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SIMPLE FORMULAE FOR THE ORDER α_s^3 QCD CORRECTIONS TO THE REACTION $p + \bar{p} \to Q + \bar{Q} + X$

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The order α_s^3 QCD corrections to the parton–parton cross sections contributing to heavy flavour production in hadron–hadron collisions are discussed. We construct simple formulae which should yield reasonable approximations to the exact order α_s^3 results. Then we examine the differences between the predictions of the exact and approximate formulae in the case of the reaction $p + \bar{p} \to Q + \bar{Q} + X$ where Q is either the b or the t quark.

1. Introduction

During the last few years a great deal of progress has been made in the calculation of higher order corrections to inclusive and semi-inclusive processes in the framework of perturbative QCD [1]. At the present moment it seems that most of the first-order α_s corrections to $n \to m$ parton reactions with $n + m \leqslant 4$ have now been completed. For some processes one has even been able to extend these calculations beyond the first order of α_s. Examples are the quantity R defined in the reaction $e^+ + e^- \to X$, where X denotes any hadronic final state, which is now completely known up to order α_s^3 [2], and the K-factor in the Drell–Yan process for which a partial result exists in order α_s^2 [3]. The expressions for these corrections are very complicated especially when the Born cross section is already

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of order α_s or higher. It is clear that it will be very laborious to calculate radiative corrections to parton reactions beyond the first order of α_s except for a few special cases. This statement also applies to the order α_s corrections to Born processes which involve more than four particles, like multi-jet production [4].

The results of the calculations mentioned above can be summarized as follows. First, the size of the corrections can be rather large; a feature which can be mainly attributed to soft and virtual gluon contributions. Second, it turns out very often that the order α_s corrected distributions only differ slightly in shape from the lowest order ones. This indicates that the theoretical K-factor is only a slowly varying function of the various kinematical variables in the reaction, such as the transverse momenta or rapidities. A very useful approach is therefore to construct approximate formulae which describe the exact corrections reasonably well in the relevant regions of phase space accessible to experiment. This approach will only work when the K-factor does not show too much structure on the level of the hadronic cross sections. Further it is clear that any attempt to construct approximate formulae will only be successful if the following conditions are satisfied. First the theoretical and experimental uncertainties have to be so large that the difference between the exact and the approximate corrections will hardly be distinguishable. Second the approximation has to contain those terms which dominate the order α_s correction and can be generalized to higher orders. Fortunately the first condition is very often satisfied in strong interaction physics because of the large systematical and statistical errors. In addition we also have the theoretical uncertainties which can be attributed to the running coupling constant and the input parton distribution functions. In many examples it turns out that the second condition also holds. As has already been mentioned above the bulk of the large correction can be attributed to soft and virtual contributions. In QCD the leading soft gluon term always exponentiates, and sometimes this also holds for the leading virtual part (e.g. the π^2 term in the Drell–Yan K-factor [3]. Other important terms are the large logarithms which arise when some kinematical variables become very large with respect to a fixed mass. This phenomenon appears for instance in heavy flavour production [5, 6] in the case that the c.m. energy or the transverse momentum becomes much larger than the heavy quark mass. Since the coefficients of these large logarithms are determined by the Altarelli–Parisi splitting functions [7] they can be easily determined in higher orders by using renormalization group methods.

The considerations outlined above have inspired some authors to construct approximate formulae for various types of processes via renormalization group methods. Examples are direct photon production $p + \bar{p} \rightarrow \gamma + X$ [8] and the process $p + \bar{p} \rightarrow W + \gamma + X$ [9]. Analogous suggestions have been made for heavy flavour production [10–12] which we want to examine here. Our paper will be organized as follows. In sect. 2 we discuss the various production mechanisms which give the main contributions to the higher order corrections to the total parton–parton cross sections. These contributions dominate in specific regions of phase space so we concentrate on understanding which terms in the parton–parton double differential cross sections integrate up to yield the correct behaviour of the total parton–parton cross section. We then construct formulae which give fair approximations to the order α_s corrections to the double differential distributions. In sect. 3 we test just how good these approximations are by comparing exact and approximate results for the parton–parton cross sections defined in the $\overline{\text{MS}}$ and DIS schemes. Finally in sect. 4 we turn our attention to hadron–hadron collisions. We start by discussing some details of parton–parton fluxes for hadron–hadron collisions. Then we present results for heavy flavour production in the reaction $p + \bar{p} \to Q + \bar{Q} + X$, where Q is either a b quark or a t quark, at the energies of the CERN and Fermilab colliders.

2. Approximate formulae

Let us start by considering the renormalized parton–parton differential cross section $d\sigma_{ij \to fX}$ for a one-particle inclusive process (X denotes any inclusive state). This $d\sigma_{ij \to fX}$ is obtained by calculating all the Feynman diagrams up to a given order in QCD perturbation theory and implementing a renormalization scheme to remove any ultraviolet divergences. Then the infrared divergences are cancelled between the virtual diagrams and the bremsstrahlung diagrams. The resulting expressions for $d\sigma_{ij \to fX}$ is usually a long and complicated function of the kinematical variables s, t_1, and u_1 which are denoted by

$$s = (p_1 + p_2)^2, \qquad t_1 = (p_1 - q_1)^2 - m^2, \qquad u_1 = (p_2 - q_1)^2 - m^2 \tag{1}$$

(see fig. 1) where m is the mass of the detected particle f indicated by the momentum q_1. Due to higher order QCD corrections the renormalized

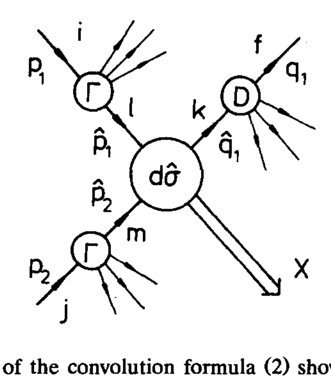


Fig. 1. Symbolic representation of the convolution formula (2) showing the splitting functions Γ, the differential cross section for the hard scattering $\hat{\sigma}$ and the fragmentation function D. The arrows denote the momenta assignments on the parton lines.

parton–parton cross section also contains collinear divergences (mass singularities) which have to be regulated in an universal way for all parton–parton processes. Therefore in addition to the kinematical variables listed in eq. (1) $d\sigma_{ij \to fX}$ also depends on initial and final state mass regulators p_i^2 and q_i^2, respectively. The mass factorization is achieved by writing $d\sigma_{ij \to fX}$ as a convolution integral over products of splitting and/or fragmentation functions with the so-called reduced parton differential cross section $d\hat{\sigma}_{lm \to kX}$ which is free of collinear divergences. The relationship between the renormalized parton cross section and the reduced cross section is called the mass factorization formula. In the case of a one-particle inclusive process it reads as follows (for the notation see fig. 1)

$$s^2 \frac{d^2\sigma_{ij \to fX}(s, t_1, u_1, p_1^2, p_2^2, q_1^2)}{dt_1\, du_1}$$
$$= \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \int_0^1 \frac{dx_3}{x_3^2} \Gamma_{li}(x_1, p_1^2, Q^2) \Gamma_{mj}(x_2, p_2^2, Q^2)$$
$$\times \hat{s}^2 \frac{d^2\hat{\sigma}_{lm \to kX}(\hat{s}, \hat{t}_1, \hat{u}_1, Q^2)}{d\hat{t}_1\, d\hat{u}_1} D_{fk}(x_3, q_1^2, Q^2). \tag{2}$$

In the following we will write this equation in a shorthand fashion as

$$d\sigma_{ij \to fX} = \Gamma_{li} \otimes \Gamma_{mj} \otimes d\hat{\sigma}_{lm \to kX} \otimes D_{fk}. \tag{3}$$

The collinear divergences are absorbed in the initial and final state splitting (fragmentation) functions which are denoted by Γ_{li}, Γ_{mj} and D_{fk} respectively. All the quantities in this formula can be expanded in the strong coupling constant as follows:

$$s^2 \frac{d^2\sigma_{ij}(s, t_1, u_1, p_1^2, p_2^2, q_1^2)}{dt_1\, du_1} = \sum_{n=0}^{\infty} s^2 \frac{d^2\sigma_{ij}^{(n)}(s, t_1, u_1, p_1^2, p_2^2, q_1^2)}{dt_1\, du_1},$$
$$\hat{s}^2 \frac{d^2\hat{\sigma}_{lm}(\hat{s}, \hat{t}_1, \hat{u}_1, Q^2)}{d\hat{t}_1\, d\hat{u}_1} = \sum_{n=0}^{\infty} \hat{s}^2 \frac{d^2\hat{\sigma}_{lm}^{(n)}(\hat{s}, \hat{t}_1, \hat{u}_1, Q^2)}{d\hat{t}_1\, d\hat{u}_1},$$
$$\Gamma_{li}(x, p^2, Q^2) = \sum_{n=0}^{\infty} \Gamma_{li}^{(n)}(x, p^2, Q^2),$$
$$D_{fk}(x, p^2, Q^2) = \sum_{n=0}^{\infty} D_{fk}^{(n)}(x, p^2, Q^2). \tag{4}$$

where $d\sigma_{ij}^{(n)}$, $d\hat{\sigma}_{lm}^{(n)}$, $\Gamma_{li}^{(n)}$ and $D_{fk}^{(n)}$ represent the order $\alpha_s^{(n)}$ parts of the collinear-singular cross section, the collinear-finite reduced cross section, the splitting function and the fragmentation function, respectively. Our aim is to calculate the coefficients in the power series expansion of the reduced cross section. This is done by iteration order-by-order in perturbation theory so that for example $d\hat{\sigma}^{(1)}(\hat{s}, \hat{t}_1, \hat{u}_1, Q^2)$ is expressed as $d\sigma^{(1)}(s, t_1, u_1, p_1^2, p_2^2, q_1^2)$ minus the convolution integrals of the splitting and/or fragmentation functions $\Gamma^{(1)}(x_1, p_1^2, Q^2)$, $\Gamma^{(1)}(x_2, p_2^2, Q^2)$, $D^{(1)}(x_3, q_1^2, Q^2)$ with the lowest order $d\hat{\sigma}^{(0)}(\hat{s}, \hat{t}_1, \hat{u}_1, Q^2)$. In this way we construct, order-by-order in perturbation theory the reduced differential cross section $d\hat{\sigma}(\hat{s}, \hat{t}_1, \hat{u}_1, Q^2)$ which is then folded with parton structure functions to predict results for one-particle inclusive distributions in hadron–hadron collisions.

Due to the arbitrariness of the mass factorization scheme, $d\hat{\sigma}$ as well as Γ and D depend on the mass factorization scale Q^2, which has to be chosen in such a way that it does not depend on x_i, otherwise we would have to explicitly integrate over this additional x_i dependence when performing the integrals in eq. (2). The kinematical variables corresponding to the reduced cross section are denoted by

$$\hat{s} = (\hat{p}_1 + \hat{p}_2)^2, \qquad \hat{t}_1 = (\hat{p}_1 - \hat{q}_1)^2 - m^2, \qquad \hat{u}_1 = (\hat{p}_2 - \hat{q}_1)^2 - m^2. \tag{5}$$

The relation between the momenta without the hat in (1) and with the hat in (5) is given by (see fig. 1)

$$\hat{p}_1 = x_1 p_1, \qquad \hat{p}_2 = x_2 p_2, \qquad \hat{q}_1 = \frac{1}{x_3} q_1. \tag{6}$$

In this way the invariants in (1) and (5) are related by

$$\hat{s} = x_1 x_2 s, \qquad \hat{t}_1 = \frac{x_1}{x_3} t_1, \qquad \hat{u}_1 = \frac{x_2}{x_3} u_1. \tag{7}$$

The starting point of the derivation of the various approximations to the reduced parton differential cross section is the mass factorization formula (2).

We now consider heavy flavour production in QCD, which proceeds by the following two reactions [13] in the Born approximation,

$$\mathrm{q} + \bar{\mathrm{q}} \to \mathrm{Q} + \bar{\mathrm{Q}}, \qquad \mathrm{g} + \mathrm{g} \to \mathrm{Q} + \bar{\mathrm{Q}}. \tag{8),(9}$$

Since in lowest order the splitting functions are

$$\Gamma_{ij}^{(0)}(x, p^2, Q^2) = \delta_{ij}\delta(1 - x), \tag{10}$$

with analogous results for $D_{ij}^{(0)}$, so we have

$$s^2 \frac{d^2\hat{\sigma}_{ij}^{(0)}(s,t_1,u_1)}{dt_1\,du_1} = s^2 \frac{d^2\sigma_{ij}^{(0)}(s,t_1,u_1)}{dt_1\,du_1}. \tag{11}$$

The zeroth-order reduced parton cross section for reaction (8) therefore equals that of the corresponding Born cross section

$$s^2 \frac{d^2\hat{\sigma}^{(0)}_{q\bar{q}\to QX}}{dt_1\,du_1} = \delta(s+t_1+u_1)\frac{\pi\alpha_s^2}{N}C_F\left[\frac{t_1^2+u_1^2}{s^2}+\frac{2m^2}{s}+\frac{\epsilon}{2}\right], \tag{12}$$

where $\epsilon = n-4$. The zeroth-order reduced parton cross section for the reaction (9) also equals that for Born process

$$s^2 \frac{d^2\hat{\sigma}^{(0)}_{gg\to QX}}{dt_1\,du_1} = \delta(s+t_1+u_1)\frac{\pi\alpha_s^2}{2(N^2-1)^2}\left[C_0 B_0(s,t_1,u_1) + C_K B_K(s,t_1,u_1)\right], \tag{13}$$

where B_0 and B_K are given by

$$B_0(s,t_1,u_1) = \frac{t_1^2+u_1^2}{s^2}B_{\mathrm{QED}}(s,t_1,u_1), \qquad B_K(s,t_1,u_1) = -B_{\mathrm{QED}}(s,t_1,u_1), \tag{14}$$

and B_{QED} is defined by

$$B_{\mathrm{QED}}(s,t_1,u_1) = \frac{t_1}{u_1}+\frac{u_1}{t_1}+\frac{4m^2s}{t_1u_1}\left(1-\frac{m^2s}{t_1u_1}\right)+\epsilon\left(-1+\frac{s^2}{t_1u_1}\right)+\epsilon^2\frac{s^2}{4t_1u_1}. \tag{15}$$

The colour coefficients are defined by

$$C_0 = N(N^2-1), \qquad C_K = (N^2-1)/N, \qquad C_{\mathrm{QED}} = (N^4-1)/N^2, \tag{16}$$

where C_{QED} will be needed later.

In the next-to-leading-order (NLO) one has the following processes

$$q+\bar{q} \to Q+\bar{Q}+g, \tag{17}$$

$$g+g \to Q+\bar{Q}+g, \tag{18}$$

$$g+q(\bar{q}) \to Q+\bar{Q}+q(\bar{q}), \tag{19}$$

as well as the virtual corrections to the Born reactions in (8) and (9). Up to order α_s the parton cross sections corresponding to the processes in (17)–(19) can be expressed with the help of eq. (2) in the following way. The order α_s corrected parton cross section for the quark-fusion process (17) is

$$\mathrm{d}\sigma^{(1)}_{q\bar{q}\to QX} = \Gamma^{(1)}_{qq}(\epsilon, \mu^2, Q^2) \otimes \mathrm{d}\hat{\sigma}^{(0)}_{q\bar{q}\to QX} + \Gamma^{(1)}_{\bar{q}\bar{q}}(\epsilon, \mu^2, Q^2) \otimes \mathrm{d}\hat{\sigma}^{(0)}_{q\bar{q}\to QX} + \mathrm{d}\hat{\sigma}^{(0)}_{q\bar{q}\to QX} \otimes D^{(1)}_{QQ}(m^2, Q^2) + \mathrm{d}\hat{\sigma}^{(1)}_{q\bar{q}\to QX} . \tag{20}$$

We can invert this relation to write the equation for $\mathrm{d}\hat{\sigma}^{(1)}_{q\bar{q}\to QX}$ as

$$\mathrm{d}\hat{\sigma}^{(1)}_{q\bar{q}\to QX} = \mathrm{d}\sigma^{(1)}_{q\bar{q}\to QX} - \Gamma^{(1)}_{qq}(\epsilon, \mu^2, Q^2) \otimes \mathrm{d}\hat{\sigma}^{(0)}_{q\bar{q}\to QX} - \Gamma^{(1)}_{\bar{q}\bar{q}}(\epsilon, \mu^2, Q^2) \otimes \mathrm{d}\hat{\sigma}^{(0)}_{q\bar{q}\to QX} - \mathrm{d}\hat{\sigma}^{(0)}_{q\bar{q}\to QX} \otimes D^{(1)}_{QQ}(m^2, Q^2), \tag{21}$$

showing the cancellation of the collinear singularities from the renormalized cross section $\mathrm{d}\sigma^{(1)}$. In a case where there are no collinear divergences in the reaction then we can immediately identify

$$\mathrm{d}\sigma^{(1)}_{q\bar{q}\to QX} = \mathrm{d}\hat{\sigma}^{(1)}_{q\bar{q}\to QX} . \tag{22}$$

In such cases we will not write a hat on the $\mathrm{d}\sigma$ to emphasize that no collinear divergences were removed.

If the square of the heavy quark mass m^2 is of the order of magnitude of the large kinematical variables s, t_1 and u_1 we do not need any fragmentation function D in eqs. (20) or (21) since it belongs to $\mathrm{d}\hat{\sigma}^{(1)}_{q\bar{q}\to QX}$. Note that in order α_s we only have one nontrivial Γ_{ij} per term in eq. (21). The reduced cross section $\mathrm{d}\hat{\sigma}$ in eq. (2) depends on m^2, as well as the kinematical variables listed in eq. (7). However, at present hadron collider energies m^2 can become much smaller than s, t_1 or u_1 in the case of bottom or charm production. Therefore the m^2 dependent terms in $\mathrm{d}\hat{\sigma}$ will become logarithmically enhanced due to potential collinear divergences if m^2 would be put equal to zero. The coefficients of these logarithms can be inferred from the mass singular terms in the functions Γ and D. In order α_s the splitting functions are

$$\Gamma^{(1)}_{ij}(x, p^2, Q^2) = \frac{\alpha_s}{2\pi}\left[P_{ij}(x)\ln\frac{Q^2}{p^2} + f_{ij}(x)\right], \tag{23}$$

with analogous expressions for $D^{(1)}_{ij}$. The coefficients of the mass singular terms in eq. (23) are the Altarelli–Parisi splitting functions [7]. For convenience we will list them below:

$$P_{gg}(x) = C_A\left[\frac{2}{(1-x)_+} + \frac{2}{x} - 4 + 2x - 2x^2\right] + \tfrac{1}{2}\beta_0\delta(1-x),$$

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \tfrac{3}{2}\delta(1-x)\right],$$

$$P_{gq}(x) = C_F\left[\frac{1+(1-x)^2}{x}\right],$$

$$P_{qg}(x) = T_f\left[x^2 + (1-x)^2\right]. \tag{24}$$

Here β_0 denotes the lowest order coefficient of the β function

$$\beta_0 = \tfrac{11}{3}C_A - \tfrac{4}{3}T_f n_f, \tag{25}$$

and n_f is the number of active flavours. The forms of the f_{ij} depend upon the choice of the factorization scheme. Standard schemes are $\overline{\text{MS}}$ and DIS [5, 6, 11]. In QCD the colour factors C_A, C_F and T_F are given by

$$C_A = N, \qquad C_F = \frac{N^2-1}{2N}, \qquad T_f = \tfrac{1}{2}, \tag{26}$$

with $N = 3$ for SU(3). The choice of the number of flavours n_f will be discussed later.

The original calculations [5, 6] of the processes in (17)–(19) were set up in such a way that the initial state gluons and quarks were taken to be massless, i.e. $p_i^2 = 0$, in (2). This means that we had to regulate the collinear divergences by using the method of n-dimensional regularization, implying that $\ln(Q^2/p^2)$ in (23) is replaced as follows:

$$\ln\frac{Q^2}{p^2} \to \frac{2}{\epsilon} + \gamma_E - \ln 4\pi + \ln\frac{Q^2}{\mu^2}, \tag{27}$$

where γ_E (the Euler constant) and μ^2 (the parameter introduced to keep the coupling constant dimensionless in n-dimensions) are artifacts of n-dimensional regularization. (Note that this parameter μ^2 should not be confused with the renormalization and factorization mass scales which unfortunately are often denoted by μ^2.) In the case that the heavy quark mass takes the role of a regulator

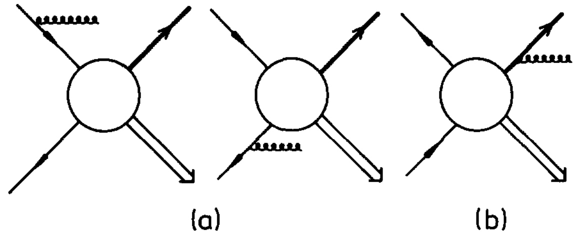


Fig. 2. Feynman diagrams showing (a) initial state gluon bremsstrahlung and (b) final state quark fragmentation contributions to the $q\bar{q}$ channel. The arrows denote the flow of charge.

mass in Γ we can simply put $p^2 = m^2$ in eq. (23). While inspecting eq. (20) we discern two types of production mechanisms in the order α_s contribution to $d\sigma^{(1)}_{q\bar{q}\to QX}$. They are initial state gluon bremsstrahlung (first and second terms in (20), see fig. 2a) and final state quark fragmentation (third term in (20) see fig. 2b), which will be referred to as mechanisms ISGB and FSQF, respectively.

The calculation of the approximate formulae for heavy flavour production will proceed as follows. First, it is clear that the initial state collinear divergences which are regularized by n-dimensional regularization have to be factored out from $d\sigma^{(1)}_{q\bar{q}}$. Second, the collinear divergences regularized by m^2 have to be kept since m^2 stands for a genuine mass. In general one cannot say too much about the order α_s reduced cross section $d\hat{\sigma}^{(1)}$ except for the case of initial state gluon bremsstrahlung. Since the splitting functions of the latter mechanism behave like $1/(1-x)_+$ (see P_{gg} and P_{qq} in eq. (24)) we can expect terms of the type $s_4^{-1}\ln^i(s_4/m^2)$ where $s_4 = s + t_1 + u_1$ in the reduced cross section. Notice that for the Born reaction $s_4 = 0$ [see eqs. (12) and (13)] so that the limit $s_4 \to 0$ in the two-to-three-body process represents the region where the gluon gets soft. This type of term is not present in the gq and $g\bar{q}$ channels since the relevant splitting function P_{gq} does not contain singular terms as $x \to 1$. The terms of the type $s_4^{-1}\ln^i(s_4/m^2)$ dominate the threshold region $s \to 4m^2$ where the soft matrix element is obtained from the eikonal approximation with a cut-off Δ on the upper limit of s_4 integration, where Δ is much smaller than m^2, s, t_1 and u_1. The $1/\epsilon^2$ and $1/\epsilon$ terms are then dropped since either they are cancelled by the virtual pieces in the cross section or they are removed by mass factorization. Then the $s_4^{-1}\ln^i(s_4/m^2)$ terms, where $i = 0, 1$ can be simply inferred from the correspondence relation between them and the $\ln^{i+1}(\Delta/m^2)$ terms. This procedure has been very extensively used in ref. [6].

Another important term which dominates the threshold region of $d\hat{\sigma}^{(1)}$ is the so-called Coulomb singularity, which appears in graphs where gluons are exchanged between heavy quarks. An explicit calculation of the heavy quark vertex function allows us to find these terms which behave like $\pi^2/\sqrt{1-4m^2/s}$. Starting with the quark–antiquark fusion process in (17) and using the $\overline{MS}$ factorization scheme which is equivalent to putting $f_{ij}(x)$ equal to zero in (23), we find the following contribution from the ISGB mechanism:

$$s^2\frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}}}{dt_1\,du_1}\bigg|^{\overline{MS}}_{ISGB} = \frac{\alpha_s^3(Q^2)C_F}{N}\Bigg[\Bigg\{C_F\Bigg(4\ln\frac{s_4}{m^2}+8\ln\frac{u_1}{t_1}+2\ln\frac{m^2s}{t_1u_1}-2$$

$$-2\frac{1-2m^2/s}{\sqrt{1-4m^2/s}}\ln x+2\ln\frac{m^2}{Q^2}\Bigg)$$

$$+C_A\Bigg(-3\ln\frac{u_1}{t_1}+\frac{1-2m^2/s}{\sqrt{1-4m^2/s}}\ln x-\ln\frac{m^2s}{t_1u_1}\Bigg)\Bigg\}\frac{1}{s_4}$$

$$+\Bigg\{C_F\Bigg(2\ln^2\frac{\Delta}{m^2}+\Bigg(8\ln\frac{u_1}{t_1}+2\ln\frac{m^2s}{t_1u_1}-2-2\frac{1-2m^2/s}{\sqrt{1-4m^2/s}}\ln x\Bigg)\ln\frac{\Delta}{m^2}$$

$$+\ln\frac{m^2}{Q^2}\ln\frac{\Delta^2}{(s+t_1)(s+u_1)}\Bigg)$$

$$+C_A\Bigg(-3\ln\frac{u_1}{t_1}+\frac{1-2m^2/s}{\sqrt{1-4m^2/s}}\ln x-\ln\frac{m^2s}{t_1u_1}\Bigg)\ln\frac{\Delta}{m^2}$$

$$+\frac{\pi^2}{2}(C_F-\tfrac{1}{2}C_A)\frac{1}{\sqrt{1-4m^2/s}}\Bigg\}\delta(s_4)\Bigg]\Bigg[\frac{t_1^2+u_1^2}{s^2}+\frac{2m^2}{s}\Bigg], \qquad (28)$$

where x and s_4 are defined by

$$x=\frac{1-\sqrt{1-4m^2/s}}{1+\sqrt{1-4m^2/s}}, \qquad s_4=s+t_1+u_1 \qquad (29)$$

and Q^2 is the mass factorization scale which is left over after the initial state collinear divergences, indicated by ϵ^{-1} in (27) are removed from the parton cross section. In eq. (28) the hard contributions proportional to $1/s_4$ have to be integrated from $\Delta\leqslant s_4\leqslant s_{4\max}$ so that the total result is finite in the limit Δ goes to zero. The running coupling constant is determined in the $\overline{MS}$ scheme where the heavy flavours are decoupled when the momenta go to zero (for a discussion of this point see refs. [5, 11]). The renormalization scale will be chosen to be equal to the mass factorization scale Q^2 so that the α_s in the above and subsequent formulae are functions of Q^2. Note that this formula is not symmetric under $t\leftrightarrow u$ so that there is an angular asymmetry, which leads to an asymmetry in the rapidity distribution. To be explicit we define $t_1=(p_{\bar{q}}-p_Q)^2-m^2$, $u_1=(p_q-p_Q)^2-m^2$

and choose θ, the polar angle of the outgoing heavy quark with respect to the z axis along the direction of the light quark.

Choosing the $f_{ij}(x)$ in eq. (23) appropriate for the DIS factorization scheme we find

$$s^2 \frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}}}{dt_1\,du_1}\bigg|^{\mathrm{DIS}}_{\mathrm{ISGB}} = s^2 \frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}}}{dt_1\,du_1}\bigg|^{\overline{\mathrm{MS}}}_{\mathrm{ISGB}} + \frac{\alpha_s^3(Q^2)C_F^2}{N}\Bigg[\bigg\{-\ln\frac{s_4^2}{(s+t_1)(s+u_1)}+\frac{3}{2}\bigg\}\frac{1}{s_4}$$
$$+\bigg\{-\tfrac{1}{2}\ln^2\frac{\Delta}{(s+t_1)}-\tfrac{1}{2}\ln^2\frac{\Delta}{(s+u_1)}+\tfrac{3}{4}\ln\frac{\Delta^2}{(s+t_1)(s+u_1)}$$
$$+\tfrac{9}{2}+\frac{\pi^2}{3}\bigg\}\delta(s_4)\Bigg]\bigg[\frac{t_1^2+u_1^2}{s^2}+\frac{2m^2}{s}\bigg]. \quad (30)$$

The contributions from the other mechanism, i.e. FSQF, can be derived from eqs. (2) and (7). Substituting eqs. (12) and (23) in eq. (2) the third term in eq. (20) becomes

$$s^2 \frac{d^2\sigma^{(1)}_{q\bar{q}}}{dt_1\,du_1}\bigg|_{\mathrm{FSQF}} = \frac{1}{2}\frac{\alpha_s^3(Q^2)C_F^2}{N}\ln\frac{m_T^2}{m^2}\Bigg\{\bigg[\frac{s^2+(t_1+u_1)^2}{-s(t_1+u_1)}\bigg]\frac{1}{s_4}$$
$$+\bigg\{\frac{3}{2}+2\ln\frac{\Delta}{s}\bigg\}\delta(s_4)\Bigg\}\bigg(\frac{t_1^2+u_1^2}{(t_1+u_1)^2}+\frac{2m^2}{s}\bigg). \quad (31)$$

Since the potential collinear divergence appearing in eq. (31) is represented by m^2 instead of ϵ^{-1} the mass factorization scale Q^2 is not related to the one mentioned below eq. (29). Here it has been chosen to be equal to t_1u_1/s. The reason is twofold. This expression is invariant under scale transformations with respect to the initial state momenta [see eq. (7)], i.e.

$$\frac{t_1u_1}{s} = \frac{\hat{t}_1\hat{u}_1}{\hat{s}} = p_t^2 + m^2 \equiv m_T^2 \quad (32)$$

and also it does not introduce unwanted asymptotic behaviour for large s in the total parton–parton cross section. We will come back to this point later on. (Notice that t_1u_1/s is not invariant under scale transformations with respect to the final state momentum of the detected particle. Therefore we should not include additional terms which arise by convoluting the fragmentation function with the corresponding reduced cross section.) Finally we want to make the following important remark. Eq. (31) contains final state soft gluon terms of the type $1/s_4$

and $\ln(\Delta/m^2)$ which have already been incorporated into the ISGB result (28). The latter formula is built up out of all soft gluon terms in initial as well as final states. Simply adding eqs. (28) and (31) would imply double counting with respect to the final state soft gluon terms. Since it will turn out that the FSQF mechanism gives a rather small contribution to the cross section we will not include eq. (31) in our final approximation.

Proceeding in an analogous way with the gluon–gluon fusion process (18) we get

$$\begin{aligned} d\sigma^{(1)}_{gg\to QX} &= \Gamma^{(1)}_{gg}(\epsilon,\mu^2,Q^2) \otimes d\hat{\sigma}^{(0)}_{gg\to QX} + \Gamma^{(1)}_{gg}(\epsilon,\mu^2,Q^2) \otimes d\hat{\sigma}^{(0)}_{gg\to QX} \\ &\quad + \Gamma^{(1)}_{Qg}(m^2,Q^2) \otimes d\hat{\sigma}^{(0)}_{Qg\to QX} + \Gamma^{(1)}_{Qg}(m^2,Q^2) \otimes d\hat{\sigma}^{(0)}_{gQ\to QX} \\ &\quad + d\hat{\sigma}^{(0)}_{gg\to gX} \otimes D^{(1)}_{Qg}(m^2,Q^2) + d\hat{\sigma}^{(0)}_{gg\to QX} \otimes D^{(1)}_{QQ}(m^2,Q^2) + d\hat{\sigma}^{(1)}_{gg\to QX}\,. \end{aligned} \tag{33}$$

The reduced parton cross sections in (33) are [14]

$$\begin{aligned} s^2\frac{d^2\hat{\sigma}^{(0)}_{Qg\to QX}}{dt_1\,du_1} &= \delta(s+t_1+u_1)\frac{\pi\alpha_s^2}{2(N^2-1)^2} \\ &\quad \times\left[NC_0\left(\frac{2st_1}{u_1^2}-1\right)+NC_K\left(3-\frac{2st_1}{u_1^2}\right)-C_{QED}\right]B_{QED}(u_1,t_1,s)\,, \\ s^2\frac{d^2\hat{\sigma}^{(0)}_{gQ\to QX}}{dt_1\,du_1} &= s^2\frac{d^2\hat{\sigma}^{(0)}_{Qg\to QX}}{dt_1\,du_1}(t_1\leftrightarrow u_1)\,, \\ s^2\frac{d^2\hat{\sigma}^{(0)}_{gg\to gX}}{dt_1\,du_1} &= \delta(s+t_1+u_1)\frac{4\pi\alpha_s^2}{(N^2-1)^2}NC_0\left[3-\frac{st_1}{u_1^2}-\frac{su_1}{t_1^2}-\frac{t_1u_1}{s^2}\right], \end{aligned} \tag{34}$$

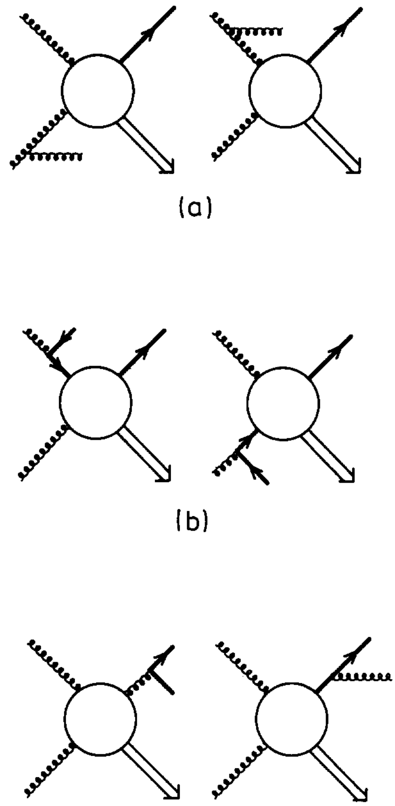
where the kinematical variables in the last reaction are those of massless particles. Since the incoming gluons are massless the initial state splitting functions Γ depend on ϵ, cf. (27). In this channel there are four types of production mechanisms. They are given by initial state gluon bremsstrahlung (first and second term in eq. (33), see fig. 3a), flavour excitation (third and fourth terms in eq. (33), see fig. 3b), gluon splitting (fifth term in eq. (33), see fig. 3c) and final state quark fragmentation (penultimate term in eq. (33), see fig. 3d). They will be referred to as mechanisms ISGB, FE, GS and FSQF respectively. For the gluon–gluon fusion process we will proceed in the same way as for the $q\bar{q}$ case. For convenience we will split the double differential cross section into three parts (see ref. [6])

$$s^2\frac{d^2\hat{\sigma}^{(1)}_{gg}}{dt_1\,du_1} = s^2\frac{d^2\hat{\sigma}^{(1)}_{gg,0}}{dt_1\,du_1} + s^2\frac{d^2\hat{\sigma}^{(1)}_{gg,K}}{dt_1\,du_1} + s^2\frac{d^2\hat{\sigma}^{(1)}_{gg,QED}}{dt_1\,du_1}\,. \tag{35}$$

In the $\overline{\rm MS}$ scheme the ISGB mechanism provides us with the following contribution, where all the $\ln^{i+1}(\Delta/m^2)$ terms are explicitly given in appendix C of ref. [6]

$$s^2 \frac{d^2\hat{\sigma}^{(1)}_{gg,0}}{dt_1\,du_1}\bigg|^{\overline{\rm MS}}_{\rm ISGB} = \frac{1}{4}\frac{\alpha_s^3(Q^2)NC_0}{(N^2-1)^2}\Bigg[\Bigg\{\frac{t_1^2+u_1^2}{s^2}\left(8\ln\frac{s_4}{m^2} - 2 + 2\ln\frac{(1+x)^2}{x} + 4\ln\frac{m^2}{Q^2}\right)$$
$$-4\frac{t_1^2}{s^2}\ln\frac{-t_1}{m^2} - 4\frac{u_1^2}{s^2}\ln\frac{-u_1}{m^2}\Bigg\}\frac{1}{s_4} + \Bigg\{\frac{t_1^2+u_1^2}{s^2}\Bigg(4\ln^2\frac{\Delta}{m^2} - 2\ln\frac{\Delta}{m^2}$$
$$+2\ln\frac{(1+x)^2}{x}\ln\frac{\Delta}{m^2} + 2\ln\frac{m^2}{Q^2}\ln\frac{\Delta^2}{(s+t_1)(s+u_1)}\Bigg)$$
$$-\left(4\frac{t_1^2}{s^2}\ln\frac{-t_1}{m^2} + 4\frac{u_1^2}{s^2}\ln\frac{-u_1}{m^2}\right)\ln\frac{\Delta}{m^2}\Bigg\}\delta(s_4)\Bigg]B_{\rm QED}(s,t_1,u_1)$$
$$+\frac{1}{2}\frac{\alpha_s^3(Q^2)NC_0}{(N^2-1)^2}\ln\frac{m^2}{Q^2}\left\{\left(\frac{s+t_1}{u_1^2}\right)\left(\frac{t_1^2+(s+t_1)^2}{s^2}\right)\right.$$
$$\left.\times B_{\rm QED}\left(-\frac{u_1 s}{s+t_1}, -\frac{t_1u_1}{s+t_1}, u_1\right) + (t_1 \leftrightarrow u_1)\right\}, \tag{36}$$

$$s^2 \frac{d^2\hat{\sigma}^{(1)}_{gg,K}}{dt_1\,du_1}\bigg|^{\overline{\rm MS}}_{\rm ISGB} = \frac{1}{4}\frac{\alpha_s^3(Q^2)NC_K}{(N^2-1)^2}\Bigg[\Bigg\{-8\ln\frac{s_4}{m^2} + \frac{6t_1^2+6u_1^2+8t_1u_1}{s^2}$$
$$+2\frac{(s-2m^2)(t_1^2+u_1^2)}{s^3\sqrt{1-4m^2/s}}\ln x - 4\ln\frac{m^2}{Q^2}\Bigg\}\frac{1}{s_4} + \Bigg\{-4\ln^2\frac{\Delta}{m^2}$$
$$+\left(\frac{6t_1^2+6u_1^2+8t_1u_1}{s^2} + 2\frac{(s-2m^2)(t_1^2+u_1^2)}{s^3\sqrt{1-4m^2/s}}\ln x\right)\ln\frac{\Delta}{m^2}$$
$$-2\ln\frac{m^2}{Q^2}\ln\frac{\Delta^2}{(s+t_1)(s+u_1)}\Bigg\}\delta(s_4)\Bigg]B_{\rm QED}(s,t_1,u_1)$$
$$-\frac{1}{2}\frac{\alpha_s^3(Q^2)NC_K}{(N^2-1)^2}\ln\frac{m^2}{Q^2}\left\{\left(\frac{s+t_1}{u_1^2}\right)\times B_{\rm QED}\left(-\frac{u_1 s}{s+t_1}, -\frac{t_1u_1}{s+t_1}, u_1\right)\right.$$
$$\left.+(t_1\leftrightarrow u_1)\right\} + \frac{1}{16}\frac{\alpha_s^3(Q^2)NC_K}{(N^2-1)^2}\frac{\pi^2}{\sqrt{1-4m^2/s}}$$
$$\times\left\{10 + 2\frac{(t_1-u_1)^2}{s^2} - 80\frac{m^4}{t_1u_1} + 128\frac{m^8}{t_1^2u_1^2}\right\}\delta(s_4). \tag{37}$$



(a)

(b)

(c) (d)

Fig. 3. Feyman diagrams showing (a) initial state gluon bremsstrahlung, (b) flavour excitation, (c) gluon splitting, and (d) final state quark fragmentation contributions to the gg channel. The arrows denote the flow of charge.

Since the mass factorization parts (the coefficients of $\ln(m^2/Q^2)$) in eqs. (36), (37) originate from the convolution of the gluon splitting function (23) with the lowest order parton cross section (13), we discern two types of terms which dominate. They are represented by the factors $1/(1-x)_+$ and $1/x$, i.e. the region where the emitted gluon and the internal gluon become soft respectively. The first term gives rise to the soft gluon part of the mass factorization piece already discovered in the $q\bar{q}$ subprocess in (28). The $1/x$ term leads to the double t_1 or u_1 channel poles in the cross section which will also be present in the FE and GS mechanisms. This pole is responsible for the constant behaviour of the total parton–parton cross section as $s \to \infty$. The exact coefficients of the $\ln(m^2/Q^2)$ terms can be found in (A.1)–(A.4) given in appendix A, but we will only use the approximate expressions here.

Finally we have

$$s^2 \frac{d^2\hat{\sigma}^{(1)}_{gg,QED}}{dt_1\,du_1}\Bigg|^{\overline{MS}}_{ISGB} = \frac{1}{4}\frac{\alpha_s^3(Q^2)C_{QED}}{(N^2-1)^2}\left[\left\{-2-2\frac{s-2m^2}{s\sqrt{1-4m^2/s}}\ln x\right\}\frac{1}{s_4}\right.$$
$$\left.+\left\{-2-2\frac{(s-2m^2)}{s\sqrt{1-4m^2/s}}\ln x \ln\frac{\Delta}{m^2}\right\}\delta(s_4)\right]B_{QED}(s,t_1,u_1)$$
$$+\frac{1}{16}\frac{\alpha_s^3(Q^2)C_{QED}}{(N^2-1)^2}\frac{\pi^2}{\sqrt{1-4m^2/s}}\left\{-4+64\frac{m^4}{t_1u_1}-128\frac{m^8}{t_1^2u_1^2}\right\}\delta(s_4). \tag{38}$$

For the definitions of x and s_4 see eq. (29). The choice of the scale in the running coupling constant has been discussed above.

From the FSQF mechanism we obtain the expression,

$$s^2\frac{d^2\hat{\sigma}^{(1)}_{gg}}{dt_1\,du_1}\Bigg|_{FSQF} = \frac{1}{8}\frac{\alpha_s^3(Q^2)}{(N^2-1)^2}\ln\frac{m_T^2}{m^2}$$
$$\times\left\{NC_0\frac{t_1^2+u_1^2}{(t_1+u_1)^2}+NC_K\left(2\frac{t_1u_1}{(t_1+u_1)^2}-3\right)+C_{QED}\right\}$$
$$\times\left[\left\{\frac{s^2+(t_1+u_1)^2}{-s(t_1+u_1)}\right\}B_{QED}\left(s,-\frac{st_1}{t_1+u_1},-\frac{su_1}{t_1+u_1}\right)\frac{1}{s_4}\right.$$
$$\left.+\left\{\frac{3}{2}+2\ln\frac{\Delta}{s}\right\}B_{QED}(s,t_1,u_1)\delta(s_4)\right], \tag{39}$$

where the choice of the mass factorization scale $Q^2=m_T^2$ has been defined in eq. (32). As in the $q\bar{q}$ case we will neglect this contribution due to double counting of the soft gluon terms. Fortunately eq. (39) only contributes a negligible amount to the final cross section.

The next two mechanisms which determine the large plateau effect in the parton cross section (see fig. 4 in ref. [5] and figs. 6 and 7 in ref. [6]) are given by the following expressions. Choosing the scale as in eq. (39), i.e. $Q^2=m_T^2$, the first mechanism (FE) provides us with the result

$$s^2 \frac{d^2\hat{\sigma}_{gg}^{(1)}}{dt_1\,du_1}\bigg|_{FE} = \frac{1}{8}\frac{\alpha_s^3(Q^2)}{(N^2-1)^2}\ln\frac{m_T^2}{m^2}$$

$$\times\left[NC_0\left(1+\frac{2s(s+u_1)}{u_1^2}\right)+NC_K\left(-3-\frac{2s(s+u_1)}{u_1^2}\right)+C_{QED}\right]$$

$$\times\left[\frac{t_1^2+s_4^2}{t_1(s+u_1)^2}\right]B_{QED}\left(-\frac{t_1u_1}{s+u_1},t_1,-\frac{st_1}{s+u_1}\right)+(t_1\leftrightarrow u_1),\quad (40)$$

and the second one (GS) equals

$$s^2 \frac{d^2\hat{\sigma}_{gg}^{(1)}}{dt_1\,du_1}\bigg|_{GS} = \frac{\alpha_s^3(Q^2)NC_0}{(N^2-1)^2}\ln\frac{m_T^2}{m^2}\left[3+\frac{t_1(t_1+u_1)}{u_1^2}+\frac{u_1(t_1+u_1)}{t_1^2}-\frac{t_1u_1}{(t_1+u_1)^2}\right]$$

$$\times\left[\frac{(t_1+u_1)^2+s_4^2}{-s^2(t_1+u_1)}\right].\quad (41)$$

The double pole terms $1/t_1^2$ and $1/u_1^2$ lead to the large plateau present in the parton cross section $\hat{\sigma}_{gg}$ at large s where $\hat{\sigma}_{gg}\to 1/m^2$ (see (2.6) in ref. [5]). From the exact form of $\hat{\sigma}_{gg}$ it follows that the factorization scale Q^2 has to be chosen in such a way that the good asymptotic behaviour of the parton cross section will be preserved. Therefore we took $Q^2 = t_1u_1/s = m_T^2$ which has the additional advantage that it is invariant under scale transformations of initial state momenta as is indicated in eq. (32). Notice that the choice $Q^2 = s$ is wrong since it will lead to an asymptotic behaviour $\hat{\sigma}_{gg}\to(1/m^2)\ln(s/m^2)$ which is not shown by the exact calculation.

Finally the order α_s corrected parton cross section for the gluon–quark(antiquark) fusion process (19) is

$$d\sigma_{gq\to QX}^{(1)} = \Gamma_{gq}^{(1)}(\epsilon,\mu^2,Q^2)\otimes d\hat{\sigma}_{gg\to QX}^{(0)} + \Gamma_{\bar{q}g}^{(1)}(\epsilon,\mu^2,Q^2)\otimes d\hat{\sigma}_{\bar{q}q\to QX}^{(0)}$$

$$+\Gamma_{Qg}^{(1)}(m^2,Q^2)\otimes d\hat{\sigma}_{qQ\to QX}^{(0)} + d\hat{\sigma}_{gq\to gX}^{(0)}\otimes D_{Qg}^{(1)}(m^2,Q^2) + d\hat{\sigma}_{gq\to QX}^{(1)}.\quad (42)$$

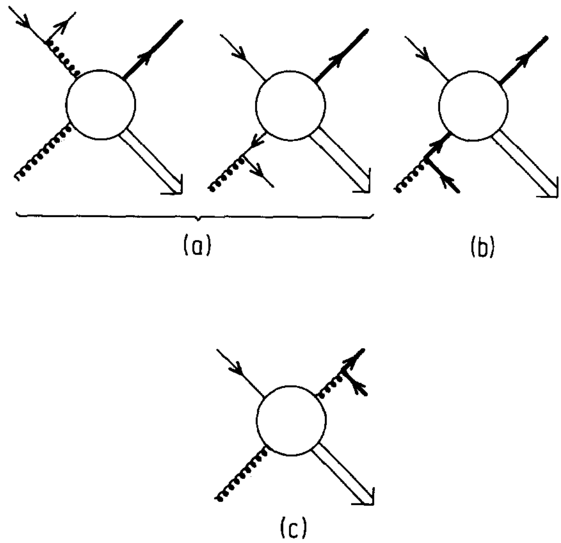


Fig. 4. Feyman diagrams showing (a) initial state quark bremsstrahlung, (b) flavour excitation, and (c) gluon splitting contributions to the gq channel. The arrows denote the flow of charge.

The Born cross sections $d\sigma_{qQ \to QX}$ and $d\sigma_{gq \to gX}$ in eq. (42) are given by

$$s^2 \frac{d\sigma^{(0)}_{qQ \to QX}}{dt_1\, du_1} = \delta(s + t_1 + u_1) \frac{\pi \alpha_s^2}{N} C_F \left[\frac{s^2 + u_1^2}{t_1^2} + \frac{2m^2}{t_1} \right],$$

$$s^2 \frac{d\sigma^{(0)}_{gq \to gX}}{dt_1\, du_1} = \delta(s + t_1 + u_1) \frac{\pi \alpha_s^2}{2(N^2 - 1)^2}$$

$$\times \left[NC_0 \left(\frac{2su_1}{t_1^2} - 1 \right) + NC_K \left(3 - \frac{2su_1}{t_1^2} \right) - C_{\mathrm{QED}} \right] \left[\frac{s^2 + u_1^2}{u_1 s} \right]. \quad (43)$$

Here there are three types of production mechanisms. The first and second terms in eq. (42) are the equivalent of ISGB where the emitted gluon is replaced by a light quark (see fig. 4a) so we will call them ISQR for initial state quark radiation. The third term represents flavour excitation (FE) (see fig. 4b) while the fourth term represents gluon splitting (GS) (see fig. 4c). The splitting function $\Gamma^{(1)}_{gq}$ in (23) contains a $1/x$ term, which was also present in $\Gamma^{(1)}_{gg}$ discussed below (40). Since this part leads to the double pole term $1/t_1^2$, which dominates the total parton–parton cross section as $s \to \infty$, we included it in our approximation, yielding

$$s^2 \frac{d^2\hat{\sigma}^{(1)}_{gq}}{dt_1\, du_1}\bigg|_{ISQR} = \frac{1}{2} \frac{\alpha_s^3(Q^2) C_F}{(N^2-1)^2} \ln \frac{m^2}{Q^2} \left(\frac{s+u_1}{t_1^2}\right) \left(C_0 \frac{u_1^2 + (s+u_1)^2}{s^2} - C_K\right)$$

$$\times B_{QED}\left(-\frac{st_1}{s+u_1}, t_1, -\frac{t_1 u_1}{s+u_1}\right). \tag{44}$$

Inclusion of terms not proportional to $\ln(m^2/Q^2)$ would make this result scheme dependent. Such terms are, however, very small in both the $\overline{\text{MS}}$ and DIS schemes so we will simply neglect them. The FE contribution is given by [14]

$$s^2 \frac{d^2\hat{\sigma}^{(1)}_{gq}}{dt_1\, du_1}\bigg|_{FE} = \frac{1}{4} \frac{\alpha_s^3(Q^2) C_F}{N} \ln \frac{m_T^2}{m^2}$$

$$\times \left[\frac{s^2 + (s+t_1)^2}{t_1^2} - \frac{2m^2(s+t_1)}{t_1 u_1}\right] \left[\frac{u_1^2 + s_4^2}{-u_1(s+t_1)^2}\right], \tag{45}$$

and the GS contribution reads

$$s^2 \frac{d^2\hat{\sigma}^{(1)}_{gq}}{dt_1\, du_1}\bigg|_{GS} = \frac{1}{8} \frac{\alpha_s^3(Q^2)}{(N^2-1)^2} \ln \frac{m_T^2}{m^2}$$

$$\times \left[NC_0\left(-1 - 2\frac{u_1(t_1+u_1)}{t_1^2}\right) + NC_K\left(3 + 2\frac{u_1(t_1+u_1)}{t_1^2}\right) - C_{QED}\right]$$

$$\times \left[\frac{(t_1+u_1)^2 + s_4^2}{s^2(t_1+u_1)}\right] \left[\frac{(t_1+u_1)^2 + u_1^2}{u_1(t_1+u_1)}\right]. \tag{46}$$

Note that these formulae are not symmetric under $t_1 \leftrightarrow u_1$ so that there is an angular asymmetry. Therefore we define $t_1 = (p_g - p_Q)^2 - m^2$, $u_1 = (p_q - p_Q)^2 - m^2$ and choose θ, the polar angle of the outgoing heavy quark with respect to the z axis along the direction of the light quark. All that has been mentioned above also holds for the approximate formulae for the gluon–antiquark reaction. The corresponding formulae follow from (44) to (46) by $t_1 \leftrightarrow u_1$.

Notice that in the above cross sections the exact m^2 dependence has been kept in the reduced cross sections $d\hat{\sigma}^{(0)}_{ij \to QX}$ of (20), (33) and (42). However, we have checked that for charm production the terms explicitly proportional to m^2 can be neglected at large p_T except for the ISGB mechanism since here the threshold behaviour is important. For $m = 0$ eqs. (39)–(41) are also presented in (5.3) of ref.

[11]. However, we have found some discrepancies in the latter for the FSQF and GS mechanisms. We also disagree with the mass singular logarithm $\ln\rho$ where $\rho = 4m^2/s$ since it leads to the wrong asymptotic behaviour for the parton cross section as explained above. Finally, we want to comment on another procedure to deal with heavy quark production when $m^2 \ll s$, t_1, or u_1. One can put $m = 0$ everywhere and regularize the mass singularities via n-dimensional regularization. The conventional procedure of mass factorization has then to be carried out. For this line of approach see ref. [15].

3. Parton–parton cross sections

The total parton–parton cross sections $\hat{\sigma}_{ij}(s, m^2, Q^2)$ follow from integrating the double differential cross sections given in sect. 2 over t_1 and u_1 using

$$\hat{\sigma}_{ij}(s, m^2, Q^2) = \int_{(s-\bar{s})/2}^{(s+\bar{s})/2} \mathrm{d}(-t_1) \int_{-sm^2/t_1}^{(s+t_1)} \mathrm{d}(-u_1) \frac{\mathrm{d}\hat{\sigma}_{ij}(s, t_1, u_1, Q^2)}{\mathrm{d}t_1\, \mathrm{d}u_1}, \quad (47)$$

where $\bar{s} = s\sqrt{1 - 4m^2/s}$ and Q^2 is taken to be independent of the kinematical variables s, t_1 and u_1. The parton–parton cross section can be expanded as follows [5]:

$$\hat{\sigma}_{ij}(s, m^2, Q^2) = \frac{\alpha_s^2(Q^2)}{m^2} f_{ij}^{(0)}(\eta) + \frac{4\pi\alpha_s^3(Q^2)}{m^2}\left[f_{ij}^{(1)}(\eta) + \bar{f}_{ij}^{(1)}(\eta) \ln\frac{Q^2}{m^2}\right]. \quad (48)$$

Here the functions $f_{ij}^{(0)}$, $f_{ij}^{(1)}$ and $\bar{f}_{ij}^{(1)}$ only depend on the scaling variable $\eta = s/4m^2 - 1$.

To show the comparison between the approximate formulae derived in sect. 2 and the exact results given in refs. [5,6], we discuss the parton–parton cross sections. In fig. 5 we plot $f_{gg}^{(0)}$ and $f_{gg}^{(1)}$ as functions of the variable η. The exact results from refs. [5,6] are shown as solid lines whereas those derived from the approximations (36)–(38), (40) and (41) are shown as dashed lines. As expected, the approximate formulae are obviously very good for small and large values of η. However, there is a large region $0.2 < \eta < 200$ where the comparison is poor. The dip in the exact result must be caused by interference terms which are simply not reproducible by the positive definite contributions from the approximate cross sections. In order to improve this undesirable situation one can incorporate a fudge factor, which is only a function of η. We have therefore added a multiplicative factor of $1/\sqrt{\eta + 1}$ to the ISGB result and $\eta/(\eta + 10)$ to the GS and FE results. The resulting curves are shown in fig. 6 to be much better fits to the exact result. However, it is not clear how these fudge factors change the transverse momentum and rapidity distributions of the outgoing quarks. We will return to this

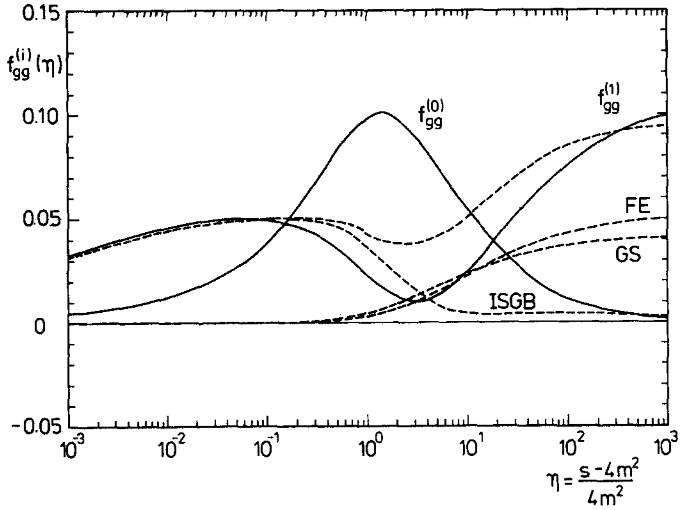


Fig. 5. The gluon–gluon contributions to the parton cross section plotted versus $\eta = (s - 4m^2)/4m^2$. The functions $f_{gg}^{(0)}$, $f_{gg}^{(1)}$ and $\bar{f}_{gg}^{(1)}$ are defined in eq. (48). Solid (dashed) lines correspond to the exact (approximate) $O(\alpha_s^3)$ results. The contributions from initial state gluon bremsstrahlung (ISGB), flavour excitation (FE) and gluon splitting (GS) to the approximate $f_{gg}^{(1)}$ are shown separately. A solid line, $f(\eta) = 0$, is shown for comparison.

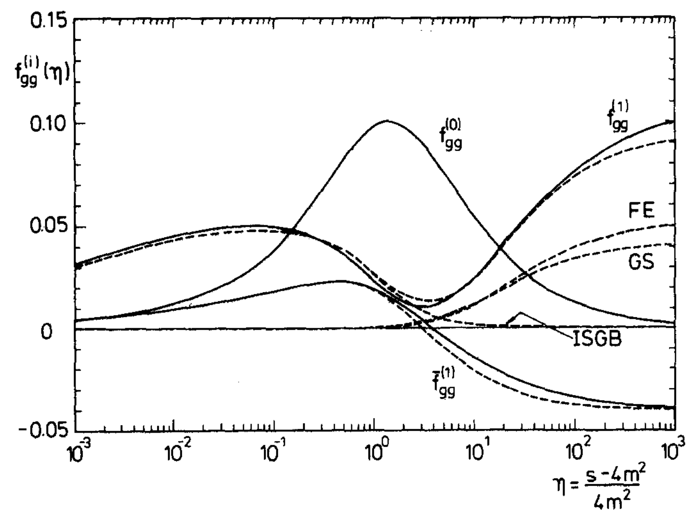


Fig. 6. Same as fig. 5. The approximate contributions to $f_{gg}^{(1)}$ are multiplied by the damping factors mentioned in the text.

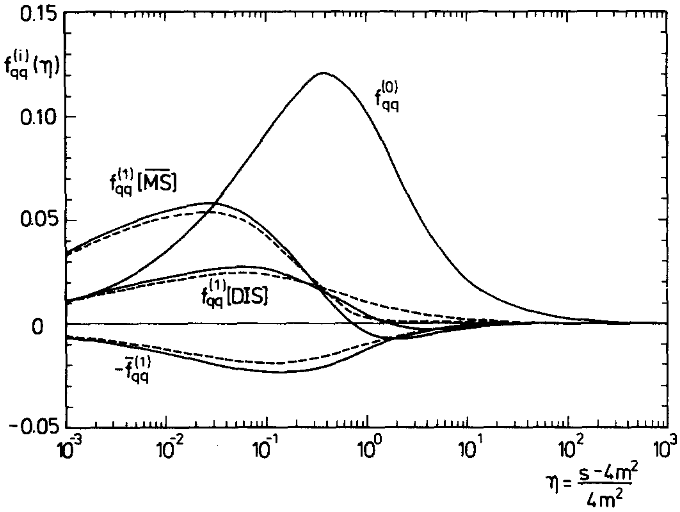


Fig. 7. The quark–antiquark contributions to the parton cross section plotted versus $\eta = (s-4m^2)/4m^2$. The approximate contributions to $f^{(1)}_{q\bar{q}}$ are multiplied by the damping factor $1/(\eta+1)$. Notation as in fig. 5.

question in a later paper. In fig. 6 we also show the exact (from appendix A) and approximate results for $\bar{f}^{(1)}_{gg}$.

In fig. 7 we show the corresponding results for the η dependence of $f^{(0)}_{q\bar{q}}$, $f^{(1)}_{q\bar{q}}$ and $\bar{f}^{(1)}_{q\bar{q}}$. The exact results for the $q\bar{q}$-channel from ref. [5] are given by the solid lines while the approximate results from (28) and (30) are given by the dashed ones. In this case the approximate result for ISGB has also been multiplied by $1/\sqrt{\eta+1}$. Therefore the approximate result fits the exact one rather well for both the $\overline{\text{MS}}$ and DIS schemes when η is small. In the region $0.3 < \eta < 12$ the approximate formulae for $f^{(1)}_{q\bar{q}}$ in both schemes are poor due to the fact that the exact results have negative regions which cannot be approximated by positive definite cross sections. This contrasts to the gg channel where the interference effects reduced the cross section but did not make it negative. From eq. (47) we note that any comparison between the sizes of the Born result $f^{(0)}_{q\bar{q}}$ and the higher order term $f^{(1)}_{q\bar{q}}$ involves multiplying the latter by the factor $4\pi\alpha_s$ which could be as large as 3. Hence the differences between the exact and approximate results are actually larger than fig. 7 suggests. Although the Born term is still the dominant contribution in this region of η we do expect to find differences between the exact and approximate results for hadron–hadron cross sections.

Finally, in fig. 8 we show $f^{(1)}_{gq}$ and $\bar{f}^{(1)}_{gq}$ in the gq channel. The exact results for the gq channel from ref. [5] are given by the solid lines while the approximate results from (44) to (46) are given by the dashed ones. In this case we have also added a factor of $\eta/(\eta+10)$ to the GS and FE contributions so that they fit the

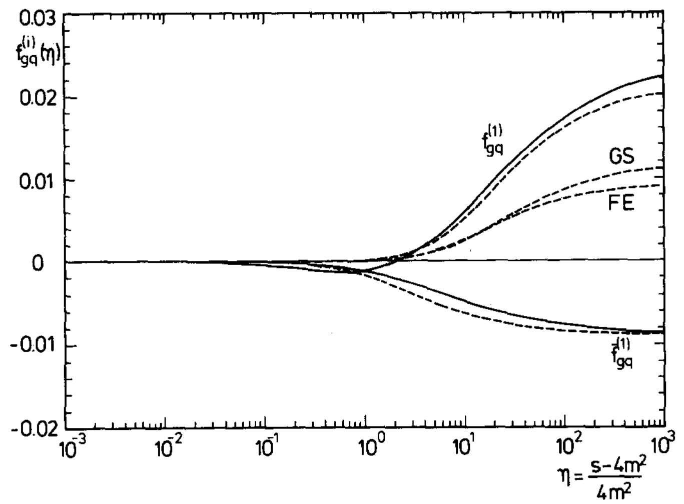


Fig. 8. The gluon–quark contributions to the parton cross section plotted versus $\eta = (s - 4m^2)/4m^2$. The approximate contributions to $f_{gq}^{(1)}$ are multiplied by the damping factor $\eta/(\eta + 10)$. Notation as in fig. 5.

exact results better above $\eta = 4$. The negative piece in the exact result below $\eta = 2$ cannot be properly approximated for the same reasons as mentioned above.

We should comment here that the ratio of the asymptotic behaviours of the approximations for $f_{gg}^{(1)}$ to $f_{gq}^{(1)}$ as $s \to \infty$ is $2N/C_F = 9/2$ in agreement with the ratio expected for the exact results given in eqs. (19) and (20) of ref. [5].

4. Hadron–hadron cross sections

The hadronic reaction in which heavy flavours are produced is given by

$$p(P_1) + \bar{p}(P_2) \to Q(Q_1) + \bar{Q}(Q_2) + X, \tag{49}$$

where p and $\bar{p}$ denote the proton and anti-proton respectively. The quantity X stands for all the final hadronic states which we sum over so that the above process is inclusive with respect to the outgoing hadrons. We use capital letters for the momenta and invariants of the proton and the antiproton to distinguish them from those of the quarks, antiquarks and gluons.

The cross section for reaction (49), as a convolution of the parton–parton cross sections $\hat{\sigma}_{ij}(s, m^2, Q^2)$ and the parton flux functions is given by

$$\sigma(S, m^2, Q^2) = \sum_{i,j} \int_{\tau_-}^{1} \frac{d\tau}{\tau} \Phi_{ij}(\tau, Q^2) \hat{\sigma}_{ij}(\tau S, m^2, Q^2) \tag{50}$$

where $\tau_- = 4m^2/S$. Note that the hadron–hadron cross section is still Q^2 dependent since the $\hat{\sigma}_{ij}$ have only been calculated up to a finite order in α_s. As mentioned in sect. 3 Q^2 is both the renormalization and mass factorization scale and is independent of any kinematical variables. The parton–parton flux function Φ_{ij} is usually defined as follows:

$$\Phi_{ij}(\tau, Q^2) = \tau \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, F_i^p(x_1, Q^2) F_j^{\bar{p}}(x_2, Q^2) \delta(x_1 x_2 - \tau). \qquad (51)$$

Note that the variable τ in both the above formulae is defined by $\tau = s/S = 4m^2(\eta + 1)/S$. In order to resolve more carefully the threshold region where $s \to 4m^2$ and therefore $\eta \to 0$, we switch integration variables in eq. (50) from τ to $\log_{10} \eta$. Then we find that the hadron–hadron cross section becomes

$$\sigma(S, m^2, Q^2) = \sum_{i,j} \int_{-\infty}^{A} \mathrm{d}(\log_{10} \eta) F_{ij}(\eta, m^2/S, Q^2) \hat{\sigma}_{ij}(\eta, m^2, Q^2), \qquad (52)$$

where $A = \log_{10}((S - 4m^2)/4m^2)$. The relation between Φ_{ij} and F_{ij} is

$$F_{ij}(\eta, m^2/S, Q^2) = \frac{\eta}{1+\eta} (\ln 10) \Phi_{ij}(\tau, Q^2). \qquad (53)$$

Since τ depends on both η and m^2/S, F_{ij} depends on η, m^2/S and Q^2. Also, due to the jacobian factor $\eta/(\eta + 1)$ it vanishes when $\eta \to 0$. The integrations in the above formula were carried out using the program VEGAS [16].

The definitions of the running coupling constant α_s, the reduced cross section $\hat{\sigma}_{ij}(s, m^2, Q^2)$ and the parton structure functions $F_i^{p(\bar{p})}(x_1, Q^2)$ are all scheme dependent once one goes beyond the lowest order in perturbation theory. To be exact as to our inputs let us discuss them in order.

To make comparison with the results of refs. [5, 11, 17] we will choose their renormalization scheme for the running coupling constant. Here all the heavy fermions are decoupled in the limit when the momenta entering the fermion loop contribution go to zero. Hence we drop all the terms in σ_{V+S} containing the heavy fermion masses of the type $\ln(m_f^2/Q^2)$ (see (6.1) in ref. [6]), and we use the two loop corrected coupling constant defined in the $\overline{\mathrm{MS}}$ scheme [17] by

$$\alpha_S(Q^2, n_f) = \frac{1}{b_f \ln(Q^2/\Lambda^2)} \left[1 - \frac{b_f' \ln\ln(Q^2/\Lambda^2)}{b_f \ln(Q^2/\Lambda^2)}\right], \qquad (54)$$

where b_{f} and b'_{f} are given by

$$b_{\mathrm{f}} = \frac{33-2n_{\mathrm{f}}}{12\pi}, \qquad b'_{\mathrm{f}} = \frac{153-19n_{\mathrm{f}}}{2\pi(33-2n_{\mathrm{f}})}. \tag{55}$$

This formula is valid for top production with $\Lambda = \Lambda_5$ and $n_{\mathrm{f}} = 5$. For bottom and charm production we need α_{S} for four and three flavours, respectively. So that there is continuity across the b and c thresholds we define

$$\begin{aligned}
\alpha_{\mathrm{S},5}(Q^2) &= \alpha_{\mathrm{S}}(Q^2,5),\\
\alpha_{\mathrm{S},4}^{-1}(Q^2) &= \alpha_{\mathrm{S}}^{-1}(Q^2,4) + \alpha_{\mathrm{S}}^{-1}(m_{\mathrm{b}}^2,5) - \alpha_{\mathrm{S}}^{-1}(m_{\mathrm{b}}^2,4),\\
\alpha_{\mathrm{S},3}^{-1}(Q^2) &= \alpha_{\mathrm{S}}^{-1}(Q^2,3) + \alpha_{\mathrm{S}}^{-1}(m_{\mathrm{c}}^2,4) + \alpha_{\mathrm{S}}^{-1}(m_{\mathrm{b}}^2,5)\\
&\quad - \alpha_{\mathrm{S}}^{-1}(m_{\mathrm{b}}^2,4) - \alpha_{\mathrm{S}}^{-1}(m_{\mathrm{c}}^2,3),
\end{aligned} \tag{56}$$

so that

$$\begin{aligned}
\alpha_{\mathrm{S}}(Q^2) &= \alpha_{\mathrm{S},5}(Q^2)\theta(Q^2-m_b^2) + \alpha_{\mathrm{S},4}(Q^2)\theta(m_b^2-Q^2)\theta(Q^2-m_c^2)\\
&\quad + \alpha_{\mathrm{S},3}(Q^2)\theta(m_c^2-Q^2).
\end{aligned} \tag{57}$$

This is so even in the calculation of the lowest order Born approximation so that we can compare our results at the same α_{s} against those in ref. [11].

The results for the reduced parton cross sections presented earlier were calculated in both $\overline{\mathrm{MS}}$ and DIS schemes. In order to compute the hadronic cross section we will need the corresponding parton structure functions parametrized in one of these schemes. The transformations to go from one scheme to the other in the case of the parton cross sections are given in ref. [11] for the exact calculation and in sect. 2 for our approximation. Here we will compare our approximation with the exact calculation in the DIS scheme only. Therefore we have chosen the new DFLM structure functions [18] which correspond to the latter scheme. This shall be sufficient to see how good our approximation is, since the effect of changing scheme is well known and easy to incorporate (in accordance with the comments made in sect. 1). Further we take the factorization scale Q^2 to be identical to the renormalization scale present in the running coupling constant (57).

We now study the case of proton–antiproton scattering. To see which regions in η give the most important contributions to the total cross section we examine the parton flux functions by using the DFLM structure function parametrization set 2 with $\Lambda_5 = 173$ MeV. In fig. 9 we present plots of the flux functions $F_{\mathrm{q\bar{q}}}(\eta, m^2/S, Q^2)$, $F_{\mathrm{gg}}(\eta, m^2/S, Q^2)$ and $F_{\mathrm{gq}}(\eta, m^2/S, Q^2)$ for the hypothetical case of top–antitop quark production at $\sqrt{S} = 1.8$ TeV and $Q = m = 80$ GeV. This is a

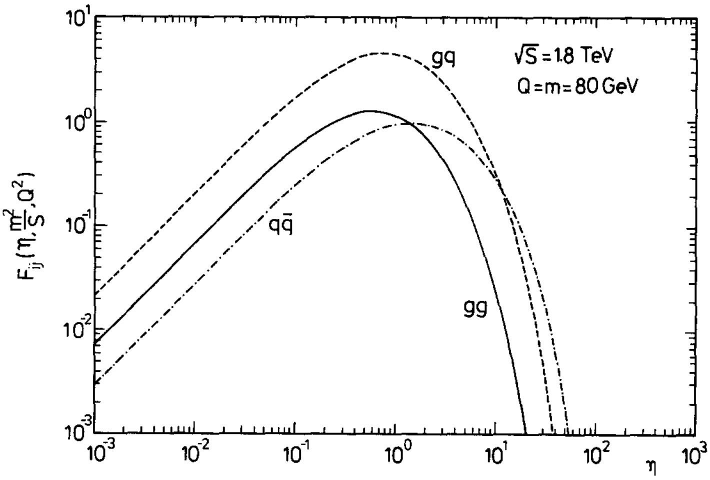


Fig. 9. The parton flux $F_{ij}(\eta, m^2/S, Q^2)$ as defined in eq. (53) versus η for $Q = m = 80$ GeV at $\sqrt{S} = 1.8$ TeV. The DFLM structure function parametrization set 2 with $\Lambda_5 = 173$ MeV is used here.

situation where there is not much phase space for the production of the heavy quark–antiquark pair. The most important region is clearly $0.1 < \eta < 10$, which is precisely where our approximate results are poorest. The situation does not improve substantially when we examine the flux functions for the production of a bottom quark with $Q = m = 5$ GeV at $\sqrt{S} = 630$ GeV. The plot in fig. 10 shows that the most important region is now $0.1 < \eta < 100$. Again this is not a region where our approximations are very good.

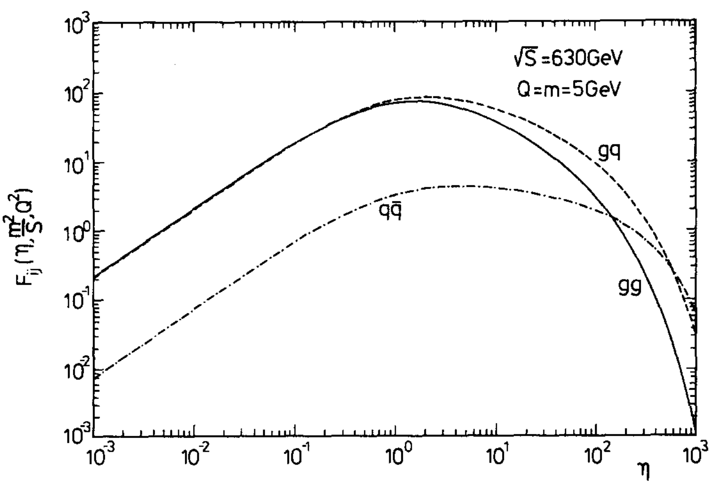


Fig. 10. The parton flux $F_{ij}(\eta, m^2/S, Q^2)$ as defined in eq. (53) versus η for $Q = m = 5$ GeV at $\sqrt{S} = 630$ GeV. The DFLM structure function parametrization set 2 with $\Lambda_5 = 173$ MeV is used here.

TABLE 1
Total cross section for top quark production ($m_t = 40$ GeV) in $p\bar{p}$ collisions at $\sqrt{S} = 630$ GeV

	σ[pb]			
	gg	$q\bar{q}$	$gq(\bar{q})$	Sum
Born	164.6	332.9	0	497.5
Exact $O(\alpha_s^3)$	147.1	61.9	−10.6	198.4
Approx $O(\alpha_s^3)$	195.8	99.6	22.2	317.6
Approx $O(\alpha_s^3)$ fudged	154.0	87.8	5.1	246.9

The results of the exact $O(\alpha_s^3)$ calculations and the approximate formulae with and without the fudge factors are compared. The contributions from the gluon–gluon, quark–antiquark, and gluon–quark subprocesses are given separately. Also given are the respective Born contributions. The mass scale $Q = m_t$, and the DFLM structure function parametrization set 3 is used with $\Lambda_5 = 250$ MeV.

Next we present some results for the total cross section of heavy quark production in proton–antiproton collisions. In table 1 we give the numbers for a hypothetical heavy quark with mass $m = 40$ GeV at $\sqrt{S} = 630$ GeV by using DFLM structure function parametrization set 3 with $\Lambda_5 = 250$ MeV. These cross sections are no longer of experimental relevance due to the present limit on the mass of the t-quark but they allow comparison with the results of ref. [17]. Notice that the contribution from the $q\bar{q}$ Born cross section is the dominant one but the corrections from the $q\bar{q}$ and gg channels are not negligible in this case. The results from the approximate formulae without the fudge factor are very poor. However, when we include the fudge factor the agreement in the gg channel becomes satisfactory, but the agreement in the $q\bar{q}$ and gq channels is still not good enough due to the negative regions in the interference terms. The discrepancy is only acceptable when comparing to the total cross sections including all the channels.

In table 2 we present the corresponding numbers for a top quark with mass $m_t = 120$ GeV produced at the CDF in Fermilab (the c.m. energy is $\sqrt{S} = 1.8$ TeV). The DFLM structure function set 2 with $\Lambda_5 = 173$ MeV is used. These numbers show roughly the same features as those remarked above for table 1 because the ratio $m/\sqrt{S}$ is about the same. However, the Born contribution in the $q\bar{q}$ channel is even more important than before.

In table 3 we give results for the case of bottom quark production ($Q = m_b = 4.75$ GeV) at the CERN collider ($\sqrt{S} = 630$ GeV) using again DFLM structure function set 2 with $\Lambda_5 = 173$ MeV. Here one sees that the dominant contributions come from the gg channel. Without the fudge factor our results are clearly unacceptable. However, with the inclusion of the fudge factor they are reasonably good. Therefore one can rely on our approximation for the gg channel which provides an error of less than 10% to the total cross section. In the gq channel the contribution becomes positive and our approximation agrees much better with the exact result.

TABLE 2
Same as table 1 but for top production ($m_t = 120$ GeV) at Fermilab collider c.m. energy $\sqrt{S} = 1.8$ TeV

	σ[pb]			
	gg	$q\bar{q}$	$gq(\bar{q})$	Sum
Born	6.50	19.33	0	25.83
Exact $O(\alpha_s^3)$	4.96	2.98	−0.45	7.49
Approx $O(\alpha_s^3)$	6.40	4.66	0.79	11.85
Approx $O(\alpha_s^3)$ fudged	5.16	4.12	0.19	9.47

The DFLM structure function parametrization set 2 with $\Lambda_5 = 173$ MeV is used.

The reason is that the important region in the flux function is now at larger η where our approximations are good.

We conclude our discussion of the results for the total $p\bar{p}$ cross section with the following comments. The approximate formulae given in sect. 2 (with the inclusion of the fudge factors) yield results for heavy flavour cross sections which are accurate to within 20% over a wide range of c.m. energies and heavy quark masses. In view of experimental uncertainties (cf. the UA1 results on b-quark production in ref. [19]) this is certainly reasonable. However, without the addition of the fudge factors our results would not be acceptable. This calls into doubt whether the resummation of the large logarithmic terms can be used to generate the even higher order corrections to heavy flavour production cross sections. It is clearly important to examine this question in more detail for the heavy quark differential distributions, since it is here that various Monte Carlo generators use some version of our approximate formulae. A detailed comparison of the transverse momentum

TABLE 3
Same as table 1 but for bottom production ($m_b = 4.75$ GeV) at $\sqrt{S} = 630$ GeV

	σ[μb]			
	gg	$q\bar{q}$	$gq(\bar{q})$	Sum
Born	7.15	0.35	0	7.50
Exact $O(\alpha_s^3)$	6.98	0.05	0.69	7.72
Approx $O(\alpha_s^3)$	12.04	0.14	1.46	13.64
Approx $O(\alpha_s^3)$ fudged	7.61	0.11	0.77	8.49

The DFLM structure function parametrization set 2 with $\Lambda_5 = 173$ MeV is also used.

and rapidity distributions for heavy quark production in $p\bar{p}$ collisions using both the exact and the approximate formulae is being prepared.

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Appendix A

In this appendix we give the terms which must be added to the formulae in ref. [6] when we change renormalization and factorization scales. In that paper we chose the simplest possibility, namely we set the renormalization scale in the coupling constant equal to the factorization scale in the amplitude and chose them to have the value m^2. If both the renormalization and factorization scale are set equal to Q^2, then we must add terms proportional to $\ln(m^2/Q^2)$. These are

$$s^2\left(\frac{d^2\hat{\sigma}^{(1)}_{gg,0}}{dt_1\,du_1}\right)^{H} = \frac{1}{4}\frac{\alpha_S^3 N C_0}{(N^2-1)^2}\left(\ln\frac{m^2}{Q^2}\right)\left[\left\{\frac{(s+t_1)^2+u_1^2}{(s+t_1)(s+t_1+u_1)} - \frac{(s+t_1)^2+(s+t_1+u_1)^2}{(s+t_1)u_1} - \frac{u_1^2+(s+t_1+u_1)^2}{(s+t_1)^2}\right\}\right.$$
$$\left.\times\left\{\frac{t_1^2+(s+t_1)^2}{t_1u_1}\right\}\left(\frac{t_1^2+(s+t_1)^2}{s^2(s+t_1)} - \frac{4m^2}{su_1} + \frac{4m^4}{t_1u_1^2}\right) + (t_1 \leftrightarrow u_1)\right] \tag{A.1}$$

which must be added to (6.16) in ref. [6],

$$s^2\left(\frac{d^2\hat{\sigma}^{(1)}_{gg,K}}{dt_1\,du_1}\right)^{H} = -\frac{1}{4}\frac{\alpha_S^3 N C_K}{(N^2-1)^2}\left(\ln\frac{m^2}{Q^2}\right)\left[\left\{\frac{(s+t_1)^2+u_1^2}{(s+t_1)(s+t_1+u_1)} - \frac{(s+t_1)^2+(s+t_1+u_1)^2}{(s+t_1)u_1} - \frac{u_1^2+(s+t_1+u_1)^2}{(s+t_1)^2}\right\}\right.$$
$$\left.\times\left(\frac{t_1^2+(s+t_1)^2}{t_1u_1(s+t_1)} - \frac{4m^2 s}{t_1u_1^2} + \frac{4m^4s^2}{t_1^2u_1^3}\right) + (t_1 \leftrightarrow u_1)\right] \tag{A.2}$$

which must be added to eq. (6.17) in ref. [6]. Then there are corresponding pieces in the S + V terms, namely

$$s^2\left(\frac{d^2\hat{\sigma}^{(1)}_{gg,0}}{dt_1\,du_1}\right)^{V+S} = \frac{1}{4}\frac{\alpha_S^3 N C_0}{(N^2-1)^2}\left(\ln\frac{m^2}{Q^2}\right)\delta(s+t_1+u_1)$$
$$\times\left(2\ln\frac{\Delta}{(s+t_1)} + 2\ln\frac{\Delta}{(s+u_1)}\right)\left(1-\frac{2t_1u_1}{s^2}\right)$$
$$\times\left(\frac{t_1}{u_1}+\frac{u_1}{t_1}+\frac{4m^2s}{t_1u_1}\left(1-\frac{m^2s}{t_1u_1}\right)\right), \qquad \text{(A.3)}$$

and

$$s^2\left(\frac{d^2\hat{\sigma}^{(1)}_{gg,K}}{dt_1\,du_1}\right)^{V+S} = -\frac{1}{4}\frac{\alpha_S^3 N C_K}{(N^2-1)^2}\left(\ln\frac{m^2}{Q^2}\right)\delta(s+t_1+u_1)$$
$$\times\left(2\ln\frac{\Delta}{(s+t_1)} + 2\ln\frac{\Delta}{(s+u_1)}\right)\left(\frac{t_1}{u_1}+\frac{u_1}{t_1}+\frac{4m^2s}{t_1u_1}\left(1-\frac{m^2s}{t_1u_1}\right)\right), \qquad \text{(A.4)}$$

which have to be added to eq. (6.20) in ref. [6].

Finally, if one does not decouple the heavy fermion loops one has to add

$$s^2\left(\frac{d^2\hat{\sigma}^{(1)}_{gg,f}}{dt_1\,du_1}\right)^{V+S} = -\frac{\alpha_S}{2\pi}\sum_{f=H}\tilde{\beta}_{0,f}\ln\frac{m^2}{Q^2}s^2\frac{d^2\sigma^{(0)}_{gg}}{dt_1\,du_1} \qquad \text{(A.5)}$$

with $\tilde{\beta}_{0,f} = -2/3$ to eq. (6.21) in ref. [6]. This last term can be removed by replacing the running coupling constant according to eq. (6.27) in ref. [6].

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