

# CP-violating correlations in the reaction $e^+e^- \rightarrow$ hadrons

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**Abstract.** We investigate CP-violating observables involving jets in the process  $e^+e^- \rightarrow$  hadrons. These observables are sensitive to sources of CP-violation beyond the standard model. We use an effective Lagrangian approach to parametrize possible new CP-violating interactions. Bounds on electric and weak dipole moments of leptons and quarks are discussed.

## 1 Introduction

In this paper we study the following question: how can we search for a possible violation of the discrete symmetry CP (C: charge conjugation, P: parity) from observations on the reaction

$$e^+ + e^- \rightarrow \text{hadrons.} \quad (1.1)$$

We will consider the case where the original  $e^+$  and  $e^-$  beams are unpolarized and that jet-variables are measured for the final state. This is a continuation of work on CP-violation in Z-decays [1] where the general motivation for this type of studies was explained and an extensive list of references was given. The reaction

$$e^+ + e^- \rightarrow Z \rightarrow \text{hadrons} \quad (1.2)$$

is a special case of the reaction (1.1) and has already been discussed in detail in [1]. In the present paper we consider the reaction (1.1) away from the Z-resonance. Some new features arise there, especially  $t$ -quark production at sufficiently high energy. We also include some new results on the Z peak using invariant mass cuts which complement the work of [1]. Our study will cover c.m. energies below the Z, i.e. the energy range of the storage rings PEP, PETRA and TRISTAN, as well as c.m. energies above the Z where for instance LEP 2 is planned to operate. Above the thresholds for  $W^+W^-$  and ZZ pair production hadronic decays of two vector bosons will contribute to (1.1). In our paper we will not discuss these channels since possible new effects, including CP-violating ones, for  $e^+e^- \rightarrow W^+W^-$  and  $e^+e^- \rightarrow ZZ$  have already been studied ([2] and references cited therein). We stress,

however, that the general considerations of Sect. 2 below remain valid also for this case.

The plan of the paper is as follows. In Sect. 2 we define CP-odd observables. In Sect. 3 we discuss possible CP-odd interactions contributing to the reaction (1.1) and give the results of our calculations. Section 4 contains our conclusions. In an appendix we collect some formulae.

## 2 CP-odd observables

At high c.m. energies the reaction (1.1) is characterized by the production of hadronic jets. It is thus natural to look for CP-odd observables involving jets [1, 3]. We propose to study exclusive or inclusive jet production, i.e.

$$e^+(\mathbf{p}_+) + e^-(\mathbf{p}_-) \rightarrow \text{jet}(\mathbf{k}_1, \alpha_1) + \dots + \text{jet}(\mathbf{k}_n, \alpha_n), \quad (2.1)$$

$$e^+(\mathbf{p}_+) + e^-(\mathbf{p}_-) \rightarrow \text{jet}(\mathbf{k}_1, \alpha_1) + \dots + \text{jet}(\mathbf{k}_n, \alpha_n) + X, \quad (2.2)$$

$(n = 1, 2, 3, \dots).$

Here  $\mathbf{p}_\pm, \mathbf{k}_i$  are the momenta of  $e^\pm$  and of the jets, respectively,  $\alpha_i$  denote other jet properties one would like to consider, e.g., the invariant mass, the flavour or charge of a jet. We will not consider spin observables in this paper. For inclusive observations  $X$  denotes the rest of the final state.

We consider the reactions (2.1), (2.2) in the c.m. system for unpolarized  $e^+$  and  $e^-$  beams. The initial state is then described by a CP-invariant density matrix. As a general prerequisite for CP-studies with jets we require that no C- and P-biases are introduced by the jet finding algorithm and the detector setup as explained in [1]. We write the cross section for the processes (2.1), (2.2) as

$$d\sigma = \frac{1}{2}(s(s - 4m_e^2))^{-1/2} d\Gamma R(\mathbf{p}_+; \mathbf{k}_1, \alpha_1, \dots, \mathbf{k}_n, \alpha_n). \quad (2.3)$$

Here  $m_e$  is the electron mass,  $s = (p_+ + p_-)^2$ , and for the exclusive reaction (2.1)

$$d\Gamma^{(e)} = \prod_{i=1}^n \frac{d^3k_i}{(2\pi)^3 2k_i^0} (2\pi)^4 \delta^4(p_+ + p_- - k_1 - \dots - k_n), \quad (2.4)$$

$$R^{(e)}(\mathbf{p}_+; \mathbf{k}_1, \alpha_1, \dots, \mathbf{k}_n, \alpha_n) = \sum_{\text{spins}} |\langle \text{jet}(\mathbf{k}_1, \alpha_1), \dots, \text{jet}(\mathbf{k}_n, \alpha_n) | \mathcal{F} | e^+(\mathbf{p}_+) e^-(\mathbf{p}_-) \rangle|^2, \quad (2.5)$$

where  $\mathcal{T}$  is the  $T$ -matrix. For the inclusive reaction (2.2) we have

$$d\Gamma^{(i)} = \prod_{i=1}^n \frac{d^3k_i}{(2\pi)^3 2k_i^0} \quad (2.6)$$

$$\begin{aligned} R^{(i)}(\mathbf{p}_+; \mathbf{k}_1, \alpha_1, \dots, \mathbf{k}_n, \alpha_n) \\ = \sum'_{\text{spins}} \sum_X (2\pi)^4 \delta^4(p_+ + p_- - k_1 - \dots - k_n - k_X) \\ \cdot |\langle \text{jet}(\mathbf{k}_1, \alpha_1), \dots, \text{jet}(\mathbf{k}_n, \alpha_n), X | \mathcal{T} | e^+(\mathbf{p}_+) e^-(\mathbf{p}_-) \rangle|^2. \end{aligned} \quad (2.7)$$

In (2.3) and in the following we generically write  $d\sigma$ ,  $d\Gamma$  and  $R$  in formulae which are valid for both the exclusive and inclusive cases.

We now list the conditions which the cross section or  $R$  must satisfy if invariance under the  $CP$ , the time reversal  $T$  or the  $CPT$  transformation is assumed. The transformations  $T$  and  $CPT$  give useful restrictions only if  $\text{Im } \mathcal{T} \equiv (\mathcal{T} - \mathcal{T}^\dagger)/(2i)$  is assumed to be negligible. Sources of  $\text{Im } \mathcal{T} \neq 0$  are initial and final state interactions and finite width effects in propagators of unstable particles, i.e. self interactions.

The  $CP$ -transformation of the jet states reads

$$CP|\text{jet}(\mathbf{k}_i, \alpha_i)\rangle = |\text{jet}(-\mathbf{k}_i, \bar{\alpha}_i)\rangle, \quad (i = 1, \dots, n) \quad (2.8)$$

where  $\bar{\alpha}_i$  indicates that all particles in the jet have to be replaced by the corresponding antiparticles. This leads to the  $CP$ -invariance condition

$$R(\mathbf{p}_+; \mathbf{k}_1, \alpha_1, \dots, \mathbf{k}_n, \alpha_n) = R(\mathbf{p}_+; -\mathbf{k}_1, \bar{\alpha}_1, \dots, -\mathbf{k}_n, \bar{\alpha}_n). \quad (2.9)$$

$T$ -invariance and  $\text{Im } \mathcal{T} = 0$  imply

$$R(\mathbf{p}_+; \mathbf{k}_1, \alpha_1, \dots, \mathbf{k}_n, \alpha_n) = R(-\mathbf{p}_+; -\mathbf{k}_1, \alpha_1, \dots, -\mathbf{k}_n, \alpha_n). \quad (2.10)$$

$CPT$ -invariance and  $\text{Im } \mathcal{T} = 0$  imply

$$R(\mathbf{p}_+; \mathbf{k}_1, \alpha_1, \dots, \mathbf{k}_n, \alpha_n) = R(-\mathbf{p}_+; \mathbf{k}_1, \bar{\alpha}_1, \dots, \mathbf{k}_n, \bar{\alpha}_n). \quad (2.11)$$

In the following we will always assume  $CPT$ -invariance to hold. We can then draw some simple conclusions from (2.8)–(2.11).

One jet inclusive distributions:

The  $CP$ -invariance condition (2.9) implies here

$$R^{(i)}(\mathbf{p}_+; \mathbf{k}_1, \alpha_1) = R^{(i)}(\mathbf{p}_+; -\mathbf{k}_1, \bar{\alpha}_1). \quad (2.12)$$

Thus a nonzero asymmetry:

$$As = \frac{R^{(i)}(\mathbf{p}_+; \mathbf{k}_1, \alpha_1) - R^{(i)}(\mathbf{p}_+; -\mathbf{k}_1, \bar{\alpha}_1)}{R^{(i)}(\mathbf{p}_+; \mathbf{k}_1, \alpha_1) + R^{(i)}(\mathbf{p}_+; -\mathbf{k}_1, \bar{\alpha}_1)} \quad (2.13)$$

is an indicator of  $CP$ -violation. This is the general type of  $CP$ -test discussed in [4] and [5]. But due to rotational invariance  $R(\mathbf{p}_+; \mathbf{k}_1, \alpha_1)$  depends only on  $|\mathbf{k}_1|$ ,  $|\mathbf{p}_+|$  and the angle between  $\mathbf{p}_+$  and  $\mathbf{k}_1$ . It follows then from (2.11) that a nonzero asymmetry (2.13) can only be generated if  $CP$  is violated and  $\text{Im } \mathcal{T} \neq 0$ .

Integrated cross sections:

Let us consider the integrated exclusive or inclusive  $n$ -jet cross section

$$\sigma(e^+ + e^- \rightarrow \alpha_1 + \dots + \alpha_n) = \int d\sigma \quad (2.14)$$

where  $d\sigma$  is given by (2.3). For experimental and theoretical reasons cuts in phase space are unavoidable. Let us assume these cuts to be  $P$ - and  $C$ -symmetric and symmetric under a (proper) rotation exchanging the  $e^+$  and  $e^-$  beams, i.e. a rotation transforming  $\mathbf{p}_+$  into  $-\mathbf{p}_+$ . Cuts on relative angles between the jets or the “ $y$ -cuts” (see below) together with forward–backward symmetric cuts on angles of jets with respect to the beam direction are of this type. With such cuts the  $CP$ -invariance condition (2.9) leads to

$$\sigma(e^+ + e^- \rightarrow \alpha_1 + \dots + \alpha_n) = \sigma(e^+ + e^- \rightarrow \bar{\alpha}_1 + \dots + \bar{\alpha}_n), \quad (2.15)$$

but due to (2.11) this relation can again only be violated if a  $CP$ -violating interaction is present and  $\text{Im } \mathcal{T} \neq 0$ .

The proposals in [6] to compare  $\sigma(e^+ e^- \rightarrow Z \rightarrow b\bar{s})$  to  $\sigma(e^+ e^- \rightarrow Z \rightarrow \bar{b}s)$  (where  $b, s$  stand for the corresponding quark jets) are  $CP$ -tests of the general type (2.15).

As shown in [1]  $CP$ -violation cannot be tested in the 2-jet exclusive reaction if no polarisation-observables are considered. In the following we will, therefore, concentrate on 2-jet inclusive and 3-jet exclusive reactions. In both cases, to leading order in the coupling constants we are then dealing with the reaction

$$e^+(p_+) + e^-(p_-) \rightarrow \bar{q}(k_+) + q(k_-) + G(k) \quad (2.16)$$

( $q$ : quark,  $G$ : gluon) at the parton level. We will study signals of  $CP$ -violation for the following two cases:

1. Flavour tagged quark jets. This is feasible for  $c$ -,  $b$ -, and in the future presumably also for  $t$ -quarks. A suitable  $CP$ -odd observable is the tensor (cf. [1]):

$$T_{ij} = \hat{k}_{+i} \hat{n}_j - \hat{k}_{-i} \hat{n}_j + (i \leftrightarrow j) \quad (2.17)$$

where  $i, j = 1, 2, 3$  are Cartesian vector indices and

$$\hat{\mathbf{k}}_{\pm} = \frac{\mathbf{k}_{\pm}}{|\mathbf{k}_{\pm}|}, \quad \hat{\mathbf{n}} = \frac{\mathbf{k}_+ \times \mathbf{k}_-}{|\mathbf{k}_+ \times \mathbf{k}_-|}. \quad (2.18)$$

Any nonzero expectation value

$$\langle T_{ij} \rangle = \frac{\int d\sigma T_{ij}}{\int d\sigma} \quad (2.19)$$

would signal  $CP$ -violation. We note that the expectation value (2.19) can be nonzero even in the absence of final state interactions.

2. No flavour tagging of the jets. The jets are ordered according to the magnitude of the jet momentum. Thus we require  $|\mathbf{k}_1| > |\mathbf{k}_2|$  for 2-jet inclusive observations and  $|\mathbf{k}_1| > |\mathbf{k}_2| > |\mathbf{k}_3|$  for the 3-jet exclusive case. Suitable  $CP$ -odd variables are the tensors

$$\tilde{T}_{ij}^a = \hat{k}_{ai} \hat{n}_j + \hat{k}_{aj} \hat{n}_i \quad (2.20)$$

where  $a = 1, 2$  (incl.) or  $a = 1, 2, 3$  (excl.),  $\hat{\mathbf{k}}_a = \mathbf{k}_a/|\mathbf{k}_a|$ , and where the oriented normal to  $\mathbf{k}_1, \mathbf{k}_2$  is defined by

$$\hat{\mathbf{n}} = \frac{\mathbf{k}_1 \times \mathbf{k}_2}{|\mathbf{k}_1 \times \mathbf{k}_2|}. \quad (2.21)$$

Nonzero expectation values  $\langle \tilde{T}_{ij}^a \rangle$  would again be signals of  $CP$ -violation which do not require  $\text{Im } \mathcal{T} \neq 0$ .

### 3 The $CP$ -violating effective Lagrangian and numerical results

In this section we discuss possible  $CP$ -violating interactions which could contribute to the process (2.16). Consider first the standard model (SM). As in [1] we can easily estimate that  $CP$ -violation through the phase in the Kobayashi–Maskawa matrix leads to  $CP$ -odd asymmetries at most of order  $10^{-7}$  which are surely too small to be observable. Therefore we turn to possible sources of  $CP$ -violation outside of the standard model. The general philosophy and procedure is as in [1]. We consider only contact interactions corresponding to operators of dimension  $d \leq 6$  at the level of the tree approximation. The effective  $CP$ -violating Lagrangian relevant for (2.16) is then

$$\begin{aligned} \mathcal{L}_{CP}(x) &= \sum_{\psi=q,e} \left( -\frac{i}{2} d_\psi \bar{\psi}(x) \sigma^{\mu\nu} \gamma_5 \psi(x) F_{\mu\nu}(x) \right. \\ &\quad - \frac{i}{2} \tilde{d}_\psi \bar{\psi}(x) \sigma^{\mu\nu} \gamma_5 \psi(x) (\partial_\mu Z_\nu(x) - \partial_\nu Z_\mu(x)) \\ &\quad \left. + (f_{V\psi} \bar{\psi}(x) \gamma^\nu \psi(x) + f_{A\psi} \bar{\psi}(x) \gamma^\nu \gamma_5 \psi(x)) Z^\mu(x) F_{\mu\nu}(x) \right) \\ &\quad + \sum_q \left( -\frac{i}{2} d'_q \bar{q}(x) T^a \sigma^{\mu\nu} \gamma_5 q(x) G_{\mu\nu}^a(x) \right. \\ &\quad + (h_{Vq} \bar{q}(x) T^a \gamma^\nu q(x) + h_{Aq} \bar{q}(x) T^a \gamma^\nu \gamma_5 q(x)) Z^\mu(x) G_{\mu\nu}^a(x) \\ &\quad + \tilde{h}_{1q} \bar{e}(x) e(x) \bar{q}(x) i \gamma_5 q(x) \\ &\quad + \tilde{h}_{2q} \bar{e}(x) i \gamma_5 e(x) \bar{q}(x) q(x) \\ &\quad \left. + \tilde{h}_{3q} e^{\mu\nu\rho\sigma} \bar{e}(x) \sigma_{\mu\nu} e(x) \bar{q}(x) \sigma_{\rho\sigma} q(x) \right). \end{aligned} \quad (3.1)$$

Here  $q = u, d, s, c, b, t$ .

The notation is as in (4.1) of [1]. Compared to the  $Z^0$ -case discussed in [1] we have three additional coupling terms in (3.1) with corresponding coupling constants  $\tilde{h}_{iq}$ , ( $i = 1, 2, 3$ ).

We are interested in  $CP$ -odd asymmetries which have to arise from interference terms between the  $CP$ -conserving SM-amplitudes and the  $CP$ -violating amplitudes due to (3.1). The lowest order SM diagrams for reaction (2.16) are shown in Fig. 1. The corresponding amplitudes conserve the electron and quark chiralities. Considering unpolarized  $e^+ e^-$  beams and no polarization observation on the jets we find that electron (quark) chirality changing terms in (3.1) contribute proportional to  $m_e$  ( $m_q$ ) to  $CP$ -odd observables. In the following we neglect the electron mass. We are then left with the contributions from the electric, weak and chromoelectric quark dipole moments  $d_q, \tilde{d}_q, d'_q$  and from the 4-point couplings  $f_{V, Ae}, f_{V, Aq}, h_{V, Aq}$ . We have calculated the cross section for (2.16) taking into account the amplitudes from the SM diagrams of Fig. 1 and their interference with the amplitudes arising in tree approximation from the above couplings. For the  $Z$ -propagator we use a Breit–Wigner form with a finite width  $\Gamma_Z$  (cf. the Remark 3 below). The result will be expressed in terms of dimensionless coupling constants  $\tilde{d}_\psi, \tilde{f}_{V\psi}, \tilde{f}_{A\psi}$  ( $\psi = e, q$ ),  $\tilde{h}_{Vq}, \tilde{h}_{Aq}$  defined in (5.1) of [1].

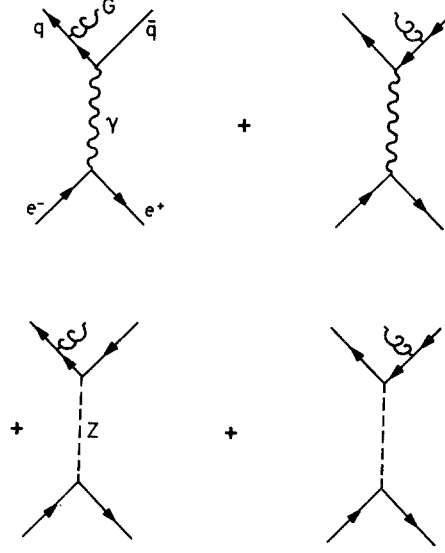


Fig. 1. Lowest order SM diagrams for  $e^+ e^- \rightarrow \bar{q} q G$

#### 3.1 The exclusive 3-jet reaction with flavour tagging

For the exclusive reaction (2.16) we obtain the following expression for  $R^{(e)}$  as defined in (2.3), (2.5) (cf. (3.5) of [1]):

$$\begin{aligned} R^{(e)}(\mathbf{p}_+; \mathbf{k}_+, \bar{q}, \mathbf{k}_-, q, \mathbf{k}, G) &= a(E_+, E_-) + \hat{\mathbf{p}}_+ \\ &\quad \cdot [\hat{\mathbf{k}}_+ b_1(E_+, E_-) + \hat{\mathbf{k}}_- b_2(E_+, E_-) + \hat{\mathbf{n}} b_3(E_+, E_-)] \\ &\quad + (\hat{p}_+ \hat{p}_+ - \frac{1}{3} \delta_{ij}) [(\hat{k}_+ \hat{k}_+ - \frac{1}{3} \delta_{ij}) c_1(E_+, E_-) \\ &\quad + (\hat{k}_- \hat{k}_- - \frac{1}{3} \delta_{ij}) c_2(E_+, E_-) \\ &\quad + (\hat{k}_+ \hat{k}_- + \hat{k}_+ \hat{k}_- - \frac{2}{3} \delta_{ij} \hat{\mathbf{k}}_+ \cdot \hat{\mathbf{k}}_-) c_3(E_+, E_-) \\ &\quad + (\hat{k}_+ \hat{n}_j + \hat{k}_+ \hat{n}_i) c_4(E_+, E_-) \\ &\quad + (\hat{k}_- \hat{n}_j + \hat{k}_- \hat{n}_i) c_5(E_+, E_-)]. \end{aligned} \quad (3.2)$$

Here we use the c.m.s variables (2.18) and  $E_+, E_-$  are the energies of  $\bar{q}, q$ , respectively. The explicit analytical expressions for  $a, \dots, c_5$  are given in the appendix. Alternatively,  $R^{(e)}$  can be expressed as follows:

$$R^{(e)}(\mathbf{p}_+; \mathbf{k}_+, \bar{q}, \mathbf{k}_-, q, \mathbf{k}, G) = L^{\mu\nu} H_{\mu\nu} \quad (3.3)$$

where the lepton tensor  $L^{\mu\nu}$  and the hadron tensor  $H_{\mu\nu}$  are defined as in [7]. When calculating the Feynman diagram contributions it proves convenient to first express the results in a covariant basis in terms of nine invariant structure functions  $H_1, \dots, H_9$  whose relation to  $a, \dots, c_5$  is given in the appendix. The general  $CP$ -invariance conditions for  $a, \dots, c_5$  following from (2.9) are as in Table 1 of [1].

We note that the  $CPT$ -condition (2.11) valid for  $\text{Im } \mathcal{T} = 0$  is not fulfilled by our  $R^{(e)}$  (cf. (A.1)–(A.6) in the appendix) since we use a Breit–Wigner form with finite width  $\Gamma_Z$  for the  $Z$ -propagator. However, these seemingly  $CPT$ -violating finite  $Z$ -width effects occur only in the  $CP$ -odd terms in  $b_1, b_2, c_1, c_2, c_3$ . This is consistent with studies of final state interaction effects in the framework of the SM: As shown in [7, 8] order  $\alpha_s^2$  QCD final state

**Table 1.** Numerical values for  $X_q(s)$  and  $Y_q(s)$  (3.7) ( $q = c, b, t$ ) for some values of the c.m. energy  $\sqrt{s}$  and of the cut parameter  $y_{\text{cut}}$  (3.4). For the quark masses we use  $m_c = 1.5 \text{ GeV}$ ,  $m_b = 4.5 \text{ GeV}$ ,  $m_t = 120 \text{ GeV}$

$\sqrt{s} = 35 \text{ GeV}$			
$y_{\text{cut}}$	0.04	0.08	0.12
$Y_c$	$-7.02 \cdot 10^{-5}$	$-1.05 \cdot 10^{-4}$	$-1.35 \cdot 10^{-4}$
$X_c$	$+3.04 \cdot 10^{-5}$	$+4.56 \cdot 10^{-5}$	$+5.84 \cdot 10^{-5}$
$Y_b$	$-2.51 \cdot 10^{-4}$	$-3.72 \cdot 10^{-4}$	$-4.77 \cdot 10^{-4}$
$X_b$	$-5.42 \cdot 10^{-5}$	$-8.04 \cdot 10^{-5}$	$-1.03 \cdot 10^{-4}$
$\sqrt{s} = 91 \text{ GeV}$			
$y_{\text{cut}}$	0.006	0.01	0.12
$Y_c$	$-2.05 \cdot 10^{-2}$	$-2.52 \cdot 10^{-2}$	$-9.48 \cdot 10^{-2}$
$X_c$	0	0	0
$Y_b$	$-1.60 \cdot 10^{-2}$	$-1.96 \cdot 10^{-2}$	$-7.30 \cdot 10^{-2}$
$X_b$	0	0	0
$\sqrt{s} = 110 \text{ GeV}$			
$y_{\text{cut}}$	0.006	0.01	0.12
$Y_c$	$-2.24 \cdot 10^{-2}$	$-2.75 \cdot 10^{-2}$	$-1.04 \cdot 10^{-1}$
$X_c$	$-5.31 \cdot 10^{-4}$	$-6.52 \cdot 10^{-4}$	$-2.46 \cdot 10^{-3}$
$Y_b$	$-2.16 \cdot 10^{-2}$	$-2.65 \cdot 10^{-2}$	$-9.89 \cdot 10^{-2}$
$X_b$	$+2.56 \cdot 10^{-4}$	$+3.02 \cdot 10^{-4}$	$+1.17 \cdot 10^{-3}$
$\sqrt{s} = 200 \text{ GeV}$			
$y_{\text{cut}}$	0.001	0.01	0.12
$Y_c$	$-1.78 \cdot 10^{-2}$	$-4.04 \cdot 10^{-2}$	$-1.52 \cdot 10^{-1}$
$X_c$	$-1.06 \cdot 10^{-3}$	$-2.41 \cdot 10^{-3}$	$-9.05 \cdot 10^{-3}$
$Y_b$	$-2.94 \cdot 10^{-2}$	$-6.60 \cdot 10^{-2}$	$-2.48 \cdot 10^{-1}$
$X_b$	$+8.76 \cdot 10^{-4}$	$+1.96 \cdot 10^{-3}$	$+7.39 \cdot 10^{-3}$
$\sqrt{s} = 500 \text{ GeV}$			
$y_{\text{cut}}$	0.06	0.12	0.18
$Y_t$	$-6.27 \cdot 10^{-2}$	$-3.20 \cdot 10^{-1}$	$-5.12 \cdot 10^{-1}$
$X_t$	$-4.55 \cdot 10^{-3}$	$-2.32 \cdot 10^{-2}$	$-3.72 \cdot 10^{-2}$

interaction effects do not contribute to  $\text{Im} \mathcal{F}$  in 3 jet events for massless quarks. Also there are no contributions to  $\text{Im} \mathcal{F}$  from finite  $Z$  width effects for 3 jet events to order  $\alpha_s$ . For 4 jet events to order  $\alpha_s^2$  the finite  $Z$  width effects on  $\text{Im} \mathcal{F}$  are next to next to leading order and thus quite small [9].

The calculation of the expectation value of the  $CP$ -odd tensor (2.19) is now straightforward. We use the following ‘‘semirealistic’’ [10] invariant mass cuts in phase space: For all jets,  $a, b$  we require

$$y_{ab} \equiv \frac{m_{ab}^2}{s} = \frac{(k_a + k_b)^2}{s} \geq y_{\text{cut}} \quad (3.4)$$

where  $y_{\text{cut}}$  is a fixed number as specified below. These cuts respect rotational invariance which allows us to write

(2.19) in the form

$$\langle T_{ij} \rangle = K_q(s) (\hat{p}_{+i} \hat{p}_{+j} - \frac{1}{3} \delta_{ij}) \quad (3.5)$$

where

$$K_q(s) = \frac{2}{5N} \int d\Gamma (1 - \hat{\mathbf{k}}_+ \cdot \hat{\mathbf{k}}_-) (c_4(E_+, E_-) - c_5(E_-, E_+)), \quad (3.6)$$

and  $N$  is a normalization factor determined from  $\langle 1 \rangle = 1$ . Inserting  $c_4, c_5$  as given in the appendix we obtain

$$K_q(s) = X_q(s) \hat{h}_{Aq} + Y_q(s) (\hat{h}_{Aq} g_{Vq} - \hat{h}_{Vq} g_{Aq}). \quad (3.7)$$

Here  $g_{Vq}, g_{Aq}$  are the SM  $Z$ -quark coupling constants (cf. [1]). Numerical results for  $X_q(s)$  and  $Y_q(s)$  are given in Table 1, where we used the following values for the SM parameters:

$$\begin{aligned} m_Z &= 91.0 \text{ GeV} \\ \Gamma_Z &= 2.6 \text{ GeV} \\ \sin^2 \theta_W &= 0.23. \end{aligned} \quad (3.8)$$

Some remarks are in order here:

1. Since the vector coupling constant

$$g_{Ve} = -(1 - 4 \sin^2 \theta_W)/2$$

of the  $Z$ -boson enters in the expression for  $c_{4,5}$  (A.6) our results are rather sensitive to the exact value of  $\sin^2 \theta_W$ . A shift of this parameter by  $\delta \sin^2 \theta_W = 0.01$  induces changes in our numbers of up to 10%.

2. A similar remark applies to the dependence on the exact value of the  $Z$ -boson mass  $m_Z$ . However, this mass is now known very precisely.

3. Away from the  $Z$ -pole, i.e. for  $|s - m_Z^2| \gg m_Z \Gamma_Z$ , the  $\gamma$  and  $Z$ -exchange diagrams contribute in the same order in the electroweak coupling constant  $\alpha$ . There, the finite  $Z$ -width  $\Gamma_Z$  is a radiative correction effect since  $\Gamma_Z/m_Z$  is of order  $\alpha$ . Calculating in leading order in  $\alpha$  we should drop  $\Gamma_Z$  in this kinematic region for consistency. However, for  $|s - m_Z^2| \lesssim m_Z \Gamma_Z$ , i.e. near the  $Z$ -pole the situation changes. There, the  $Z$ -exchange diagrams get a resonance enhancement. The  $\gamma$ -exchange diagrams enter as a radiative correction of order  $\alpha$  and should only be considered together with radiative corrections to the  $Z$ -exchange diagrams. Thus, for consistency, we should keep only the  $Z$ -exchange diagrams near the  $Z$ -pole. In this way we recover the results of [1]. Numerically  $K_q(s)$  changes only at the percent level if  $\Gamma_Z$  is set to zero away from the  $Z$ -pole and  $\gamma$ -exchange is dropped in the vicinity of the  $Z$ -pole. With this accuracy the expressions given in the appendix can therefore be used for all values of  $s$ .

4. We also give results for the production of the still hypothetical  $t$ -quarks assuming  $m_t = 120 \text{ GeV}$ . The  $t$ -quark should be particularly interesting to study, once it is found experimentally since nonstandard Higgs particles – if they exist – are a likely source of  $CP$ -violation (cf. [12] for reviews). Generically, the coupling of such particles is proportional to the quark mass.

### 3.2 The exclusive 3-jet reaction without flavour tagging

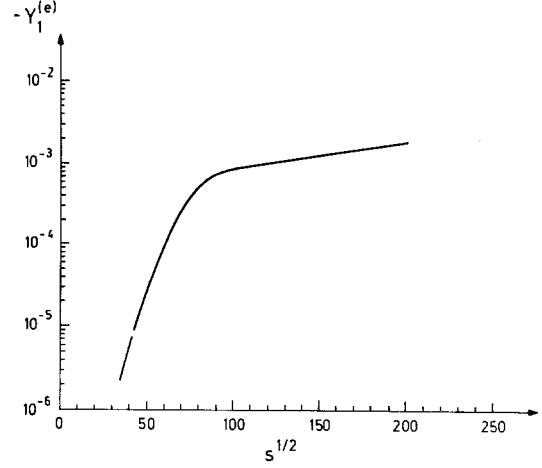
Now we give the results for the exclusive 3-jet calculation

**Table 2.** Numerical values for  $X_a^{(e)}(s)$ ,  $Y_a^{(e)}(s)$  ( $a = 1, 2, 3$ ) as defined in (3.12) for some values of the c.m. energy  $\sqrt{s}$  and of the cut parameter  $y_{\text{cut}}$  (3.4)

$\sqrt{s} = 35 \text{ GeV}$			
$y_{\text{cut}}$	0.04	0.08	0.12
$Y_1^{(e)}$	$-4.13 \cdot 10^{-6}$	$-5.38 \cdot 10^{-6}$	$-5.83 \cdot 10^{-6}$
$X_1^{(e)}$	$-6.70 \cdot 10^{-5}$	$-8.73 \cdot 10^{-5}$	$-9.45 \cdot 10^{-5}$
$Y_2^{(e)}$	$+5.11 \cdot 10^{-6}$	$+6.90 \cdot 10^{-6}$	$+7.68 \cdot 10^{-6}$
$X_2^{(e)}$	$+8.29 \cdot 10^{-5}$	$+1.12 \cdot 10^{-4}$	$+1.24 \cdot 10^{-4}$
$Y_3^{(e)}$	$-7.54 \cdot 10^{-8}$	$-7.97 \cdot 10^{-7}$	$-1.37 \cdot 10^{-6}$
$X_3^{(e)}$	$-1.22 \cdot 10^{-6}$	$-1.29 \cdot 10^{-5}$	$-2.22 \cdot 10^{-5}$
$\sqrt{s} = 91 \text{ GeV}$			
$y_{\text{cut}}$	0.006	0.01	0.12
$Y_1^{(e)}$	$-6.21 \cdot 10^{-4}$	$-7.56 \cdot 10^{-4}$	$-1.91 \cdot 10^{-3}$
$X_1^{(e)}$	0	0	0
$Y_2^{(e)}$	$+7.33 \cdot 10^{-4}$	$+8.99 \cdot 10^{-4}$	$+2.52 \cdot 10^{-3}$
$X_2^{(e)}$	0	0	0
$Y_3^{(e)}$	$+1.23 \cdot 10^{-4}$	$+1.18 \cdot 10^{-4}$	$-4.49 \cdot 10^{-4}$
$X_3^{(e)}$	0	0	0
$\sqrt{s} = 110 \text{ GeV}$			
$y_{\text{cut}}$	0.006	0.01	0.12
$Y_1^{(e)}$	$-7.77 \cdot 10^{-4}$	$-9.46 \cdot 10^{-4}$	$-2.39 \cdot 10^{-3}$
$X_1^{(e)}$	$+6.90 \cdot 10^{-4}$	$+8.40 \cdot 10^{-4}$	$+2.12 \cdot 10^{-3}$
$Y_2^{(e)}$	$+9.16 \cdot 10^{-4}$	$+1.12 \cdot 10^{-3}$	$+3.15 \cdot 10^{-3}$
$X_2^{(e)}$	$-8.14 \cdot 10^{-4}$	$-9.99 \cdot 10^{-4}$	$-2.80 \cdot 10^{-3}$
$Y_3^{(e)}$	$+1.53 \cdot 10^{-4}$	$+1.49 \cdot 10^{-4}$	$-5.61 \cdot 10^{-4}$
$X_3^{(e)}$	$-1.36 \cdot 10^{-4}$	$-1.32 \cdot 10^{-4}$	$+4.99 \cdot 10^{-4}$
$\sqrt{s} = 200 \text{ GeV}$			
$y_{\text{cut}}$	0.001	0.01	0.12
$Y_1^{(e)}$	$-8.40 \cdot 10^{-4}$	$-1.85 \cdot 10^{-3}$	$-4.68 \cdot 10^{-3}$
$X_1^{(e)}$	$+1.87 \cdot 10^{-3}$	$+4.14 \cdot 10^{-3}$	$+1.05 \cdot 10^{-2}$
$Y_2^{(e)}$	$+9.75 \cdot 10^{-4}$	$+2.20 \cdot 10^{-3}$	$+6.17 \cdot 10^{-3}$
$X_2^{(e)}$	$-2.18 \cdot 10^{-3}$	$-4.92 \cdot 10^{-3}$	$-1.38 \cdot 10^{-2}$
$Y_3^{(e)}$	$+2.32 \cdot 10^{-4}$	$+2.91 \cdot 10^{-4}$	$-1.10 \cdot 10^{-3}$
$X_3^{(e)}$	$-5.17 \cdot 10^{-4}$	$-6.51 \cdot 10^{-4}$	$+2.46 \cdot 10^{-3}$
$\sqrt{s} = 500 \text{ GeV}$			
$y_{\text{cut}}$	0.001	0.01	0.12
$Y_1^{(e)}$	$-4.26 \cdot 10^{-3}$	$-9.40 \cdot 10^{-3}$	$-2.38 \cdot 10^{-2}$
$X_1^{(e)}$	$+1.16 \cdot 10^{-2}$	$+2.56 \cdot 10^{-2}$	$+6.47 \cdot 10^{-2}$
$Y_2^{(e)}$	$+4.95 \cdot 10^{-3}$	$+1.12 \cdot 10^{-2}$	$+3.13 \cdot 10^{-2}$
$X_2^{(e)}$	$-1.35 \cdot 10^{-2}$	$-3.04 \cdot 10^{-2}$	$-8.52 \cdot 10^{-2}$
$Y_3^{(e)}$	$+1.18 \cdot 10^{-3}$	$+1.48 \cdot 10^{-3}$	$-5.58 \cdot 10^{-3}$
$X_3^{(e)}$	$-3.20 \cdot 10^{-3}$	$-4.03 \cdot 10^{-3}$	$+1.52 \cdot 10^{-2}$

using as jet ordering criterium  $|\mathbf{k}_1| > |\mathbf{k}_2| > |\mathbf{k}_3|$ :

$$R^{(e)}(\mathbf{p}_+; \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = a(E_1, E_2)$$



**Fig. 2.** The quantity  $-Y_1^{(e)}(s)$  (3.12) as function of the c.m. energy  $\sqrt{s}$  in GeV for a cut value  $y_{\text{cut}} = 0.01$

$$\begin{aligned}
& + \hat{\mathbf{p}}_+ [\hat{\mathbf{k}}_1 b_1(E_1, E_2) + \hat{\mathbf{k}}_2 b_2(E_1, E_2) + \hat{\mathbf{n}} b_3(E_1, E_2)] \\
& + (\hat{p}_{+i} \hat{p}_{+j} - \frac{1}{3} \delta_{ij}) [(\hat{k}_{1i} \hat{k}_{1j} - \frac{1}{3} \delta_{ij}) c_1(E_1, E_2) \\
& + (\hat{k}_{2i} \hat{k}_{2j} - \frac{1}{3} \delta_{ij}) c_2(E_1, E_2) \\
& + (\hat{k}_{1i} \hat{k}_{2j} + \hat{k}_{1j} \hat{k}_{2i} - \frac{2}{3} \delta_{ij} \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2) c_3(E_1, E_2) \\
& + (\hat{k}_{1i} \hat{n}_j + \hat{k}_{1j} \hat{n}_i) c_4(E_1, E_2) \\
& + (\hat{k}_{2i} \hat{n}_j + \hat{k}_{2j} \hat{n}_i) c_5(E_1, E_2)]. \tag{3.9}
\end{aligned}$$

Here  $E_{1,2}$  are the energies of jets 1, 2 and  $\hat{\mathbf{n}}$  is defined in (2.21). The  $CP$ -invariance conditions for  $a, \dots, c_5$  follow easily from (2.9). They are identical to the corresponding conditions for the 3-photon decay of positronium (cf. [11]). The explicit expressions for  $a, \dots, c_5$  are given in the appendix. Here we neglect the quark masses and sum over the quark flavours  $u, d, s, c, b$ . Using cuts in phase space as in (3.4) we obtain for the expectation value of the tensor (2.20):

$$\langle \tilde{T}_{ij}^a \rangle_{\text{excl}} = \tilde{K}_a(s) (\hat{p}_{+i} \hat{p}_{+j} - \frac{1}{3} \delta_{ij}) \tag{3.10}$$

where  $a = 1, 2, 3$ , and

$$\tilde{K}_a(s) = \frac{2}{5N} \int d\Gamma ((\hat{\mathbf{k}}_a \cdot \hat{\mathbf{k}}_1) c_4(E_1, E_2) + (\hat{\mathbf{k}}_a \cdot \hat{\mathbf{k}}_2) c_5(E_1, E_2)) \tag{3.11}$$

and  $N$  is again determined from the normalization condition. Inserting the expressions of the appendix for  $c_4, c_5$  we obtain

$$\tilde{K}_a(s) = X_a^{(e)}(s) g_{V_e} \sum_q \hat{h}_{Aq} Q_q + \bar{Y}_a^{(e)}(s) \sum_q (\hat{h}_{Aq} g_{V_q} - \hat{h}_{Vq} g_{Aq}). \tag{3.12}$$

Numerical results for  $X_a^{(e)}, Y_a^{(e)}$  are shown in Table 2 and Fig. 2 where we use for the SM parameters (3.8).

As one can see from the formulas in the appendix the energy dependence and the dependence on the cut parameter  $y_{\text{cut}}$  do not mix for massless quarks. Using another value for  $y_{\text{cut}}$  means a multiplication of the results by a certain factor which is independent of  $\sqrt{s}$ . Therefore we can easily show a ‘generic’ picture of the energy dependence of our results (Fig. 2).

**Table 3.** Numerical values for  $X_1^{(i)}(s)$ ,  $Y_1^{(i)}(s)$  as defined in (3.15) for some values of the c.m. energy  $\sqrt{s}$  and of the cut parameter  $y_{\text{cut}}$  (3.4)

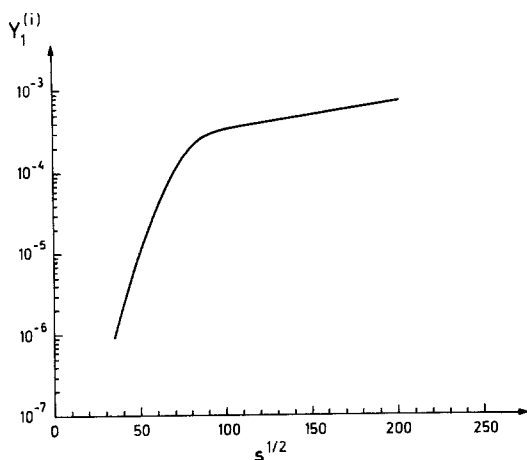
$\sqrt{s} = 35 \text{ GeV}$			
$y_{\text{cut}}$	0.04	0.08	0.12
$Y_1^{(i)}$	$+1.71 \cdot 10^{-6}$	$+2.30 \cdot 10^{-6}$	$+2.56 \cdot 10^{-6}$
$X_1^{(i)}$	$+2.77 \cdot 10^{-5}$	$+3.73 \cdot 10^{-5}$	$+4.15 \cdot 10^{-5}$
$\sqrt{s} = 91 \text{ GeV}$			
$y_{\text{cut}}$	0.006	0.01	0.12
$Y_1^{(i)}$	$+2.44 \cdot 10^{-4}$	$+3.00 \cdot 10^{-4}$	$+8.39 \cdot 10^{-4}$
$X_1^{(i)}$	0	0	0
$\sqrt{s} = 110 \text{ GeV}$			
$y_{\text{cut}}$	0.006	0.01	0.12
$Y_1^{(i)}$	$+3.05 \cdot 10^{-4}$	$+3.75 \cdot 10^{-4}$	$+1.05 \cdot 10^{-3}$
$X_1^{(i)}$	$-2.71 \cdot 10^{-4}$	$-3.33 \cdot 10^{-4}$	$-9.33 \cdot 10^{-4}$
$\sqrt{s} = 200 \text{ GeV}$			
$y_{\text{cut}}$	0.001	0.01	0.12
$Y_1^{(i)}$	$+3.25 \cdot 10^{-4}$	$+7.34 \cdot 10^{-4}$	$+2.06 \cdot 10^{-3}$
$X_1^{(i)}$	$-7.26 \cdot 10^{-4}$	$-1.64 \cdot 10^{-3}$	$-4.59 \cdot 10^{-3}$

### 3.3 The inclusive 2-jet reaction

For the 2-jet inclusive reaction we use the jet ordering  $|\mathbf{k}_1| > |\mathbf{k}_2|$ , neglect again the quark masses, and sum over  $u, d, s, c, b$  quarks. We have then (cf. (2.7)):

$$\begin{aligned}
 R^{(i)}(\mathbf{p}_+; \mathbf{k}_1, \mathbf{k}_2) &= 2\pi\delta(k'^2) \cdot (R^{(e)}(\mathbf{p}_+; \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}') + R^{(e)}(\mathbf{p}_+; \mathbf{k}_1, \mathbf{k}', \mathbf{k}_2) \\
 &\quad + R^{(e)}(\mathbf{p}_+; \mathbf{k}', \mathbf{k}_1, \mathbf{k}_2))|_{\mathbf{k}' = -\mathbf{k}_1 - \mathbf{k}_2} \quad (3.13)
 \end{aligned}$$

where  $R^{(e)}$  is given in (3.9). Note that  $R^{(e)} \equiv 0$  if its



**Fig. 3.** The quantity  $Y_1^{(i)}(s)$  (3.15) as function of the c.m. energy  $\sqrt{s}$  in GeV for a cut value  $y_{\text{cut}} = 0.01$

arguments do not satisfy the ordering criterium of Sect. 3.2. The result for the tensor correlation (2.20) using the cuts (3.4) can be expressed as

$$\langle \tilde{T}_{ij}^a \rangle_{\text{incl}} = \tilde{K}_a^{(i)}(s) (\hat{p}_+ \hat{p}_{+j} - \frac{1}{3} \delta_{ij}) \quad (3.14)$$

where  $a = 1, 2$ , and

$$\tilde{K}_a^{(i)}(s) = X_a^{(i)}(s) g_{Ve} \sum_q \hat{h}_{Aq} Q_q + Y_a^{(i)}(s) \sum_q (\hat{h}_{Aq} g_{Vq} - \hat{h}_{Vq} g_{Aq}). \quad (3.15)$$

In our leading order calculation only 3 jet events contribute. This leads – as is easily seen – to  $\tilde{K}_1^{(i)}(s) = \tilde{K}_2^{(i)}(s)$ . Numerical results for  $X_1^{(i)}$ ,  $Y_1^{(i)}$  are presented in Table 3 and Fig. 3.

## 4 Conclusion

In this paper we discussed possible  $CP$ -violating effects in  $e^+e^- \rightarrow$  hadrons for the exclusive 3-jet and the inclusive 2-jet final states. We defined  $CP$ -odd tensor correlations which should not be difficult to measure experimentally. We parametrized possible new  $CP$ -violating interactions in terms of an effective Lagrangian. Useful limits for the  $CP$ -violating coupling constants occurring therein can be obtained from measurements of the  $CP$ -odd tensor correlations. The bounds obtainable at the  $Z$ -pole have already been discussed in [1]. Higher energies are in principle more favourable since we are looking for nonrenormalizable interactions. However, statistics in the continuum will be lower. As an illustration we estimate that with  $10^5$  3jet events at  $\sqrt{s} = 200 \text{ GeV}$  (500 GeV) limits on the dimensionless coupling parameters  $\hat{h}_{Vq}$ ,  $\hat{h}_{Aq}$  of order 0,3 (0,06) could be obtained for  $q = u, d, s, c, b$ . The top quark couplings can, of course, only be investigated at energies above the  $Z$ -pole given the present experimental indications that  $m_t > m_Z/2$ . Finally we stress that our approach to parametrize possible new effects at high energies through some effective coupling constants is a very conservative one. Surprising new phenomena are not excluded in a new energy domain and  $CP$ -odd observables as discussed here are a useful tool to look for them.

We now discuss how our results relate to measurements of other  $CP$ -odd effects. Without concrete model assumptions no bounds on our flavour *diagonal* couplings (3.1) can be obtained from the observed flavour *changing*  $CP$  violation effects in the neutral  $K$  meson system. Within the effective Lagrangian approach new  $CP$ -odd couplings govern the flavour changing (FC) interactions and can compensate any contributions induced from the flavour diagonal terms (3.1) to  $CP$ -odd FC amplitudes. Note that in general loop diagrams involving the couplings (3.1) will also contribute to the renormalization of the FC couplings. We should stress that for reasons explained in [1] we do *not* assume invariance of  $\mathcal{L}_{CP}$  (3.1) under the weak isospin group  $SU(2)$ . Thus, in our approach “new physics”  $Z$  and  $W$  couplings are unrelated. We note, however, that we can always restore  $SU(2)$  invariance in (3.1) by invoking suitable new Higgs fields with corresponding vacuum expectation values. In this way the operators in (3.1) could arise from  $SU(2)$ -invariant

ones – with in general even higher dimensions – involving new higgses.

The existing bounds on electric dipole moments (e.d.ms) of particles [13] and on  $T$ -odd nuclear forces [14] are more relevant for our work. Assuming  $CPT$  invariance they give directly bounds on  $CP$  violation effects. As discussed in Sect. 3 the e.d.m. and the weak dipole moment (w.d.m.) of the electron ( $d_e, \tilde{d}_e$ ) do not contribute to our correlations if we neglect  $m_e$ . Also the quark electric and weak dipole moments do not contribute to the tensor correlations (2.17), (2.20) in our calculation as we see from (3.6), (3.11), (A.6), (A.12). On the other hand the couplings  $h_{Vq}, h_{Aq}$  in (3.1) will in general contribute to the e.d.ms of hadrons, especially the neutron. However, if we try to derive bounds on  $h_{Vq}, h_{Aq}$  from dipole moments we face the same problem as explained for FC interactions above. Without a concrete model any contribution from the  $h$ -couplings in (3.1) to an e.d.m. can be compensated by adjusting the explicit quark e.d.m. coupling constants in (3.1). A similar situation arises if we try to bound the  $h$ -couplings using the analysis of  $T$ -odd nuclear forces [14].

Let us finally make some remarks on bounds obtainable for e.d.ms  $d_\psi$  and w.d.ms  $\tilde{d}_\psi$  of leptons and quarks ( $\psi = l, q$ ) from  $e^+e^- \rightarrow$  hadrons. Here we are assuming that the  $CP$ -odd form factors in the  $\psi\psi\gamma$  and  $\psi\psi Z$  vertices are well approximated by their values at  $s = 0$ , i.e. by the d.ms in the complete  $s$ -range considered. The contribution of a w.d.m. to the width  $\Gamma(Z \rightarrow \bar{\psi}\psi)$  has been given in (5.8) of [1]. Blaming any deviation of the experimental values from the SM theoretical results on w.d.m. contributions we obtain upper limits for  $|\tilde{d}_\psi|$ . As an illustration we took recent data from two LEP experiments [15–17] and obtained limits as shown in Table 4. Of course, these are not direct determinations of the  $\tilde{d}_\psi$  since no measurement of a  $CP$ -odd effect is involved and compensations with other new physics effects reducing the partial widths cannot be excluded. Such a possibility is quite realistic [18]. But the numbers in Table 4 should give an idea at which level direct searches for w.d.ms become especially interesting.

We consider now the total cross section  $\sigma(e^+e^- \rightarrow \bar{\psi}\psi)$  away from the  $Z$ -resonance. The contribution of  $d_\psi$  and  $\tilde{d}_\psi$  to  $\sigma$  is

$$\Delta\sigma(e^+e^- \rightarrow \bar{\psi}\psi) = \frac{1}{6} \alpha N_c^\psi \left(1 - \frac{4m_\psi^2}{s}\right)^{3/2} \cdot (|D_{V\psi}(s)|^2 + |D_{A\psi}(s)|^2), \quad (4.1)$$

where

$$D_{V\psi}(s) = d_\psi - \frac{g_{Ve}\tilde{d}_\psi s}{2 \sin\theta_w \cos\theta_w (s - m_Z^2 + im_Z\Gamma_Z)},$$

$$D_{A\psi}(s) = -\frac{g_{Ae}\tilde{d}_\psi s}{2 \sin\theta_w \cos\theta_w (s - m_Z^2 + im_Z\Gamma_Z)} \quad (4.2)$$

and  $N_c^\psi = 1(3)$  for  $\psi =$  lepton(quark). Equating (4.1) to the possible deviation of the experimental and SM values for  $\sigma$  bounds a combination of  $d_\psi$  and  $\tilde{d}_\psi$ . As an illustration we took the data on  $\mu^+\mu^-$  and  $\tau^+\tau^-$  production from [19] and obtained the following results for 1 s.d. bounds: At  $\sqrt{s} = 35$  GeV:

$$\Delta\sigma(e^+e^- \rightarrow \mu^+\mu^-) \leq 1.0 \text{ pb},$$

$$\Delta\sigma(e^+e^- \rightarrow \tau^+\tau^-) \leq 3.0 \text{ pb},$$

$$(|d_\mu - 0.008\tilde{d}_\mu|^2 + |0.10\tilde{d}_\mu|^2)^{1/2} \leq 10^{-16} \text{ e cm}$$

$$(|d_\tau - 0.008\tilde{d}_\tau|^2 + |0.10\tilde{d}_\tau|^2)^{1/2} \leq 1.7 \cdot 10^{-16} \text{ e cm}. \quad (4.3)$$

For hadron production we used the data from [20] where  $\sigma(e^+e^- \rightarrow$  hadrons) is well fitted by the SM expression, but with a rather large value for  $\alpha_s$ . The “true” SM value can be estimated to be  $\approx 1$  s.d. lower. Thus we take here twice the statistical and systematic errors quoted in [20]

**Table 5.** Numerical values for the quantities  $w_{iq}(s)$  where  $i = 1, \dots, 4$  and  $q = c, b, t$ . The quark masses are as in Table 1. The  $y$ -cut is  $y_{\text{cut}} = 0.01 + m_q^2/s$

	$w_{1q}$	$w_{2q}$	$w_{3q}$	$w_{4q}$
$\sqrt{s} = 35 \text{ GeV}$				
$c$	$-0.31 \cdot 10^{-4}$	$+0.51 \cdot 10^{-6}$	$-0.10 \cdot 10^{-3}$	$-0.15 \cdot 10^{-5}$
$b$	$+0.30 \cdot 10^{-3}$	$-0.50 \cdot 10^{-5}$	$+0.21 \cdot 10^{-3}$	$+0.50 \cdot 10^{-5}$
$\sqrt{s} = 91 \text{ GeV}$				
$c$	0	$+0.54 \cdot 10^{-4}$	$+0.41 \cdot 10^{-3}$	$-0.11 \cdot 10^{-2}$
$b$	0	$-0.12 \cdot 10^{-3}$	$+0.58 \cdot 10^{-3}$	$+0.81 \cdot 10^{-3}$
$\sqrt{s} = 110 \text{ GeV}$				
$c$	$+0.14 \cdot 10^{-3}$	$+0.41 \cdot 10^{-4}$	$+0.48 \cdot 10^{-2}$	$-0.12 \cdot 10^{-2}$
$b$	$-0.38 \cdot 10^{-3}$	$-0.12 \cdot 10^{-3}$	$-0.29 \cdot 10^{-2}$	$+0.11 \cdot 10^{-2}$
$\sqrt{s} = 200 \text{ GeV}$				
$c$	$+0.15 \cdot 10^{-3}$	$+0.18 \cdot 10^{-4}$	$+0.17 \cdot 10^{-1}$	$-0.17 \cdot 10^{-2}$
$b$	$-0.74 \cdot 10^{-3}$	$-0.88 \cdot 10^{-4}$	$-0.15 \cdot 10^{-1}$	$+0.28 \cdot 10^{-2}$
$\sqrt{s} = 500 \text{ GeV}$				
$t$	$+0.64 \cdot 10^{-2}$	$+0.63 \cdot 10^{-3}$	$+0.98 \cdot 10^{-1}$	$-0.42 \cdot 10^{-2}$

**Table 4.** Bounds (1 s.d.) for weak dipole moments. The second column gives  $\Delta\Gamma = \Gamma_{\text{exp}} - \Gamma_{\text{SM}}$  where we took the experimental mean value plus the 1 s.d. error and subtracted the minimal value of  $\Gamma_{\text{SM}}$ , as quoted in the references

Channel	$\Delta\Gamma(\text{MeV})$	Ref.	Bound (1 s.d.)
$Z \rightarrow e^+e^-$	4.8	[15]	$ \tilde{d}_e  \leq 4.5 \cdot 10^{-17} \text{ e cm}$
$Z \rightarrow \mu^+\mu^-$	5.6	[16]	$ \tilde{d}_\mu  \leq 4.9 \cdot 10^{-17} \text{ e cm}$
$Z \rightarrow \tau^+\tau^-$	10.0	[16]	$ \tilde{d}_\tau  \leq 6.5 \cdot 10^{-17} \text{ e cm}$
$Z \rightarrow$ hadrons	161	[16]	$\left(\sum_{q=u,d,s,c,b}  \tilde{d}_q ^2\right)^{1/2} \leq 1.5 \cdot 10^{-16} \text{ e cm}$
$Z \rightarrow \bar{c}c$	73	[17]	$ \tilde{d}_c  \leq 1.0 \cdot 10^{-16} \text{ e cm}$
$Z \rightarrow \bar{b}b$	58	[17]	$ d_b  \leq 9.1 \cdot 10^{-17} \text{ e cm}$

as an estimate for  $\Delta\sigma$  and get the following results:

$$\text{At } \sqrt{s} = 43.3 \text{ GeV:}$$

$$\Delta\sigma(e^+e^- \rightarrow \text{hadrons}) \leq 14.8 \text{ pb,}$$

$$\left[ \sum_{q=u,d,s,c,b} (|d_q - 0.014\tilde{d}_q|^2 + |0.173\tilde{d}_q|^2) \right]^{1/2} \leq 2.1 \cdot 10^{-16} \text{ e cm.} \quad (4.4)$$

In [21] a bound  $|d_c| \leq 10^{-16} \text{ e cm}$  was derived using the same method with a slightly smaller error estimate  $\Delta\sigma$ . But they did not consider the possible presence of a w.d.m.  $\tilde{d}_c$ . With our results of Table 4 we see now that indeed the contributions of w.d.ms can be neglected in (4.3) and (4.4) and the bounds given there can be interpreted as bounds on the e.d.ms. The bound on the e.d.m. of the muon from (4.3) is not competitive with  $|d_\mu| \leq 10^{-18} \text{ e cm}$  obtained from the muon ( $g-2$ ) experiments, cf. [13, 21].

$CP$ -odd observables for high energy reactions to look directly for e.d.ms have been discussed in [22] and for e.d.ms and w.d.ms in [1, 23]. Another  $CP$ -odd observable of this sort can be constructed in analogy to (3.18) of [1] for exclusive 3-jet events with flavour tagging (Sect. 3.1). The expectation value of  $\hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_- = |\hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_-| \hat{\mathbf{n}}$  (cf. (2.18)) must vanish if  $CP$  is conserved. With our  $CP$ -odd couplings (3.1) we find for quarks  $q = c, b, t$

$$\langle \hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_- \rangle = I_q(s) \frac{\mathbf{p}_+}{|\mathbf{p}_+|}, \quad (4.5)$$

where

$$\begin{aligned} I_q(s) &= \int d\Gamma^{(e)} \sqrt{1 - (\hat{\mathbf{k}}_+ \cdot \hat{\mathbf{k}}_-)^2} (b_3(E_+, E_-) + b_3(E_-, E_+)) \\ &\quad \cdot [6 \int d\Gamma^{(e)} a(E_+, E_-)]^{-1} \\ &= \left( \frac{d_q m_Z}{e} \right) w_{1q}(s) + \left( \frac{\tilde{d}_q m_Z}{e} \right) w_{2q}(s) \\ &\quad + \hat{h}_{Vq} w_{3q}(s) + \hat{h}_{Aq} w_{4q}(s). \end{aligned} \quad (4.6)$$

Values for  $w_{iq}(s)$  ( $i = 1, \dots, 4$ ) are given in Table 5. For  $s = m_Z^2$  this complements the results of [1] using now invariant mass cuts. For  $q = c, b$  the correlation (4.5) is not very sensitive to the quarks' e.d.m. and w.d.m. due to a helicity suppression factor  $m_q/\sqrt{s}$  multiplying their contribution in  $I_q(s)$ . However, for the top quark, once it can be produced in  $e^+e^-$  collisions, the correlation (4.5) should be of interest.

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## Appendix

Here we give the explicit expressions for the structure functions  $a, \dots, c_5$  defined in (3.2) and calculated as explained in Sect. 3:

$$\begin{aligned} a(E_+, E_-) &= \frac{128}{3} \frac{e^4 g_s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \left\{ \left( \frac{Q_q^2 (s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}{s^2} - \frac{Q_q}{2 \sin^2 \theta_W \cos^2 \theta_W} (g_{Vq} g_{Ve}) \left( 1 - \frac{m_Z^2}{s} \right) \right) \right. \\ &\quad \cdot \left( \frac{(E_+^2 + E_-^2)s}{E_3^2 - (E_+ - E_-)^2} - m_q^2 \left( \frac{2E_3 \sqrt{s}}{E_3^2 - (E_+ - E_-)^2} + \frac{(E_3^2 + (E_+ - E_-)^2)s}{(E_3^2 - (E_+ - E_-)^2)^2} \right) \right) \\ &\quad - m_q^4 \frac{4E_3^2}{(E_3^2 - (E_+ - E_-)^2)^2} + \frac{(g_{Ve})^2 + (g_{Ae})^2}{16 \sin^4 \theta_W \cos^4 \theta_W} \left( (g_{Vq}^2 + g_{Aq}^2) \frac{(E_+^2 + E_-^2)s}{E_3^2 - (E_+ - E_-)^2} \right. \\ &\quad - m_q^2 \left( g_{Vq}^2 \frac{3E_3^4 + 4E_3^3 E_+ + 4E_3^3 E_- + 4E_3^2 E_+ E_- + (E_+^2 - E_-^2)^2}{(E_3^2 - (E_+ - E_-)^2)^2} \right. \\ &\quad \left. \left. + g_{Aq}^2 \frac{-2E_3^4 + 4E_3^3 E_+ + 4E_3^3 E_- + 8E_3^2 (E_+^2 + E_-^2) - 2(E_+^2 - E_-^2)^2}{(E_3^2 - (E_+ - E_-)^2)^2} \right) \right. \\ &\quad \left. - m_q^4 ((g_{Vq})^2 - 2(g_{Aq})^2) \frac{4E_3^2}{(E_3^2 - (E_+ - E_-)^2)^2} \right\}, \end{aligned} \quad (A.1)$$

$$\begin{aligned} b_{1,2}(E_+, E_-) &= \frac{\sqrt{s}}{2} \frac{|\mathbf{k}_{+,-}|}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \left\{ \left( \frac{e^4 g_s^2}{8 \sin^4 \theta_W \cos^4 \theta_W} (g_{Vq} g_{Aq} g_{Ve} g_{Ae}) - \frac{e^4 g_s^2 Q_q}{4 \sin^2 \theta_W \cos^2 \theta_W} (g_{Aq} g_{Ae}) \left( 1 - \frac{m_Z^2}{s} \right) \right) \right. \\ &\quad \cdot \left( \pm \frac{256 E_\pm \sqrt{s}}{E_3^2 - (E_+ - E_-)^2} \mp \frac{512 m_q^2 E_3}{(E_3^2 - (E_+ - E_-)^2) (E_3 \mp (E_+ - E_-))} \right) \\ &\quad + \left( \frac{e^3 g_s^2 d_q}{4 \sin^2 \theta_W \cos^2 \theta_W} \frac{m_Z \Gamma_Z}{s} (g_{Vq} g_{Ae}) \right) \left( + \frac{256 m_q E_3 \sqrt{s}}{E_3^2 - (E_+ - E_-)^2} \right) \\ &\quad + \left( \frac{e^3 g_s^2 Q_q \tilde{d}_q}{2 \sin \theta_W \cos \theta_W} \frac{m_Z \Gamma_Z}{s} (g_{Ae}) \right) \left( - \frac{256 m_q E_3 \sqrt{s}}{E_3^2 - (E_+ - E_-)^2} \right) \\ &\quad \left. + \left( \frac{e^3 g_s Q_q}{2 \sin \theta_W \cos \theta_W} \frac{m_Z \Gamma_Z}{s} (h_{Aq} g_{Ae}) \right) \left( - \frac{32 M_{1,2} \sqrt{s}}{E_3^2 - (E_+ - E_-)^2} + \frac{64 m_q^2 E_3 \sqrt{s}}{E_3^2 - (E_+ - E_-)^2} \right) \right\}, \end{aligned} \quad (A.2)$$



$$\begin{aligned}
b_3(E_+, E_-) &= \frac{\sqrt{s}}{2} \frac{|\mathbf{k}_+ \times \mathbf{k}_-|}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \\
&\cdot \left\{ \left( \frac{e^3 g_s^2 d_q}{4 \sin^2 \theta_W \cos^2 \theta_W} \left( 1 - \frac{m_Z^2}{s} \right) (g_{Aq} g_{Ae}) \right) \right. \\
&\cdot \left( -\frac{256 m_q E_3}{E_3^2 - (E_+ - E_-)^2} \right) \\
&+ \left( \frac{e^3 g_s^2 \tilde{d}_q}{8 \sin^3 \theta_W \cos^3 \theta_W} (g_{Aq} g_{Ae} g_{Ve}) \right) \\
&\cdot \left( +\frac{512 m_q E_3}{E_3^2 - (E_+ - E_-)^2} \right) \\
&+ \left( \frac{e^3 g_s Q_q}{2 \sin \theta_W \cos \theta_W} \left( 1 - \frac{m_Z^2}{s} \right) (h_{Vq} g_{Ae}) \right) \\
&\cdot \left( -\frac{256 m_q^2 E_3}{E_3^2 - (E_+ - E_-)^2} \right. \\
&\left. - \frac{64(E_3(E_+ + E_-) - (E_+ - E_-)^2) \sqrt{s}}{E_3^2 - (E_+ - E_-)^2} \right) \\
&+ \left( \frac{e^3 g_s}{8 \sin^3 \theta_W \cos^3 \theta_W} (h_{Aq} g_{Aq} - h_{Vq} g_{Vq}) (g_{Ae} g_{Ve}) \right) \\
&\cdot \left( -\frac{128(E_3(E_+ + E_-) - (E_+ - E_-)^2) \sqrt{s}}{E_3^2 - (E_+ - E_-)^2} \right) \\
&+ \left( \frac{e^3 g_s}{8 \sin^3 \theta_W \cos^3 \theta_W} (h_{Aq} g_{Aq} + h_{Vq} g_{Vq}) (g_{Ae} g_{Ve}) \right) \\
&\cdot \left. \left( \frac{512 m_q^2 E_3}{E_3^2 - (E_+ - E_-)^2} \right) \right\}, \tag{A.3}
\end{aligned}$$

$$\begin{aligned}
c_{1,2}(E_+, E_-) &= \frac{s}{4} \frac{|\mathbf{k}_{+,-}|^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \\
&\cdot \left\{ \left( e^4 g_s^2 Q_q^2 \frac{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}{s^2} - \frac{e^4 g_s^2 Q_q}{2 \sin^2 \theta_W \cos^2 \theta_W} \right) \right. \\
&\cdot (g_{Vq} g_{Ve}) \left( 1 - \frac{m_Z^2}{s} \right) + \frac{e^4 g_s^2}{16 \sin^4 \theta_W \cos^4 \theta_W} \\
&\cdot \left( (g_{Vq})^2 + (g_{Aq})^2 \right) \left( (g_{Ve})^2 + (g_{Ae})^2 \right) \\
&\cdot \left( \frac{128}{E_3^2 - (E_+ - E_-)^2} - \frac{256 m_q^2}{(E_3 \mp (E_+ - E_-))^2 s} \right) \\
&+ \left( \frac{e^3 g_s^2 d_q}{4 \sin^2 \theta_W \cos^2 \theta_W} \frac{m_Z \Gamma_Z}{s} (g_{Aq} g_{Ve}) \right) \\
&\cdot \left( \mp \frac{256 m_q}{(E_3 \mp (E_+ - E_-)) \sqrt{s}} \right) \\
&+ \left( \frac{e^3 g_s Q_q}{2 \sin \theta_W \cos \theta_W} \frac{m_Z \Gamma_Z}{s} (h_{Vq} g_{Ve}) \right) \\
&\cdot \left. \left( \pm \frac{128(E_3 + E_\pm)}{(E_3 \pm (E_+ - E_-))} \mp \frac{256 m_q^2}{(E_3 \mp (E_+ - E_-)) \sqrt{s}} \right) \right\}, \tag{A.4}
\end{aligned}$$

$$\begin{aligned}
c_3(E_+, E_-) &= \frac{s}{4} \frac{|\mathbf{k}_+| |\mathbf{k}_-|}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \\
&\cdot \left\{ \left( e^4 g_s^2 Q_q^2 \frac{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}{s^2} - \frac{e^4 g_s^2 Q_q}{2 \sin^2 \theta_W \cos^2 \theta_W} \right) \right. \\
&\cdot (g_{Vq} g_{Ve}) \left( 1 - \frac{m_Z^2}{s} \right) + \frac{e^4 g_s^2}{16 \sin^4 \theta_W \cos^4 \theta_W} \\
&\cdot \left( (g_{Vq})^2 + (g_{Aq})^2 \right) \left( (g_{Ve})^2 + (g_{Ae})^2 \right) \\
&\cdot \left( \frac{256 m_q^2}{(E_3^2 - (E_+ - E_-)^2) s} \right) \\
&+ \left( \frac{e^3 g_s^2 d_q}{4 \sin^2 \theta_W \cos^2 \theta_W} \frac{m_Z \Gamma_Z}{s} (g_{Aq} g_{Ve}) \right) \\
&\cdot \left( -\frac{256 m_q (E_+ - E_-)}{(E_3^2 - (E_+ - E_-)^2) \sqrt{s}} \right) \\
&+ \left( \frac{e^3 g_s Q_q}{2 \sin \theta_W \cos \theta_W} \frac{m_Z \Gamma_Z}{s} (h_{Vq} g_{Ve}) \right) \\
&\cdot \left( \frac{64(E_+ - E_-)(E_3 - E_+ - E_-)}{E_3^2 - (E_+ - E_-)^2} \right. \\
&\left. - \frac{256 m_q^2 (E_+ - E_-)}{(E_3^2 - (E_+ - E_-)^2) \sqrt{s}} \right) \left. \right\}, \tag{A.5}
\end{aligned}$$

$$\begin{aligned}
c_{4,5}(E_+, E_-) &= \frac{s |\mathbf{k}_{+,-}| |\mathbf{k}_+ \times \mathbf{k}_-|}{4} \frac{e^3 g_s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \\
&\cdot \left( \frac{Q_q}{2 \sin \theta_W \cos \theta_W} \left( 1 - \frac{m_Z^2}{s} \right) (h_{Aq} g_{Ve}) \right) \\
&- \frac{1}{8 \sin^3 \theta_W \cos^3 \theta_W} (h_{Aq} g_{Vq} - h_{Vq} g_{Aq}) \left( (g_{Ae})^2 + (g_{Ve})^2 \right) \\
&\cdot \left( \pm \frac{64}{E_3 \pm (E_+ - E_-)} \right). \tag{A.6}
\end{aligned}$$

Here

$$\begin{aligned}
M_1 &= E_3^3 + E_3^2 E_+ + 3E_3^2 E_- + E_3 E_+^2 + 4E_3 E_+ E_- \\
&\quad - E_3 E_-^2 - 3E_3^+ + 3E_+^2 E_- + 3E_+ E_-^2 - 3E_-^3, \\
M_2 &= E_3^3 + E_3^2 E_- + 3E_3^2 E_+ + E_3 E_-^2 + 4E_3 E_- E_+ \\
&\quad - E_3 E_+^2 - 3E_-^3 + 3E_-^2 E_+ + 3E_- E_+^2 - 3E_+^3 \\
E_3 &= \sqrt{s - E_+ - E_-}, \tag{A.7}
\end{aligned}$$

$Q_q$  is the charge of  $q$  in units of the positron charge  $e$  ( $e > 0$ ) and  $g_{V\psi}, g_{A\psi}$  are the usual  $Z - \psi$  ( $\psi = l, q$ ) coupling constants (cf. [1]). The relation of  $a, \dots, c_5$  to the structure functions  $H_1, \dots, H_9$  defined in [7] is as follows:

$$\begin{aligned}
a &= -H_1 + \frac{1}{3} |\mathbf{k}_+|^2 H_2 + \frac{1}{3} |\mathbf{k}_-|^2 H_3 + \frac{2}{3} \mathbf{k}_+ \cdot \mathbf{k}_- H_4 \\
b_1 &= \sqrt{s} |\mathbf{k}_+| H_6 \\
b_2 &= \sqrt{s} |\mathbf{k}_-| H_7 \\
b_3 &= -|\mathbf{k}_+ \times \mathbf{k}_-| H_5 \\
c_1 &= -|\mathbf{k}_+|^2 H_2 \\
c_2 &= -|\mathbf{k}_-|^2 H_3
\end{aligned}$$

$$c_3 = -|\mathbf{k}_+||\mathbf{k}_-|H_4$$

$$c_4 = -\sqrt{s}|\mathbf{k}_+||\mathbf{k}_+ \times \mathbf{k}_-|H_8$$

$$c_5 = -\sqrt{s}|\mathbf{k}_-||\mathbf{k}_+ \times \mathbf{k}_-|H_9.$$

For the 3-jet exclusive reaction without flavour tagging we find for the structure functions of  $R^{(e)}$  defined in (3.9) for vanishing quark masses:

$$\begin{aligned} a(E_1, E_2) = & \frac{128}{3} \frac{e^4 g_s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \sum_q \left( Q_q^2 \frac{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}{s^2} - \frac{Q_q}{2 \sin^2 \theta_W \cos^2 \theta_W} (g_{Vq} g_{Ve}) \left( 1 - \frac{m_Z^2}{s} \right) \right. \\ & \left. + \frac{1}{16 \sin^4 \theta_W \cos^4 \theta_W} ((g_{Vq})^2 + (g_{Aq})^2)((g_{Ve})^2 + (g_{Ae})^2) \right) \\ & \cdot 2s \left( \frac{(E_1^2 + E_2^2)}{E_3^2 - (E_1 - E_2)^2} + \frac{(E_1^2 + E_3^2)}{E_2^2 - (E_1 - E_3)^2} + \frac{(E_2^2 + E_3^2)}{E_1^2 - (E_2 - E_3)^2} \right), \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} b_1(E_1, E_2) = & -\frac{16 E_1 m_Z \Gamma_Z}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \frac{e^3 g_s g_{Ae}}{\sin \theta_W \cos \theta_W} \sum_q Q_q h_{Aq} \\ & \cdot \left\{ \frac{M_1(E_1, E_2)}{E_3^2 - (E_1 - E_2)^2} + \frac{M_1(E_1, E_3) - M_1(E_3, E_1)}{E_2^2 - (E_1 - E_3)^2} - \frac{M_1(E_3, E_2)}{E_1^2 - (E_3 - E_2)^2} \right\}, \\ b_2(E_1, E_2) = & b_1(E_2, E_1), \\ b_3(E_1, E_2) = & 0. \end{aligned} \quad (\text{A.9})$$

Here  $M_1(E_1, E_2)$  is given by  $M_1$  (A.7) with  $E_{+,-}$  replaced by  $E_{1,2}$ ,  $M_1(E_1, E_3)$  etc. are obtained by corresponding permutations of  $E_1, E_2, E_3$ .

$$\begin{aligned} c_1(E_1, E_2) = & \frac{s}{4} \frac{E_1^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \sum_q \left\{ \left( e^4 g_s^2 Q_q^2 \frac{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}{s^2} - \frac{e^4 g_s^2 Q_q}{2 \sin^2 \theta_W \cos^2 \theta_W} (g_{Vq} g_{Ve}) \left( 1 - \frac{m_Z^2}{s} \right) \right. \right. \\ & \left. \left. + \frac{e^4 g_s^2}{16 \sin^4 \theta_W \cos^4 \theta_W} ((g_{Vq})^2 + (g_{Aq})^2)((g_{Ve})^2 + (g_{Ae})^2) \right) \right. \\ & \left. \cdot 256 \left( \frac{1}{E_3^2 - (E_1 - E_2)^2} + \frac{2}{E_2^2 - (E_3 - E_1)^2} + \frac{1}{E_1^2 - (E_2 - E_3)^2} \right) \right\}, \\ c_2(E_1, E_2) = & c_1(E_2, E_1), \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} c_3(E_1, E_2) = & \frac{s}{4} \frac{E_1 E_2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} 256 \sum_q \left\{ e^4 g_s^2 Q_q^2 \frac{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}{s^2} - \frac{e^4 g_s^2 Q_q}{2 \sin^2 \theta_W \cos^2 \theta_W} g_{Vq} g_{Ve} \left( 1 - \frac{m_Z^2}{s} \right) \right. \\ & \left. + \frac{e^4 g_s^2}{16 \sin^4 \theta_W \cos^4 \theta_W} ((g_{Vq})^2 + (g_{Aq})^2)((g_{Ve})^2 + (g_{Ae})^2) \right\} \left( \frac{1}{E_1^2 - (E_2 - E_3)^2} + \frac{1}{E_2^2 - (E_1 - E_3)^2} \right), \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned} c_{4,5}(E_1, E_2) = & \frac{s E_{1,2} |\mathbf{k}_1 \times \mathbf{k}_2|}{4} \frac{e^3 g_s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \sum_q \left( \frac{Q_q}{2 \sin \theta_W \cos \theta_W} \left( 1 - \frac{m_Z^2}{s} \right) (h_{Aq} g_{Ve}) \right. \\ & \left. - \frac{1}{8 \sin^3 \theta_W \cos^3 \theta_W} (h_{Aq} g_{Vq} - h_{Vq} g_{Aq}) ((g_{Ae})^2 + (g_{Ve})^2) \right) \\ & \cdot 128 \left( \frac{2}{E_1 - E_2 \pm E_3} \mp \frac{1}{E_1 + E_2 - E_3} + \frac{1}{E_1 - E_2 \mp E_3} \right). \end{aligned} \quad (\text{A.12})$$

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