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Random walk approximation in a chiral Yukawa-model and global symmetries*

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Abstract. The fermion propagator is investigated in a chiral Yukawa-model with explicit mirror fermions applying the random walk approximation to the hopping parameter expansion. It is shown that the global $SU(2)_L \otimes SU(2)_R$ symmetry breaking due to the mass splitting within fermion doublets does influence the critical behaviour of the fermion spectrum in the continuum limit. In particular, in the case of a mirror pair of split doublets, where $SU(2)_L \oplus SU(2)_R$ is broken to $SU(2)_L$, no evidence is found for a dynamical spectrum doubling at infinitely strong bare Yukawa-couplings, in contrast to the case with degenerate doublets and $SU(2)_L \otimes SU(2)_R$ symmetry.

1 Introduction

Non-perturbative lattice formulations of chiral Yukawa-models have to deal with the consequence of the Nielsen–Ninomiya theorem [1], which implies the mirror doubling of the fermion spectrum on the lattice. The mirror doubling, i.e. the existence of fermion pairs with the same quantum numbers but opposite chirality, is usually realized by "fermion doubler" states in different parts of the Brillouin-zone of momentum space. The doubling pattern depends on the specific lattice formulation. For instance, for Wilson-fermions [2] the "doubling" actually means the existence of 16 states associated to every fermion field component. In vector-like theories, such as QCD, in the continuum limit 15 out of these states are removed from the physical spectrum by a mass of order 1 in lattice units (infinite in physical units). That is, only one fermion state per field component remains. In the case of chiral Yukawa-models relevant in the electroweak sector of the standard model the situation is different: the lattice formulation becomes more transparent if an explicit mirror doubling is introduced at the level of field components. In a Wilson-like formulation [3] this implies the existence of 32 states on the lattice. In the perturbative continuum limit 30 out of these states are removed by a chiral invariant off-diagonal Wilson-term. The remaining 2 states correspond to a "minimal doubling" (actually true doubling) of the original chiral spectrum. Keeping mirror pairs of states in a general continuum limit is important because of the existence of phases with spontaneously broken mirror symmetry, where the mirror states are split in mass and are mixed with each other [3]. Such phases will probably also appear in the alternative formulation of chiral Yukawa-models without mirror fermion fields [4]. In such a formulation the masses of the mirror fermion doublers are tuned to be of order 1 in lattice units by a Wilson–Yukawa coupling term involving the scalar Higgs-field. Due to the expected universality of the fixed points governing the continuum limit the two formulations with and without explicit mirror fermion fields may be equivalent in the large cut-off limit.

In a formulation with explicit mirror fields the problem of decoupling of mirror fermion partners in the continuum limit can also be investigated. Without chiral gauge fields the decoupling can be achieved, for instance, by introducing an additional Higgs-field with vacuum expectation value of order 1 in lattice units and by tuning the Yukawa-couplings appropriately. Another possibility, proposed by Borrelli et al. [5], can be explored also in lattice perturbation theory by requiring that in the continuum limit the off-diagonal mass mixing between fermion and mirror fermion and the coupling of the mirror fermion to the Higgs-field vanish. This way of decoupling is facilitated by the extension of the global symmetry in the "target" continuum theory implying Ward-identities.

The decoupling of mirror partners in the presence of chiral gauge fields seems, however, more difficult. The extra doublet with vacuum expectation value $O(1)$ does not help because its coupling to the gauge fields removes the gauge fields, too, from the physical spectrum. The decoupling by zero mixing and zero Yukawa-coupling is not possible either, because of the gauge coupling of the mirror states. Borrelli et al. proposed a way of decoupling

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also in chiral gauge theories [6], which works in 1-loop perturbation theory, but the price to pay is the loss of gauge invariance and a very large number of tuned bare parameters. Moreover, it is not clear whether this approach can be extended to higher loops and/or to a non-perturbative situation. Therefore, further non-perturbative studies of chiral Yukawa-models and their gauged counterparts are necessary and useful.

In the present paper we continue the study of chiral Yukawa-models with explicit mirror fermions by the random walk approximation to the non-perturbative hopping parameter expansion. This approach turned out to be useful in the sigma model with Wilson-fermions and broken chiral symmetry [7]. There it was shown that in the infinitely strong bare Yukawa-coupling limit mirror fermions are produced dynamically. This means that the Wilson procedure to remove the species doublers is not completely successfull in that region of the parameter space. This is different from the small coupling region, where lattice perturbation theory suggests that all the doublers are removed. In other words, in general, perturbative arguments are not sufficient to guarantee the decoupling of doublers. One could perhaps argue that in the standard model strong bare Yukawa-coupling are not relevant. Nevertheless, Yukawa-couplings get stronger and stronger at higher energy scales, therefore even a fermion with a mass around 200 GeV may correspond to a very strong Yukawa-coupling at a cut-off scale near the Planck-mass. Therefore, it is possible that in the standard model strong bare Yukawa-couplings have a physical relevance.

The random walk approximation was applied also in the PhD Thesis of Wagner [8] for the investigation of the spectrum in the $SU(2)_L \otimes SU(2)_R$ symmetric model with a miror pair of fermion doublets [3]. In this work an 8th order hopping parameter expansion was evaluated for the masses and couplings at vanishing scalar hopping parameter, infinite bare quartic scalar coupling and equal Yukawa-couplings of the fermion and mirror fermion. General theorems about mirror doubling of the fermion spectrum were also proven. At infinitely large bare Yukawa-couplings the random walk approximation showed an additional dynamical doubling of the spectrum, in accordance with the general theorems. It would be useful to confront this result with the perturbative decoupling proposal [5]. This is, however, not directly possible because in [5] a lepton doublet was considered, which has a smaller global symmetry due to the mass splitting between the charged lepton and the neutrino. More generally, one can rise the question whether the larger $SU(2)_L \otimes SU(2)_R$ global symmetry of degenerate doublets is important from the point of view of fermion spectrum doubling. Therefore, here we consider a model of a mirror pair of fermion doublets with mass splitting. This has only a $SU(2)_L$ global symmetry which can be gauged as in the standard electroweak model. The fermion proparator will be considered in the random walk approximation and the doubling of the fermion spectrum in the continuum limit will be investigated. Compared to [8] a further generalization is that, besides the infinitely strong bare quartic coupling limit $\lambda \to \infty$, the other extreme, namely $\lambda \to 0$, will also be considered. In the next section the model will be defined. In Sect. 3, the random walk approximation to the fermion propagator will be derived both for infinite and zero bare quartic coupling. The case of a zero mass neutrino without a right handed component will also be separately considered. The last section contains some concluding remarks.

2 The model

The field content of the model is given by the fermion doublet field ψ, the mirror doublet field χ and the scalar field ϕ. The scalar field may be considered either as an $SU(2)_L$-doublet or as a four-vector under $O(4) \equiv SU(2)_L \oplus SU(2)_R$. In the $O(4)$-notation the lattice field is ϕ_{Sx}, where x is the space-time point and $S = 0, 1, 2, 3$. This can also be represented by a $2 \otimes 2$ matrix φ_x as

$$\varphi_x \equiv \phi_{0x} + i\phi_{sx}\tau_s \equiv \phi_{Sx}\sigma_S \equiv \begin{pmatrix} \tilde{F}_1 & F_1 \\ \tilde{F}_2 & F_2 \end{pmatrix} \tag{1}$$

where $\tau_s (s = 1, 2, 3)$ is a Pauli-matrix, $\sigma_S \equiv (1, i\tau_s)$ and $F_A (A = 1, 2)$ is the doublet field and $\tilde{F}_A \equiv \varepsilon_{AB} F_B^+$. The lattice action can be written as

$$S = \sum_x \Big\{ \mu_\phi \phi_{Sx}\phi_{Sx} + \lambda(\phi_{Sx}\phi_{Sx})^2 - \kappa \sum_\mu \phi_{Sx+\hat{\mu}}\phi_{Sx} - K \sum_\mu [(\bar{\psi}_{x+\hat{\mu}}\gamma_\mu \psi_x) + (\bar{\chi}_{x+\hat{\mu}}\gamma_\mu \chi_x)] - rK \sum_\mu [(\bar{\chi}_{x+\hat{\mu}}\psi_x) + (\bar{\psi}_{x+\hat{\mu}}\chi_x)] + \bar{\Psi}_x Q(\phi_x) \Psi_x \Big\}. \tag{2}$$

Here we use slightly different notations than in [3], in particular, the normalization freedom of the fermion fields is exploited in order to have a common hopping parameter K for the fields ψ and χ. In addition we introduced $\Psi \equiv (\psi_R, \psi_L, \chi_R, \chi_L)$ and the block matrix

$$Q(\phi) \equiv \begin{pmatrix} 0 & (\varphi G)^+ & 0 & \mu_H \\ (\varphi G) & 0 & \mu & 0 \\ 0 & \mu & 0 & (\varphi H) \\ \mu_H & 0 & (\varphi H)^+ & 0 \end{pmatrix} \tag{3}$$

where the $2 \otimes 2$ submatrices are:

$$\mu = \begin{pmatrix} \mu_0 & 0 \\ 0 & \mu_0 \end{pmatrix} \quad \mu_H = \begin{pmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix} \quad G = \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \quad H = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix}. \tag{4}$$

In the same way as in [3], the transformation properties of ψ and χ are such that $SU(2)_L$ acts on ψ_L and χ_R and $SU(2)_R$ on ψ_R and χ_L, that is χ is the mirror partner of ψ. In this case, for $\mu_1 \neq \mu_2$ or $G_1 \neq G_2$ or $H_1 \neq H_2$ the global $SU(2)_L \otimes SU(2)_R$ symmetry is broken to $SU(2)_L$, similarly to a mirror-doubled standard fermion family. Using the normalization freedom of the scalar field the single site scalar part of the action can be written in a suitable form:

$$S_\phi = \sum_x \{\phi_{Sx}\phi_{Sx} + \lambda(\phi_{Sx}\phi_{Sx} - 1)^2\}. \tag{5}$$

In the limit $\lambda \to \infty$ the length of the scalar field is frozen to $\phi_{Sx}\phi_{Sx} = 1$. Equation (5) corresponds to the $\kappa = 0$ limit of the scalar action which we shall investigate in this paper. As discussed in [7], this particularly simple limit is well suited for the hopping parameter expansion because all the states, including the scalar boson, are represented by fermionic composites. (The general case could only be studied in a double expansion in powers of K and κ.) Since the fermion interactions reproduce the scalar coupling, too, it is expected that the $\kappa = 0$ model is in the same universality class as $\kappa \neq 0$, therefore the qualitative properties of the model can be studied at $\kappa = 0$.

3 Random walk approximation to the fermion propagator

We want to examine whether the fermions χ and ψ are parity doubled or not. To this end we calculate the fermion propagator in the random walk approximation and determine the value of the fermion hopping parameter K for which there is a pole in the propagator at zero momentum. We then examine whether for this K there also exist poles in the propagator for momenta at the other corners of the Brillouin-zone.

The hopping parameter expansion consists in expanding the part of the exponential in the functional integral containing the kinetic terms. The result is a sum of connected single-site expectation values (clusters), where the different clusters are held together by the links corresponding to the hopping terms.

The random walk approximation is taking into account only paths of a special type [7] which can be summed up by a recursion relation. In our case the recursion relation reads:

$$G(k) = [1 - (M + F)K_0]^{-1}(M + F) \tag{6}$$

where M is the connected two-operator single site vev and $F = \Sigma_\mu F_\mu \exp(-ik_\mu)$. Here F_μ is the contribution of the graph shown in Fig. 1. In order to calculate F we need the connected four-operator single site vev matrix, as well as the free fermion propagator K_μ. (K_0 in (6) is obtained from K_μ as $K_0 = \Sigma_\mu K_\mu \exp(-ik_\mu)$. Note that we have restricted ourselves to the case of vanishing scalar hopping parameter κ, that is, we considered non-propagating scalar fields, therefore there is no need to take into account scalar field propagators. One will first do the fermionic integration giving rise to the matrix Q^{-1}. It turns out to be of the form:

$$Q^{-1} = \begin{pmatrix} 0 & A^+ & 0 & E \\ A & 0 & F & 0 \\ 0 & F & 0 & B \\ E & 0 & B^+ & 0 \end{pmatrix}, \quad A \equiv \sigma_S \phi_S g, \quad B \equiv \sigma_S \phi_S h. \tag{7}$$

If we define

$$\Pi_i \equiv \mu_0 \mu_i - G_i H_i \phi^2, \quad g_i \equi -\frac{H_i}{\Pi_i}, \quad h_i \equiv -\frac{G_i}{\Pi_i}, \quad (i = 1, 2) \tag{8}$$

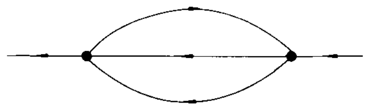

Fig. 1. The graph defining the contribution F_μ in (6)

then the matrices g and h are given by

$$g = \begin{pmatrix} g_1 & 0 \\ 0 & g_2 \end{pmatrix} \quad h = \begin{pmatrix} h_1 & 0 \\ 0 & h_2 \end{pmatrix}. \tag{9}$$

Further notations in (7) are:

$$E = \begin{pmatrix} \mu_0/\Pi_1 & 0 \\ 0 & \mu_0/\Pi_2 \end{pmatrix}$$

$$F = \begin{pmatrix} (\mu_1 F_2 \tilde{F}_2/\Pi_1 + \mu_2 F_1 \tilde{F}_1/\Pi_2) 1/\phi^2 & (\mu_2/\Pi_2 - \mu_1/\Pi_1) F_1 F_2^*/\phi^2 \\ (\mu_2/\Pi_2 - \mu_1/\Pi_1) F_2 F_1^*/\phi^2 & (\mu_1 F_2 \tilde{F}_2/\Pi_1 + \mu_2 F_1 \tilde{F}_1/\Pi_2) 1/\phi^2 \end{pmatrix}. \tag{10}$$

It is easily seen that the determinant of Q is equal to

$$\det Q = (\Pi_1 \Pi_2)^4 = (\mu_0 \mu_1 - G_1 H_1 \phi^2)^4 (\mu_0 \mu_2 - G_2 H_2 \phi^2)^4 \equiv P^4. \tag{11}$$

The two-operator vev is given by

$$\langle \psi_a \bar{\psi}_b \rangle = \frac{\int [d\phi] \det Q(\phi) e^{-S_\phi} Q_{ab}^{-1}(\phi)}{\int [d\phi] \det Q(\phi) e^{-S_\phi}}. \tag{12}$$

The four-operator vev is equal to

$$\langle \psi_a \bar{\psi}_b \psi_c \bar{\psi}_d \rangle = \frac{\int [d\phi] \det Q(\phi) e^{-S_\phi} [Q_{ab}^{-1}(\phi) Q_{cd}^{-1}(\phi) - Q_{ad}^{-1}(\phi) Q_{cb}^{-1}(\phi)]}{\int [d\phi] \det Q(\phi) e^{-S_\phi}}. \tag{13}$$

The propagator K_μ may be seen by inspection to read:

$$K_\mu = \begin{pmatrix} \gamma_\mu & 0 & 0 & r \\ 0 & \gamma_\mu & r & 0 \\ 0 & r & \gamma_\mu & 0 \\ r & 0 & 0 & \gamma_\mu \end{pmatrix} \tag{14}$$

The contribution F_μ of the graph in Fig. 1 is:

$$F_\mu = \frac{\int [d\phi][d\phi'] \det Q(\phi) \det Q(\phi') e^{-S_\phi} e^{-S_{\phi'}} I_\mu(\phi)}{\int [d\phi)[d\phi') \det Q(\phi) \det Q(\phi') e^{-S_\phi} e^{-S_{\phi'}}}$$

$$I_\mu(\phi) \equiv \phi_S \phi'_{S'} \phi_T \phi'_{T'} [Q_S^{-1} K_\mu Q_{S'}^{-1} K_{-\mu} Q_T^{-1} K_\mu Q_{T'}^{-1} - \mathrm{Tr}(Q_S^{-1} K_\mu Q_{S'}^{-1} K_{-\mu}) Q_T^{-1} K_\mu Q_{T'}^{-1}]. \tag{15}$$

In order to proceed one should perform the scalar integration. We shall examine here two extreme case, namely $\lambda = \infty$ and $\lambda = 0$.

3.1 *The case* $\lambda = \infty$

In this case the length of the scalar field is frozen to unity, so we are left with easy $SU(2)$- integrations. The final result for $G(k)$ at $k_\mu = 0$ or π is:

$$G(k) = \begin{pmatrix} 0 & 0 & 0 & G_s \\ 0 & 0 & G_d & 0 \\ 0 & G_d & 0 & 0 \\ G_s & 0 & 0 & 0 \end{pmatrix} \tag{16}$$

where

$$G_s = \begin{pmatrix} B_1/A_1 & 0 \\ 0 & B_2/A_2 \end{pmatrix}, \quad G_d = \begin{pmatrix} B_3/A_3 & 0 \\ 0 & B_3/A_3 \end{pmatrix} \tag{17}$$

and

$$B_i = \frac{K^3}{16}(8 - \hat{k}^2)\alpha_i + \mu_i', \quad A_i = 1 - Kr(8 - \hat{k}^2)B_i, \quad i = 1,2,3;$$

$$\mu_1' = \frac{\mu_0}{\Pi_1}, \quad \mu_2' = \frac{\mu_0}{\Pi_2}, \quad \mu_3' = \frac{\mu_1}{2\Pi_1} + \frac{\mu_2}{2\Pi_2}. \tag{18}$$

Here in $\Pi_{1,2}$ one should set $\phi^2 = 1$ and $\hat{k}_\mu$ is defined, as usual, by $\hat{k}_\mu \equiv 2\sin k_\mu/2$. Furthermore, α_i is defined by the following relations:

$$\begin{aligned} \alpha_1 &= 8rg_1h_1[3g_2^2 + 3h_2^2 - (5r^2+1)g_2h_2 \\ &\quad + 2g_1^2 + 2h_1^2 - 4r^2g_1h_1], \quad \alpha_2 = \alpha_1(1 \leftrightarrow 2), \\ \alpha_3 &= 4r(g_1h_2 + g_2h_1)(g_1g_2 + h_1h_2) \\ &\quad - 4r(g_1^2h_2^2 + g_2^2h_1^2 - 2r^2g_1h_1g_2h_2) \\ &\quad + 8r(g_1h_1 + g_2h_2)(g_1^2 + h_1^2 - 2r^2g_1h_1 \\ &\quad + g_2^2 + h_2^2 - 2r^2g_2h_2). \end{aligned} \tag{19}$$

In the expressions for $g_{1,2}$ and $h_{1,2}$ one should set $\phi^2 = 1$. The value of critical K is determined by demanding that A_i, $i = 1,2,3$ vanish at $k_\mu = 0$. If the critical K values for $i = 1,2,3$ are different, the smallest one matters, and the other states are decoupled from the physical spectrum in the continuum limit (at least in the random walk approximation). In order to obtain a common critical value for $i = 1,2,3$ the Yukawa-coupling and off-diagonal mass mixing parameters have to satisfy some relations (see below). Thus we observe that the momenta lying at the other fifteen corners of the Brillouin-zone in general do not give zero A_i, so there is no additional dynamical doubling in the general case.

It is interesting to display the expressions for α_i in two special cases. The value of the parameter r is easily seen not to have any particular physical importance, therefore we set it equal to 1 in what follows. In that case for ($G_1 = G_2 = G, H_1 = H_2 = H, \mu_0 = \mu_1 = \mu_2$) it turns out that $g_1 = g_2 = g$ and $h_1 = h_2 = h$ and $\alpha_1 = \alpha_2 = \alpha_3 = 40gh(g-h)^2$. This agrees with the results in Ref. [8], therefore, in the limit of infinite Yukawa-couplings ($\mu_0 = \mu_1 = \mu_2 = 0$) we get a dynamically doubled spectrum. In another special case ($G_2 = H_2 = 0$), we have $g_2 = h_2 = 0$ and $\alpha_1 = 16g_1h_1(g_1 - h_1)^2$, $\alpha_2 = 0$, $\alpha_3 = \alpha_1/2$. Therefore, at the critical $K = K_{cr}$ determined by the vanishing of A_i, ($i = 1,2,3$), we are forced to tune $\mu_{1,2}$ and μ_0 to non-zero values. As it can be seen from the structure of the equations determining K_{cr}, for $\mu_{0,1,2} \neq 0$ the K_{cr} values for all other corners of the Brillouin-zone are larger, therefore dynamical fermion doubling does not occur.

3.2 *The case* $\lambda = 0$

In this case, in order to perform the scalar integration, we have to invoke Gaussian integrals. To simplify the calculations, we set here the parameter r equal to 1 from the beginning. For $G(k)$ at $k_\mu = 0$ and $k_\mu = \pi$ we end up with a result of the same form as above. The functions μ_1' and α_1 are now given by

$$\begin{aligned} \mu_1' &= \frac{1}{I_{\det}}[\mu_0^2\mu_2\gamma I_0 - \mu_0^2\mu_2(2\beta_1 + \beta_2)I_A \\ &\quad + \mu_0G_2H_2(\beta_1 + 2\beta_2)I_B - \mu_0G_2H_2\alpha I_C] \end{aligned} \tag{20}$$

respectively,

$$\begin{aligned} \alpha_1 &= \frac{1}{I_{\det}^2}[3G_1H_1(G_2 - H_2)^2(\alpha I_A - \beta I_B + \gamma I_C)^2 \\ &\quad + 2G_1H_1(G_1 - H_1)^2(\mu_0^2\mu_2^2I_A - 2\mu_0\mu_2 \\ &\quad \cdot G_2H_2I_B + G_2^2H_2^2I_C)]^2 \end{aligned} \tag{21}$$

μ_2' and α_2 are obtained from the corresponding expressions for μ_1', respectively, α_1 by interchanging the indices 1 and 2. We also have

$$\mu_3' = \frac{1}{2\mu_0 I_{\det}}[2\gamma^2I_0 - 3\beta\gamma I_A + (\beta^2 + 2\alpha\gamma)I_B - \alpha\beta I_C] \tag{22}$$

α_3 is given by $\alpha_3 = E_{12} + E_{21}$, where E_{21} is obtained from E_{12} by interchanging the indices 1 and 2 and

$$\begin{aligned} E_{12} &= \frac{(G_1 - H_1)(G_2 - H_2)}{2I_{\det}^2}\{G_1H_2[(\gamma I_A - 2\beta_1I_B + \alpha I_C)^2 \\ &\quad + 2\beta_1(\beta_1 - \beta_2)I_AI_C] \\ &\quad + 4\beta_1G_2H_1(G_1H_2 - G_2H_1)I_BI_C\} \\ &\quad + (G_2 - H_2)^2\{G_1H_1(\gamma I_A - \beta I_B + \alpha I_C)^2 \\ &\quad + G_2H_2(\mu_0^2\mu_1^2I_A - 2\mu_0\mu_1G_1H_1I_B + G_1^2H_1^2I_C)^2\}. \end{aligned} \tag{23}$$

The symbols appearing in these equations are defined as

$$\begin{aligned} &\alpha \equiv G_1H_1G_2H_2, \quad \beta_1 \equiv \mu_0\mu_1G_2H_2, \quad \beta_2 \equiv \mu_0\mu_2G_1H_1, \\ &\beta = \beta_1 + \beta_2, \quad \gamma = \mu_0^2\mu_1\mu_2, \quad I_0 \equiv \int d\phi e^{-\phi^2}P, \\ &I_A \equiv \int d\phi e^{-\phi^2}P^2\phi^2, \quad I_B \equiv \int d\phi e^{-\phi^2}P^2\phi^4, \\ &I_C \equiv \int d\phi e^{-\phi^2}P^2\phi^6, \quad I_{\det} \equiv \int d\phi e^{-\phi^2}P^4 \end{aligned} \tag{24}$$

P is defined in (11). If we consider the limits ($G_1 = G_2 = G$, $H_1 = H_2 = H$) and ($G_2 = H_2 = 0$), we find qualitatively the same results as in the case $\lambda = \infty$. Thus is seems that the dynamical fermion doubling near the critical point does not depend on the value of λ.

3.3 *No right handed neutrino*

The results of the previous subsections cannot immediately be applied to the case when $\mu_2 = G_2 = H_2 = 0$, that is when the second state in the doublet has zero mass and is completely decoupled from the rest. This is a simple special case, but we consider it explicitly here, because it occurs in the standard model in the lepton doublet and it was considered by Borrelli et al. in [5]. From the technical point view, some of the above formulae become singular in this limit, therefore it is not easy to immediately infer the results. It is better to work them out separately. The results have the same form as above. In order to find the critical hopping parameter K one has to look for the zeros of A_i, $i=1,2,3$.

Let us first consider the case with $\lambda=\infty$. In this case $\phi^2=1$ and A_i is given by

$$A_i \equiv 1-K(8-\hat{k}^2)\left[\frac{K^3}{16}(8-\hat{k}^2)\alpha_i+\mu_i''\right] \tag{25}$$

where

$$\alpha_1=16g_1h_1(g_1-h_1)^2,\quad \alpha_2=0,\quad \alpha_3=8g_1h_1(g_1-h_1)^2,$$
$$\mu_1''=\frac{-\mu_0}{G_1H_1-\mu_0\mu_1},\quad \mu_2''=0,\quad \mu_3''=\frac{G_1H_1-2\mu_0\mu_1}{2\mu_0(G_1H_1-\mu_0\mu_1)},$$
$$g_1=\frac{H_1}{G_1H_1-\mu_0\mu_1},\quad h_1=\frac{G_1}{G_1H_1-\mu_0\mu_1}. \tag{26}$$

It follows that $A_2=1$ permanently.

The case with $\lambda=0$ is similar but, instead of Eq. (25) we have

$$A_i \equiv 1-K(8-\hat{k}^2)\left[K^3\frac{I_n^2}{I_d^2}(8-\hat{k}^2)\alpha_i+\mu_i''\right] \tag{27}$$

where

$$\alpha_1=16\mu_0^4G_1H_1(G_1-H_1)^2,\quad \alpha_2=0,$$
$$\alpha_3=8\mu_0^4G_1H_1(G_1-H_1)^2,$$
$$\mu_1''=-\frac{1}{I_d}\mu_0^2[24G_1^3H_1^3-18G_1^2H_1^2\mu_0\mu_1$$
$$+6G_1H_1\mu_0^2\mu_1^2-\mu_0^3\mu_1^3],\quad \mu_2''=0,$$
$$\mu_3''=\frac{1}{I_d}[60G_1^4H_1^4-60G_1^3H_1^3\mu_0\mu_1+9G_1^2H_1^2\mu_0^2\mu_1^2$$
$$-25G_1H_1\mu_0^3\mu_1^3+\mu_0^4\mu_1^4]$$
$$I_d=\mu_0[120G_1^4H_1^4-96G_1^3H_1^3\mu_0\mu_1+36G_1^2H_1^2\mu_0^2\mu_1^2$$
$$-8G_1H_1\mu_0^3\mu_1^3+\mu_0^4\mu_1^4]$$
$$I_n=\frac{\mu_0^2}{2}[12G_1^2H_1^2-6G_1H_1\mu_0\mu_1+\mu_0^2\mu_1^2]. \tag{28}$$

Comparing these formulae to those in the previous subsection it is clear that the qualitative behaviour in the case of the standard lepton doublet is the same as in the general split doublet with different non-zero masses (like the quark doublets in the standard model).

4 Concluding remarks

The main result of this paper is that the dynamical fermion doubling phenomenon in the chiral $SU(2)$ Yukawa-model with explicit mirror fermion fields seems to depend on the mass splitting within doublets. In the case of split doublets with reduced symmetry in the random walk approximation to the hopping parameter expansion no evidence is found for dynamical fermion doubling. This is in contrast to the case of degenerate doublets, where dynamical fermion doubling seems to occur [8]. This shows the non-trivial rõle played by the global symmetries, which can very well affect the phase structure in any Yukawa-model with strong couplings. The important consequence for non-perturbative studies of Yukawa-models is that at some point the global $SU(2)_L \otimes SU(2)_R$ symmetry breaking has to be considered as well.

A new aspect of our calculation with respect to [8] is that, besides infinitely strong bare quartic coupling, we also considered the case of zero bare quartic coupling. In both cases at infinitely strong bare Yukawa coupling we found the same qualitative behaviour. In particular, the problem of dynamical fermion doubling seems to be qualitatively the same both for $\lambda=\infty$ and $\lambda=0$. This is a non-trivial point, because in the presence of Yukawa-couplings there is no general reason why the bare quartic scalar self-coupling could not influence the physical picture even qualitatively.

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