Using squeezed light to improve the sensitivity of terrestrial axion production and detection experiments

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It is shown that the use of squeezed light can enhance the sensitivity of experiments which look for invisible axions using the "shining light through walls" technique. The maximal improvement in sensitivity depends on the amount of squeezing that can be applied to the vacuum.

1. Introduction

Physics at a large scale Λ may manifest itself at lower mass-scales through the presence of nearly massless weakly interacting bosons. The prototypical example of this is the invisible axion [1–3]. This particle was invented to solve the strong *CP* problem and at the same time avoid the tight limits from the non-observation of the decays J/ψ, $\Upsilon\to\gamma+$ axion. The attempt to avoid laboratory limits succeeded very well. Consequently all presently known limits on the invisible axion mass-scale Λ come from the sky in one form or another. The energy loss of the sun and of red giants [4,5] due to axion emission would be compatible with observations when $\Lambda > 10^9$ GeV. The recent supernova SN1987A gave rise to more stringent bounds [6,7]: $\Lambda > 10^{10}$ GeV. A recent experiment [8] has put correlated upper limits on the density of axions in the galactic halo and on the scale Λ. Finally cosmological considerations put an upper limit $\Lambda < 10^{12}$ GeV [9–11].

Apart from these astrophysical and cosmological constraints it is of obvious importance to try to limit the axion scale by entirely terrestrial experiments. A very promising experiment was recently proposed by van Bibber et al [12]. These authors exploited the fact that in all axion models a coupling $\mathscr{L}_{\rm int} = gaF_{\mu\nu}\tilde{F}^{\mu\nu}$ between the axion field a and two photons fields appears. The coupling constant g is model dependent but roughly equal to the inverse scale Λ^{-1} times the fine structure constant α. In an external magnetic field this term induces a mixing between photons and axions. The proposed experiment, in its simplest form, consists of shining a powerful laser into a magnetic field where some photons are converted into axions. Then the photon beam is blocked by an absorber (which is transparent to axions). The axions then traverse a second magnetic field where the inverse process takes place: some of them are reconverted to photons which can be detected. In the largest version the experiment is supposedly sensitive to coupling constants $g^{-1} \approx 10^{11}$ GeV.

This method seems to be limited by the fact that the number of reconverted photons that has to be produced in order to detect a signal has to be at least one. In this paper it is shown how to circumvent this limit. Put less cryptically, it is shown how to modify the proposal of ref. [12] to enhance the signal to noise ratio. The technique proposed below makes use of squeezed light. Squeezed states of harmonic oscillators are states in which a coordinate can be known with high accuracy at the cost of increasing the fluctuations in the conjugate momentum. The accuracy with which the coordinate can be known is truly arbitrary and can be better than the magnitude of the vacuum fluctuations. Even before squeezed light was produced in the laboratory [13], its use to reduce the quantum fluctuations in the interferometric detec-

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tion of gravitational waves was suggested [14].

Below a short description of coherent and squeezed states will be given in section 2. A discussion of the proposal of ref. [12] fills section 3. The language used here sets the stage for section 4 where it is shown how to increase the signal over noise ratio using squeezed light. Conclusions are drawn in section 5.

Although the word axion is used throughout this paper, nothing depends critically on its spin and/or parity. Slight modifications of the scheme proposed here make it applicable to particles with other spin–parity than 0^-.

2. Coherent and squeezed states

Consider a single mode of the electromagnetic field, created by the operator $a^\dagger([a, a^\dagger]=1)$. A coordinate and its canonically conjugate momentum are now defined by

$$q=\frac{a+a^\dagger}{\sqrt{2}}, \quad p=\mathrm{i}\frac{a^\dagger-a}{\sqrt{2}}, \tag{2.1}$$

which obey the uncertainty relation

$$\Delta q\Delta p\geqslant\tfrac{1}{2}. \tag{2.2}$$

Coherent states are a specific set of states for which the equality sign in eq. (2.2) is true. They are most conveniently described using the displacement operator

$$D(\alpha)=\exp(\alpha a^\dagger-\alpha^* a) \tag{2.3}$$

(where α is a complex number), which has the properties

$$D(\alpha)^\dagger a^\dagger D(\alpha)=a^\dagger+\alpha^*, \quad D(\alpha)^\dagger=D(-\alpha). \tag{2.4}$$

The coherent state $|\alpha\rangle$ is obtained from the ground state by applying the displacement operator

$$|\alpha\rangle=D(\alpha)|0\rangle. \tag{2.5}$$

In the coherent state $|\alpha\rangle$ we have

$$\Delta q=\Delta p=\frac{1}{\sqrt{2}}. \tag{2.6}$$

Evolution in time with the hamiltonian $H=\omega(a^\dagger a+\frac{1}{2})$ takes one coherent state to another

$$\exp(-\mathrm{i}Ht)D(\alpha)\exp(\mathrm{i}Ht)=D(\alpha\exp(-\mathrm{i}\omega t)), \tag{2.7}$$

The class of squeezed states is a wider class which is obtained when one applies the squeeze operator [15]

$$\Sigma(z)=\exp[\tfrac{1}{2}(za^2-z^*a^{\dagger 2})] \tag{2.8}$$

to a coherent state. In eq. (2.8) z is an arbitrary complex number $z=r\exp(\mathrm{i}\theta)$. The squeezed state $|\alpha, z\rangle$ is defined as

$$|\alpha, z\rangle=\Sigma(z)D(\alpha)|0\rangle. \tag{2.9}$$

The squeeze operator generates a Bogoliubov transformation

$$\Sigma(z)^\dagger a^\dagger\Sigma(z)=a^\dagger\cosh r-\exp(\mathrm{i}\theta)a\sinh r. \tag{2.10}$$

Time evolution takes squeezed states to squeezed states

$$\exp(-\mathrm{i}Ht)\Sigma(z)\exp(\mathrm{i}Ht)=\Sigma(\exp(2\mathrm{i}\omega t)z). \tag{2.11}$$

The uncertainties in the coordinate and its conjugate momentum are

$$\Delta q=\frac{1}{\sqrt{2}}|\cosh r-\exp(\mathrm{i}\theta)\sinh r|,$$

$$\Delta p=\frac{1}{\sqrt{2}}|\cosh r+\exp(\mathrm{i}\theta)\sinh r|, \tag{2.12}$$

and their product equals

$$\Delta q\Delta p=\tfrac{1}{2}\sqrt{1+\sin^2\theta\sinh^2 2r}. \tag{2.13}$$

From these formulae one learns that squeezed states are in general not minimal uncertainty packets, but that any squeezed state has this property four times per period of the oscillator. And also that the values of Δq and Δp are unequal, varying in time and that their minimal values can be arbitrarily small. This last property is in marked contrast to coherent states and is the reason why squeezed states may be used in high precision measurements.

3. Shining light through a wall

It is a general feature of invisible axion models that the axion possesses a coupling to two photons which is induced by the ABJ anomaly

$$\mathscr{L}_{\mathrm{int}}=gaF_{\mu\nu}\tilde{F}^{\mu\nu}=-4ga\boldsymbol{E}\cdot\boldsymbol{B}. \tag{3.1}$$

This coupling will induce a mixing between photons and axions when an external electromagnetic field is present. This conversion occurs when one scatters photons off a magnetic field which had a nonvanishing component along the direction of the electric polarization vector of the photons. When, in addition the magnetic field is stationary and does not vary in directions transverse to the incoming photons, the axions produced have the same energy and their momenta point in the same direction as the incoming photons. Under these circumstances one can restrict one's attention to one mode of the electromagnetic field and the corresponding mode of the axion field. The S-matrix of a magnetic field can than be expressed as

$$S_B = \exp\{ib[\exp(-i\beta)a_\gamma^\dagger a_a + \exp(i\beta)a_a^\dagger a_\gamma]\}, \tag{3.2}$$

where the coupling $b\exp(-i\beta)$ depends on the axion to photon coupling g, the strength of the magnetic field and the geometry. The simplest experiment proposed by ref. [12] is shown in fig. 1 and can now be described as follows. One shines a laser beam from input port 1 into the magnetic field B_1. The photons are after passage through the magnetic field reflected out by a mirror M. (It is conceptually easier to think of a mirror, which can be described by a unitary S-matrix, than of an absorber which has a complicated internal behaviour.) The axions do not sense the mirror and go straight through. Nothing, i.e. the electromagnetic vacuum is inserted in input port 2. Some of the axions are converted back into photons in the second magnetic field.

The in-state consists of coherent laser light

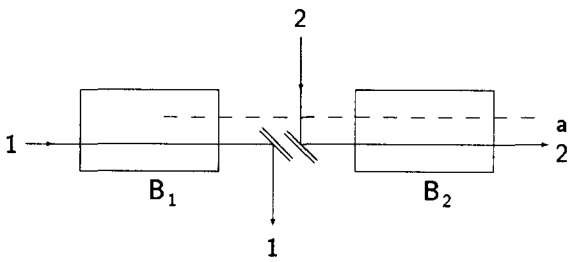

Fig. 1. The simplest experiment to produce and detect axions. (The two boxes are magnetic fields and the dashed line stands for the excited axion mode.)

$$|\text{in}\rangle = D_1(\alpha)|0\rangle = \exp(\alpha a_1^\dagger - \alpha^* a_1)|0\rangle \tag{3.3}$$

with photon number

$$\langle \text{in}|a_1^\dagger a_1|\text{in}\rangle = |\alpha|^2. \tag{3.4}$$

The out-state is found by the action of the two consecutive magnetic fields on the in-state

$$\begin{aligned}|\text{out}\rangle &= S_{B_2}S_{B_1}|\text{in}\rangle\\ &= \exp\{ib_2[\exp(-i\beta_2)a_2^\dagger a_a\\ &\quad + \exp(i\beta_2)a_a^\dagger a_2]\}\\ &\quad \times\exp\{ib_1[\exp(-i\beta_1)a_1^\dagger a_a\\ &\quad + \exp(i\beta_1)a_a^\dagger a_1]\}|\text{in}\rangle.\end{aligned} \tag{3.5}$$

The number of photons seen at output port 2 is

$$\langle \text{out}|a_2^\dagger a_2|\text{out}\rangle = |\alpha|^2\sin^2 b_1\sin^2 b_2, \tag{3.6}$$

as can be calculated using (2.4), (3.3) and (3.5). Comparison with formula (5) in ref. [12], where the same result was obtained using a semiclassical method, allows one to identify

$$b \approx \tfrac{1}{2}gBL, \tag{3.7}$$

where L is the length of the magnetic field.

When measuring over a longer period of time this method is limited by fluctuations in the noise of the photon detector. In case one measures during a time T, using a laser with output $\dot N$ photons per unit time and a photon detector which has on average a time T_0 between two noise pulses, then the number of pulses above background is $b_1^2 b_2^2 \dot N T$. The fluctuation in the number of background pulses is however $\sqrt{T/T_0}$, according to the Poisson distribution. The signal to noise ratio is therefore $b_1^2 b_2^2 \dot N\sqrt{TT_0}$, which only grows as the square root of time.

To overcome this difficulty, ref. [12] proposed to "homodyne from a local loscillator". What is meant is shown in fig. 2. The experiment is similar to the one shown in fig. 1, but the beam which enters input port 1 is split into two separate beams by the beam splitting mirror M_1 before entering the magnetic field. One beam enters the magnetic field and, after conversion and reconversion to and from axions, emerges drastically reduced from the second magnet. The other beam, the local oscillator, is led around the experiment. Finally both beams are recombined by the beam splitter M_2.

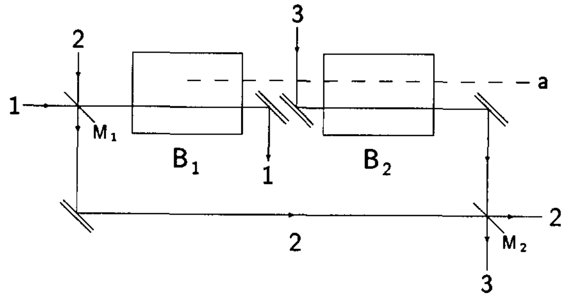

Fig. 2. Axion detection using a local oscillator.

To describe this experiment by an S-matrix it is useful to consider the S-matrix of a single beam splitting mirror, such as M_1 in fig. 2. This mirror couples the two modes, coming towards the mirror, which are created by $a_{1\,\mathrm{in}}^\dagger$ and $a_{2\,\mathrm{in}}^\dagger$ respectively, to the two modes leaving the mirror which are created by $a_{1\,\mathrm{out}}^\dagger$, $a_{2\,\mathrm{out}}^\dagger$. Assuming that the S-matrix is unitary, commutes with the hamiltonian of the electromagnetic field and that all incoming photons scatter independently, one can deduce

$$S_{\mathrm{M}_1} a_{i\,\mathrm{in}}^\dagger S_{\mathrm{M}_1}^\dagger = m_{ij} a_{j\,\mathrm{out}}^\dagger \quad (i,j=1,2)\,. \tag{3.8}$$

Further assumptions, which are not strictly necessary but convenient, can be made. When the mirror M_1 divides the intensity of one oncoming beam exactly over the two output ports, is symmetric under reflections which interchanges both faces of the mirror and finally is invariant under the combined operation of time-reversal and a 180° rotation in the plane of the mirror, then one can show

$$S_{\mathrm{M}_1} a_1^\dagger S_{\mathrm{M}_1}^\dagger = \frac{a_1^\dagger + \mathrm{i} a_2^\dagger}{\sqrt{2}}\,, \quad S_{\mathrm{M}_1} a_2^\dagger S_{\mathrm{M}_1}^\dagger = \frac{a_2^\dagger + \mathrm{i} a_1^\dagger}{\sqrt{2}}\,, \tag{3.9}$$

where the indices "in" and "out" have been dropped.

The S-matrix for the entire experiment of fig. 2 is now

$$S = S_{\mathrm{M}_2} S_{B_2} S_{B_1} S_{\mathrm{M}_1}\,, \tag{3.10}$$

and the in-state is again coherent laser light from port 1

$$|\mathrm{in}\rangle = D_1(\alpha)\,|0\rangle\,. \tag{3.11}$$

When the axions do not couple the only beam entering the second beam splitter is the local oscillator, beam number 2. In that case the number of photons in output port 2 will be equal to the number in output port 3. The axion signal is therefore the difference between the number of photons leaving these two output ports

$$\begin{aligned}&\langle \mathrm{out}|a_2^\dagger a_2 - a_3^\dagger a_3|\mathrm{out}\rangle\\ &= \langle 0|D_1(\alpha)^\dagger S^\dagger (a_2^\dagger a_2 - a_3^\dagger a_3) S D_1(\alpha)|0\rangle\\ &= -\sin b_1 \sin b_2 \cos(\beta_1-\beta_2)\,|\alpha|^2\,.\end{aligned} \tag{3.12}$$

At first sight the improvement over the first experiment seems dramatic: the signal is proportional to $b_1 b_2$ instead of to $b_1^2 b_2^2$ as in (2.6). The catch resides in the statistical fluctuations in $a_2^\dagger a_2 - a_3^\dagger a_3$. They are given by

$$\begin{aligned}&\langle \mathrm{out}|(a_2^\dagger a_2 - a_3^\dagger a_3)^2|\mathrm{out}\rangle\\ &\quad - \langle \mathrm{out}|a_2^\dagger a_2 - a_3^\dagger a_3|\mathrm{out}\rangle^2\\ &= \tfrac{1}{2}|\alpha|^2\,.\end{aligned} \tag{3.13}$$

So when one measures during a time T with $\dot{N}$ photons entering the experiment per unit time, the ratio of the signal $b_1 b_2 \cos(\beta_1-\beta_2)\dot{N}T$ to the noise $\sqrt{\dot{N}T/2}$ is $b_1 b_2 \cos(\beta_1-\beta_2)\sqrt{2\dot{N}T}$. Therefore the time one has to spend before the signal to noise ratio becomes one in the second experiment is nearly equal to the time one has to spend in the first experiment before one axion has reconverted to a photon. The advantage of the second scheme is however that the noise in the photodetectors gets overwhelmed by the statistical fluctuations in the count rate. One can therefore use imperfect detectors and still achieve the sensitivity the first experiment can reach when it is equipped with ideal noiseless photodetectors.

The signal depends on the phases β_1, β_2 which were associated with the magnetic fields. At first sight this may seem to be a problem, but it is not. All the phase differences between the two routes through the experiment, whether they arise from differences in refractive index, axion mass, imperfections in the mirrors or deliberate phase shifts, may be absorbed in the phase $\beta_1-\beta_2$. Should one observe a signal, then one can use this phase to determine the mass of the axion. To do this one has to change the distance between the two magnets and observe the signal. Changing this distance by an amount x, will cause an additional relative phase between the two legs of the interferometer $(\omega - \sqrt{\omega^2 - m_{\mathrm{a}}^2})x$.

4. Squeezing light through a wall

In the experiment proposed by ref. [12], two input ports remain unused. Analogous to ref. [14] one might consider squeezed vacuum as input for these two ports. The in-state is then

$$|\mathrm{in}\rangle = D_1(\alpha)\Sigma_2(z_2)\Sigma_3(z_3)|0\rangle \quad (z_n = r_n \exp(i\theta_n)) . \tag{4.1}$$

The S-matrix remains given by eq. (3.10). It is now a simple matter to calculate the signal

$$\langle \mathrm{out}|a_2^\dagger a_2 - a_3^\dagger a_3|\mathrm{out}\rangle = -\sin b_1 \sin b_2 \cos(\beta_1-\beta_2)(|\alpha|^2 - \sinh^2 r_2) \tag{4.2}$$

and the noise

$$\begin{aligned}&\langle \mathrm{out}|(a_2^\dagger a_2 - a_3^\dagger a_3)^2|\mathrm{out}\rangle - \langle \mathrm{out}|a_2^\dagger a_2 - a_3^\dagger a_3|\mathrm{out}\rangle^2\\ &= \tfrac{1}{2}\{|\alpha|^2[\cosh^2 r_3 + \sinh^2 r_3\\ &-2\sinh r_3 \cosh r_3 \cos(2\arg\alpha+\theta_3)]\\ &+\sinh^2 r_2 \cosh^2 r_3 + \sinh^2 r_3 \cosh^2 r_2\\ &-2\cosh r_2 \sinh r_2 \cosh r_3 \sinh r_3 \cos(\theta_2-\theta_3)\\ &+\sinh^2 r_3\} .\end{aligned} \tag{4.3}$$

In this last expression the terms due to axion interactions have been dropped. Since the number of photons in the in-state is large, the main contribution to (4.3) comes from the first two lines. By carefully tuning the phase of θ_3 one can achieve that $\cos(2\arg\alpha+\theta_3)=1$. This implies that the noise is reduced by a factor e^{-r_3}. The improved signal to noise ratio now implies a sensitivity in the coupling constant g which is a factor $e^{r_3/2}$ better.

It is therefore not necessary to squeeze the light in input port 2. The squeeze factor of port 2 influences the sum of output ports 2 and 3:

$$\begin{aligned}&\langle \mathrm{out}|a_2^\dagger a_2 + a_3^\dagger a_3|\mathrm{out}\rangle\\ &= \tfrac{1}{2}(|\alpha|^2+\sinh^2 r_2)(1+\sin^2 b_1 \sin^2 b_2)\\ &+\cos^2 b_2 \sinh^2 r_3 .\end{aligned} \tag{4.4}$$

The fluctuations in this quantity are given by

$$\begin{aligned}&\langle \mathrm{out}|(a_2^\dagger a_2 + a_3^\dagger a_3)^2|\mathrm{out}\rangle - \langle \mathrm{out}|a_2^\dagger a_2 + a_3^\dagger a_3|\mathrm{out}\rangle^2\\ &= \tfrac{1}{2}\{|\alpha|^2[\cosh^2 r_2\\ &+\cosh r_2 \sinh r_2 \cos(2\arg\alpha+\theta_2)]\\ &+(\cosh^2 r_2 + \tfrac{1}{2})\sinh^2 r_2 + 4\cosh^2 r_3 \sinh^2 r_3\} .\end{aligned} \tag{4.5}$$

These fluctuations can therefore be enhanced, but not reduced, by squeezing the input in port 2, but they do not carry an interesting signal.

It was also noted by the authors of ref. [12] that one could count the number of photons disappearing in the first magnetic field. In principle the sensitivity of this scheme is comparable to the sensitivity one can achieve using a local oscillator. In case one uses the squeezed input state (4.1) one finds

$$\langle \mathrm{out}|a_1^\dagger a_1|\mathrm{out}\rangle = \tfrac{1}{2}\cos^2 b_1(|\alpha|^2+\sinh^2 r_2) \tag{4.6}$$

for the mean number of photons emerging from the first magnetic field and

$$\begin{aligned}&\langle \mathrm{out}|(a_1^\dagger a_1)^2|\mathrm{out}\rangle - \langle \mathrm{out}|a_1^\dagger a_1|\mathrm{out}\rangle^2\\ &= \tfrac{1}{2}|\alpha|^2[\cosh^2 r_2\\ &+\sinh r_2 \cosh r_2 \cos(2\arg\alpha+\theta_2)]\\ &+\tfrac{1}{2}(\cosh^2 r_2 + \tfrac{1}{2})\sinh^2 r_2\end{aligned} \tag{4.7}$$

gives the size of the fluctuations. From eqs. (4.6) and (4.7) it can be seen that squeezed light can *not* be used to enhance the signal to noise ratio when one counts the number of photons disappearing.

5. Conclusions

In the above it has been shown that the use of squeezed light may be used to increase the sensitivity of "shining light through walls" type of axion searches. It remains to be seen however, how large the quantitative impact of this idea is. Producing squeezed light proved to be possible only rather recently [13] and with a modest squeeze factor. One might hope that light with a squeeze factor of ten, i.e. light in which the fluctuations in one coordinate are reduced by a factor of ten, becomes available in the next few years. Applying this light to the axion experiment would give an increment in sensitivity of $\sqrt{10}$. The authors of ref. [12] estimated that using one kilometer of 6.6 T magnets and a 100 kW laser emitting 1 eV photons, one could be sensitive to coupling constants g down to $10^{-11}\mathrm{GeV}^{-1}$. The same experiment, but equipped with squeezed light is now sensitive down to $3\times10^{-12}\mathrm{GeV}$. The same increment in sensitivity can be had by increasing the magnetic field strength or the size of the magnetic field by a factor three, or the output power of the laser times the duration of the experiment by a factor 100.

It is clear that the use of squeezed light is not the single idea that makes a large jump in sensitivity possible, but it is conceivable that at some time the use of squeezed light is the cheapest way to stretch terrestrial axion searches to their limit.

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