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Chiral fermions in 2d quantum gravity

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Abstract. Two theories of chiral fermions coupled to different quantum gravities in two dimensions are studied. One employs Jackiw's ansatz for classical gravity by introducing an auxiliary scalar, the other is based on the induced quantum gravity of Polyakov, which has no classical analogue. By investigating a localized theory of the effective action we show that in both cases a limited number of fermions of either chirality may couple consistently. It is stressed that the Weyl variable has to be quantized properly, which is related to recent work done on non-critical strings.

1 Introduction

Several years ago, Jackiw and Rajaraman [1] showed that the chiral Schwinger model, a two dimensional (2d) anomalous model of a chiral fermion coupled to a U(1) gauge field, can be consistently quantized. The reason is that a classically frozen variable is turned into a physical quantum field by the chiral U(1) anomaly and thereby restores the symmetry. Although this aspect has not yet been fully investigated in Polyakov's path integral approach [2] to string theory, it is reasonable to expect that the Weyl symmetry may also be restored even for non-critical dimensions by considering the Weyl variable as a dynamical one. First affirmative results have been derived recently by Fujikawa et al. [3]. Another important aspect of Polyakov's approach is that it allows to define 2d quantum gravity through the Weyl anomaly without any classical gravity.

A different approach towards 2d gravity was proposed by Jackiw [4], who introduced an auxiliary scalar to define a nontrivial classical gravity action. Following this approach together with the basic idea of Ref. [1], Li [5] and Fukuyama and Kamimura [6] tried to find out whether it is possible to couple chiral fermions consistently to gravity. They both promoted the inevitable Lorentz variable, which arises through the Lorentz anomaly [7], to be a quantum field, but gave contradictory answers to the consistency question. However, apart from the difference between their procedures which leads to the contradiction, both analysis share a crucial drawback in that the Weyl anomaly which comes in through the Faddeev–Popov ghost sector has not been taken into account.

In this paper we (re)investigate the two approaches towards 2d gravity interacting with chiral fermions. In Sect. 2 we adopt Jackiw's gravity in order to settle the above mentioned question. It is shown that the algebra of generators of surface deformations, which can be used to define a physical space, closes without Schwinger terms. However, combined with the demand of positivity for the Weyl and Lorentz variables, we get a constraint on the number of chiral fermions. Then, in Sect. 3, we adopt Polyakov's gravity and show that the algebra closes likewise but the allowed number of chiral fermions is slightly different from the previous model with Jackiw's gravity. Remarkably, the allowed number is identical to the one obtained in our previous work [8], where the consistency of the theory has been demonstrated by exploiting the conformal gauge in the path-integral formalism. Section 4 contains the conclusions.

2 Chiral fermions coupled to Jackiw's gravity

Following Jackiw [4], let us first define gravity in two dimensions by the action,★

$$I_G = -\frac{1}{16\pi G}\int dx\sqrt{-g}N(R+2\Lambda), \tag{1}$$

where N is an auxiliary field. Fermionic matter is

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★ Notation: $\gamma^0=\sigma_1$, $\gamma^1=i\sigma_2$, $\gamma^5=-\gamma^0\gamma^1=\sigma_3$, $\varepsilon^{01}=1$, $\sigma^{ab}=\frac{1}{4}[\gamma^a,\gamma^b]=-\frac{1}{2}\varepsilon^{ab}\gamma^5$, $\gamma^5\psi_{R,L}=\pm\psi_{R,L}$

introduced by

$$I_{\mathrm{F}} = \int dx \sqrt{-g}\hbar \frac{i}{2} e^{\mu}_{a} (\bar{\psi}\gamma^{a} \overleftrightarrow{\partial}_{\mu} \psi), \tag{2}$$

where ψ denotes a set of n_{R} right-handed and n_{L} left-handed Weyl fermions. The metric is given by the lapse and shift functions η_0 and η_1, respectively, together with the Weyl variable ϕ:

$$g_{\mu\nu} = e^{\phi} \begin{pmatrix} \eta_0^2 - \eta_1^2 & -\eta_1 \\ -\eta_1 & -1 \end{pmatrix}. \tag{3}$$

By introducing the Lorentz variable F, the zweibein is parametrized as

$$e^{a}_{\mu} = \begin{pmatrix} \cosh\frac{F}{2} & -\sinh\frac{F}{2} \\ -\sinh\frac{F}{2} & \cosh\frac{F}{2} \end{pmatrix} e^{\phi/2} \begin{pmatrix} \eta_0 & \eta_1 \\ 0 & 1 \end{pmatrix}_{\mu a}. \tag{4}$$

Then the spin connection reads

$$\begin{aligned} \omega_{\mu} &= \varepsilon^{ab} e^{\nu}_{a} \nabla_{\mu} e_{b\nu} \equiv \partial_{\mu} F + \tilde{\omega}_{\mu}, \\ \tilde{\omega}_1 &= \frac{1}{\eta_0}(\dot{\phi} - 2\eta_1' - \eta_1 \phi'), \\ \tilde{\omega}_0 &= \eta_1 \tilde{\omega}_1 + 2\eta_0' + \phi' \eta_0, \end{aligned} \tag{5}$$

where dot and primes mean time and space derivatives, respectively. As is well-known, one may either work with a local Lorentz invariant or a general coordinate invariant regularization for the fermionic effective action [7]. Here we adopt the second alternative given by Leutwyler [12]:

$$\begin{aligned} e^{iI_{\mathrm{eff}}/\hbar} &= \int d\psi\, d\bar{\psi}\, e^{iI_{\mathrm{F}}/\hbar}, \\ I_{\mathrm{eff}} &= \frac{1}{48\pi} \int dx \left\{ \frac{\hbar}{4} \sqrt{-g} R \frac{1}{\sqrt{-g}\nabla^2} \right. \\ &\quad \cdot (\alpha\sqrt{-g}R + \beta\sqrt{-g} g^{\mu\nu} \nabla_{\mu}\omega_{\nu}) \\ &\quad \left. + \mu\sqrt{-g} + \frac{a'}{2}\sqrt{-g} g^{\mu\nu}\omega_{\mu}\omega_{\nu} \right\}, \end{aligned} \tag{6}$$

where $\alpha = n_{\mathrm{R}} + n_{\mathrm{L}}$, $\beta = n_{\mathrm{R}} - n_{\mathrm{L}}$, and two arbitrary parameters μ and a' represent regularization ambiguities. The effective action can be derived easily by use of perturbation theory [9], up to the μ-term. The nonlocal term in I_{eff} is eliminated by introducing a scalar field φ

$$\begin{aligned} I_{\mathrm{loc}} &= \int dx \sqrt{-g} \{ \tfrac{1}{2} g^{\mu\nu} \partial_{\mu}\varphi \partial_{\nu}\varphi + \mu \\ &\quad + \varphi(cR + b g^{\mu\nu} \nabla_{\mu}\omega_{\nu}) + \tfrac{1}{2} a g^{\mu\nu} \omega_{\mu}\omega_{\nu} \}. \end{aligned} \tag{7}$$

The parameters a, b, and c can be deduced by comparing I_{eff} with the effective action derived from I_{loc} [5]:

$$\begin{aligned} b^2 &= -\frac{\hbar}{96\pi}(m_{\mathrm{A}} - m_{\mathrm{G}}), \quad c^2 = -\frac{\hbar}{96\pi}(m_{\mathrm{A}} + m_{\mathrm{G}}), \\ a &= a' + b^2, \end{aligned} \tag{8a}$$

with the algebraic and geometric means of $(1 - n_{\mathrm{R}})$ and $(1 - n_{\mathrm{L}})$:

$$m_{\mathrm{A}} = \frac{(1 - n_{\mathrm{R}}) + (1 - n_{\mathrm{L}})}{2}, \quad m_{\mathrm{G}} = \sqrt{(1 - n_{\mathrm{R}})(1 - n_{\mathrm{L}})}. \tag{8b}$$

b and c could be exchanged, but as we shall see the choice (8a) turns out to be a suitable one. Note that b and c are imaginary for $m_{\mathrm{A}} > m_{\mathrm{G}}$. In this case we may redefine φ as $\varphi \to i\varphi$, so that I_{loc} remains real.

For the combined system $I = I_{\mathrm{loc}} + I_{\mathrm{G}}$, we have

$$\begin{aligned} I &= \int dx \{ \eta_0^{-1}(\dot{\phi} - 2\eta_1' - \phi'\eta_1)(\dot{\hat{N}} - \eta_1 \hat{N}') - (2\eta_0' + \phi'\eta_0)\hat{N}' \\ &\quad + 2\eta_0 \hat{N} \Lambda e^{\phi} + \mu\eta_0 e^{\phi} + \tfrac{1}{2}\eta_0^{-1}[(\dot{\varphi} - \varphi'\eta_1)^2 - \varphi'^2\eta_0^2] \\ &\quad + b\eta_0^{-1}[-(\dot{\varphi} - \eta_1\varphi')(\omega_0 - \eta_1\omega_1) + \eta_0^2\omega_1\varphi'] \\ &\quad + \tfrac{1}{2} a \eta_0^{-1}[(\omega_0 - \omega_1\eta_1)^2 - \eta_0^2\omega_1^2] \}, \end{aligned} \tag{9}$$

where we defined $\hat{N} \equiv N - c\varphi$, and the factor $1/16\pi G$ has been absorbed in N. With the canonical momenta of the variables φ, $\hat{N}$, ϕ, and F, this may be recast into

$$I = \int dx \{ \dot{\hat{N}} P_{\hat{\mathrm{N}}} + \dot{\phi} P_{\phi} + \dot{\varphi} P\varphi + \dot{F} P_{\mathrm{F}} - (\eta_0 H + \eta_1 T) \}, \tag{10}$$

where (with μ absorbed into Λ)

$$\begin{aligned} H &= -b\varphi' F' - 2\hat{N}'' + \hat{N}'\phi' + (P_{\phi} - b\varphi') P_{\hat{\mathrm{N}}} \\ &\quad - 2(\hat{N} + c\varphi)\Lambda e^{\phi} - P_{\mathrm{F}}\phi' + 2P_{\mathrm{F}}' + \tfrac{1}{2}P_{\varphi}^2 + \tfrac{1}{2}\varphi'^2 \\ &\quad + \frac{1}{2(a - b^2)}(P_{\mathrm{F}} + bP_{\varphi})^2 + \tfrac{1}{2}a(P_{\hat{\mathrm{N}}} + F')^2, \\ T &= P_{\phi}\phi' - 2P_{\phi}' + P_{\hat{\mathrm{N}}}\hat{N}' + P_{\mathrm{F}} F' + P_{\varphi}\varphi'. \end{aligned} \tag{11}$$

H and T are generators of surface deformations and satisfy closed algebras classically [10]. H and T are transformed canonically into a form where afterwards the singularities connected to the quantum nature of the fields can easily be dealt with. By means of the generating functional

$$\begin{aligned} W &= (\tilde{P}_{\varphi} + \sqrt{a}\tilde{P}_{\hat{\mathrm{N}}})\varphi + (\tilde{P}_{\mathrm{F}} - b\tilde{P}_{\varphi} - \sqrt{a}b\tilde{P}_{\hat{\mathrm{N}}})F \\ &\quad + (\tilde{P}_{\phi} - \sqrt{a}\tilde{P}_{\varphi} - a\tilde{P}_{\hat{\mathrm{N}}} - aF' + b\varphi')\phi + \tilde{P}_{\hat{\mathrm{N}}}\hat{N} \end{aligned} \tag{12a}$$

and the rescaling

$$\tilde{F} \to \sqrt{a - b^2}^{\,-1} \tilde{F}, \quad \tilde{P}_{\mathrm{F}} \to \sqrt{a - b^2}\,\tilde{P}_{\mathrm{F}}, \tag{12b}$$

we arrive at (omitting all tildes)

$$\begin{aligned} H_{\pm} &= H \pm T \\ &= \tfrac{1}{2}(P_{\mathrm{F}} \pm F')^2 + \tfrac{1}{2}(P_{\varphi} \pm \varphi')^2 + 2\sqrt{a - b^2}(P_{\mathrm{F}} \pm F')' \\ &\quad + 2(-b \pm \sqrt{a})(P_{\varphi} \pm \varphi')' + (P_{\phi} \pm \hat{N}')(P_{\hat{\mathrm{N}}} \pm \phi') \\ &\quad \mp 2(P_{\phi} \pm \hat{N}')' + 2\sqrt{a}(\sqrt{a} \mp b)(\pm P_{\mathrm{N}} + \phi')' \\ &\quad - 2\left[\hat{N} + (c - \sqrt{a})\varphi + c\sqrt{a}\phi + \frac{cb}{\sqrt{a - b^2}} F \right] \Lambda e^{\phi}. \end{aligned} \tag{13}$$

$a - b^2 \geqq 0$ is required for the rescaling of F and P_{F} in order to ensure that the kinetic term of F has the correct sign.

Defining [6]

$$\sigma_\pm = \frac{1}{\sqrt{2}}(\phi \pm \hat{N}), \quad \pi_\pm = \frac{1}{\sqrt{2}}(P_\phi \pm P_{\hat{N}}), \tag{14}$$

we get the quantum expression

$$\begin{aligned} H_\pm = {} & \frac{1}{2}:(P_F \pm F')^2: + \frac{1}{2}:(P_\varphi \pm \varphi')^2: + \frac{1}{\beta_F}(P_F \pm F')' \\ & + \frac{1}{\beta_\varphi}(P_\varphi \pm \varphi')' + \frac{1}{2}:(\pi_\pm \pm \sigma'_\pm)^2: - \frac{1}{2}:(\pi_\mp \mp \sigma'_\mp)^2: \\ & + \frac{1}{\beta_L^\pm}(\pi_\pm \pm \sigma'_\pm)' + \frac{1}{\beta_M^\pm}(\pi_\mp \mp \sigma'_\mp)' \\ & - 2:\left[\frac{1}{\sqrt{2}}(c_+\sigma_+ - c_-\sigma_-) + (c - \sqrt{a})\varphi \right. \\ & \left. + \frac{cb}{\sqrt{a-b^2}} F\right] A e^{\sqrt{2}(p_+\sigma_+ + p_-\sigma_-)}:, \end{aligned} \tag{15a}$$

with

$$\begin{aligned} &\frac{1}{\beta_L^\pm} = \mp\sqrt{2}[1 + \sqrt{a}(\pm b - \sqrt{a})], \\ &\frac{1}{\beta_M^\pm} = \mp\sqrt{2}[1 - \sqrt{a}(\pm b - \sqrt{a})], \\ &\frac{1}{\beta_\varphi} = 2(-b \pm \sqrt{a}), \\ &\frac{1}{\beta_F} = 2\sqrt{a - b^2}, \\ &c_\pm = 1 \pm c\sqrt{a}, \\ &p_\pm : \text{arbitrary parameters.} \end{aligned} \tag{15b}$$

In (15a) colons do not necessarily mean normal ordering, but only that the coincident-point limit can be taken in terms of the following prescription [13]. Starting with the quantum expression $A \equiv \Pi \pm \Phi'$ for a generic canonical conjugate pair Π and Φ, one defines the product $A(x)A(y)$ by

$$\begin{aligned} & A(x)A(y) + A(y)A(x) \\ & \quad =: A(x)A(y): + :A(y)A(x): - \frac{2\hbar\zeta_1}{\pi(x-y)^2}, \end{aligned} \tag{16}$$

where ζ_1 is an arbitrary constant, as only the form of the singularity can be inferred from a dimensional analysis. Another relation we need is

$$\begin{aligned} & m^2[A(x)e^{\beta\sigma(y)} + e^{\beta\sigma(y)}A(x)] \\ & \quad = M^2[:A(x)e^{\beta\sigma(y)}: + :e^{\beta\sigma(y)}A(x):] - \frac{\hbar\beta\zeta_2}{\pi(x-y)} M^2 :e^{\beta\sigma(y)}:. \end{aligned} \tag{17}$$

Again, ζ_2 is an arbitrary constant and M a renormalized mass.

The generator of surface deformations should fulfill the following algebra,

$$\begin{aligned} & \frac{1}{i\hbar}[H_\pm(x), H_\pm(y)]_{ET} \\ & \quad = \pm 2(H_\pm(x) + H_\pm(y))\delta'(x-y) \pm \mathscr{C}\delta'''(x-y), \\ & \frac{1}{i\hbar}[H_+(x), H_-(y)]_{ET} = 0. \end{aligned} \tag{18}$$

We have to demand a vanishing central charge $\mathscr{C}$ so that the closure of the classical algebra, i.e., the invariance under surface deformations, is kept on the quantum level. By using (16) and (17), we obtain constraints on the coefficients (15b) analogous to the ones derived in Ref. 6. They lead to $a = 0$; however, the central charge $\mathscr{C}$ does not vanish. Here one has to employ the freedom to introduce an quantum ambiguity in $\beta_{\varphi,F,L,M}$ [13], which renders $\mathscr{C} = 0$ when properly adjusted. The demand for a non-ghost F-field reads $a - b^2 \geqq 0$, which now becomes

$$-b^2 = \frac{\hbar}{96\pi}(m_A - m_G) \geqq 0. \tag{19}$$

It is obvious that a physical F just excludes a physical φ because the latter has to be imaginary for an imaginary b. This poses no problem because φ is an auxiliary field which may not be physical by itself. Note that the choice (8a) for b and c ensures that the Lorentz variable disappears for $n_R = n_L$, if we set $a = 0$.

The analysis up to this point, however, falls short in an important point: the Weyl anomaly of the whole system has not been taken into account yet. The main contribution to the Weyl anomaly comes in through the Faddeev–Popov ghost sector, which is inevitable when the gravity is quantized. From this we effectively get an additional action $26I_L$, which turns into the Liouville action if the conformal gauge is chosen [2, 8, 15]. In our analysis it is convenient to use the expression for arbitrary coordinates,

$$26I_L = -26\frac{\hbar}{96\pi}\int dx\sqrt{-g}R\frac{1}{\sqrt{-g}\nabla^2}\sqrt{-g}R. \tag{20}$$

This has to be added to I_{eff} (6), which results in a change of coefficients b and c through m_A and m_G. The Weyl anomalies of all participating scalar fields should also be taken into account (see Ref. 8). Since each field contributes a "+1" to $n_{R,L}$ in (8b), we get b and c from (8a) with

$$m_A = \frac{(23 - n_R) + (23 - n_L)}{2}, \quad m_G = \sqrt{(23 - n_R)(23 - n_L)}. \tag{21}$$

Consequently, we obtain a constraint on the number of chiral fermions from (19):

$$n_R \leqq 23, \quad n_L \leqq 23. \tag{22}$$

3 Chiral fermions coupled to Polyakov's induced gravity

Next, we will perform the same analysis for a model with Polyakov's induced gravity, which is reached simply by setting $N=1$. Then, for the constraints (11) we have

$$H=-b\varphi' F'+2c\varphi''-c\varphi'\phi'-P_{\mathrm{F}}\phi'+2P'_{\mathrm{F}}$$
$$+\frac{1}{2}aF'^2+\frac{1}{2}P_\varphi^2+\frac{1}{2}\varphi'^2+\frac{1}{2(a-b^2)}(P_{\mathrm{F}}+bP_\varphi)^2$$
$$-2\Lambda e^{\phi}-\frac{1}{2}\frac{a-b^2}{a(a-b^2+c^2)}$$
$$\cdot\left(-b\varphi'+aF'+\frac{cb}{a-b^2}P_{\mathrm{F}}+\frac{ca}{a-b^2}P_\varphi+P_\phi\right)^2,$$
$$T=P_\phi\phi'-2P'_\phi+P_{\mathrm{F}}F'+P_\varphi\varphi'. \tag{23}$$

The generating functional for the canonical transformation corresponding to (12a) is

$$W=\tilde{P}_\varphi\varphi+(\tilde{P}_F-b\tilde{P}_\varphi)F$$
$$+\left(\tilde{P}_\phi-\frac{cb}{a-b^2}\tilde{P}_{\mathrm{F}}-c\tilde{P}_\varphi-aF'+b\varphi'\right)\phi. \tag{24a}$$

In addition to (12b) for $\tilde{F}$ and $\tilde{P}_{\mathrm{F}}$, we rescale

$$\tilde{\phi}\to k^{-1}\tilde{\phi},\quad \tilde{P}_\phi\to k\tilde{P}_\phi,\quad k\equiv\sqrt{-\frac{a(a-b^2+c^2)}{a-b^2}}. \tag{24b}$$

Then the quantum generators $H_\pm$ read (again all tildes are omitted)

$$H_\pm=\tfrac{1}{2}{:}(P_{\mathrm{F}}\pm F')^2{:}+\tfrac{1}{2}{:}(P_\phi\pm\phi')^2{:}$$
$$+\tfrac{1}{2}{:}(P_\varphi\pm\varphi')^2{:}-2\Lambda{:}e^{p\phi}{:}\pm 2(c\mp b)(P_\varphi\pm\varphi')'$$
$$+2\frac{a-b^2\pm cb}{\sqrt{a-b^2}}(P_{\mathrm{F}}\pm F')'-2k(\pm P_\phi+\phi')'. \tag{25}$$

In contrast to the previous case, the Λ-term is trivial, so that a quantum ambiguity introduced for the coefficients of the last three terms is sufficient to fulfil the algebra (18) with $\mathscr{C}=0$, and there is no need to put $a=0$. However, from the demand for non-ghost F and ϕ, the following conditions arise:

$$a-b^2\geqq 0,\quad k^2\geqq 0, \tag{26}$$

with

$$m_{\mathrm{A}}=\frac{(24-n_{\mathrm{R}})+(24-n_{\mathrm{L}})}{2},\quad m_{\mathrm{G}}=\sqrt{(24-n_{\mathrm{R}})(24-n_{\mathrm{L}})}. \tag{27}$$

This results in the constraint

$$n_{\mathrm{R}}\leqq 24,\quad n_{\mathrm{L}}\leqq 24, \tag{28}$$

which coincides with the result found in Ref. 8.

What happens if we freeze the Weyl variable by a gauge fixing condition? Then we have a nonvanishing central charge

$$\mathscr{C}=4k^2. \tag{29}$$

If only Dirac fermions couple ($b^2=0$, $n_{\mathrm{R}}=n_{\mathrm{L}}=n_{\mathrm{d}}$), the F-field does not appear ($a=0$) and we are left with $\mathscr{C}=(\hbar/12\pi)(26-n_{\mathrm{d}})$, which is the familiar result. For chiral matter one may set $a=0$ to reach $\mathscr{C}=0$; then $n_{\mathrm{R,L}}\leqq 25$ ($n_{\mathrm{R}}\neq n_{\mathrm{L}}$) is the condition for a vanishing central charge. These observations seem to have a close relationship to recent developments in non-critical string theory [3, 14, 15], where it has been pointed out that the conformal anomaly vanishes even in non-critical dimensions if the Weyl variable is quantized properly.

4 Conclusions

In conclusion one can say that it is possible to quantize chiral gravity consistently if no more than a limited number of fermions of either chirality couple. For this it is not decisive whether 2d gravity is defined as Polyakov's induced gravity [2] or as Jackiw's classical model with the help of an auxiliary scalar N [4]. The number of fermions that are allowed to couple, however, depends on the ansatz chosen, as this auxiliary scalar contributes to the Weyl anomaly of the matter sector. Using Jackiw's ansatz, one has to mix the Weyl variable ϕ with the fields N and φ to get the generators of surface deformations $H_\pm$ in the form (15). While this makes the introduction of quantum corrections straightforward, it obscures the role played by ϕ. When the closure of the surface deformation algebra is required on the quantum level, the quantization procedure which we employed imposes a constraint on the parameters a, b and c. Another constraint is obtained from the positivity requirement of the kinetic term of F, resulting in (22).

In the case $N=1$, the constraint on the fermionic matter (6) comes about by the coefficients of the kinetic terms of F and of ϕ. This situation has already been found in our previous paper [8], where we employed the conformal gauge and therefore had no need to use a localized action. As has been shown in Refs. 8, 9, the chiral quantum gravity closely resembles the chiral Schwinger model [1]. There the breakdown of the chiral U(1) symmetry is prevented by turning a classically frozen variable into a dynamical one. On the other hand, in chiral quantum gravity we have to deal with two broken symmetries: the local Lorentz (or, alternatively, general coordinate) symmetry and the Weyl symmetry. Indeed, not only the Lorentz variable F, but also the Weyl variable ϕ has to be treated as a dynamical quantum field.

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References

1. R. Jackiw, R. Rajaraman: Phys. Rev. Lett. 54 (1985) 1219
2. A.M. Polyakov: Phys. Lett. 103B (1981) 207, 211
3. K. Fujikawa: Nucl. Phys. B291 (1987) 583; K. Fujikawa, T. Inagaki, H. Suzuki: Phys. Lett. 213B (1989) 279; K. Fujikawa, N. Nakazawa, H. Suzuki: Phys. Lett. 221B (1989) 289; K. Fujikawa, T. Inagaki, H. Suzuki: Nucl. Phys. B332 (1990) 499
4. R. Jackiw. In: Quantum theory of gravity, S. Christensen (ed), p. 403; Bristol: Adam Hilger 1984; Nucl. Phys. B252 (1985) 343
5. K. Li: Phys. Rev. D34 (1986) 2292; see also F.A. Schaposnik, H. Vucetich: Int. J. Mod. Phys. A2 (1987) 1755

6. T. Fukuyama, K. Kamimura: Phys. Lett. 200B (1988) 75
7. L. Alvarez-Gaume, E. Witten: Nucl. Phys. B234 (1983) 269
8. T. Berger, I. Tsutsui: Nucl. Phys. B335 (1990) 245
9. T. Berger, I. Tsutsui: DESY-preprint 90-052, Hamburg, May 1990
10. The form (11) agrees with Li's result [5] up to an unimportant error there, but it is different from the one of Fukuyama and Kamimura [6]
11. W.A. Bardeen, B. Zumino: Nucl. Phys. B244 (1984) 421
12. H. Leutwyler: Phys. Lett. 153B (1985) 65
13. E. D'Hoker, R. Jackiw: Phys. Rev. D26 (1982) 3517
14. A.M. Polyakov: Mod. Phys. Lett. A2 (1987) 893; V.G. Knizhnik, A.M. Polyakov, A.B. Zamolodchikov: Mod. Phys. Lett. A3 (1988) 819
15. F. David: Mod. Phys. Lett. A3 (1988) 1651; J. Distler, H. Kawai: Nucl. Phys. B321 (1989) 509; J. Polchinski: Nucl. Phys. B324 (1989) 123