COHERENT PRODUCTION OF LIGHT SCALAR OR PSEUDOSCALAR PARTICLES IN BRAGG SCATTERING

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X-rays penetrating crystals can produce light scalar or pseudoscalar particles via the Primakoff effect. We compute the intensity of scalar and pseudoscalar particles in the reflected beam of a Bragg reflection. We then estimate the sensitivity of a realistic experiment, using synchrotron radiation and a double crystal spectrometer, which can test for the existence of light scalar and pseudoscalar particles.

New interactions with a characteristic mass scale M can manifest themselves at energies much smaller than M through pseudo-Goldstone bosons, whose masses and couplings to "light" particles scale as $1/M$. Two particularly interesting examples, which arise in minimal extensions of the standard model, are the axion and dilation. The axion [1,2], the pseudo-Goldstone boson of a spontaneously broken chiral Peccei–Quinn symmetry invented to solve the strong CP problem, acquires its mass through the vacuum expectation value of the chiral anomaly; the dilaton, a Brans–Dicke type scalar [3], which arises in theories with spontaneously broken scale invariance [4], obtains its mass from the vacuum expectation value of the conformal anomaly [5,6]. Both scalar particles, axion and dilaton, appear together if the standard model is the low energy limit of a theory with spontaneously broken superconformal invariance [7].

Light scalar particles can be produced and detected by means of the Primakoff process (cf. fig. 1), i.e., the mixing with photons in an external electromagnetic field. Following Sikivie [8], various suggestions have been made to search for light scalars by means of external magnetic fields [9,10]. Here we will explore the feasibility to make use of the strong electric fields seen by X-rays penetrating crystals for photon-scalar conversion. The basic idea is illustrated in fig. 2: The incident X-ray is reflected from a crystal under a Bragg angle Θ_B; the reflected beam contains scalar particles which are produced in the crystal via the Primakoff effect; only the scalar particles penetrate the absorber and produce in a second Bragg reflection the outgoing electromagnetic wave which is detected.

Let us now calculate the intensity of the final photon beam. To be specific we concentrate on the pseu-

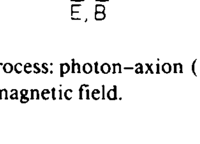

Fig. 1. Primakoff process: photon–axion (dilaton) conversion in an external electromagnetic field.

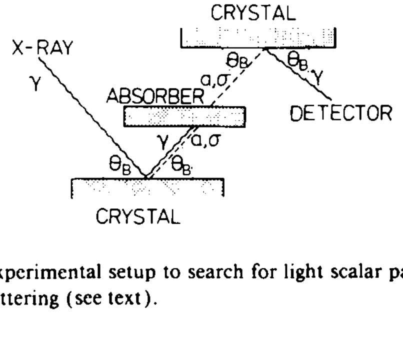

Fig. 2. Experimental setup to search for light scalar particles in Bragg scattering (see text).

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doscalar axions whose electromagnetic interactions are described by the lagrangian density

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(\partial_\mu a)^2 - \tfrac{1}{2}m_a^2 a^2 - \frac{1}{4M}F_{\mu\nu}\tilde{F}^{\mu\nu}a\,. \tag{1}$$

The corresponding field equations read

$$(\Box + m_a^2)a = -\frac{1}{M}\boldsymbol{E}\cdot\boldsymbol{B}\,, \tag{2a}$$

$$\nabla\times\boldsymbol{B} - \frac{\partial}{\partial t}\boldsymbol{E} = -\frac{1}{M}\left(\boldsymbol{B}\frac{\partial}{\partial t}a - \boldsymbol{E}\times\nabla a\right), \tag{2b}$$

$$\nabla\cdot\boldsymbol{E} = \frac{1}{M}\boldsymbol{B}\cdot\nabla a\,. \tag{2c}$$

We note that in models where the two-photon coupling of axions is generated through the triangle anomaly, M is about two orders of magnitude larger than the mass scale f of the spontaneous symmetry breaking.

In the electrostatic field of a screened Coulomb potential,

$$\boldsymbol{E} = -\nabla\phi\,,\quad \phi = \frac{Ze}{4\pi r}\exp(-r/r_0)\,, \tag{3}$$

where Z is the nuclear charge and r_0 the screening length, an incoming plane electromagnetic wave,

$$\boldsymbol{B}(t,\boldsymbol{x}) = \boldsymbol{B}^{(0)}\exp[\mathrm{i}(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})]\,, \tag{4}$$

generates an outgoing spherical axion wave

$$\dot{a}(t,\boldsymbol{x}) \equiv \frac{\partial}{\partial t}a(t,\boldsymbol{x}) = \frac{F_a(2\Theta)}{4\pi M}\boldsymbol{e}_r\cdot\boldsymbol{B}^{(0)}\frac{1}{r}\exp[\mathrm{i}(\omega t - kr)]\,, \tag{5a}$$

where

$$F_a(2\Theta) = k^2\int \mathrm{d}^3x\,\phi(\boldsymbol{x})\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x})\,,$$

$$\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}\,,\quad \boldsymbol{k}' = k\boldsymbol{e}_r\,,\quad k = |\boldsymbol{k}|\,,\quad \boldsymbol{e}_r = \boldsymbol{x}/r\,,$$

$$2\Theta = \sphericalangle(\boldsymbol{k},\boldsymbol{k}')\,. \tag{5b}$$

Here we have neglected the axion mass. In the case of non-vanishing mass $k' = (k^2 - m_a^2)^{1/2}$. For a screened Coulomb potential one easily verifies:

$$F_a(2\Theta) = \frac{Zek^2}{(1/r_0)^2 + 2k^2(1-\cos 2\Theta)}\,. \tag{6}$$

It is convenient to define an average electric field by

$$\bar{E}(k,2\Theta) \equiv \frac{1}{kd^3}F_a(2\Theta)\,, \tag{7}$$

where d is the lattice spacing of a cubic lattice.

Eq. (5a) yields for the differential cross section of unpolarized photon-axion conversion:

$$\frac{\mathrm{d}\sigma_a}{\mathrm{d}\Omega} = \frac{1}{32\pi^2M^2}F_a^2(2\Theta)\sin^2 2\Theta\,. \tag{8}$$

The corresponding Thomson cross section for electromagnetic scattering by an atom reads [11]

$$\frac{\mathrm{d}\sigma_\gamma}{\mathrm{d}\Omega} = \left(\frac{\alpha}{m}\right)^2 F_\gamma^2(2\Theta)\frac{1+\cos^2 2\Theta}{2}\,, \tag{9a}$$

with

$$F_\gamma(2\Theta) = \frac{1}{e}\int \mathrm{d}^3x\,\rho(\boldsymbol{x})\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x})\,,$$

$$F_\gamma(0) = Z\,. \tag{9b}$$

Here m is the electron mass, F_γ is the atomic structure factor and ρ is the electron charge density of the atom. The formfactors F_a and F_γ satisfy the relation

$$F_a(2\Theta) = \frac{ek^2}{q^2}[z - F_\gamma(2\Theta)]\,. \tag{10}$$

Inserting this equation into eq. (8) yields an expression for the differential photon-axion cross section which has previously been derived by Raffelt [12]. We note that for a neutral atom, contrary to the Thomson cross section, the photon–axion cross section reaches its maximum at $2\Theta \cong \pi/2$ and vanishes in the forward direction.

Given the scattering amplitudes for elastic photon scattering and photon–axion conversion by a single atom the coherent scattering by a crystal can be calculated using "Darwin's dynamical theory" [11]. Here the first step is to compute the wave scattered from a single layer of pointlike scattering centers. Interference of the outgoing spherical waves leads to reflected and transmitted plane waves. In the case of Thomson scattering one finds for the two polarizations parallel and perpendicular to the scattering plane (cf. ref. [11], fig. 3) of reflected (R) and transmitted (T) waves:

$$E^{(\mathrm{R})}_{\parallel(\perp)}(t,\boldsymbol{x}) = \mathrm{i}\rho_{\parallel(\perp)}(2\Theta)E^{(0)}_{\parallel(\perp)}\exp[\mathrm{i}(\omega t - \boldsymbol{t}'\cdot\boldsymbol{x})]\,, \tag{11a}$$

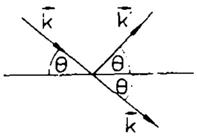

Fig. 3. Scattering plane and Bragg angle.

$$E^{(\mathrm{T})}_{\parallel(\perp)}(t,\boldsymbol{x})=[1+\mathrm{i}\rho_{\parallel(\perp)}(0)]\times E^{(0)}_{\parallel(\perp)}\exp[\mathrm{i}(\omega t-\boldsymbol{k}\cdot\boldsymbol{x})]\,,\tag{11b}$$

where

$$\rho_{\parallel}(2\Theta)=\frac{\alpha F_{\gamma}(2\Theta)N_{\mathrm{s}}\lambda}{m\sin\Theta}\,|\cos 2\Theta|\,,\tag{12a}$$

$$\rho_{\perp}(2\Theta)=\frac{\alpha F_{\gamma}(2\Theta)N_{\mathrm{s}}\lambda}{m\sin\Theta}\equiv\rho(2\Theta)\,.\tag{12b}$$

In eq. (11b) we have neglected the attenuation of the transmitted wave. In eq. (12) N_{s} denotes the number of scattering centers per unit area.

The scattering amplitude for photon–axion conversion can be read off from eq. (5). For the inverse process one obtains from eqs. (2):

$$\boldsymbol{B}(t,\boldsymbol{x})=-\frac{F_{\mathrm{a}}(2\Theta)}{4\pi M}\,\boldsymbol{e}_r\times(\boldsymbol{e}_r\times\hat{\boldsymbol{k}})\times\dot{a}^{(0)}\frac{1}{r}\exp[\mathrm{i}(\omega t-\boldsymbol{k}\cdot\boldsymbol{x})]\,,$$

$$\hat{\boldsymbol{k}}=\boldsymbol{k}/|\boldsymbol{k}|\,.\tag{13}$$

In eqs. (5) and (13) only the polarization contributes where the magnetic field is parallel to the scattering plane, i.e., $\boldsymbol{B}=\boldsymbol{B}_{\parallel}$. After integration over one layer of atoms one obtains from eqs. (5) and (13) for the reflected waves

$$\dot{a}^{(\mathrm{R})}(t,\boldsymbol{x})=\mathrm{i}\xi(2\Theta)B^{(0)}_{\parallel}\exp[\mathrm{i}(\omega t-\boldsymbol{k}'\cdot\boldsymbol{x})]\,,\tag{14a}$$

$$\boldsymbol{b}^{(\mathrm{R})}_{\parallel}(t,\boldsymbol{x})=\mathrm{i}\xi(2\Theta)\dot{a}^{(0)}\exp[\mathrm{i}(\omega t-\boldsymbol{k}'\cdot\boldsymbol{x})]\,,\tag{14b}$$

where

$$\xi(2\Theta)=\frac{F_{\mathrm{a}}(2\Theta)N_{\mathrm{s}}\lambda}{4\pi M\sin\Theta}\sin 2\Theta\,.\tag{15}$$

The transmitted waves are not modified since the forward scattering amplitude vanishes.

From eqs. (11) and (14) we can now calculate the intensities of the outgoing axion (photon) waves produced by the photon–axion (anion–photon) conversion in a crystal. Let the layers of atoms be located at $z=0$, $z=-d$, ..., $z=-nd$. The amplitudes of transmitted and reflected waves at the nth layer are $B^{(\mathrm{T})}_{\parallel}\equiv A_{n+1}$, $B^{(\mathrm{R})}_{\parallel}\equiv B_n$, $\dot{a}^{(\mathrm{T})}\equiv C_{n+1}$, $\dot{a}^{(\mathrm{R})}\equiv D_n$ (cf. fig. 4). From (11) and (14) we then obtain the set of coupled equations ($\rho=\rho(2\Theta)$, $\rho_0=\rho F_{\gamma}(0)/F_{\gamma}(2\Theta)$, $\xi=\xi(2\Theta)$):

$$B_n=\mathrm{i}\rho\exp(-2\mathrm{i}n\phi)A_n+(1+\mathrm{i}\rho_0)B_{n+1}+\mathrm{i}\xi\exp(-2\mathrm{i}n\phi)C_n\,,\tag{16a}$$

$$A_{n+1}=(1+\mathrm{i}\rho_0)A_n+\mathrm{i}\rho\exp(2\mathrm{i}n\phi)B_{n+1}+\mathrm{i}\xi\exp(2\mathrm{i}n\phi)D_{n+1}\,,\tag{16b}$$

$$C_{n+1}=\mathrm{i}\xi\exp(2\mathrm{i}n\phi)B_{n+1}+C_n\,,\tag{16c}$$

$$D_n=\mathrm{i}\xi\exp(-2\mathrm{i}n\phi)A_n+D_{n+1}\,,\tag{16d}$$

where $\phi=kd\sin\Theta$. Clearly, for $\phi=\pi$ the Bragg condition is fulfilled and one has constructive interference. In the case $\xi=0$ eqs. (16) reduce to the equations well known from ordinary Thomson scattering (cf. ref. [11]).

In order to obtain the intensity of the outgoing electromagnetic wave after the double scattering shown in fig. 2 one has to compute from eqs. (16) first the ratio D_0/A_0 with the boundary condition $C_0=0$, and second the ratio B_0/C_0 with the boundary condition $A_0=0$. The calculation can be carried out as for Thomson scattering (ref. [11]) and one obtains up to terms of order $\rho\xi$, ξ^2:

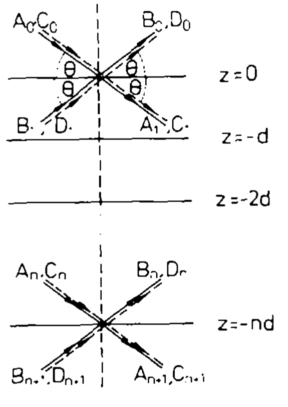

Fig. 4. Beams of photons and scalar particles in the crystal: A(B) = transmitted (reflected) photon beam, C(D) = transmitted (reflected) axion/dilaton beam.

$$\frac{D_0}{A_0}=\frac{B_0}{C_0}=\frac{\mathrm{i}\xi}{1-\tau\exp(-2\mathrm{i}\phi)}, \tag{17a}$$

where

$$\tau=\kappa\pm[\kappa^2-\exp(2\mathrm{i}\phi)]^{1/2}, \quad |\tau|<1,$$

$$\kappa=\frac{1}{2(1+i\rho_0)}[(1+\mathrm{i}\rho_0)^2+\rho^2+\exp(2\mathrm{i}\phi)]. \tag{17b}$$

It is instructive to express the ratio (17a) in terms of the deviation of the scattering angle Θ from the Bragg angle Θ_B defined by

$$2d\sin\Theta_B\left(1-\frac{1-n}{\sin^2\Theta_B}\right)=m\lambda, \quad m=1,2,..., \tag{18}$$

where

$$n=1-\frac{\rho_0\lambda}{2\pi d}\sin\Theta, \tag{19}$$

is the index of refraction. After some algebra one obtains (cf. ref. [11])

$$\frac{D_0}{A_0}=\frac{\xi}{\rho}\frac{\mathrm{i}\Delta}{\mathrm{i}(\Delta_0+\delta)\pm(\Delta^2-\delta^2)^{1/2}},$$

$$\delta=\Theta-\Theta_B, \tag{20}$$

where

$$\Delta=\frac{1}{\pi}\frac{\lambda}{l}\frac{1}{\sin 2\Theta_B},$$

$$\Delta_0=\Delta\frac{F_\gamma(0)}{F_\gamma(2\Theta_B)}, \tag{21}$$

$$l^{-1}=\frac{\alpha}{m}N\lambda F_\gamma(2\Theta_B). \tag{22}$$

l is the penetration depth of the X-ray into the crystal, and N denotes the number of scattering centers per unit volume. From eq. (20) it is obvious that the ratio D_0/A_0 is strongly peaked for scattering angles Θ in the range $[\Theta_B-\Delta, \Theta_B+\Delta]$, which is the width of an ordinary Bragg peak.

Within the width of the Bragg peak the transition probability for photon–axion and axion–photon conversion takes a simple form. From eqs. (7), (12), (15), (17), (20) and (22) one obtains

$$P=\left|\frac{D_0}{A_0}\right|^2=\left|\frac{B_0}{C_0}\right|^2=\left(\frac{\xi}{\rho}\right)^2$$

$$=\left(\frac{\bar{E}l}{2M}\sin 2\Theta\right)^2. \tag{23}$$

This means that the photon–axion transition probability is essentially determined by two parameters: the average electric field $\bar{E}$ seen by photons in the crystal and the penetration depth l into the crystal. Eq. (23) is analogous to the expression obtained for the photon–axion transition probability in an external magnetic field [10].

What sensitivity with respect to the mass scale M can be reached in a realistic experiment? Typical values for penetration depth and Bragg angle are $l\sim 1$ µm and $\Theta\sim 10°$; for the average electric field $\bar{E}$ (cf. eq. (7)) one finds for $Z=10$, $d=2$ Å and $r_0=(0.1, 0.5, 1.0)$ Å [13] the values $\bar{E}=(0.07, 1.0, 1.8)$ keV^2. Note that the average electric field $\bar{E}$ strongly depends on the screening length r_0 which is smaller than the lattice spacing d. We emphasize that this microscopic electric field is much stronger than the macroscopic magnetic field attainable with dipole magnets (1 Tesla ~ 200 eV^2). For a photon energy $\omega\sim 10$ keV the width of the Bragg peak is $\Delta\sim 10^{-4}$. An intense source of X-rays will be provided by the European Synchrotron Radiation Facility. By means of undulators one expects to achieve a brightness of $\Phi\sim 10^{18}$/s (0.1% BW); the divergence of the beam is given by $\delta\sim\gamma^{-1}=m_e/E_e\sim 10^{-4}$ for 5 GeV electrons [13]. From $\mathrm{d}\lambda/\lambda=\cot\Theta\,\mathrm{d}\Theta$ and $\mathrm{d}\Theta\sim 10^{-4}$ one finds that photons within a bandwidth $\mathrm{d}\lambda/\lambda\sim 10^{-3}$ contribute to the Bragg reflection. Hence the number of photons N^{obs} in the final state is given by the product of the brightness Φ, the probability P^2 of the photon–axion–photon transition and the running time T. Realistic requirements are $N^{\mathrm{obs}}=10$ and $T=100$ d. From eq. (23) we then obtain for the mass scale M:

$$M>1\times 10^3\,\mathrm{GeV}\left(\frac{\bar{E}}{1\,\mathrm{keV}^2}\frac{l}{1\,\mu m}\frac{\sin 2\Theta}{0.34}\right)$$

$$\times\left(\frac{\Phi}{10^{18}/\mathrm{s}\ 0.1\%\,\mathrm{BW}}\frac{T}{100\,d}\frac{10}{N^{\mathrm{obs}}}\right)^{1/4}. \tag{24}$$

This lower bound will be slightly decreased if finite detection efficiency and temperature effects, i.e., the

Debye–Waller factor, are taken into account. In the case of nonvanishing axion mass the axions are emitted under an angle $\tilde{\Theta}_B < \Theta_B$. In principle the experiment is sensitive to masses $m_a < \omega \sin \Theta_B$, where ω is the photon energy.

In addition to the Primakoff effect light scalar particles can also be produced in a Compton-type process via their direct coupling to electrons. For a Yukawa coupling strength $g \sim m/f$ we expect the production cross section for scalars [#1] to be of the same order of magnitude or even larger than the Primakoff cross section, whereas for pseudoscalars the cross section will be suppressed by $(v/c)^2$, where v is the electron velocity. If the two-photon coupling of the scalar particles is radiatively generated through the triangle anomaly. The strongest bound on the mass scale f of spontaneous symmetry breaking could come from this Compton-type process. However, further investigations are necessary in order to clarify under what conditions this scattering can take place coherently.

Instead of Bragg scattering one can of course also consider Laue scattering, where the penetration depth is much larger. For 100 keV photons and scattering angle $\Theta \sim 1°$ one can achieve $l \sim 1$ cm [13]. This would improve the lower bound (24) on the mass scale M by three orders of magnitude up to $\sim 10^6$ GeV. Such an experiment would clearly be very interesting. It remains to be seen, however, whether the effective electric field $\bar{E}$ which appears in eq. (23) is the same as for Bragg scattering. A detailed calculation will be published elsewhere.

We conclude that the proposed Bragg scattering experiment can test for the existence of light scalar particles with masses up to 10 keV and a mass scale for the two-photon coupling up to 10^3 GeV; in Laue scattering it may be possible to reach even 10^6 GeV of the interaction mass scale. This range of parameters has not yet been explored by other laboratory experiments [14,10] and is not excluded by astrophysical bounds from the Sun which apply to scalars with masses below ~ 1 keV [12,15]. An interesting laser experiment for photon–axion conversion is an external magnetic field, which has recently been proposed [10], is sensitive to interaction mass scales $M > 5 \times 10^8$ GeV, possibly even $M > 1 \times 10^{11}$ GeV, but only to masses below ~ 1 eV. Both experiments, as well as the recent proposal based on the Mößbauer effect [16], cannot exceed a range of parameters which appears to be almost excluded by astrophysical bounds inferred from helium burning stars [17], which apply to scalars with masses below ~ 10 keV. However, since laboratory experiments are independent of models of stellar evolution, they can nevertheless significantly contribute to our present knowledge about masses and interaction strengths of very light scalar particles, and complement astrophysical considerations.

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[#1] Here "scalars" denotes particles with $J^P = 0^+$. Otherwise in this paper "scalar" refers to particles with $J^P = 0^+$ and/or $J^P = 0^-$.

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