ^2 prescription".) The value of x_s calculated from this set of parameters is given the same weight; when the entire parameter space is integrated over, this yields a probability distribution for the prediction of x_s in the four-generation SM. That is, the probability of finding x_s with the value x_s^0 is

$$P(x_s^0) = \frac{\int dz_i \exp[-\chi^2(z_i)/2]\,\delta(x_s(z_i) - x_s^0)}{\int dz_i \exp[-\chi^2(z_i)/2]}, \tag{13}$$

where the z_i are the parameters in the space. (The shape of the distribution is, of course, somewhat dependent on the assumption that all values of m_t, $m_{t'}$, B_K and $f_{B_d}^2 B_{B_d}$ are equally probable within their allowed ranges.) From this, the fraction of the parameter space which has $x_s < x_d$ (or $x_s < 2$ or $x_s \sim 35$) is easily obtained.

In this paper the results using both prescriptions will be presented. There turns out, however, to be little difference between them. The parameter space is sampled using a Monte Carlo lottery technique. 10^7 sets of (s_k, ϕ_l, m_t, $m_{t'}$) are generated, consistent with experimental limits on the $|V_{ij}|$, and tested against ΔM_K, $|\epsilon|$ and x_d.

Let us first consider the 90% CL prescription. s_x is taken to be equal to 0.22. Values for $|V_{cd}|$ and $|V_{cb}|$ are generated according to [10]

$$|V_{cd}| = 0.21 \pm 0.03, \quad |V_{cb}| = 0.046 \pm 0.010, \tag{14}$$

except that values more than 1.64σ (90% CL) from the central values are not allowed. $|V_{cs}|$ and $|V_{ub}|$ are generated in the ranges

$$0.66 \leqslant |V_{cs}| \leqslant \sqrt{1 - |V_{cd}|^2 - |V_{cb}|^2}, \quad 0 \leqslant |V_{ub}| \leqslant 0.2|V_{cb}|. \tag{15}$$

From the V_{ij}, values for s_y, s_z, s_t and s_w are obtained. Random values of s_u and the ϕ_l are also generated, as are m_t and $m_{t'}$. Each set of points is tested to see if it passes the constraints for ΔM_K, $|\epsilon|$ and x_d. This was done for two cases: first, when the upper bound on $m_{t'}$ was the unitarity bound (400 GeV), and second, using the bound from the ρ parameter (200 GeV). The number of sets which had $x_s < x_d$ was counted, as were those with $x_s/x_d < 2.2$ (minimum SM value) and $x_s/x_d > 71.4$ (maximum SM value).

The results are as follows. For $m_{t'} \leqslant 400$ GeV, of the 10^7 initial sets, ~ 12000 satisfied the constraints from ΔM_K, $|\epsilon|$ and x_d. Of these,

$$0.09\% \text{ had } x_s < x_d, \quad 0.7\% \text{ had } x_s/x_d < 2.2, \quad 3.3\% \text{ had } x_s/x_d > 71.4. \tag{16}$$

In the case of $m_{t'} \leqslant 200$ GeV, ~ 16000 sets of points passed the ΔM_K, $|\epsilon|$ and x_d tests. Here,

$$0.03\% \text{ had } x_s < x_d, \quad 0.2\% \text{ had } x_s/x_d < 2.2, \quad 0.3\% \text{ had } x_s/x_d > 71.4. \tag{17}$$

From these numbers, it is obvious that, even with four generations, small values of x_s are quite unlikely. I find that $x_s < x_d$ less than 0.1% of the time, which clearly contradicts the results of ref. [9]. In fact, most of the parameter space predicts x_s/x_d in the SM range. And, consistent with expectations, those sets of points in which larger values of $m_{t'}$ are allowed predict $x_s/x_d > 71.4$ more often.

Similar results are obtained when the χ^2 prescription is used. In this case, the parameters are generated as in the 90% CL prescription, except that $|V_{cd}|$ and $|V_{cb}|$ are not restricted to be $\leqslant 1.64\sigma$ from their central values. Since we have no other information, $|V_{cs}|$ and $|V_{ub}|$ are still taken to be in their 90% CL ranges. Probability distributions are obtained as detailed above for the two cases $m_t \leqslant m_{t'} \leqslant 400$ GeV and $m_t \leqslant m_{t'} \leqslant 200$ GeV. These are shown in fig. 2. The curves are clearly quite similar. For the $m_{t'} \leqslant 400$ GeV distribution, of the total area under the curve,

$$0.1\% \text{ has } x_s < x_d, \quad 1.2\% \text{ has } x_s < 2, \quad 7.0\% \text{ has } x_s > 35. \tag{18}$$

In the second ($m_{t'} \leqslant 200$ GeV) distribution,

$$0.07\% \text{ has } x_s < x_d, \quad 0.6\% \text{ has } x_s < 2, \quad 1.1\% \text{ has } x_s > 35. \tag{19}$$

Again, small values of x_s are disfavoured – at most 0.1% of the parameter space predicts $x_s < x_d$. Furthermore, $x_s > 35$ is more likely than $x_s < 2$ (particularly for $m_{t'} \leqslant 400$ GeV), which is precisely what one expects. However, the three-generation prediction (eq. (3)) is largely reproduced even with four generations.

Another area which is of great interest is CP violation in the B system. Because of mixing, an initial B^0 or $\bar{B}^0$ state will evolve in time into a mixture of B^0 and $\bar{B}^0$. A nonzero value of the asymmetry

$$A_f = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to \bar{f})} \tag{20}$$

will indicate CP violation. The most interesting CP asymmetries are those in which the final state f is purely hadronic and a CP eigenstate ($\bar{f} = \pm f$) [21,22]. In this case, the asymmetries measure the quantity

$$\alpha_f = -\operatorname{Im}\left(\rho_f \frac{p}{q}\right), \tag{21}$$

where

$$\rho_f = \frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)}, \quad \frac{q}{p} \simeq \sqrt{\frac{M_{12}^*}{M_{12}}}, \tag{22,23}$$

where M_{12} is the off-diagonal element of the B_q^0–$\bar{B}_q^0$ mixing matrix ($\Delta M_q = 2|M_{12}|$).

In the SM, for each of B_d and B_s, there are only two distinct classes of asymmetries – those which involve the quark-level decays $b \to c\bar{c}s$, $c\bar{c}d$ (Cabibbo allowed), and those which have $b \to u\bar{u}s$, $u\bar{u}d$ (Cabibbo suppressed) [22]. Although the CP asymmetries in Cabibbo suppressed B_d and B_s decays may take any value in the three-

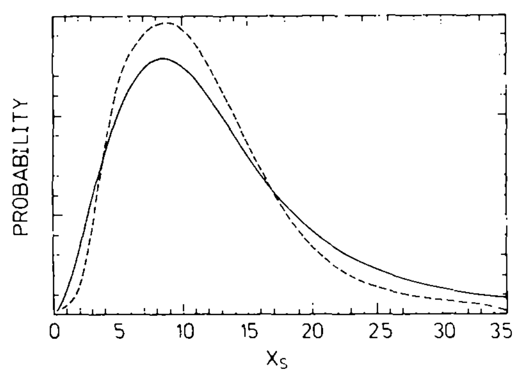

Fig. 2. The probability distribution for the four-generation standard model prediction of x_s. The solid (dashed) line is for $m_{t'} < 400$ GeV (200 GeV).

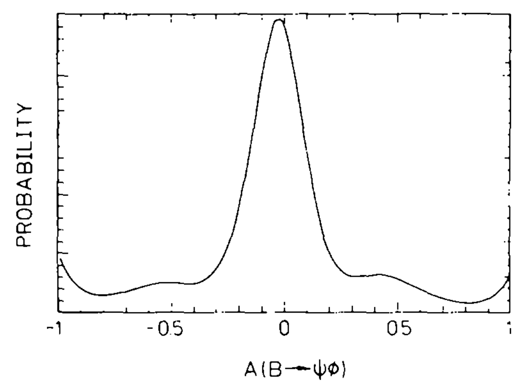

Fig. 3. The probability distribution for the four-generation standard model prediction of the CP asymmetry in the decay $B_s \to \Psi\varphi$.

generation SM, the asymmetry in Cabibbo allowed B_d decays (e.g. $B_d \to \Psi K_S$, D^+D^-) is predicted to be positive. Furthermore, the CP violation in Cabibbo allowed B_s decays (e.g. $B_s \to \Psi\phi$, ΨK_S) should be $\simeq$ zero!

These simple predictions should be altered in models with four generations [23]. First of all, because of the two extra phases in the CKM matrix, one would guess that all values of the CP asymmetries in Cabibbo allowed B_d and B_s decays would be possible. Secondly, since the phases of the matrix elements V_{cd} and V_{cs} are, in general neither equal to one another nor zero, the asymmetries $A(B_d \to \Psi K_S)$ and $A(B_d \to D^+D^-)$ are not expected to be equal (and similarly for $A(B_s \to \Psi\phi)$ and $A(B_s \to \Psi K_S)$). However, it would be interesting to see if these expectations are indeed born out, and how much of the parameter space predicts unequal asymmetries in the above decay modes.

As in the case of x_s, the parameter space is sampled using one of the prescriptions described earlier. The limits on $m_{t'}$ are taken to be $m_t \leqslant m_{t'} \leqslant 400$ GeV. For Cabibbo allowed B_d decays, the 90% CL prescription is used. I find that, as expected, all values of the asymmetry in $B_d \to \Psi K_S$ are allowed, with about equal probability, so that roughly half of the parameter space predicts $A(B_d \to \Psi K_S) < 0$. However, a difference in the asymmetries in $B_d \to \Psi K_S$ and $B_d \to D^+D^-$ is rather rare. Only about 3.5% of the parameter space fulfills the requirement that

$$\left|\frac{A(B_d \to \Psi K_S) - A(B_d \to D^+D^-)}{A(B_d \to \Psi K_S)}\right| \geqslant 10\%. \tag{24}$$

Thus, although negative values of $A(B_d \to \Psi K_S)$ are quite likely, the SM expectation $A(B_d \to \Psi K_S) \simeq A(B_d \to D^+D^-)$ is reproduced in most of the four-generation parameter space.

For Cabibbo allowed B_s decays, it is convenient to use the χ^2 prescription. A probability distribution of the four-generation prediction for $A(B_s \to \Psi\phi)$ is obtained as in eq. (13). This is shown in fig. 3. As expected, all values are allowed, although the distribution is still peaked at 0. $|A(B_s \to \Psi\phi)| > 0.1$ occurs in $\sim 60\%$ of the parameter space, and $\sim 40\%$ of the space gives an asymmetry larger than 20%. In other words, a large fraction of the four-generation parameter space predicts the asymmetry in Cabibbo allowed B_s decays to be significantly different from 0. I also find that $\sim 12\%$ of the allowed points predict $A(B_s \to \Psi\phi)$ and $A(B_s \to \Psi K_S)$ to differ by more than 10%. However, this is rather uninteresting, since a nonzero measurement of either of these asymmetries would by itself indicate new physics.

In conclusion, although essentially any value of B_s^0–$\overline{B}_s^0$ mixing can be accommodated in models with a fourth generation, small values of x_s are unlikely – they occur in a very small region of the allowed four-generation parameter space. At most $\sim 1\%$ of the parameter space predicts $x_s < 2$ (the minimum value in the SM). And $x_s < x_d$ in at most 0.1% of the space, in sharp contrast to the claim of Hewett and Rizzo [9]. In fact, four-generation models are more likely to give larger values of x_s than the SM, in accord with naive expectations. It is more probable that CP violating hadronic decay asymmetries will show effects of a fourth generation. A much larger region of the parameter space predicts non-SM values for these asymmetries. About half of the space predicts the asymmetry in $B_d \to \Psi K_S$ to be negative. (However, only about 3.5% of the allowed points yield values for the asymmetries $A(B_d \to \Psi K_S)$ and $A(B_d \to D^+D^-)$ which differ from one another by more than 10% (they are equal in the SM).) Furthermore, the asymmetry in the Cabibbo allowed B_s decay $B_s \to \Psi\phi$ is found to be larger than 20% in $\sim 40\%$ of the parameter space (it is $\simeq 0$ in the SM). Thus, although a measurement of x_s is not likely to reveal the existence of a fourth generation, measurements of CP violation in the B system may well do so.

I wish to thank Paul Langacker for helpful discussions, and for the hospitality of the University of Pennsylvania, where part of this work was done.

References

[1] See, e.g., G. Belanger et al., DESY preprint DESY 88-181 (1988), in: Proc. DPF Summer Study on High energy physics in the 1990's (Snowmass, CO, 1988), to appear;
P. Krawczyk, D. London and H. Steger, Nucl. Phys. B 321 (1989) 1.
[2] E.A. Paschos and U. Türke, Phys. Rep. 178 (1989) 145.
[3] A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. B 238 (1984) 529; B 245 (1984) 369;
J. Hagelin, Nucl. Phys. B 193 (1981) 123.
[4] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.
[5] For a recent analysis, see C.S. Kim and A.D. Martin, Phys. Lett. B 225 (1989) 186.
[6] C. Bernard et al., Phys. Rev. D 38 (1988) 3540.
[7] D. London and D. Wyler, DESY preprint DESY 89-099 (1989).
[8] I.I. Bigi and A.I. Sanda, Phys. Lett. B 194 (1987) 307;
W.S. Hou and A. Soni, Phys. Lett. B 196 (1987) 92;
A. Datta, E.A. Paschos and U. Türke, Phys. Lett. B 196 (1987) 376.
[9] J.L. Hewett and T.G. Rizzo, Mod. Phys. Lett. A 3 (1988) 975.
[10] Particle Data Group, G.P. Yost et al., Review of particle properties, Phys. Lett. B 204 (1988) 1.
[11] F.J. Botella and L.-L. Chau, Phys. Lett. B 168 (1986) 97.
[12] P. Sinervo, talk Lepton-photon Symp. (Stnaford, CA, 1989).
[13] U. Amaldi et al., Phys. Rev. D 36 (1987) 1385.
[14] M.S. Chanowitz, M.A. Furman and I. Hinchliffe, Nucl. Phys. B 153 (1979) 402;
[15] K. Kleinknecht, Proc. XXIV Intern. Conf. on High energy physics, eds. R. Kotthaus and J.H. Kühn (Springer, Berlin, 1988) p. 98.
[16] F.J. Gilman and M.B. Wise, Phys. Rev. D 27 (1983) 1128.
[17] W.A. Kaufman, H. Steger and Y.P. Yao, Mod. Phys. Lett. A 3 (1988) 1479.
[18] NA31 Collab., H. Burkhardt et al., Phys. Lett. B 206 (1988) 169.
[19] B. Winstein, talk Lepton-photon Symp. (Stanford, CA, 1989).
[20] D. London, DESY preprint DESY 89-048 (1989).
[21] A. Carter and A.I. Sanda, Phys. Rev. Lett. 45 (1980) 952; Phys. Rev. D 23 (1981) 1567;
I.I. Bigi and A.I. Sanda, Nucl. Phys. B 193 (1981) 85; B. 281 (1987) 41;
I. Dunietz and J. Rosner, Phys. Rev. D 34 (1986) 1404;
Y. Azimov, V. Khoze and M. Uraltsev, Yad. Fiz. 45 (1987) 1412;
D. Du, I. Dunietz and D. Wu, Phys. Rev. D 34 (1986) 3414.
[22] P. Krawczyk, D. London, R.D. Peccei and H. Steger, Nucl. Phys. B 307 (1986) 19.
[23] M. Tanimoto, Y. Suetaka and K. Senba, Z. Phys. C 40 (1988) 539;
M. Tanimoto, Ehime University preprint EHU-07-88 (1988);
T. Hasuike, T. Hattori, T. Hayashi and S. Wakaizumi, University of Tokushima preprint TOKUSHIMA 89-01 (1989).