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QCD Plasma Parameters from the S-Matrix

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The one-loop damping rate of color oscillations in a QCD plasma has previously been found to be gauge dependent, with even different gauge independent frameworks yielding differing results. In this paper manifestly gauge independent QCD plasma parameters are derived from the structure functions of on-shell scattering amplitudes of test particles which are sufficiently heavy so as to permit the definition of an S-matrix in the usual sense despite the presence of a plasma, and suppressing on-shell infrared divergencies in the limit of infinite masses. This provides an alternative to the usual linear response analysis based on the gluon propagator. The one-loop result coincides with the one based on Cornwall's "gauge invariant propagator." It is pointed out that higher-loop corrections will modify the one-loop result at the same order of magnitude, and an approximation towards inclusion of higher-loop effects is considered.

1. INTRODUCTION

Quantum chromodynamics (QCD) is expected to undergo a deconfinement phase transition at a temperature $T \sim \Lambda_{\text{QCD}} \sim 200$ MeV, and at sufficiently high temperature and density asymptotic freedom should provide room for a perturbative analysis of the resulting quark–gluon plasma [1–3].

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Certain collective phenomena can be studied by a calculation of the finite temperature one-loop gluon self energy, which is recapitulated in Section 2. This reveals that static color electric fields are screened through the appearance of an electric mass $m_{el} \sim gT$, while a corresponding magnetic mass is thought to be generated non-perturbatively at the order g^2T, thereby solving the problem of infra-red divergences arising at higher loops [4]. Starting from a minimal frequency $\omega_{pl} \sim gT$ there can also be weakly damped plasma oscillations with decay rate $\gamma \sim g^2T$.

In contrast to the leading order results m_{el} and ω_{pl}, the one-loop plasmon damping constant γ, however, has turned out to be gauge fixing dependent [1, 5–12], most often even carrying a negative sign which seems to signal instability of the perturbative vacuum. The only positive results have been obtained in the temporal axial gauge [6] and in the Coulomb gauge [7, 8], where a direct connection between the gluon propagator and correlation functions for the color electric field strength can be established. In [9] the one-loop plasmon damping constant has been derived from background gauge invariant effective actions [13], which still depend on the gauge fixing parameters, and as in conventional covariant gauges [1] negative definite values have been obtained. In order to achieve manifest gauge independence, the concept of the off-shell reparametrization invariant effective action due to Vilkovisky and DeWitt [14] has been invoked [9, 10], still with negative result. Another manifestly gauge independent approach [11] based on Cornwall's "gauge invariant propagator" [15] also has led to a negative one-loop result, but differing in magnitude from the former. In the latter approach gauge independence is achieved by rearranging contributions to on-shell amplitudes such that a gauge independent propagator-like contribution is distilled out. This procedure, however, has recently been critized as being ambiguous [16].

In Section 3 we extract plasma parameters from on-shell scattering amplitudes in a different manner. We consider scattering of fictitious test particles with masses $M \gg T$ so that for these particles an S-matrix in the conventional sense can be defined. Even in QCD such a scattering amplitude exists in the limit of infinite mass M, because all problems with soft gluon bremsstrahlung from external lines disappear in this case [19]. Then we look for poles corresponding to collective modes in the analytically continued structure functions of this scattering amplitude. Thereby we again find a negative one-loop plasmon damping constant, coinciding with the particular value of Ref. [11].

However, since in this context a one-loop calculation is inaccurate (cf. Section 4), the most important aspect of our approach is that it establishes an alternative to the usual linear response analysis based on the gluon propagator. Unlike the aforementioned attempts to achieve gauge independence, we do not single out one particular gluon propagator but focus on the naturally gauge independent scattering amplitudes.

In Section 4 we finally discuss higher-loop corrections, concluding that they still can modify the one-loop result at the considered order g^2T. As an attempt towards inclusion of higher-loop effects, we consider an approximation to the dressed propagator introduced in [6, 7], finding that the modified one-loop plasmon damping constant still is gauge dependent and negative in the covariant gauges, but zero in the non-covariant ones as well as for our scattering amplitude.

In the Appendix we give the evaluation of the finite temperature gluon self energy in conventional and background Feynman gauge in the high-T expansion down to the order T^0.

2. The Finite-T One-Loop Gluon Propagator

In linear response theory [17, 22] the basic quantity from which plasma parameters are obtained usually is the propagator. In our case this is the gluon propagator

$$\begin{aligned}\langle TA^a_\mu(x)\, A^b_\nu(y)\rangle_\beta &= \mathrm{Tr}[e^{-\beta H} TA^a_\mu(x)\, A^b_\nu(y)]/\mathrm{Tr}[e^{-\beta H}] \\ &= \delta^{ab} \Delta_{\mu\nu}(x-y),\end{aligned} \tag{2.1}$$

evaluated at high temperatures $T = 1/\beta \gg \Lambda_{\mathrm{QCD}}$.

At finite temperatures, there is a preferred coordinate system, the rest frame of the plasma, and unless a non-covariant choice of gauge further reduces Lorentz symmetry, the general form of the (inverse) propagator is given by

$$\Delta^{-1}_{\mu\nu}(Q) = a(Q)\, A_{\mu\nu} + b(Q)\, B_{\mu\nu} + c(Q)\, C_{\mu\nu} + d(Q)\, D_{\mu\nu}, \tag{2.2}$$

where in momentum space ($Q_\mu = (q_0, q_i)$) we employ the following tensorial basis

$$A_{\mu\nu} = \delta_{\mu i}\left(\frac{q_i q_j}{q^2} - \delta_{ij}\right)\delta_{j\nu}, \tag{2.3}$$

$$B_{\mu\nu} = g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} - A_{\mu\nu}, \tag{2.4}$$

$$C_{\mu\nu} = \frac{1}{\sqrt{2q^2}}\left[\left(\delta_{\mu 0} - \frac{q_0 Q_\mu}{Q^2}\right) Q_\nu + Q_\mu\left(\delta_{\nu 0} - \frac{q_0 Q_\nu}{Q^2}\right)\right], \tag{2.5}$$

$$D_{\mu\nu} = \frac{Q_\mu Q_\nu}{Q^2}, \tag{2.6}$$

of which A and B are transverse with respect to Q_μ.

In the following we shall restrict ourselves to the Feynman gauge, where the bare propagator is as simple as possible. In order to be able to make out potential gauge dependences, we shall consider the conventional Feynman gauge (FG) as well as its background covariant version [13] (BFG).

In both, FG and BFG, the gluon self energy

$$\Pi_{\mu\nu} = \Delta^{-1}_{\mu\nu} - \Delta^{-1}_{(0)\mu\nu} \tag{2.7}$$

turns out to be transverse, even at finite temperature. (In the case of background covariant gauges this is a consequence of background gauge invariance, while the conventional covariant gauges in general lose transversality at finite T.) As a consequence, there are only two independent structure functions in the corresponding gluon propagator,

$$\Delta_{\mu\nu}(Q)=\frac{1}{a(Q)}A_{\mu\nu}+\frac{1}{b(Q)}B_{\mu\nu}+\frac{1}{Q^2}D_{\mu\nu}, \tag{2.8}$$

with a describing spatially transverse, and b describing spatially longitudinal modes.

In terms of the gluon self energy, $a(Q^2)$ and $b(Q^2)$ are given generally by

$$a(Q^2)=Q^2-\frac{1}{2}\left[\frac{q^iq^j}{q^2}\Pi_{ij}-\Pi_{ii}\right], \tag{2.9}$$

$$b(Q^2)=Q^2-\left[\Pi_{00}-\frac{q^iq^j}{q^2}\Pi_{ij}-\frac{Q^\mu Q^\nu}{Q^2}\Pi_{\mu\nu}\right], \tag{2.10}$$

which in the case of a transverse self energy reduces to

$$a(Q^2)=Q^2-\frac{1}{2}\left[\frac{q_0^2}{q^2}\Pi_{00}-\Pi_{ii}\right], \tag{2.11}$$

$$b(Q^2)=Q^2+\frac{Q^2}{q^2}\Pi_{00}. \tag{2.12}$$

The two relevant quantities Π_{00} and Π_{ii} are evaluated in one-loop approximation in the Appendix for the case of a purely gluonic SU(N) Yang–Mills theory. (We omit quarks as they play no role in the question of gauge dependence at one-loop order. For their contributions consult, e.g., Ref. [8].)

The leading parts of order g^2T^2 in (2.11) and (2.12) are given by the gauge independent expressions

$$a(Q^2)=Q^2-\frac{g^2NT^2}{6}\left\{\frac{q_0^2}{q^2}+\left(1-\frac{q_0^2}{q^2}\right)\frac{q_0}{2q}\ln\left(\frac{q_0+q}{q_0-q}\right)\right\}, \tag{2.13}$$

$$b(Q^2)=Q^2-\frac{g^2NT^2}{3}\left(1-\frac{q_0^2}{q^2}\right)\left(1-\frac{q_0}{2q}\ln\left(\frac{q_0+q}{q_0-q}\right)\right). \tag{2.14}$$

In the static limit $q_0\to 0$, the results are

$$\lim_{q_0\to 0}\Pi_{00}(Q)=\frac{g^2NT^2}{3}+g^2NqT\times\begin{cases}-\frac{1}{4} & \text{(FG)}\\ -\frac{1}{2} & \text{(BFG)}\end{cases}+O(\ln T), \tag{2.15}$$

$$\lim_{q_0\to 0}\Pi_{ii}(Q)=g^2NqT\times\begin{cases}-\frac{3}{8} & \text{(FG)}\\ \frac{7}{8} & \text{(BFG)}\end{cases}+O(\ln T), \tag{2.16}$$

where the leading term in (2.15) is the well-known electric screening mass, whereas (2.16) does not give rise to a magnetic mass. Note that the terms $O(T)$ are gauge dependent.

The limit of long wavelengths, $q \to 0$, is relevant for the study of plasma oscillations. Since in this limit

$$\lim_{q \to 0} \Pi_{jk} = \lim_{q \to 0} \tfrac{1}{3}\delta_{jk}\Pi_{ii}, \tag{2.17}$$

now the longitudinal and the transverse modes coincide,

$$a(q_0, 0) = b(q_0, 0) = q_0^2 + \tfrac{1}{3}\Pi_{ii}(q_0, 0), \tag{2.18}$$

and from the results of the Appendix we read off

$$\lim_{q \to 0} \frac{1}{3}\Pi_{ii} = g^2 N\left(-\frac{1}{9}T^2 + i\frac{q_0 T}{12\pi} \times \begin{cases} -5 & (\text{FG}) \\ -11 & (\text{BFG}) \end{cases} + O(\ln T)\right). \tag{2.19}$$

Determing the pole in $1/a(q_0, 0)$ or equivalently in $1/b(q_0, 0)$,

$$0 = a(q_0, 0) = b(q_0, 0) = q_0^2 - (\omega - i\gamma)^2, \tag{2.20}$$

the real part of (2.19) yields the plasma frequency

$$\omega_{\text{pl}}^2 = g^2 N T^2/9. \tag{2.21}$$

The dispersion relations for plasma oscillations with $q \neq 0$ is displayed in Fig. 1, where it is made evident by the quadratic scales for ω and q that the dispersion relations are not Lorentz invariant for which they would have to be straight lines parallel to the diagonal. In other words, the plasmon "mass" is dependent on the momentum and it approaches zero for the spatially longitudinal mode, but is a non-vanishing constant in the case of the transverse one.

The plasmon damping "constant" γ is usually defined in the long-wavelength limit and is determined by the imaginary part of q_0 in Eq. (2.20). In contrast to the plasma frequency (2.21) it turns out to be gauge dependent, and in our case even becomes negative,

$$\gamma = \frac{g^2 N T}{24\pi} \times \begin{cases} -5 & (\text{FG}) \\ -11 & (\text{BFG}) \end{cases}. \tag{2.22}$$

In view of missing higher-loop results, which, as we shall discuss later, still can modify (2.22), there have been various attempts to ameliorate the lowest order calculation by methods which promise manifest gauge independence.

Kajantie and Kapusta [6] have studied the gauge covariant correlation function

$$\langle \mathbf{E}^a(x)\, \mathbf{E}^b(y) \rangle,$$

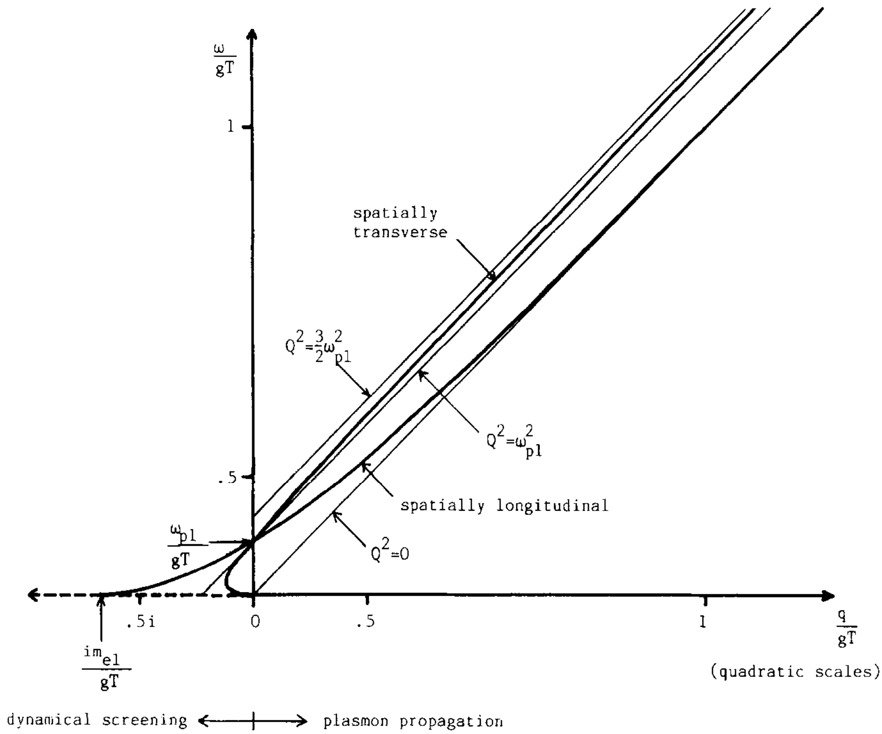

FIG. 1. High temperature dispersion relations for the spatially longitudinal and transverse gluonic modes. The enveloping straight lines correspond to the light cone and to the mass hyperboloid of mass $\sqrt{\frac{3}{2}}\,\omega_{pl}$ (note the quadratic scales).

which is simply related to the gluon self energy in the temporal gauge, and have obtained a positive one-loop damping constant $\gamma = +g^2NT/24\pi$. In response to questions concerning difficulties [23] with the temporal gauge, this result has been reproduced starting from the Coulomb gauge [7, 8], but not in covariant gauges [24]. Indeed, a gauge covariant correlation function need not be gauge fixing independent.

Alternatively, Hansson and Zahed [9] have studied background field gauge invariant effective actions[1] to extract the plasma parameters. However, in spite of (background) gauge invariance, the problem of gauge fixing dependences remains, so these authors have appealed to the concept of the reparametrization invariant effective action due to Vilkovisky and DeWitt [14], which achieves gauge invariance and gauge fixing independence even off the physical mass-shell, and which singles out the background covariantized Landau gauge. To one-loop order the latter yields a negative damping constant, $\gamma = -45g^2NT/96\pi$.

[1] It has been overlooked in Ref. [9] that also the temporal gauge can be interpreted as a background covariant gauge, as is the case for all homogeneous axial gauges in Yang–Mills theories [18].

Kobes and Kunstatter [10] have pointed out that an application of the Vilkovisky–DeWitt effective action which is compatible with linear response theory necessitates a modification of the calculation of Ref. [9] resulting in the value $\gamma = -9g^2NT/32\pi$, which still is negative, though.

Finally, Nadkarni [11] has argued in favour of Cornwall's "gauge invariant propagator" [15] which happens to coincide exactly with the one of the background covariant Feynman gauge, where value and sign of g is bound up with the QCD β-function [12]. In Cornwall's approach gauge independence is achieved by rearranging contributions to on-shell amplitudes in propagator-like and vertex-like parts. However, this procedure has been criticized recently as having ambiguities of its own [16].

In the following we will adhere to a different strategy. Instead of investigating the gluon propagator as an ingredient of a linear response analysis, we will extract the plasmon parameters from the pole structure of on-shell scattering amplitudes.

Neither the gluon potential nor even the gluon field strengths are observables, since they are not gauge invariant. A naturally gauge independent object (as opposed to the more or less artificial gauge independence of the aforementioned attempts) would be provided by a scattering amplitude. In order to be able to define an S-matrix in the usual ($T=0$) sense, we resort to scattering of fictitious test particles with mass $M \gg T$ and by employing colored ones we can probe the dynamics of the QCD plasma. (As we shall show, the infrared problem of this process can be eliminated by the limit $M \to \infty$.) By this we can deal with a simple and in-principle measurable process which should contain the information that our test particles exchange plasmons rather than $T=0$ gluons. These should then appear as poles of the scattering amplitude in the (analytically continued) momentum transfer variable.

3. QCD Plasma Parameters from an on-Shell Scattering Amplitude

On-shell scattering of super-heavy test fermions in ($T=0$) QCD has been proposed some time ago by Kummer [19] to define a manifestly gauge independent renormalization scheme, the "mass-shell momentum subtraction scheme" (MMOM). In the limit of masses $M \to \infty$ the fictitious particles disappear [25] from the "low-energy" theory after having served their purpose as "observers." Moreover, this limit provides a most simple kinematical situation where the fermions can be treated non-relativistically, with the additional advantage that the usual on-shell IR-singularities of the scattering amplitude are removed. In the cross-section these divergences are now suppressed by a factor $Q^2/M^2 \to 0$, where Q is the momentum transfer. In a sense, MMOM thus provides an alternative to the Thomson limit in QED, which is not viable in non-abelian gauge theories.

In our generalization of the original MMOM scheme to $T \neq 0$, we must not take the limit of non-relativistic test fermions, $|\mathbf{p}|/p_0 \to 0$, in the rest frame of the plasma, since this would mean $q_0 \to 0$ for the momentum transfer and we would only be able to study the static limit. Unlike the zero temperature case we cannot invoke Lorentz symmetry for analytical continuation to general Q, so we have to ensure $q_0 \neq 0$. Therefore we take $p_0, |\mathbf{p}| \gg T \gg q_0, |\mathbf{q}|$. In this limit all spin-dependent corrections will vanish. For two different fermion momenta we additionally take $|p_0 - p'_0|, |\mathbf{p} - \mathbf{p}'| \ll p_0, |\mathbf{p}|$, that is, the fermions are assumed to be non-relativistic in their center-of-mass system which may be relativistic with respect to the plasma rest frame. This is necessary for the elimination of infra-red divergencies.

The general structure of our scattering amplitude thus reduces to $g^2 j^\mu(P) j^\nu(P) \mathscr{A}_{\mu\nu}$, where $\mathscr{A}_{\mu\nu}$ is a symmetric tensor transverse to Q_μ in virtue of current conservation. As in the case of a transverse gluon self energy, $\mathscr{A}$ contains two structure functions corresponding to spatially transverse and longitudinal modes, $\mathscr{A}_{\mu\nu} = (1/a)\, A_{\mu\nu} + (1/b)\, B_{\mu\nu}$. The scattering amplitude is thus parametrized through

$$j^\mu(P)\, j^\nu(P)\, \mathscr{A}_{\mu\nu} = j^\mu j_\mu \frac{1}{b} + j_i j_j \left(\frac{q_i q_j}{q^2} - \delta_{ij} \right) \left(\frac{1}{a} - \frac{1}{b} \right). \tag{3.1}$$

In the previous section we have already obtained the contribution of the one-loop gluon propagator to (3.1). We now have to compute the remaining contributions as displayed in Fig. 2. We again perform our calculations in FG and

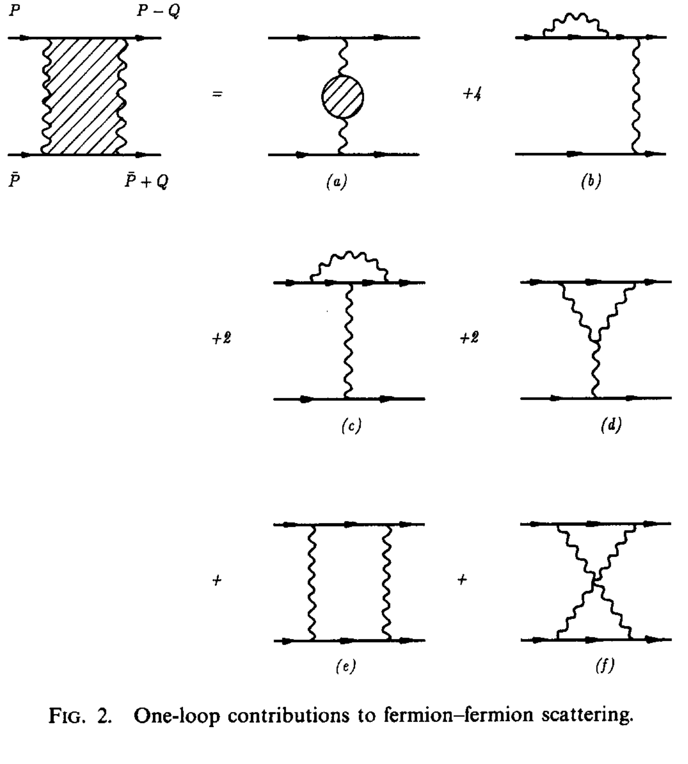

FIG. 2. One-loop contributions to fermion–fermion scattering.

BFG in order to demonstrate gauge independence of our result. In addition we have checked independence in the general covariant gauges, although we shall not give the details here.

3.1. *Fermion Self Energy*

With fermion masses $M \gg T$, the bare Fermi propagator reduces to the one at $T=0$, and we have only Bose–Einstein distribution functions

$$n(k)=\frac{1}{e^{k/T}-1}$$

associated with the gluon momenta.

The fermion self energy is thus given by

$$\begin{aligned}\Sigma(P) &= -g^2 C_{\mathrm{F}} \int \frac{d^4K}{(2\pi)^3}\,\delta(K^2)\,n(k)\,\frac{\gamma^\mu(\not{P}-\not{K}+M)\,\gamma_\mu}{(P-K)^2-M^2}\\ &= g^2 C_{\mathrm{F}} \int \frac{d^4K}{(2\pi)^3}\,\delta(K^2)\,n(k)\,\frac{2(\not{P}-\not{K}-2M)}{(P-K)^2-M^2},\end{aligned} \tag{3.2}$$

where C_{F} is the Casimir of the fundamental representation, and in order to insert Σ into the on-shell scattering amplitude, Fig. 2b, we have to expand (3.2) around the mass-shell $\not{P}=M$,

$$\Sigma(P)=\Sigma_0+\Sigma_1(\not{P}-M)+O(P^2-M^2). \tag{3.3}$$

The on-shell mass counter-term receives the temperature correction

$$\Sigma_0(P)=g^2 C_{\mathrm{F}} \int \frac{d^4K}{(2\pi)^3}\,\delta(K^2)\,n(k)\,\frac{\not{K}}{P\cdot K}, \tag{3.4}$$

which unlike at $T=0$ remains a (matrix) function of P, but vanishes in our limit p_0, $|\mathbf{p}| \gg T$. This latter conclusion is readily drawn from expression (3.4), since the integration momentum is effectively cut off for $k \gg T$ by the Bose–Einstein distribution $n(k)$.

There is, however, a non-vanishing contribution to the fermionic on-shell wave function renormalization,

$$\begin{aligned}\Sigma_1(P) &= \frac{P_\mu}{M}\frac{\partial}{\partial P_\mu}(\Sigma-\Sigma_0)\bigg|_{\not{P}=M}\\ &= g^2 C_{\mathrm{F}} \int \frac{d^4K}{(2\pi)^3}\,\delta(K^2)\,n(k)\,\frac{M^2}{(P\cdot K)^2} = g^2 C_{\mathrm{F}}\, 2\int_\mu^\infty \frac{dk}{(2\pi)^2}\,\frac{n(k)}{k},\end{aligned} \tag{3.5}$$

containing a linear infra-red divergence, which we have regulated with the cut-off μ. In contrast to Σ_0, Σ_1 is a constant independent of P and M.

3.2. *Abelian Vertex Correction*

On the mass-shell the QED-type vertex graph $\Lambda^{(A)}$ (Fig. 2c) contributes

$$\Lambda_\mu^{(A)}(P, P-Q) = -ig^3 \frac{\lambda}{2}\left(C_{\mathrm{F}} - \frac{C_{\mathrm{A}}}{2}\right) \int \frac{d^4K}{(2\pi)^3}\, \delta(K^2)\, n(k)\, \frac{(M^2 + Q^2/2)\,\gamma_\mu - K_\mu \not{K}}{P\cdot K(P-Q)\cdot K} \tag{3.6}$$

where λ is a Gell–Mann matrix, and $C_{\mathrm{A}} = N$ is the Casimir of the adjoint representation.

With p_0, $|\mathbf{p}| \gg T$ this reduces to

$$\Lambda_\mu^{(A)}(P, P-Q) \rightarrow -ig^3 \frac{\lambda}{2}\gamma_\mu \left(C_{\mathrm{F}} - \frac{C_{\mathrm{A}}}{2}\right) \int \frac{d^4K}{(2\pi)^3}\, \delta(K^2)\, n(k)\, \frac{M^2}{(P\cdot K)^2}, \tag{3.7}$$

leading to the same infra-red divergent integral as in (3.5). The infra-red divergences proportional to C_{F} combine as usual into a correspondingly infra-red divergent wave function renormalization of our test fermions, whereas the infra-red divergence proportional to C_{A} has to cancel with the remaining graphs.

3.3. *Non-Abelian Vertex Correction*

The non-abelian part of the vertex correction $\Lambda^{(N)}$ (Fig. 2d) is the only place (besides the gluon self energy) where it makes a difference whether we use FG or BFG. We distinguish these cases by the parameter

$$\sigma = \begin{cases} 0 & \text{(FG)} \\ 1 & \text{(BFG)} \end{cases}.$$

We obtain

$$\begin{aligned} \Lambda_\mu^{(N)}(P, P-Q) = ig^3 \frac{\lambda}{2}\frac{C_{\mathrm{A}}}{2} \int \frac{d^4K}{(2\pi)^3}\, \delta(K^2)\, n(k)\, \frac{1}{Q^2 - 2K\cdot Q} \\ \times \left\{ \frac{Q\cdot K}{P\cdot K(P-Q)\cdot K}\left[\not{K}(Q-2K)_\mu - (1-\sigma)\frac{1}{2}Q^2\gamma_\mu\right] \right. \\ + \frac{(2P-Q)\cdot K}{P\cdot K(P-Q)\cdot K}\left[-\not{P}(Q-2K)_\mu + \sigma\gamma_\mu \not{K}\not{Q} - \not{Q}\not{K}\gamma_\mu \right. \\ \left.\left. + (1-\sigma)\left(-2P_\mu \not{K} + \frac{1}{2}Q^2\gamma_\mu\right)\right]\right\}. \end{aligned} \tag{3.8}$$

With p_0, $|\mathbf{p}| \gg T$ this reduces to

$$\begin{aligned} \Lambda_\mu^{(N)}(P, P-Q) \rightarrow ig^3 \frac{\lambda}{2} C_{\mathrm{A}} \int \frac{d^4K}{(2\pi)^3}\, \delta(K^2)\, n(k) \\ \times \frac{1}{Q^2 - 2K\cdot Q}\frac{2}{P\cdot K}\left[\not{P}K_\mu - (1-\sigma)\,P_\mu \not{K}\right], \end{aligned} \tag{3.9}$$

where we have omitted terms proportional to Q_μ, since they are annihilated by current conservation when (3.8) is inserted into the scattering amplitude.

Using Gordon's identity, which for $q_0, |\mathbf{q}| \ll p_0, |\mathbf{p}|$ simplifies to

$$\bar{u}(P)\,\gamma_\mu u(P-Q) \to \frac{1}{M} P_\mu \bar{u}(P)\,u(P), \tag{3.10}$$

and because of our assumption that $|p_0 - p_0'|, |\mathbf{p}-\mathbf{p}'| \ll p_0, |\mathbf{p}|$, (3.9) can be replaced by

$$\Lambda_\mu^{(N)}(P, P-Q) \to ig^3 \frac{\lambda}{2} C_A \sigma \int \frac{d^4K}{(2\pi)^3} \delta(K^2)\, n(k) \frac{2\gamma_\mu}{Q^2 - 2K\cdot Q}, \tag{3.11}$$

where all dependence on the fermionic momentum P has disappeared. Moreover, (3.11) is infra-red finite.

In the case of FG ($\sigma = 0$), (3.11) vanishes, and for the case of BFG we have to evaluate

$$\int \frac{d^4K}{(2\pi)^3} \delta(K^2)\, n(k) \frac{1}{Q^2 - 2K\cdot Q} = -\frac{1}{16q} \mathscr{L}(Q), \tag{3.12}$$

where $\mathscr{L}(Q)$ is a function which has already appeared in the evaluation of the gluon self energy and which is given in the Appendix, Eqs. (A.4), (A.15)–(A.20).

We note that by adding the contribution of (3.11) to the one of the gluon propagator we have now reached independence of σ, i.e., whether we use FG or BFG, and as an interim result we have $\gamma = -5g^2NT/24\pi$, with the contributions of the box and the crossed graph (Fig. 2e, f) still to be calculated.

3.4. *Box and Crossed Graph*

The sum of the box and of the crossed graph, which we denote by Ξ, evaluated on-shell and taking the same limits as in the previous section, yields

$$\Xi(P, P-Q, \bar{P}, \bar{P}+Q)$$
$$\to g^4 C_A \frac{\lambda^a}{2} \gamma^\mu \otimes \frac{\lambda^a}{2} \gamma_\mu \int \frac{d^4K}{(2\pi)^3} \delta(K^2)\, n(k) \frac{1}{Q^2 - 2K\cdot Q} \frac{M^2}{(K\cdot P)^2}. \tag{3.13}$$

This expression is infra-red divergent. Adding to it the contributions of the abelian vertices proportional to C_A, whose infra-red divergences have not yet cancelled, gives the infra-red finite integral

$$\Xi' = g^4 C_A \int \frac{d^4K}{(2\pi)^3} \delta(K^2)\, n(k) \frac{2K\cdot Q}{Q^2(Q^2 - 2K\cdot Q)} \frac{M^2}{(K\cdot P)^2}, \tag{3.14}$$

where we have omitted the Gell-Mann and the Dirac matrices for simplicity.

Because the fermionic momentum P has not dropped out, (3.14) is more difficult to compute. Observing that in the high-T expansion its real part does not contribute at order T^2, we restrict ourselves to the evaluation of its imaginary part, and we find

$$\operatorname{Im} \Xi' = -\frac{g^4 C_A \pi}{2Q^2} \int \frac{d^3k}{(2\pi)^3} n(k) \frac{M^2}{[p_0 - \mathbf{k}\cdot\mathbf{p}/k]^2} \times \left\{ [\theta(Q^2) + \theta(-Q^2)\,\theta(\mathbf{k}\cdot\mathbf{q} - kq_0)]\,\delta\left(k - \frac{1}{2}\frac{Q^2}{q_0 - \mathbf{k}\cdot\mathbf{q}/k}\right) + \theta(-Q^2)\,\theta(\mathbf{k}\cdot\mathbf{q} + kq_0)\,\delta\left(k + \frac{1}{2}\frac{Q^2}{q_0 + \mathbf{k}\cdot\mathbf{q}/k}\right)\right\}. \tag{3.15}$$

Keeping only the leading term of the high-T expansion and assuming $q < q_0$, (3.15) can be evaluated to

$$\operatorname{Im} \Xi' = \frac{g^4 C_A T}{4\pi Q^4} \left\{ -2q_0 + \frac{\mathbf{q}\cdot\mathbf{p}}{p}\left[\frac{2p_0}{p} - \frac{P^2}{p^2} \ln \frac{p_0 + p}{p_0 - p}\right]\right\} + O(\ln T). \tag{3.16}$$

We note parenthetically that contrary to appearance the non-relativistic limit $v = p/p_0 \to 0$ of (3.16) exists, whereas the $T = 0$ part of Ξ in this limit develops a $1/v$-singularity in its imaginary part. The latter is a reflection of a well-known difficulty [20] associated with infinite-range potentials. It corresponds to a diverging phase shift and drops out when the complete scattering amplitude is squared to give the S-matrix. Here we have found that this $1/v$-singularity does not receive finite-T corrections in $O(g^4)$.

For our application we are interested in the $q \to 0$ limit of expression (3.16), and there the dependence on the fermionic momentum P disappears.

4. Discussion and Conclusion

Adding up all imaginary parts, the one coming from the gluon propagator (2.22) and those from the additional contributions to the scattering amplitude computed in Section 3, we finally obtain the gauge independent result for the one-loop plasmon damping constant

$$\gamma = ([-5 - 6\sigma] + 6\sigma - 6)\frac{g^2 NT}{24\pi} = -11\,\frac{g^2 NT}{24\pi}. \tag{4.1}$$

Without giving the details of the calculations, we note that the individual contributions to (4.1) are in a remarkable one-to-one correspondence with the real part contributions proportional to $\ln(T^2/Q^2)$ which combine with the $T = 0$ terms $\ln(Q^2/\tilde{\mu}^2)$ carrying the QCD β-function coefficient.

Our one-loop result (4.1) coincides numerically with the one obtained by Nadkarni by employing Cornwall's "gauge invariant propagator," which in turn amounts to BFG. Although the idea behind our approach and that of Nadkarni is similar, the strategy is quite different. In Cornwall's approach one uses elementary Ward identities to pinch out some of the fermionic propagators of an on-shell amplitude which then yields effective propagator-like contributions. This procedure has been criticized in [16] as being to some extent ambiguous. Our attitude on the other hand was to take the scattering amplitude seriously, and in virtue of our superheavy test particles we can claim a relation to an in-principle measureable process even at finite temperature. In fact, in our approach the individual graphs are contributing differently. For example, by constructing Cornwall's "gauge invariant propagator" in a Feynman gauge, one does not obtain contributions from the box or from the crossed graph, while we did.[2]

However, the coincidence of our result with that of Nadkarni does not yet resolve the vexatious disagreement of Refs. [6–11] on magnitude and sign of the one-loop plasmon damping constant. But, as we will now discuss and as has been already noted in [1, 2, 7, 11], a bare one-loop result is of little significance because higher loops will contribute in the same order of magnitude.

Although one may assume $g \ll 1$, in the high-T-limit gT cannot be considered as small but is the relevant mass scale set by $\omega_{\rm pl} \sim gT$. Now the imaginary part of the gluon self energy is governed by the location of the poles of the internal propagators which through self energy insertions are shifted by amounts $\sim gT$. Hence the imaginary part is modified already at lowest order. Another way to understand this is to realize that a (bare) two-loop calculation in the long-wavelength limit can produce an imaginary part proportional to g^4T^3/q_0. Evaluated at the mass scale of plasma oscillations $q_0 \sim gT$, the latter is of the same order of magnitude as the bare one-loop result $\sim g^2Tq_0$.

Summing up the relevant higher-loop contributions is certainly a non-trivial task. A first approximation would be to replace the bare propagators in the one-loop diagrams by those obtained through the bare one-loop calculation. Following Ref. [7] we will instead consider approximated dressed propagators where the self energy insertions are replaced by their (gauge independent) values on the bare gluon mass shell. From (2.9), (2.10), and (A.21) we thus start with

$$\begin{aligned} a(Q^2) &= Q^2 - \frac{1}{2}\Pi^\mu_\mu(Q^2)\Big|_{q_0=q} = Q^2 - \frac{g^2NT^2}{6} \equiv Q^2 - m_G^2, \\ b(Q^2) &= Q^2. \end{aligned} \tag{4.2}$$

This certainly will not yet give the true value of the plasmon damping constant at the order g^2T, but it may give a hint about the direction in which the one-loop result will be changed.

[2] Our original expectation even was that we might end up with the positive result obtained in [7, 8] in the Coulomb gauge, for in the previous applications at $T=0$ [19, 21] MMOM was found to just amount to using the Coulomb gauge gluon propagator.

Recomputing (2.19) in FG and BFG we find[3]

$$\lim_{q\to 0}\frac{1}{3}\Pi_{ii}=g^2N\left(-\frac{1}{9}T^2+i\frac{q_0T}{12\pi}\times\begin{cases}-2 & (\mathrm{FG})\\ -4 & (\mathrm{BFG})\end{cases}+O(\ln T)\right), \tag{4.3}$$

corresponding to a diminished but still negative (and still gauge dependent) plasmon damping constant. This reduction is largely due to the fact that with (4.2) the spatially transverse modes described by $a(Q^2)$ no longer contribute to the imaginary part because the plasma frequency is below the threshold $2m_G$ introduced into (4.2). In Coulomb gauge as well as in temporal axial gauge the analogous calculation [7] even gives a zero imaginary part of Π, because with (4.2) the propagators in these two gauges no longer contain terms proportional to $1/Q^2$ which were the only ones capable of producing an imaginary part:

$$\Delta_{00}^{\mathrm{CG}}=\frac{-1}{q^2},\qquad \Delta_{0i}^{\mathrm{CG}}=0,\qquad \Delta_{ij}^{\mathrm{CG}}=\frac{1}{Q^2-m_{\mathrm{G}}^2}\left(\frac{q_iq_j}{q^2}-\delta_{ij}\right), \tag{4.4}$$

$$\Delta_{00}^{\mathrm{TAG}}=\Delta_{0i}^{\mathrm{TAG}}=0,\qquad \Delta_{ij}^{\mathrm{TAG}}=\frac{1}{Q^2-m_{\mathrm{G}}^2}\left(\frac{q_iq_j}{q^2}-\delta_{ij}\right)-\frac{q_iq_j}{q_0^2q^2}. \tag{4.5}$$

In order to recompute also our S-matrix element in the approximation (4.2), one could simply resort to either (4.4) or (4.5) to again find a zero result for the imaginary part, corresponding to a vanishing plasmon damping constant, $\gamma=0$, although we have not checked whether gauge independence of the S-matrix still holds in the approximation (4.2) and, actually, we do not see any stringent reason for this.

At any rate these considerations show that the bare one-loop results may not yet be interpreted as a plasmon damping constant. The gauge dependences found in the poles of the bare one-loop propagator just reflect that not all contributions have been taken into account. The same is to be said about the discrepancy between the two manifestly gauge independent methods of the Vilkovisky–DeWitt effective action and of Cornwall's "gauge invariant propagator," with which our S-matrix calculation coincides numerically at one loop, since both approaches are still lacking the inclusion of the relevant higher-loop contributions. The lesson to be learned from this is simply that gauge independent does not yet mean physical.

In fact, it has recently been shown [26, 27] that a resummation of the leading order contributions will achieve gauge independence for the poles of the otherwise gauge dependent gluon propagator.

All the differing one-loop results obtained so far should thus better be viewed as

[3] The approximation (4.2) turns out to be satisfactory at least in BFG where it preserves transversality of the self energy as far as the imaginary part is concerned.

[4] Assuming that the unphysical poles $1/q_0^2$ arising in the temporal axial gauge are treated in the principal value prescription.

agreeing in giving a zero result for γ in the order gT, which in fact was the conclusion drawn in the original works [1]. Whether the final plasmon damping constant is zero also at order g^2T as suggested by the approximation (4.2) is still to be decided by a calculation which is exact at this order.

Appendix: Evaluation of the Finite-T Gluon Self Energy in Conventional and Background Feynman Gauge

In this appendix we evaluate both, the real and the imaginary part of the finite temperature one-loop gluon self energy, in the conventional Feynman gauge (FG) as well as in its background covariant version (BFG) in the high-T limit up to $O(1/T)$ following the methods of Ref. [3]. Our results supplement those of Ref. [12] for the case of BFG, and those of Ref. [3] for the case of the conventional FG, correcting some misprints in the latter.

With the following abbreviations,

$$\omega_\pm \equiv \frac{1}{2}(q_0 \pm q), \tag{A.1}$$

$$L(k) \equiv \ln\left(\frac{k+\omega_+}{k-\omega_+}\right) - \ln\left(\frac{k+\omega_-}{k-\omega_-}\right), \tag{A.2}$$

$$M(k) \equiv (k+\omega_+)(k+\omega_-)\ln\left(\frac{k+\omega_+}{k+\omega_-}\right) - (k-\omega_+)(k-\omega_-)\ln\left(\frac{k-\omega_+}{k-\omega_-}\right), \tag{A.3}$$

$$\mathscr{L}(Q) \equiv \frac{1}{\pi^2}\int_0^\infty dk\, n(k)\, L(k), \tag{A.4}$$

$$\mathscr{M}(Q) \equiv \frac{1}{\pi^2}\int_0^\infty dk\, n(k)\left\{k\left(1-\frac{q_0}{q}\ln\frac{\omega_+}{\omega_-}\right)+\frac{1}{2q}M(k)\right\}, \tag{A.5}$$

the finite-T corrections to the gluon self energy for the case of the conventional FG is given by [3]

$$\Pi_{00}^{\mathrm{FG}}(Q) = g^2N\left\{\mathscr{M}(Q) - \frac{q}{8}\mathscr{L}(Q)\right\} \tag{A.6}$$

$$\Pi_{ii}^{\mathrm{FG}}(Q) = g^2N\left\{-\frac{T^2}{3} + \mathscr{M}(Q) - \frac{5q_0^2-4q^2}{8q}\mathscr{L}(Q)\right\}, \tag{A.7}$$

and for the case of BFG

$$\Pi_{00}^{\mathrm{BFG}}(Q) = g^2N\left\{\mathscr{M}(Q) - \frac{3q}{8}\mathscr{L}(Q)\right\} \tag{A.8}$$

$$\Pi_{ii}^{\mathrm{BFG}}(Q) = g^2N\left\{-\frac{T^2}{3} + \mathscr{M}(Q) - \frac{11q_0^2-8q^2}{8q}\mathscr{L}(Q)\right\}. \tag{A.9}$$

Remarkably, all of the gauge dependence is in the coefficient of the function $\mathscr{L}(Q)$, which, as we shall see presently, does not contribute to the leading terms proportional to T^2.

With $q_0 = \omega + i\varepsilon$, $\omega > 0$, the gauge independent piece $\mathscr{M}(Q)$ is evaluated to

$$\operatorname{Re} \mathscr{M}(Q) = \frac{T^2}{3}\left[1 - \frac{\omega}{2q} \ln\left|\frac{\omega_+}{\omega_-}\right|\right] + T\theta(-Q^2)\frac{Q^2}{8q}$$
$$+ \frac{1}{\pi^2}\left[\left(\frac{\omega^3}{48q} - \frac{\omega q}{16}\right)\ln\left|\frac{\omega_+}{\omega_-}\right| - \frac{\omega^2}{24} + \frac{q^2}{9} - \frac{1}{12}q^2\left(\gamma_E + \ln\frac{|Q|}{4\pi T}\right)\right] + O(1/T), \quad \text{(A.10)}$$

$$\operatorname{Im} \mathscr{M}(Q) = \frac{\theta(-\omega_-)\,\omega}{\pi q}\int_{|\omega_-|}^{\infty} dk\, kn(k) + \frac{1}{2\pi q}\int_{|\omega_-|}^{\omega_+} dk\, n(k)(k-\omega_+)(k-\omega_-)$$
$$= T^2\frac{\pi\theta(-Q^2)\,\omega}{6q} - \frac{T}{4\pi}\left[\omega\left(1 - \frac{\omega}{2q}\ln\left|\frac{\omega_+}{\omega_-}\right|\right) + \frac{q}{2}\ln\left|\frac{\omega_+}{\omega_-}\right|\right]$$
$$+ \frac{q^2\theta(Q^2)}{24\pi} + \frac{\theta(-Q^2)}{16\pi}\,\omega\left(q - \frac{\omega^2}{3q}\right) + O(1/T), \quad \text{(A.11)}$$

where γ_E is Euler's constant and θ is the step function.

In the static limit these expressions reduce to

$$\lim_{\omega\to 0} \operatorname{Re} \mathscr{M}(Q) = \frac{T^2}{3} - \frac{qT}{8} + \frac{q^2}{\pi^2}\left[\frac{1}{9} - \frac{1}{12}\left(\gamma_E + \ln\frac{q}{4\pi T}\right)\right] + O(1/T) \quad \text{(A.12)}$$

$$\lim_{\omega\to 0} \operatorname{Im} \mathscr{M}(Q) = 0, \quad \text{(A.13)}$$

and in the long-wavelength limit

$$\lim_{q\to 0} \mathscr{M}(Q) = 0. \quad \text{(A.14)}$$

The other function $\mathscr{L}(Q)$, whose coefficient has turned out to be gauge dependent, is evaluated to

$$\operatorname{Re} \mathscr{L}(Q) = T\theta(-Q^2)$$
$$+ \frac{1}{\pi^2}\left[q\left(\gamma_E - 1 + \ln\frac{|Q|}{4\pi T}\right) + \frac{\omega}{2}\ln\left|\frac{\omega_+}{\omega_-}\right|\right] + O(1/T^2), \quad \text{(A.15)}$$

$$\operatorname{Im} \mathscr{L}(Q) = \frac{1}{\pi}\int_{|\omega_-|}^{\omega_+} dk\, n(k)$$
$$= \frac{1}{\pi}\left[T\ln\frac{e^{\omega_+/T} - 1}{e^{|\omega_-|/T} - 1} - \omega_+ + |\omega_-|\right]$$
$$= \frac{T}{\pi}\ln\left|\frac{\omega_+}{\omega_-}\right| - \frac{1}{2\pi}(\omega_+ - |\omega_-|) + O(1/T). \quad \text{(A.16)}$$

Limiting cases of interest are

$$\lim_{\omega \to 0} \text{Re} \frac{\mathscr{L}(Q)}{q} = \frac{T}{q} + \frac{1}{\pi^2} \left(\gamma_E - 1 + \ln \frac{q}{4\pi T} \right) + O(1/T^2) \tag{A.17}$$

$$\lim_{\omega \to 0} \text{Im} \frac{\mathscr{L}(Q)}{q} = 0 \tag{A.18}$$

and

$$\lim_{q \to 0} \text{Re} \frac{\mathscr{L}(Q)}{q} = \frac{1}{\pi^2} \left(\gamma_E + \ln \frac{\omega}{4\pi T} \right) + O(1/T^2) \tag{A.19}$$

$$\lim_{q \to 0} \text{Im} \frac{\mathscr{L}(Q)}{q} = \frac{2T}{\pi\omega} - \frac{1}{2\pi} + O(1/T). \tag{A.20}$$

Finally we note that whereas the limit $q \to q_0$ does not exist for $\mathscr{M}(Q^2)$ and $\mathscr{L}(Q^2)$, it exists for Π^μ_μ and is gauge independent,

$$\lim_{q \to q_0} \Pi^\mu_\mu(Q) = \frac{g^2 N T^2}{3}, \tag{A.21}$$

which moreover is the exact one-loop result, i.e., not just its high-T limit.

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