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Measuring hadronic currents and weak coupling constants in $\tau \to \nu 3\pi$

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Abstract. A theoretical description of the decay $\tau \to \nu 3\pi$ in a covariant tensor language employing the isobar model is presented. Special emphasis is devoted to the dominating decay mode into νa_1 with $a_1 \to \rho\pi$ in *S*- and *D*-wave orbital momentum eigenstates. These formulae are useful for quantitative tests of the standard model prediction for the parity violation effect recently observed by the ARGUS experiment. We emphasize the difference between these orbital angular momentum amplitudes and Born term amplitudes erroneously identified as *S*- and *D*-wave in the literature. Implications of a possible PCAC suppressed $\pi'(1300)$ as well as exotic contributions are discussed. Analysis methods for the experimental determination of these effects, the D/S ratio of the a_1 and the weak τ decay constants are presented. For the latter a new moment is introduced and the model dependence is discussed.

The study of τ lepton decays opens a wide variety of tests of the leptonic and hadronic weak charged currents. Most of the τ properties can be calculated in the standard model, and many have been tested experimentally (see e.g. the review [1]). The gross features are measured to be well in accord with standard predictions, however there are some inconsistencies between different experiments and standard model calculations ("missing one prong problem") [2]. A recent CELLO analysis [3] suggests that this discrepancy is smaller than previously thought. The argument in favour of a "problem" relies strongly on the naive isospin expectation $B(\tau \to \nu\pi^-\pi^-\pi^+) = B(\tau \to \nu\pi^-\pi^0\pi^0)$ to relate the relative badly measured latter channel to the former.

Recently the ARGUS Collaboration has presented preliminary results providing evidence for direct parity violation in τ decays into three pions [4]. Being sensitive to magnitude and sign of $g_V \cdot g_A$, their result for the first time experimentally shows that (within errors) also the τ-neutrino appears left-handed only. Based on a sample of about 6800 $\tau \to \nu\pi^-\pi^-\pi^+$ decays, a clear Dalitz plot asymmetry is observed after projecting the decay plane

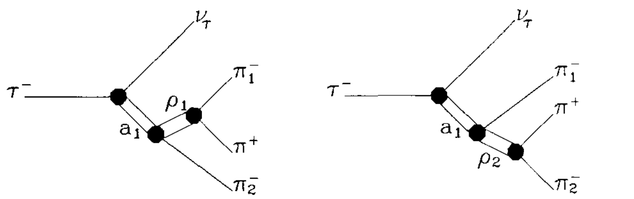

Fig. 1. Feynman diagrams for the Bose symmetrized chain decay $\tau \to \nu a_1$, $a_1 \to \rho\pi$, $\rho \to \pi\pi$

normal onto the τ direction. Such an effect has been predicted by Kühn and Wagner [5]. It is based on the fact that the (charged) three pion system is dominated by $\rho\pi$ intermediate states (see Fig. 1). The sign of the interference term between the two possibilities to form a ρ depends on the a_1 helicity. A left–right asymmetry due to this interference term indicates an a_1 helicity asymmetry and thus probes the space-time structure of the weak charged current of the third lepton generation. For the quantitative evaluation of the τ coupling constants a model for the hadronic weak current is needed. The authors of [5] use a Bose symmetric 3π amplitude which is modified by a form factor depending on $Q^2 (= m_{3\pi}^2)$, a ρ-Breit Wigner and a *p*-wave factor $(q_1 - q_+)_\mu$. Another feature of their amplitude is current conservation, i.e. it fulfills the condition $Q^\mu J_\mu = 0$. It does however not correspond to an orbital angular momentum eigenstate.

In this paper we propose another ansatz which is based on the decay chain $a_1 \to \rho\pi$ followed by $\rho \to \pi\pi$. The two possible Born term tensors for this transition are transformed to describe $L=0$ and $L=2$ transitions, respectively. Current conservation is not forced – the axial current is only partially conserved (otherwise the π^+ were stable and the decay $\tau^- \to \nu_\tau\pi^-$ forbidden). We instead use the spin 1 projection operator to get rid of the unwanted fourth vector degree of freedom. In fact, the timelike component corresponds to a pseudoscalar state, which in a phenomenological description should not have the a_1 parameters and angular distributions but might be dominated by the radial excitation $\pi'(1300)$. We also calculate amplitudes for this final state. A high statistics partial wave analysis employing these amplitudes can be used to measure the pseudoscalar component as well as the not well known D/S ratio of a_1 decay. The possible existence of a 0^- contribution would also change the isospin ratio mentioned above. After describing the principal analysis methods shortly, we resume the Dalitz plot asymmetry method to measure the ν_τ helicity [5] (and thus the τ charged current coupling constants) employed by the ARGUS Collaboration [4] in some detail to work out its model dependence.

Orbital angular momentum eigenstate amplitudes for $a_1 \to \rho\pi \to 3\pi$

We begin with the construction of the amplitude for the a_1 decay into $\rho\pi$ ($J^P = 1^+ \to 1^- 0^-$). There are two independent parity conserving Born term amplitudes for this transition [6]: $g_{\mu\nu}$ and $p_{\rho\mu} Q_\nu$ (Q_ν is the four-momentum of the $3\pi(=a_1)$ system).

These invariant amplitudes are neither eigenstates of helicity nor of orbital angular momentum. It will be shown below that the common identification of these tensors as describing S- and D-wave transitions (see e.g. [7]) is wrong. In the absence of more information a common approach in strong interaction physics is to assume the dominance of the lowest orbital angular momentum amplitude. This assumption is in no way better or worse than assuming the dominance of the lowest dimensional Born term, both are reasonable first approximations. But we criticize the use of the terms "S- and D-waves" to label amplitudes which are not eigenstates of orbital angular momentum. Note in particular that the ARGUS Dalitz plot analysis [8] also employed Born term amplitudes but named them S and D waves.

Any linear combination with Lorentz scalar coefficients is a valid amplitude:

$$T_{\mu\nu} = g \cdot g_{\mu\nu} + h \cdot p_{\rho\mu} Q_\nu. \tag{1}$$

To construct eigenstates of L we calculate the helicity amplitudes in the final state helicity system (i.e. the a_1 rest frame with the ρ moving along the $+z$ axis) using standard polarisation vectors [6]. For the description of the longitudinal outgoing ρ we take $\varepsilon_0^\nu = 1/\sqrt{p_\rho^2} \cdot (|\vec{p}_\rho|, 0, 0, E_\rho)$, i.e. we normalize using the actual as opposed to the nominal ρ mass. This is in accord with extending the usual ortho-normalisation properties and angular momentum conservation also to off-shell particle polarisation vectors, as is certainly desired in a phenomenological description of strong interactions [9]. The helicity amplitudes read:

$$D_{\lambda_{a_1}\lambda_\rho} = \varepsilon^\mu_{\lambda_{a_1}} \cdot T_{\mu\nu} \cdot \varepsilon^{*\nu}_{\lambda_\rho} \tag{2}$$

$$= -\delta^{\lambda_\rho}_{\lambda_{a_1}} \cdot \left(g \cdot \left(\delta^{+1}_{\lambda_{a_1}} + \delta^{-1}_{\lambda_{a_1}} + \delta^0_{\lambda_{a_1}} \cdot \frac{Qp_\rho}{\sqrt{Q^2 p_\rho^2}} \right) + h \cdot \delta^0_{\lambda_{a_1}} \cdot \frac{(Qp_\rho)^2 - Q^2 p_\rho^2}{\sqrt{Q^2 p_\rho^2}} \right). \tag{3}$$

In this derivation the covariant expressions $E_\rho = Qp_\rho/\sqrt{Q^2}$ and $|\vec{p}_\rho|^2 = ((Qp_\rho)^2 - Q^2 p_\rho^2)/Q^2$ are very useful. Note that the magnitude of the g-term helicity amplitude depends on the a_1 helicity and thus does not describe a simple helicity transfer as expected for an S- wave transition. We now calculate linear combinations for the description of $L=0$ and $L=2$ transitions defined by an angular distribution in terms of spherical harmonics and a threshold behaviour of $|\vec{p}_\rho|^L$ as in the case of spinless final state particles:

$$A^L(s_{a_1}, s_\rho) = \left(\begin{matrix} L & 1 \\ M & s_\rho \end{matrix} \middle| \begin{matrix} 1 \\ s_{a_1} \end{matrix} \right) \cdot |\vec{p}_\rho|^L Y^L_M(\theta^*_\rho, \phi^*_\rho). \tag{4}$$

Here the bracket expression denotes a standard Clebsch Gordan coefficient which is non-zero only for $M + s_\rho = s_{a_1}$, s being the spin projection along the z-axis. The correct angular behaviour can be achieved by demanding that the corresponding helicity amplitude ratios in the final state helicity frame are fulfilled. In this frame ($\theta^* = 0$, $s_\rho = \lambda_\rho$, $D^L_{\lambda_{a_1}, \lambda_\rho} = A^L(\lambda_{a_1}, \lambda_\rho)$) the orbital angular momentum must be perpendicular to the particle momenta (all $Y^L_M = 0$ unless $M = 0$). Inspection of the Clebsch Gordan coefficients leads to $D_{++} : D_{00} = 1:1$ for the S-wave and $D_{++} : D_{00} = 1 : -2$ for the D-wave. Setting g to 1, the coupling constant h then can be calculated from the helicity amplitudes (3). The results are:

$$T^S_{\mu\nu} = g_{\mu\nu} - \frac{1}{Qp_\rho + \sqrt{Q^2 p_\rho^2}} p_{\rho\mu} Q_\nu \tag{5}$$

$$T'^D_{\mu\nu} = g_{\mu\nu} - \frac{Qp_\rho + 2\sqrt{Q^2 p_\rho^2}}{(Qp_\rho)^2 - Q^2 p_\rho^2} p_{\rho\mu} Q_\nu. \tag{6}$$

This procedure only restricts the ratio of g and h, the overall normalisation is not given by first principles – any Lorentz scalar function can be multiplied to the complete amplitude. In particular, we can thus realize the expected $|\vec{p}_\rho|^L$ threshold behaviour of the amplitude. The necessary factors are most easily calculated in the final state helicity system contracting the tensors with transverse polarisation vectors. Whereas the S-wave in (5) is already correctly normalized (constant in $\vec{p}_\rho$), the D wave tensor has to be modified:

$$T^D_{\mu\nu} = \frac{(Qp_\rho)^2 - Q^2 p_\rho^2}{Q^2} \cdot g_{\mu\nu} - \frac{Qp_\rho + 2\sqrt{Q^2 p_\rho^2}}{Q^2} p_{\rho\mu} Q_\nu. \tag{7}$$

Results consistent with the present approach have also been found in [10].

The P-wave transition of the ρ into two pions is unique [6]: $(p_1 - p_+)_\alpha$. The complete decay matrix element for the chain decay reads:

$$J_\mu(a_1 \to \rho\pi \to \pi\pi\pi) = T_{\mu\nu} \cdot \frac{\sum_{\lambda_\rho} \varepsilon^{*\nu}_{\lambda_\rho} \varepsilon^\alpha_{\lambda_\rho}}{m_\rho^2 - p_\rho^2 - i m_\rho \Gamma_\rho} (p_1 - p_+)_\alpha \tag{8}$$

$$= T_{\mu\nu} \cdot \frac{-g^{\nu\alpha} + p_\rho^\nu p_\rho^\alpha / p_\rho^2}{m_\rho^2 - p_\rho^2 - i m_\rho \Gamma_\rho} (p_1 - p_+)_\alpha. \tag{9}$$

Here we explicitely restrict the ρ to be a spin one particle by projecting out only the physical components using the spin summation which in (9) is written in its covariant form. In this case of equal mass final state particles

$(p_1^2 = p_+^2)$ the second term in the numerator does not contribute, such that the final Bose symmetrized hadronic currents can be written:

$$J_\nu^S = \frac{-1}{m_\rho^2 - p_{\rho_1}^2 - im_\rho\Gamma_\rho} \cdot\left((p_1 - p_+)_\nu - \frac{(p_1 - p_+)Q}{Qp_{\rho_1} + \sqrt{Q^2 p_{\rho_1}^2}}\, p_{\rho_1\nu}\right) + (1 \leftrightarrow 2) \quad (10)$$

$$J_\nu^D = \frac{-1}{m_\rho^2 - p_{\rho_1}^2 - im_\rho\Gamma_\rho} \cdot\left(\frac{(Qp_{\rho_1})^2 - Q^2 p_{\rho_1}^2}{Q^2}(p_1 - p_+)_\nu - \frac{(Qp_{\rho_1} + 2\sqrt{Q^2 p_{\rho_1}^2})\cdot((p_1 - p_+)Q)}{Q^2}\, p_{\rho_1\nu}\right) + (1 \leftrightarrow 2). \quad (11)$$

The isobar model – i.e. the coherent addition of amplitudes of various subresonances without allowing for final state interactions – does not obey 3 particle unitarity, however it has been shown to be highly successful and has delivered a wealth of data on baryon and meson resonances [11–14].

Description of $\tau \to \nu 3\pi$

This hadronic axial current now has 4 independent vector components, of which only 3 are physical if one wants to describe merely the spin 1 contribution corresponding to the a_1. This is achieved by introducing the spin 1 projection operator into the contraction with the leptonic current L_μ:

$$M_{\lambda_\tau}^{(a_1 S,D)}(\tau^- \to \nu a_1 \to \nu 3\pi) = L_\mu \cdot G(Q^2) \sum_{\lambda_{a_1}} \varepsilon_{\lambda_{a_1}}^{*\mu} \varepsilon_{\lambda_{a_1}}^{\nu} J_\nu^{a_1 S,D} \quad (12)$$

$$= \sqrt{\tfrac{1}{2}} G_F \bar{u}_\nu(p_\nu)\gamma_\mu(g_V + g_A\gamma_5)u(p_{\tau^-})\cdot G(Q^2) \cdot(-g^{\mu\nu} + Q^\mu Q^\nu/Q^2)\cdot J_\nu^{a_1 S,D}. \quad (13)$$

In this equation we included a form factor $G(Q^2)$ which describes the $I^G(J^{PC}) = 1^-(1^{++})\rho\pi$ final state interaction. It is usually described by a Breit–Wigner shape (see the discussions in [15–17]) for the a_1 modified by a form factor $g(Q^2)$ which describes a possible deviation from a pointlike $W - a_1$ coupling:

$$G(Q^2) = g(Q^2)\cdot\frac{1}{m_{a_1}^2 - Q^2 - im_{a_1}\Gamma_{a_1}(Q^2)}. \quad (14)$$

The phenomenological analysis of Bowler [15] suggests that no strong deviation from pointlike coupling is necessary: If the form factor is parameterized as $g(Q^2) = (\sqrt{Q^2}/m_{a_1})^n$, then $n = 0 - 0.5$ (Bowler's x (see (28) below) is related to n via $x = 2 - 2n$).

Comparing our result with that of [5], we observe that the difference only is due to their ansatz $g_{\mu\nu}$ for the $a_1\rho\pi$ vertex. Let us emphasize again that their ansatz is neither better nor worse than an S-wave description. Their model only should be labelled "lowest dimensional Born term" instead of "S-wave". Both assumptions are reasonable first approximations. In both bases, just one of two possible amplitudes is considered, and thus both are model dependent. Only dynamical models can relate both independent amplitudes and give a unique description. In a recent paper Kühn and Santamaria [18] argue that the lowest dimensional Born term might get some justification from consistency with chiral invariance and its derivation from chiral Lagrangians [19]. Also the analytic properties of the Born terms are simpler than those of the complicated linear combinations of the orbital angular momentum eigenstates. Nevertheless, from a phenomenological point of view the latter clearly are justified as a basis for a kinematical analysis. For example, in quark model calculations [10] one calculates transition amplitudes for specific orbital angular momentum eigenstates.

In principle, the a_1 decay into three pions might also proceed through a P-wave into a pion and an even isospin $(\pi\pi)_{S\text{-wave}}$ system. This contribution is determined to be only 0.003 ± 0.003 compared to the $\rho\pi$ mode [14]. However, this result of a combined analysis of different hadronic production modes [20] relies on the existence of a second pole which strongly couples to $f_0(1400)\pi$ and is interpreted as a $3q3\bar{q}$ state. Thus, an independent analysis in τ decays is important. Also the 1:20 suppression of the D wave observed in diffractive production [21] might be due to interference with the Deck mechanism or, as a dual description, with $2q2\bar{q}$ or $3q3\bar{q}$ poles [20]. It should not be taken for granted that these numbers are also valid for the a_1 decay as observed in τ decays.

Although suppressed by PCAC, a pseudoscalar component might contribute to the hadronic current. It would naturally be dominated by the radial excitation candidate $\pi'(1300)$. This contribution corresponds to the timelike vector component in the 3π c.m.s., the Born term tensor thus is proportional to Q_ν (this is the only non-zero vector which can be constructed, since the polarisation "tensor" of a spin 0 field is just a scalar). The possible pseudoscalar hadronic currents (S-wave into $\pi(\pi\pi)_{S\text{-wave}}$ or P-wave into $\rho\pi$) can be written:

$$J_\nu^{(\pi' S)} = Q_\nu \quad (15)$$

$$J_\nu^{(\pi' P)} = Q_\nu \frac{(Q + p_2)(p_1 - p_+)}{m_\rho^2 - p_{\rho_1}^2 - im_\rho\Gamma_\rho} + (1 \leftrightarrow 2). \quad (16)$$

In the absence of detailed knowledge about the scalar state $f_0(1400)$ we don't give it a Breit–Wigner form. In the spirit of using Watson's theorem in the isobar model, one can also include the measured $I = 0$, $J = 0$ $\pi\pi$ phase shifts into the amplitude [13]. Certainly the description of the $(\pi\pi)_{S\text{-wave}}$ is one of the weakest links in the isobar model, and very different approaches (real phase space, $f_0(1400)$ Breit–Wigner, measured phase shifts) have been followed in different analyses, often without an estimate of the systematic uncertainties introduced by the special choice.

We now calculate the lepton helicity amplitudes in the τ c.m.s. with the a_1 moving along the positive z-axis. We use standard representation spinors with covariant normalisation [6]. In this calculation and the definition of the coupling constants one has to take care of the anticommutation properties of γ matrices: Our matrix element (13) is in accord with the standard model coupling constants to be $g_V = +1$, $g_A = -1$, in contrast to the amplitude of [5], which differs in the order of γ_5 and γ_μ and thus requires a positive g_A in order to couple to left-handed neutrinos only. Explicit calculation using massless neutrinos leads to

$$L_\mu(s_\tau = +\tfrac{1}{2}, \lambda_\nu = -\tfrac{1}{2}) = \sqrt{\tfrac{1}{2}} G_F \sqrt{2m_\tau E_\nu}(g_V - g_A)\cdot(1,0,0,1) \quad (17)$$

$$L_\mu(s_\tau = -\tfrac{1}{2}, \lambda_\nu = -\tfrac{1}{2}) = \sqrt{\tfrac{1}{2}} G_F \sqrt{2m_\tau E_\nu}(g_V - g_A)\cdot(0,-1,i,0) \quad (18)$$

$$L_\mu(s_\tau = +\tfrac{1}{2}, \lambda_\nu = +\tfrac{1}{2}) = \sqrt{\tfrac{1}{2}} G_F \sqrt{2m_\tau E_\nu}(g_V + g_A)\cdot(0,1,i,0) \quad (19)$$

$$L_\mu(s_\tau = -\tfrac{1}{2}, \lambda_\nu = +\tfrac{1}{2}) = \sqrt{\tfrac{1}{2}} G_F \sqrt{2m_\tau E_\nu}(g_V + g_A)\cdot(1,0,0,1). \quad (20)$$

In the standard model $(g_V + g_A = 0)$ the latter two contributions vanish. Contracting with the a_1 polarisation vectors, the helicity amplitudes $M_{s_\tau \lambda_\nu}$ are calculated to be (with $\lambda_{a_1} = s_\tau + \lambda_\nu = \lambda$ and $\pm 1/2$ abbreviated by $\pm$)

$$M_{+-} = -G_F(g_V - g_A)\sqrt{m_\tau^2 - Q^2} \cdot\left(\frac{m_\tau}{\sqrt{2Q^2}} G^{J=1}_{\lambda=0}(Q^2) + \sqrt{\frac{Q^2}{2}} G^{J=0}(Q^2)\right) \quad (21)$$

$$M_{--} = G_F(g_V - g_A)\sqrt{m_\tau^2 - Q^2}\cdot G^{J=1}_{\lambda=-1}(Q^2) \quad (22)$$

$$M_{++} = G_F(g_V + g_A)\sqrt{m_\tau^2 - Q^2}\cdot G^{J=1}_{\lambda=+1}(Q^2) \quad (23)$$

$$M_{-+} = -G_F(g_V + g_A)\sqrt{m_\tau^2 - Q^2} \cdot\left(\frac{m_\tau}{\sqrt{2Q^2}} G^{J=1}_{\lambda=0}(Q^2) + \sqrt{\frac{Q^2}{2}}\cdot G^{J=0}(Q^2)\right). \quad (24)$$

This means that in the final state helicity system the a_1 is polarized depending on the coupling constants g_V and g_A. In the standard model the helicities are -1 or 0, the relative contribution of the latter decreasing with increasing three pion mass Q^2, helicity $+1$ is forbidden. The helicity 0 contribution only depends on $g_V^2 + g_A^2$. Note that for τ^+ decays the roles of neutrino and thus a_1 helicities are reversed.

Up to now we did not take into account second class currents [22] (corresponding to states with $PC(-1)^J = 1$), which are measured and expected to be very small and only possible due to isospin violation (i.e. the $u-d$ quark mass difference). Isospin violation could also introduce an $I = 2$ component ($I = 0$ is not possible since the 3 pion system is charged). To test these expectations in a completely model independent analysis also the following (exotic) possibilities should be included: $I^G(J^{PC}) = 2^-(0^{--})$ and $2^-(1^{+-})$ (first class) and $1^-(1^{-+})$ and $2^-(1^{--})$ (second class). The Born term tensor for the decay of the latter two possibilities to $\rho\pi$ reads [6]: $T_{\mu\nu} = i\varepsilon_{\mu\nu\alpha\beta} Q^\alpha p_\rho^\beta$.

We now calculate the differential decay rate. Using the usual recurrence formula [14] the four particle Lorentz invariant phase space element can be decomposed into

$$d\,\mathrm{LIPS}_4(\tau, \nu 3\pi) = (2\pi)^{-1} d\,\mathrm{LIPS}_2(\tau, \nu a_1) \cdot d\,\mathrm{LIPS}_3(a_1, 3\pi) dQ^2 \quad (25)$$

$$= (2\pi)^{-1} \frac{m_\tau^2 - Q^2}{32\pi^2 m_\tau^2} d\cos\theta_{a_1} d\phi_{a_1} \frac{1}{1024\pi^2 Q^2} \cdot dm^2_{\pi_1\pi_+} dm^2_{\pi_2\pi_+} d\alpha\, d\cos\beta\, d\gamma\, dQ^2. \quad (26)$$

Neglecting small corrections of the order $(m_\tau/E_{\mathrm{beam}})^2$ the outgoing τ's have opposite helicities which leads to correlation phenomena [5, 1]. Taking this into account, it is in principle possible to "tag" or at least enrich the τ^- helicity through the observation of the τ^+ decay and vice versa. However, analyzing the τ decays separately, the coherence is lost and the incoherent mean of both τ helicities is observed:

$$\frac{d\Gamma}{d\,\mathrm{LIPS}_4} = \frac{1}{4m_\tau}(|M_{+-}|^2 + |M_{--}|^2 + |M_{++}|^2 + |M_{-+}|^2). \quad (27)$$

In this case we immediately consider the τ helicities in the τ *decay* helicity frame as already done above, the integration over $d\Omega_{a_1}$ then is a trivial 4π multiplication. Performing the integration in the case of the a_1 recovers the well known standard model result [23]

$$\frac{d\Gamma}{dQ^2} = \frac{G_F^2}{16\pi} \frac{(m_\tau^2 - Q^2)^2}{Q^2 m_\tau^3}(m_\tau^2 + 2Q^2) \cdot\left(\frac{m_{a_1}}{\sqrt{Q^2}}\right)^x \frac{m\Gamma_{a_1\to\rho\pi}(Q^2)}{(m_{a_1}^2 - Q^2)^2 + m_{a_1}^2 \Gamma_{a_1}^2(Q^2)}. \quad (28)$$

It is interesting to note that the first term in $(m_\tau^2 + 2Q^2)$ is due to the helicity 0 contribution and the second to helicity -1.

Prospects of a partial wave analysis

If the τ direction is measured (by reconstruction of the decay vertex), the ν momentum can be calculated using momentum conservation and the τ mass constraint. In this case, which may be possible at LEP, all of the phase space variables are known and one has a very powerful tool for partial wave analysis which would be performed in slices of Q^2. This could e.g. be done using the extended maximum likelihood technique [9, 13, 11, 12] minimizing the likelihood function

$$\mathscr{L}(\vec{a}) = -\sum_{\text{events}\, i} \ln \frac{d^n\Gamma}{d\omega^n}(\vec{\omega}_i, \vec{a}) + C\int \frac{d^n\Gamma}{d\omega^n}(\vec{\omega}, \vec{a})\varepsilon(\vec{\omega}) d\omega^n. \quad (29)$$

Here the sum is extended over all detected events in a given Q^2 bin. $\vec{\omega}$ denotes the n available phase space variables apart from Q^2 and $\vec{a}$ the fit parameters: One complex number for each spin-parity-isobar-L combination if one assumes the validity of the standard model, and one additional (real) number, e.g.

$$\gamma_{AV} = \frac{2g_V g_A}{g_V^2 + g_A^2}, \quad (30)$$

if the weak couplings are also to be measured ($\gamma_{AV} = -1$ for $V-A$, $+1$ for $V+A$, 0 for pure V or pure A). Since only relative phases are accessible, one phase in each incoherent term can be fixed to zero. Where in principle two isospin states can occur, these are distinguishable only if one considers simultaneously the decay into three charged and one charged – two neutral pions. This will not be considered here. In the extended maximum likelihood procedure the maximum information including all correlations is taken into account. In practice, the cross section of the events observed in the experiment is calculated as if they were 3 pion phase space events which are weighted according to a model cross section depending on the fit parameters $\vec{a}$. The second term in (29) is called the normalisation integral, it represents the total number of events observed in the detector (ε is the acceptance) for the actual fit parameter set $\vec{a}$. The conversion constant C has to relate the integrated width to the total number of events, it depends on the integrated luminosity of the dataset. During the fit procedure, the calculation of both terms can be simplified to a few complex multiplications [12, 13, 9].

Without vertex reconstruction the τ direction cannot be determined. Only if both τ's decay semihadronically and are fully reconstructed, the missing two neutrino momenta can be reconstructed by kinematic constraints – however only up to a twofold discrete ambiguity [5] and only if one neglects initial state radiation. This complicates a straight forward analysis considerably. While $\cos\theta_{a_1}$ still can be calculated, neither ϕ_{a_1} nor the 3 Euler angles describing the decay plane are uniquely accessible, and one has to average over both possibilities in doing any fit. However, in all cases a Dalitz plot analysis, e.g. analogous to that in [24], is feasible. Here one integrates (usually numerically with Monte Carlo methods) over all variables except s_1 and s_2, thereby partly loosing coherence and (of course) lots of information.

Determination of weak τ decay constants from a parity violating Dalitz plot asymmetry

Without any restrictions on the recoiling τ, in an $e^+e^- \to \tau^+\tau^-$ event only the 3π momentum is known, and the possible τ four momenta can only be constrained to lie on a cone around the 3π momentum vector with an opening angle which depends only on measurable kinematics [5]:

$$\cos\psi = \frac{x(m_\tau^2 + Q^2) - 2Q^2}{(m_\tau^2 - Q^2)\sqrt{x^2 - 4Q^2/s}} \quad \text{with} \quad x = \frac{E_{3\pi}}{E_{\text{beam}}}. \tag{31}$$

The ARGUS experiment [4] has used this kind of analysis to observe a parity violating asymmetry effect and was thus able to measure γ_{AV}, see (30). Using Kühn and Wagner's amplitude [5] (i.e. the lowest dimensional Born term), the preliminary value is in good accord with the standard model expectation and has a 29% relative error, it corresponds to a statistical significance of 4.2σ for the parity violating effect. In order to explore the model dependence, we shortly resume the argumentation line. The following relations are worked out in the 3π c.m.s. with the z-axis opposite to the τ momentum. Assuming that the 3 pion system can be described by an a_1 decaying into $\rho\pi$, the τ spin averaged and ν spin summed squared matrix elements (with coupling constants taken out to conform with [5]) can be written:

$$\omega = \frac{1}{G_F^2(g_V^2 + g_A^2)} |L_\mu(-g^{\mu\nu} + Q^\mu Q^\nu/Q^2)J_\nu|^2 \tag{32}$$

$$= \frac{m_\tau^2 - Q^2}{(m_{a_1}^2 - Q^2)^2 + m_{a_1}^2 \Gamma_{a_1}^2(Q^2)} \cdot \left(\frac{m_\tau^2}{Q^2} J_3^* J_3 + J_1^* J_1 + J_2^* J_2 + 2\gamma_{AV}\,\mathrm{Im}(J_2^* J_1)\right). \tag{33}$$

In this equation the J_i are the space components of the hadronic current (10), which as a reasonable starting point would be modelled pure S-wave. The $J_3^* J_3$ term corresponds to an a_1 helicity of 0, all other to ± 1. Most interesting is the last term which is parity violating and directly proportional to γ_{AV}. An explicit calculation shows that it merely depends on the $\rho_1\rho_2$ interference:

$$\mathrm{Im}(J_2^* J_1) = \frac{m_\rho \Gamma_\rho}{((m_\rho^2 - s_1)^2 + m_\rho^2 \Gamma_\rho^2)((m_\rho^2 - s_2)^2 + m_\rho^2 \Gamma_\rho^2))} \cdot (s_2 - s_1) \cdot (j_{\rho_2 1} j_{\rho_1 2} - j_{\rho_2 2} j_{\rho_1 1}) \tag{34}$$

with the Dalitz plot variables $s_i = p_{\rho_i}^2$ and j_i the (real) hadronic currents (10) with the ρ Breit Wigner factors taken out. Because $\vec{j}_{\rho_1}$ and $\vec{j}_{\rho_2}$ both are vectors in the decay plane (this is true for any D/S ratio), their cross product is proportional to $\hat{\mathbf{n}}_{3\pi}$, the unit length normal vector of the decay plane. The combination $j_{1\rho_2} j_{2\rho_1} - j_{2\rho_2} j_{1\rho_1}$ can be interpreted as z-component of $\hat{\mathbf{n}}_{3\pi}$, projected by scalar product with the τ direction unit vector: $\hat{\mathbf{n}}_{3\pi}\hat{\mathbf{p}}$. Having two likesign pions in the final state, the orientation of the decay plane normal is unique only after multiplying with a scalar function antisymmetric under pion exchange. These properties lead Kühn and Wagner [5] to propose to measure the moment

$$A_{RL}(Q^2) = \frac{\int_{Q^2\text{bin}} \int d\,\mathrm{LIPS}_4\, \omega \hat{\mathbf{n}}_{3\pi} \hat{\mathbf{p}}\, \mathrm{sign}(s_1 - s_2)}{\int_{Q^2\text{bin}} \int d\,\mathrm{LIPS}_4\, \omega} \tag{35}$$

which is directly proportional to the parity violating term and thus to γ_{AV}. Since the τ direction cannot be measured, one has to integrate over all possible τ directions in every event. Alternatively, (as done by ARGUS) one simply replaces $\hat{\mathbf{p}}$ by the measurable $\hat{\mathbf{Q}}$, i.e. the direction of the 3π system as measured in the laboratory. The mean asymmetry observed using this modified moment is smaller (and reversed in sign) than the original by $\langle \cos\psi \rangle$, but remains observable:

$$\langle \hat{\mathbf{Q}} \hat{\mathbf{n}}_{3\pi} \rangle = -\langle \hat{\mathbf{p}} \hat{\mathbf{n}}_{3\pi} \rangle \cos\psi. \tag{36}$$

Since the opening angle ψ is known in every individual event, it is possible to select on events with a large $\cos\psi$ to suppress low and negative values which are not contributing to a positive signal and thus reduce the statistical significance. In practice the integrations are replaced by summing over measured events. Theoretical expectations for the experimental observable's dependence on γ_{AV} are most easily calculated using the Monte Carlo method: Events simulated according to 3π phase space are weighted according to the model cross section and the required moment. The influence of detector acceptance and resolution effects as well as the effect of replacing $\hat{\mathbf{p}}$ by $\hat{\mathbf{Q}}$, of cuts in $\cos\psi$ and also the explicit decay model dependence is such accessible. Note that due to the direct Lorentz transformation from the laboratory into the 3π c.m.s. without going via the (unknown) τ c.m.s. a Wigner rotation is introduced. This effect can also easily be handled with the Monte Carlo method.

We now propose to use also another moment to measure γ_{AV} which takes advantage of the knowledge about the relative importance of the parity violating term depending on the measurable Dalitz plot variables:

$$A_F(Q^2) = \int\limits_{Q^2\text{bin}} \int d\,\text{LIPS}_4 \omega \hat{\mathbf{n}}_{3\pi}\hat{\mathbf{p}} \frac{2m_\rho \Gamma_\rho (s_1 - s_2)}{(2m_\rho^2 - (s_1+s_2))^2 + 4m_\rho^2 \Gamma_\rho^2} \cdot \left(\int\limits_{Q^2\text{bin}} \int d\,\text{LIPS}_4 \omega \right)^{-1}. \quad (37)$$

Analysing data using this moment has the advantage that statistical fluctuations of the majority of events, which we know can hardly contribute to the asymmetry, are damped. It is similar in spirit to Kühn and Wagner's A_D. One should not get confused by the lower numerical values of these moments (see below) compared to A_{RL}: being weighted moments $\propto (s_1 - s_2)$ instead of a simple asymmetry $\propto \text{sign}(s_1 - s_2)$ their statistical significance may easily be larger although numerically smaller. γ_{AV} can be determined by comparing the measured asymmetry moments with the predicted ones: $\gamma_{AV}(Q^2) = A_i^{\text{meas}}(Q^2)/A_i^{\text{pred}}(Q^2)$. Note that in the standard model this quantity will be negative for τ^- and positive for τ^+ decays. Of course, in a reasonable model the result should be independent of Q^2.

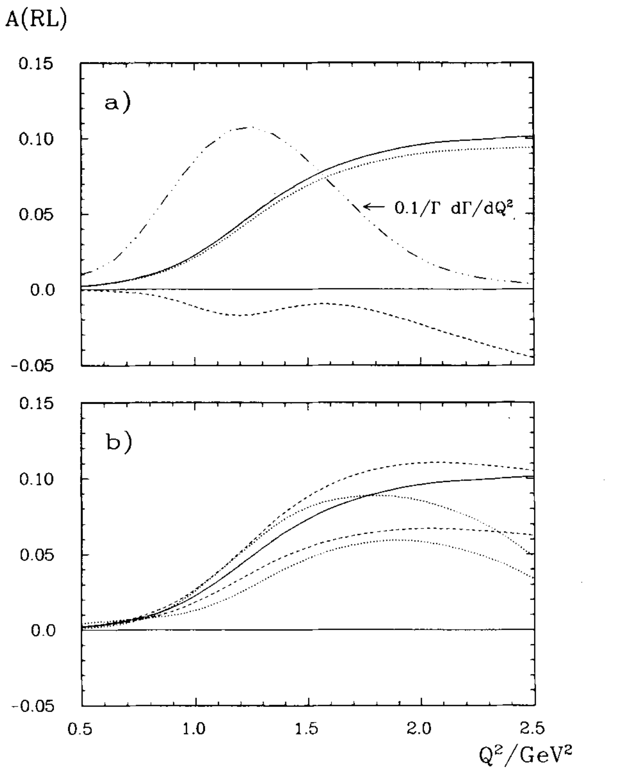

Fig. 2 a, b. Standard model prediction for the moment A_{RL} as a function of the three pion mass Q^2 (negative for τ^-, positive for τ^+). **a** Solid line: Pure $a_1 \to \rho\pi$, S-wave; dotted: calculation of [5] (i.e. lowest dimensional Born term amplitude); dashed: pure $a_1 \to \rho\pi$, D-wave. Dash-3 dotted line: Q^2 mass spectrum as observed by ARGUS [4] (not acceptance corrected). **b** Solid line: Pure $a_1 \to \rho\pi$, S-wave; dashed: 2.2% D-wave admixture (upper curve: relative sign + and vice versa); dotted: 10% $a_1 \to \varepsilon\pi$ wave admixture.

Model dependence in the determination of γ_{AV}

The determination of γ_{AV} using the Dalitz plot asymmetry depends on assumptions about the hadronic current. In this section we summarize some contributions which are easily accessible using the formalism developed in this paper.

As indicated repeatedly, the amplitude in [5] does not correspond to an $L=0$ transition but the lowest dimensional Born term. The influence on the asymmetry moments can be seen by normalising g in (1) to 1 and observing the dependence on h, or better h', the coefficient in front of the $p_{\rho_i v}$ in (10). The current in the $\rho\rho$ interference term (34) in the 3π c.m.s. can be rewritten as $j_{1\rho_2} j_{2\rho_1} - j_{2\rho_2} j_{1\rho_1} \propto (4 - (1+h')^2) \cdot \vec{p}_2 \times \vec{p}_1$. Kühn and Wagner's model ($h'=0$) leads to a factor 3 in front of the cross product. The maximum reachable value (for $h' = -1$) is 4, large positive and large negative h reduce the asymmetry and can even change its sign. Thus, it is clearly evident that a proper modelling à la (10) is important for the quantitative evaluation of the parity violation effect. The results are summarized in Figs. 2 and 3. It turns out that the S-wave matrix element (solid line in Fig. 2a) and that of [5] (dotted line) do not differ severely. However, an admixture of D-wave with either sign changes the expected asymmetry strongly. A pure D-wave as well as a pure h Born term tensor predict a few percent asymmetry with reversed sign (!) (dashed line in Fig. 2a); a 20% D-wave admixture with negative sign leads to an almost vanishing asymmetry. Note that the Isgur et al. analysis [10] of the ARGUS Dalitz plot projections [8] results in a D/S amplitude ratio of -0.14 ± 0.03 (corresponding to a relative branching ratio of 2.2%), in accord with the flux tube breaking model prediction of -0.15 (lower dashed line in Fig. 2b).

The existence of a possible a_1 decay into $(\pi\pi)_{S\text{-wave}}\pi$ can contribute to an asymmetry moment in several ways. A reliable isobar model prediction cannot be given, since the S-wave is not dominated by a relatively narrow Breit–Wigner like resonance. It is not even clear how many poles contribute to the complicated $\pi\pi$ phase behaviour. If we nevertheless assume that it can be described by a single Breit–Wigner BW_ε, we have to consider the interference between both ways to construct such an ε, and altogether 4 terms between the ρ's and ε's. The first can lead to an observable asymmetry, becoming increasingly important with Q^2 due to the high f_0 mass. The $\rho - \varepsilon$ interference terms can be written as

$$I_{\rho\varepsilon} \propto \vec{p}_1 \times \vec{p}_2 \cdot \text{Im}((h'+1)(\text{BW}^*_{\rho_1}\text{BW}_{\varepsilon_2} - \text{BW}^*_{\rho_2}\text{BW}_{\varepsilon_1}) + 2(\text{BW}^*_{\rho_1}\text{BW}_{\varepsilon_1} - \text{BW}^*_{\rho_2}\text{BW}_{\varepsilon_2})). \quad (38)$$

A large fraction of the interference cancels after integration over the whole phase space. The dotted lines in

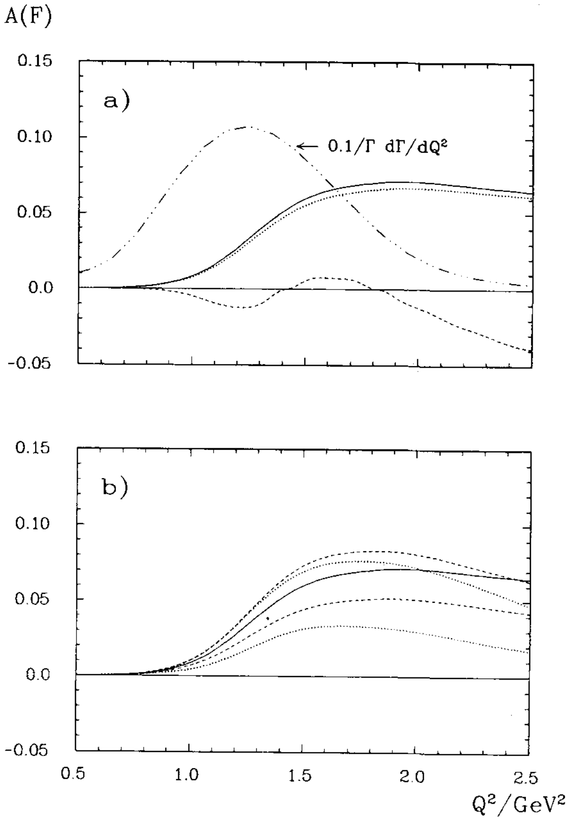

Fig. 3 a, b. Standard model prediction for the moment A_F as a function of the three pion mass Q^2. Line symbols as in Fig. 2

Fig. 2b show the effect if the $\varepsilon\pi$ branching ratio were 10% with both relative signs, corresponding to the ARGUS upper limit for non-$\rho\pi$ decays [8]. If the S-wave is not given a phase, the $\rho-\varepsilon$ interference still remains significant, leading to an even larger deviation from pure $\rho\pi$, S-wave at low Q^2.

Another model dependent feature can be seen directly from (33): A possible 0^- background to the a_1, which naturally is helicity 0, cannot contribute to the parity violating term (which is due to the helicity $+1/-1$ asymmetry). Such a background thus can only reduce the asymmetry by increasing the parity conserving part. One should also test the influence of exotic intermediate 3π states, e.g. a vector contribution like a possible hybrid or 4 quark state $\tilde{\rho}$ with quantum numbers $I^G(J^{PC})=1^-(1^{-+})$. As a second class current and because of its exotic origin it is expected to be suppressed. The symmetrized decay matrix element into $\rho\pi$ can be written as a vector along the decay plane normal, leading to a vanishing asymmetry for a pure $\tilde{\rho}$. One could however imagine a strong interference between the a_1 and the $\tilde{\rho}$. Quantitative evaluation shows that this interference has no large impact on the asymmetry term at low three pion masses, but becomes increasingly important with increasing Q^2. With a 10% relative branching ratio of the $\tilde{\rho}$ the asymmetry smoothly goes down (similar to the $a_1\to\varepsilon\pi$ admixture) and reaches 0 at $Q^2=3\,\mathrm{GeV}^2$.

Part of these model dependencies can be minimized by a consistency check between both moments types, A_{RL} as defined by Kühn and Wagner as well as our proposed moment A_F. Another good test of the importance of non-$a_1\to\rho\pi$ background is the dependence of the extracted asymmetries on the 3 pion mass Q^2: There should be no such dependence if the background is small. A large part of detector effects can be excluded, if no asymmetry is observed for τ^- and τ^+ added without changing the sign for antiparticle decays. Finally, even with infinite statistics and considering all possible intermediate states with infinite accuracy, there remains a small theoretical uncertainty due to the neglect of final state interactions in the isobar model. Only a $\tau^+\tau^-$ correlation analysis, an improved Dalitz plot analysis or even a full-fledged partial wave analysis will reduce these uncertainties and give some more insight into the hadronic currents.

Summary and conclusions

In summary, we (re-)derived a theoretical description of τ decays into $\nu 3\pi$ in a covariant tensor language using the isobar model. Special emphasis was devoted on the orbital angular momentum eigenstates of the $a_1\to\rho\pi$ decay, which frequently are mixed up with the simplest Born term amplitudes. Amplitudes are also given for other possible intermediate states. We discussed the physics perspectives of the reaction, i.e. the measurement of the weak τ decay coupling constants and the structure of the hadronic 3π current. Some analysis methods have been discussed and a new moment for the weak coupling constant determination with available data (ARGUS) has been proposed. Finally we discussed the model dependence of such a measurement and found it to be considerable (at the $+25/-50\%$ level, depending on Q^2) without a further investigation of the hadronic 3π current. Consistency checks using different moments and comparing the Q^2 evolution can help to reduce theoretical uncertainties.

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Note added in proof. During the print the finalized ARGUS analysis became available as preprint DESY 90-079.