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IS QED TRIVIAL ?

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This talk summarizes a recent lattice investigation of the chiral phase transition in non-compact QED with light dynamical Kogut-Susskind fermions done in collaboration with M. Göckeler, R. Horsley, E. Laermann, P. Rakow, R. Sommer and U.-J. Wiese. The phase transition is found to be of second order, so that we can take the continuum limit. Near the critical point the theory is shown to be well described by a Gaussian model of non-interacting scalar and pseudoscalar fields.

1. INTRODUCTION

Our belief, that elementary particle physics can be described in terms of a local, renormalizable quantum field theory, rests to a large extent on the success of perturbative QED. This success and the great appeal of gauge theories can, however, not conceal the fact that conceptually QED is still a poorly understood theory - let alone more complex non-asymptotically free gauge theories like the standard model. In spite of great calculational efforts [2], we do not even know whether QED is a consistent quantum field theory at all in the sense that it has a continuum limit, in which the cut-off can be taken to infinity. The main obstacle is that the effective charge grows as one approaches the ultraviolet region, so that perturbation theory cannot be applied.

Recent progress in lattice gauge theory and numerical methods to incorporate fermions [3] have made it possible to study the ultraviolet behavior of QED from first principles. In order that QED is a consistent field theory, the Callan-Symanzik β-function [4] must have an ultraviolet stable fixed point (zero). In lattice simulations such a fixed point will show up as a point of second (or higher) order phase transition, at which the correlation length diverges. The first step in such a study will then be to search for a transition of this kind. If the theory has a second order phase transition, the next question is what the continuum theory is like, and in particular whether it is interacting or not. The theoretical prejudice is that QED admits only a vanishing renormalized charge [5]. The true nature of the continuum theory is reflected in the critical exponents of the transition, while the effective theory is given by the renormalized action. The latter involves the renormalized charge and mass. But in general it may also include higher couplings [6].

Kogut, Dagotto and Kocic [7,8,9,10] have reported evidence for the existence of a continuous chiral phase transition in massless, non-compact QED at strong coupling. This result has been confirmed by the Edinburgh group [11]. In a recent paper [1] we have shown that the chiral phase transition is second order. We have found furthermore that the critical exponents of the transition are consistent with the exponents of a Gaussian model. In this talk I shall present the basic results of this work. Because of earlier, unjustified claims [7,8,9,10] that the chiral condensate exhibits non-trivial scaling behavior near the critical point, great importance is attached to the extrapolation of the lattice data to the chiral limit.

The remainder of the talk is organized as follows.

In sec. 2 we present the details of the calculation. The predictions of mean field theory and the Gaussian model for the chiral phase transition are summarized in sec. 3. In sec. 4 we present and analyze the lattice data. We conclude with some remarks in sec. 5.

2. LATTICE CALCULATION

We take Kogut-Susskind fermions. The lattice action for non-compact QED is $S = S_G + S_F$, where

$$S_G = \frac{\beta}{2} \sum_{x,\mu<\nu} (A_\mu(x)+A_\nu(x+\mu)-A_\mu(x+\nu)-A_\nu(x))^2, \tag{2.1}$$

$\beta = 1/e^2$, and

$$S_F = -\overline{\chi}_x(M+m)_{xy}\chi_y, \tag{2.2}$$

$$\begin{aligned} M_{xy} = & - \frac{1}{2}\sum_\mu (-1)^{x_1+\dots+x_{\mu-1}}[e^{iA_\mu(x)}\delta_{y,x+\mu} \\ & - e^{-iA_\mu(y)}\delta_{y,x-\mu}]. \end{aligned} \tag{2.3}$$

The partition function reads

$$Z = \int [d\overline{\chi}][d\chi][dA_\mu]e^{-S}. \tag{2.4}$$

This action describes four species of light Dirac fermions. For finite lattice spacings the theory has a chiral $U(1) \times U(1)$ symmetry in the limit $m \to 0$, while the $SU(4) \times SU(4)$ symmetry is only recovered in the continuum limit.

The chiral condensate

$$\langle\overline{\psi}\psi\rangle = \lim_{m\to 0}\langle\overline{\chi}\chi\rangle_\infty, \tag{2.5}$$

where

$$\langle\overline{\chi}\chi\rangle_\infty = \lim_{V\to\infty}\langle\overline{\chi}\chi\rangle \tag{2.6}$$

and V is the space-time volume of the lattice, provides an order parameter for spontaneous breaking of chiral symmetry. From the scaling behavior of $\langle\overline{\chi}\chi\rangle_\infty$ and the associated Goldstone boson mass, m_{PS}, near the phase transition point one can derive three critical exponents. The determination of these exponents will be the main subject of this talk.

We have chosen to compute the chiral condensate in two different ways. One way is by inversion of the fermion matrix M, which is the standard procedure. The other way is by means of the eigenvalue spectrum of M [12], which I shall describe briefly below. In this case it is convenient to consider the propagation of a fermion of mass $\overline{m}$ through a background gauge field configuration generated with mass m. Accordingly, we may write

$$\langle\overline{\chi}\chi\rangle_\infty(\overline{m},m) = \int_{-\infty}^{\infty} d\lambda \frac{\rho(\lambda,m)}{i\lambda+\overline{m}}, \tag{2.7}$$

where ρ is the eigenvalue density. We now define the eigenvalue number

$$N(\lambda,m) = \int_0^\lambda d\overline{\lambda}\rho(\overline{\lambda},m), \tag{2.8}$$

which counts the number of eigenvalues per unit volume between 0 and λ. Assuming that ρ is a power series at small λ, which is generally the case in the broken phase, we find

$$N(\lambda,m) = \frac{\lambda}{\pi}\langle\overline{\chi}\chi\rangle_\infty(0,m) + O(\lambda^2). \tag{2.9}$$

To compute $\langle\overline{\chi}\chi\rangle$ it is sufficient to know the lowest O(100) eigenvalues. This method has the advantage, that it only requires to extrapolate to $m = 0$, which turns out to be less ambiguous. In the quenched approximation, $m = \infty$, no extrapolation is needed at all. On finite lattices equ. (2.9) is subject to finite size corrections. For details the reader is refered to ref. 1.

3. PREDICTIONS OF MEAN FIELD THEORY AND GAUSSIAN MODEL

We shall now seek a description of spontaneous chiral symmetry breaking in terms of mean field theory. By integrating out the gauge potentials and the Grassmann fields in equ. (2.4) one arrives at the effective action

$$S_{eff}(\sigma,\pi) = \sum_x \{\eta[(\partial_\mu\sigma_x)^2 + (\partial_\mu\pi_x)^2]$$

$$
\begin{aligned}
- & \; m\sigma_x + \kappa(\sigma_x^2 + \pi_x^2) \\
+ & \; \zeta(\sigma_x^2 + \pi_x^2)^2 + ...\}, \qquad (3.1)
\end{aligned}
$$

where σ_x and π_x are elementary scalar and pseudoscalar fields with $<\sigma>=\langle\overline{\chi}\chi\rangle_\infty$. Under chiral $U(1)$ transformations σ_x and π_x transform as

$$
\begin{aligned}
\sigma_x & \longrightarrow \sigma_x\cos 2\epsilon - \pi_x \sin 2\epsilon, \\
\pi_x & \longrightarrow \pi_x\cos 2\epsilon - \sigma_x \sin 2\epsilon, \qquad (3.2)
\end{aligned}
$$

so that equ. (3.1) has the same symmetry properties as the original action. The dots in equ. (3.1) stand for higher derivatives and higher polynomials. Mean field theory assumes that $\sigma_x = \sigma$ and $\pi_x = 0$. If one considers only powers of σ up to σ^4 (which is enough for the description of a second order phase transition), then σ is given by the minimum of the effective action,

$$2\kappa\sigma + 4\zeta\sigma^3 - m = 0. \qquad (3.3)$$

The Gaussian model allows for fluctuations around the minimum action configuration. The fluctuations are treated as independent modes with Gaussian distribution. Writing $\sigma_x = \sigma + \overline{\sigma}_x$ and $\pi_x = \overline{\pi}_x$, we thus obtain

$$
\begin{aligned}
S_{eff}(\sigma,\pi) &= \sum_x \{\eta[(\partial_\mu\overline{\sigma}_x)^2 + (\partial_\mu\overline{\pi}_x)^2] \\
&+ (\kappa + 6\zeta\sigma^2)\overline{\sigma}_x^2 + (\kappa + 2\zeta\sigma^2)\overline{\pi}_x^2 \\
&- m\sigma + \kappa\sigma^2 + \zeta\sigma^4\}. \qquad (3.4)
\end{aligned}
$$

This gives the Goldstone boson mass

$$m_{PS}^2 = \frac{\kappa + 2\zeta\sigma^2}{\eta}. \qquad (3.5)$$

It is assumed that the coefficients η, κ and ζ are analytic functions of β.

In the chiral limit $m \to 0$ we obtain

$$\sigma = \begin{cases} \sqrt{-\frac{\kappa}{2\zeta}} & \kappa \text{ negative}, \\ 0 & \text{else}, \end{cases} \qquad (3.6)$$

so that we may write $\kappa = \overline{\kappa}(\beta - \beta_c)$, where β_c is the critical coupling. This leads to the critical behavior

$$\sigma = \propto (\beta_c - \beta)^{\frac{1}{2}} \text{ for } \beta_c > \beta, \qquad (3.7)$$

$$m_{PS} \propto (\beta - \beta_c)^{\frac{1}{2}} \text{ for } \beta > \beta_c. \qquad (3.8)$$

At $\beta = \beta_c$ (but $m > 0$) we furthermore find

$$\sigma \propto m^{\frac{1}{3}}. \qquad (3.9)$$

Equations (3.7)-(3.9) reflect the standard critical exponents, $\beta = 1/2$, $\nu = 1/2$ and $\delta = 3$, of a Gaussian model.

4. DETERMINATION OF CRITICAL EXPONENTS

We shall now locate the position of the chiral phase transition and determine its critical exponents. The procedure will be to show that our lattice data are consistent with the predictions of the Gaussian model, which implies mean field critical exponents.

4.1. Quenched QED

We begin our investigation with quenched QED. Though the photons do not interact with each other, we may have spontaneous chiral symmetry breaking and fermion-antifermion bound states due to strong fluctuations of the fields. One can also argue that mean field theory is applicable here. Our interest in this case was raised by Miransky's proposal [13] of non-trivial scaling behavior of $\langle\overline{\psi}\psi\rangle$ near the critical point and the subsequent work of Kogut, Dagotto and Kocic [7,8,9,10], which supported the underlying *collapse of the wave function* picture. If we find the same exponents in the quenched approximation as in the full theory, we could argue furthermore that the theory with a single Dirac fermion should show the same behavior.

I will first discuss the data that have been obtained with the standard method. This gives me the possibility to compare both our and Kogut et al.'s data and to pinpoint the origin of the discrepancy. The data are shown in figs. 1 and 2. Our results are on the 16^4 and 22^4 lattice and are marked by open symbols. The data of Kogut et al. [10] are on the 10^4 lattice and are marked by solid symbols. Note that the raw data obtained by the two groups are in good agreement. The

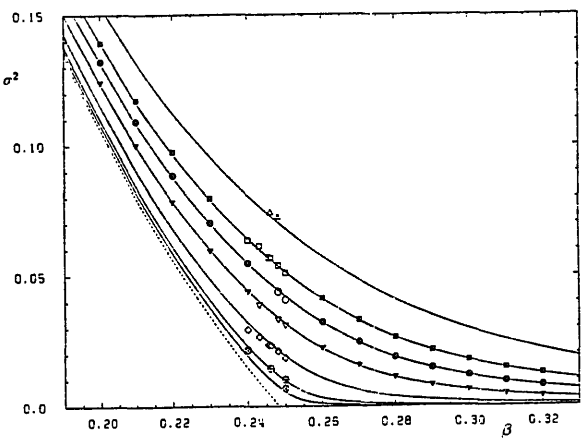

Figure 1: The chiral condensate σ^2 ($\sigma = \langle\overline{\chi}\chi\rangle$) as a function of β in quenched QED for masses between $m = 0.002$ (bottom) and $m = 0.06$ (top). An open symbol indicates data from ref. 1, a solid symbol from ref. 9. The solid lines are a mean field fit. The dashed curve is the extrapolation to $m = 0$.

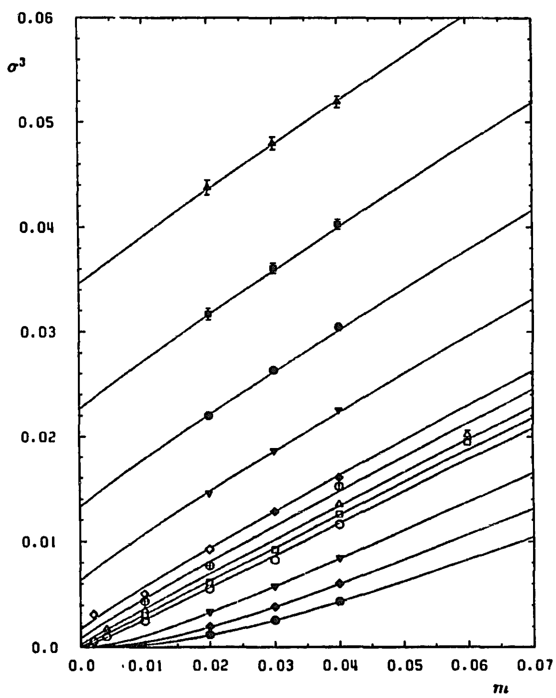

Figure 2: The chiral condensate σ^3 as a function of m in quenched QED for couplings between $\beta = 0.28$ (bottom) and $\beta = 0.20$ (top). An open symbol indicates data from ref. 1, a solid symbol from ref. 9. The solid lines are a mean field fit.

solid lines are a fit of the mean field relation (3.3) to the combined data. For details of the fit see ref. 1. We find that the data are very well described by mean field theory. Figure 2 shows clearly that $\langle\overline{\chi}\chi\rangle \propto m^{\frac{1}{3}}$ at the critical point. Kogut et al. have computed $\langle\overline{\chi}\chi\rangle$ first [8] at two and later [10] at three mass values and done a linear or quadratic extrapolation to $m = 0$, which is not justified as we see. The critical coupling comes out to be $\beta_c = 0.2482(1)$, which is substantially lower than the value quoted elsewhere [8,10]. This is due to the fact that we do not find a *Miransky tail* [13].

Let me now turn to the results obtained by means of the eigenvalue spectrum. These are shown in fig. 3. In this case the lattice sizes are 12^4 and 16^4. The solid lines are again a mean field fit. This involves solving equ. (3.3) for the eigenvalue density ρ. We find very good agreement of our data with mean field theory also

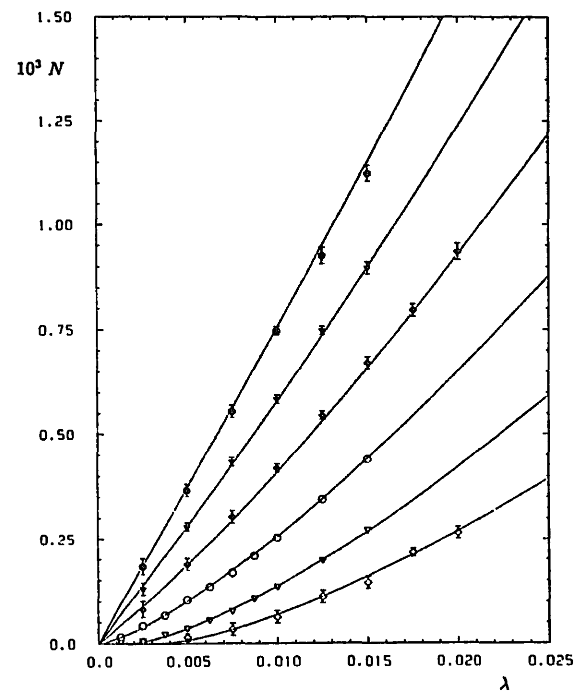

Figure 3: The eigenvalue number N as a function of λ in quenched QED for couplings between $\beta = 0.27$ (bottom) and $\beta = 0.22$ (top). The solid lines are a mean field fit.

here. The critical coupling is $\beta_c = 0.2495(6)$, which is consistent with the previous value. This gives us confidence that our interpretation is correct.

4.2. Dynamical QED

We shall now consider the theory with one set of dynamical Kogut-Susskind fermions. We have used the hybrid Monte Carlo algorithm to simulate the field configurations [3]. So far our calculations are done on 8^4 and 12^4 lattices. In this case we have also computed the Goldstone boson mass, so that we are testing all three critical exponents here.

I first like to discuss the results for $\langle\bar{\chi}\chi\rangle(m,m)$ and m_{PS} as obtained by the standard method. Our data are shown in figs. 4 - 6. The solid lines are a fit of the Gaussian model to the data. We find good agreement between the data and the model. Figure 5 shows, even without a fit, that $\langle\bar{\chi}\chi\rangle \propto m^{\frac{1}{3}}$ at the critical point. The critical coupling is $\beta_c = 0.1950(2)$.

We have computed the chiral condensate via the eigenvalue spectrum in this case as well. This gives us $\langle\bar{\chi}\chi\rangle_\infty(0,m)$. For this quantity one can also derive

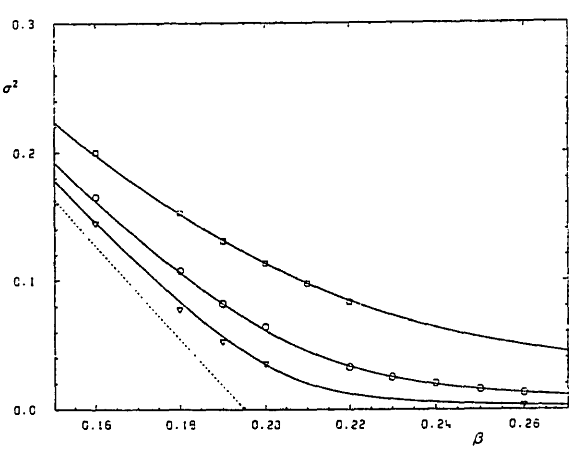

Figure 4: The chiral condensate σ^2 ($\sigma = \langle\bar{\chi}\chi\rangle(m,m)$) as a function of β in dynamical QED for masses between $m = 0.02$ (bottom) and $m = 0.09$ (top). The solid lines are a fit of the Gaussian model. The dashed curve is the extrapolation to $m = 0$.

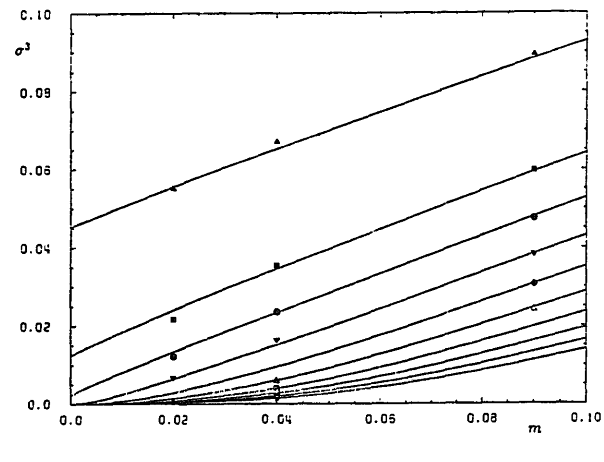

Figure 5: The chiral condensate σ^3 as a function of m in dynamical QED for couplings between $\beta = 0.26$ (bottom) and $\beta = 0.16$ (top). The solid lines are a fit of the Gaussian model.

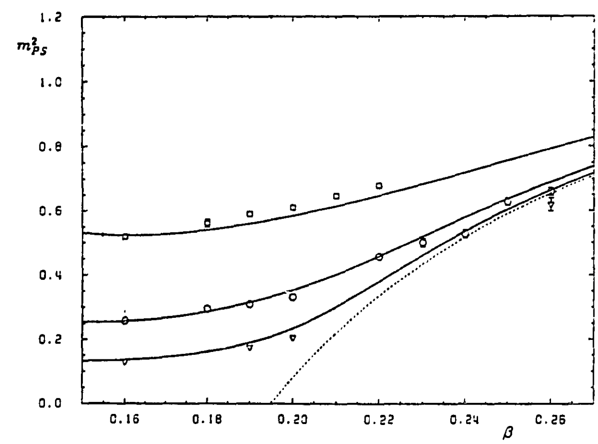

Figure 6: The Goldstone boson mass m_{PS}^2 as a function of β in dynamical QED for masses between $m = 0.02$ (bottom) and $m = 0.09$ (top). The solid lines are a fit of the Gaussian model. The dashed curve is the extrapolation to $m = 0$.

a mean field equation like equ. (3.3). The data are shown in fig. 7 together with the mean field fit. As expected, $\langle\bar{\chi}\chi\rangle_\infty(0,m)$ is much closer to the chiral limit than $\langle\overline{\chi}\chi\rangle(m,m)$ is, which reduces the ambiguity in the extrapolation of the data to $m = 0$ to a minimum. Again, we find that the data are very well described by mean field theory. The critical coupling comes out to be $\beta_c = 0.1948(8)$, which agrees with the previous value.

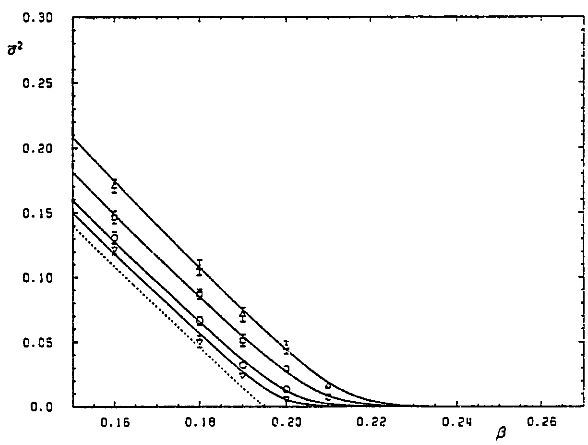

Figure 7: The chiral condensate $\overline{\sigma}^2$ ($\overline{\sigma} = \langle\overline{\chi}\chi\rangle_\infty(0,m)$) as a function of β in dynamical QED for masses between $m = 0.02$ (bottom) and $m = 0.09$ (top). The solid lines are a mean field fit. The dashed curve is the extrapolation to $m = 0$.

5. CONCLUSIONS

Is QED trivial? The agreement of the data with the Gaussian model suggests that the continuum theory is non-interacting. It cannot be excluded though that the behavior will change at correlation lengths much larger than we were able to probe. We can also not claim yet that our results are able to rule out a very small, non-vanishing renormalized charge at the critical point, as the results do rule out larger charges. It is of great importance now to compute the renormalized charge and also the renormalized mass down to $\beta = \beta_c$ and $m = 0$ in order to settle this question.

If this behavior persists, our QED must have at least one more relevant, dimensionless coupling (ζ ?) in order to be a consistent field theory. This would call into question the whole concept of constructing renormalizable quantum field theories on the basis of classical Lagrangians, at least for non-asymptotically free theories.

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