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Monte Carlo investigation of the transition effect

J. del Peso * and E. Ros
DESY, 2000 Hamburg 52, FRG

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We review the status of the transition effect in electromagnetic sampling calorimeters using the Monte Carlo shower generator EGS4. We study the energy, thickness, charge number and density dependence of this effect, considering both the influence of the absorber and the readout medium. We propose a parametrization for the electron sampling fraction valid for absorber thicknesses around $1 X_0$. Finally we also review the experimental situation.

1. Introduction

The transition effect in electromagnetic sampling calorimeters was observed experimentally long time ago [1] and reproduced by analytical [2] and Monte Carlo [3] calculations. Apart from some specific questions like calibration with cosmic rays, this effect is of no real practical use as far as electromagnetic calorimeters are concerned. It plays, however, an important role in the mechanisms which lead to compensation in hadron calorimetry. This aspect of the transition effect has been fully realized and discussed in previous reports [4].

The effect consists in a reduction of the electron sampling fraction as compared to the sampling fraction of minimum ionizing particles and is present whenever a high-Z material is used as absorber and a low-Z material as readout medium. The origin of the effect has been explained (see ref. [4], for example) and its dependence with the Z value of the absorber has been investigated with Monte Carlo calculations [3,4]. In this work we would like to review the present status both from the Monte Carlo and the experimental side. In addition we present new Monte Carlo calculations on thickness dependence and on the influence of the readout medium and propose a parametrization for the electron sampling fraction.

2. Parametrization of the transition effect

In the following we only consider sampling calorimeters of the sandwich type with absorber layer thickness t and readout layer thickness s. The sampling fraction for electrons is defined as

$$e = E_s/E,$$

where E_s is the visible energy in the readout medium and E the energy of the showering particle. The sampling fraction for a minimum ionizing particle (mip) is

$$\text{mip} = s\epsilon_s/(s\epsilon_s + t\epsilon_t),$$

where $\epsilon_s = (dE/dx)_s$ and $\epsilon_t = (dE/dx)_t$ are the ionization energy losses per unit length in the readout and absorber respectively, as given in ref. [5]. Since the energy of the electron shower is deposited in the calorimeter mostly by low-energy electrons, one would naively expect that $e/\text{mip} \approx 1$. In fact e/mip can be much smaller than 1 and this effect is called "transition effect" *.

The effect can be explained in the following way (see fig. 1): the electrons inside the shower produce many low energy photons (with typical energies in the MeV region) when traversing the absorber. These low-energy photons lead to low-energy electrons by Compton, photoelectric or pair-production effects **. These low-energy electrons have a short range and stay mostly in the absorber. In other words, in addition to ionization losses, ϵ_t, we should consider radiation losses r_t, so the electron sampling fraction is

$$\epsilon = s\epsilon_s/[s\epsilon_s + t(\epsilon_t + r_t)],$$

and therefore

$$\frac{e}{\text{mip}} \approx \frac{1}{1 + r_t/\epsilon_t},$$

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* This name was proposed in ref. [2] (Pinkau), but the origin and magnitude of the effect was only understood later. For this reason it has been proposed a different name: migration effect [4] (Brückmann et al.) In this article we keep, however, the traditional name of "transition effect".

** The Compton cross section is proportional to Z, the photoelectric effect to Z^5 and the pair production to Z^2. This dependence on Z explains the strong absorption of radiation by the absorber.

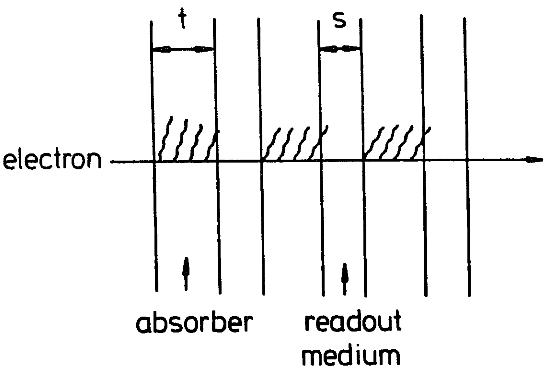

Fig. 1. Graphic explanation of the transition effect in a sandwich calorimeter of absorber thickness t and active medium thickness s. The electrons produced in the shower lose additional energy by radiation in the absorber. The low-energy component of this radiation stays in the absorber.

assuming that the energy deposition in the readout is much smaller than in the absorber. As r_t is proportional to Z_t^2 and ϵ_t to Z_t, we finally obtain

$$\frac{e}{\text{mip}} \approx \frac{1}{1+aZ_t} \qquad (Z_t \gg Z_s),$$

a being a constant and Z_t and Z_s the charge numbers of absorber and readout respectively. In the following we will see how well this formula, derived from a very simple model, reproduces the Monte Carlo calculations.

3. Tuning the EGS4 parameters

We have investigated the dependence of e/mip on various parameters which have to be specific to generate showers with the EGS4 Monte Carlo program [6]. As pointed out elsewhere [7,8], the result of EGS4 calculations depend, sometimes very significantly, on parameters like the electron and photon cutoff energies ϵ_e and ϵ_γ and the step-size parameter ESTEPE. In fig. 2 we show the electron sampling fraction as a function of ϵ_e for 1 GeV electron showers entering lead–scintillator sampling calorimeters with the following absorber and readout thicknesses:

a) $t = 5.0$ mm, $s = 5.0$ mm,
b) $t = 0.2$ mm, $s = 5.0$ mm,
c) $t = 5.0$ mm, $s = 0.2$ mm.

We observe that e/mip depends on ϵ_e, especially for very thin readout layers, and therefore low electron cutoff energies have to be used in order to obtain precise results. A similar study can be performed with the photon cutoff energy. In the following we have always used the values $\epsilon_e = 711$ keV, corresponding to 200 keV electron kinetic energy, and $\epsilon_\gamma = 10$ keV.

The dependence on the step-size parameter ESTEPE can be studied in a similar way. A detailed analysis can be found in ref. [8]. In the following we have taken the recommended values [6] of 0.3% and 1% for ESTEPE in the absorber and readout media, respectively.

4. Energy dependence

Since the mip sampling fraction is an energy-independent quantity, the energy dependence of e/mip reflects the energy dependence of the electron sampling fraction e and the linearity of the calorimeter response.

We have generated electron showers in an infinitely large lead–scintillator calorimeter with absorber and readout thicknesses $t = 5$ mm and $s = 5$ mm, the energy ranging from 10 MeV to 100 GeV (see table 1 and fig. 3a). We observe a drop in the calorimeter response below energies around 500 MeV. Such an energy dependence is common to all sampling calorimeters irrespec-

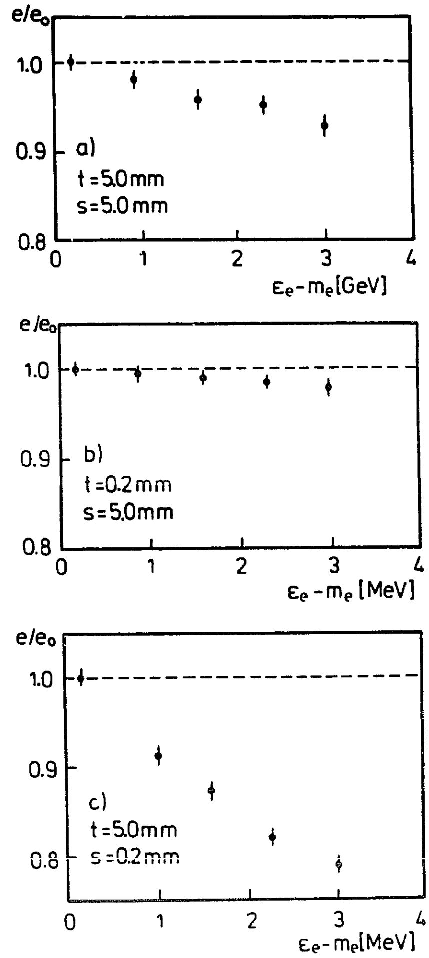

Fig. 2. Dependence of the electron sampling fraction e on the electron energy cutoff ϵ_e used in the Monte Carlo calculation. The electron sampling fraction is normalized to the value e_0 obtained for a cutoff $\epsilon_e - m_e = 200$ keV. Three sampling structures of lead-scintillator calorimeters are considered (a, b and c described in the figure). The electron energy is 1 GeV in all cases.

Table 1
Electron sampling fraction as a function of energy for a lead–scintillator calorimeter with absorber and readout layer thickness equal to 5 mm. The sampling fraction is normalized to the value obtained at 100 GeV

E [GeV]	e/e_{100} [%]
0.01	14.4±0.5
0.02	52.1±0.5
0.05	88.1±0.4
0.10	93.8±0.4
0.20	96.3±0.4
0.50	98.2±0.3
1.0	99.0±0.3
2.0	99.0±0.2
5.0	100.0±0.2
10.0	99.8±0.1
20.0	99.8±0.1
50.0	100.0±0.1

tive of absorber and readout thicknesses: at low enough energies the sampling fraction for electrons drops to zero since most energy is deposited in the first absorber layer, whereas at high enough energies it becomes constant. Of course, the energy above which the response becomes linear (or equivalently, the sampling fraction and e/mip are energy independent) depends strongly on the absorber thickness. In fig. 3b we indicate the drop in the electron sampling fraction as a function of incident energy for lead–scintillator calorimeters with absorber thickness between 0.5 and $5X_0$ (the radiation length for lead is $1X_0 = 5.6$ mm) and a constant readout thickness $s = 5$ mm. We conclude from these calculations that e/mip ratios can be safely computed at 1 GeV provided that the absorber thickness does not exceed 1 to $2X_0$.

Table 2
e/mip ratio as a function of absorber thickness for lead–scintillator calorimeters with readout layer thickness equal to 5 mm

t [mm]	e/mip
0.2	0.92±0.00
0.3	0.89±0.00
0.4	0.86±0.00
0.5	0.84±0.00
0.6	0.82±0.00
0.8	0.79±0.00
1.0	0.77±0.00
1.5	0.73±0.00
2.0	0.70±0.00
3.0	0.67±0.01
4.0	0.66±0.01
5.0	0.65±0.01
6.0	0.64±0.01
8.0	0.63±0.01
10.0	0.62±0.01
15.0	0.58±0.02
20.0	0.56±0.02
30.0	0.51±0.03
40.0	0.36±0.03
50.0	0.32±0.03
60.0	0.23±0.03
80.0	0.10±0.02

5. Thickness dependence

In order to investigate the thickness dependence of the transition effect, we have generated 1 GeV electron showers in lead–scintillator calorimeters with fixed readout thickness ($s = 5$ mm) and absorber thickness t varying from 0.2 to 100 mm. The result of these calculations is shown in table 2 and fig. 4a. For very thin absorber layers the transition effect is small, as expected

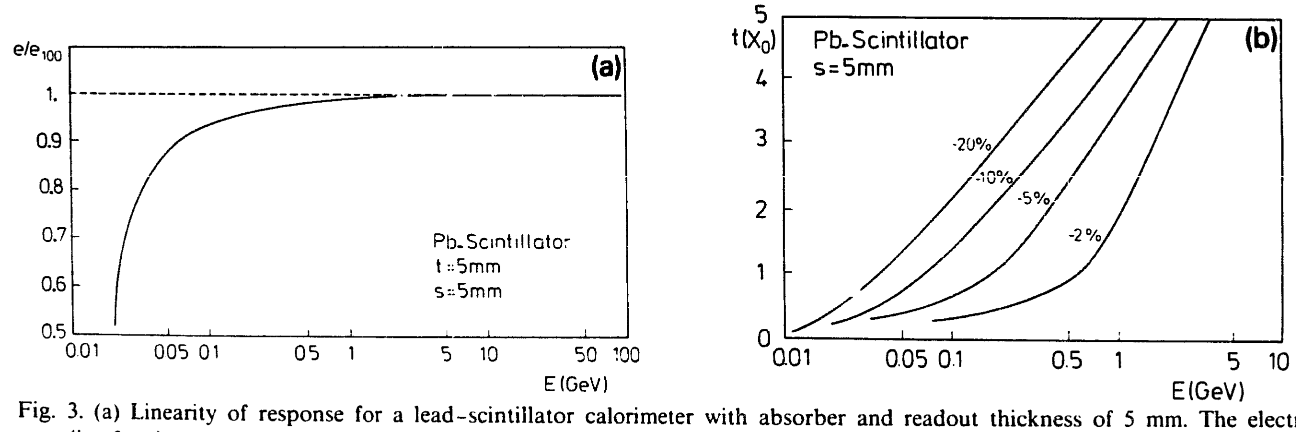

Fig. 3. (a) Linearity of response for a lead–scintillator calorimeter with absorber and readout thickness of 5 mm. The electron sampling fraction e normalized to the value obtained at 100 GeV, e_{100}, is plotted versus electron energy E in the range of 20 MeV to 100 GeV. (b) Reduction in electron sampling fraction for lead–scintillator calorimeter of absorber thickness t and readout thickness $s = 5$ mm as a function of the electron energy.

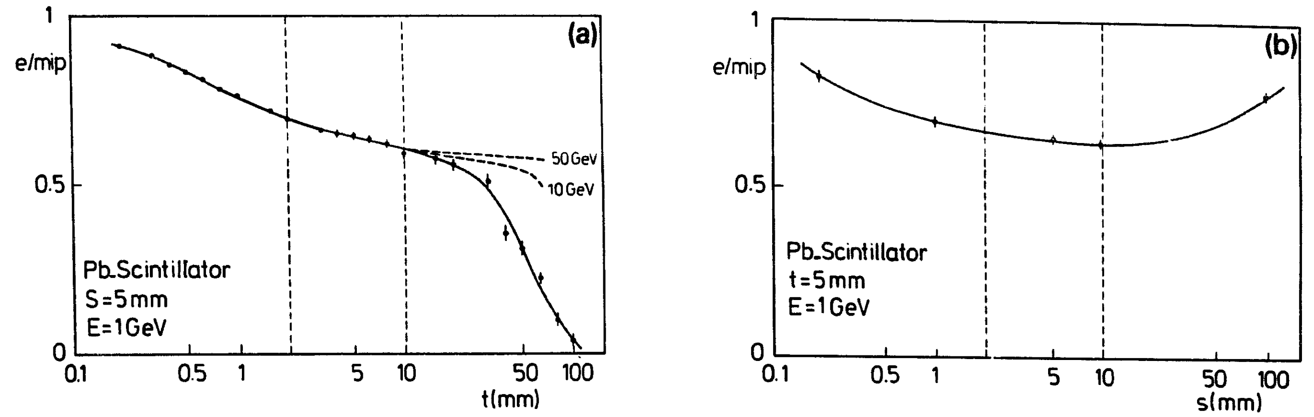

Fig. 4. (a) Absorber-thickness dependence of e/mip for 1 GeV electrons entering lead–scintillator calorimeters with fixed readout thickness $s = 5$ mm. The curves for 10 and 50 GeV are also indicated (dashed lines) for t above 10 mm. The region of practical interest for the absorber, between 2 and 10 mm, is also indicated. (b) Readout-thickness dependence of e/mip for 1 GeV electrons entering lead–scintillator calorimeters with fixed absorber thickness $t = 5$ mm. The region of practical interest for the readout, between 2 and 10 mm, is also indicated.

since low-energy electrons can easily escape from them. As the layer thickness increases, e/mip decreases, first logarithmically but finally in an abrupt way since the shower is increasingly absorbed by the first calorimeter layer. This effect observed for very thick absorber layers ($t > 4X_0$) can be called "geometrical transition effect" and is very energy dependent contrary to the "physical transition effect" discussed in section 2, which is rather energy independent as discussed previously. Most practical calorimeters have an absorber thickness varying from 2 to 10 mm. In this range the transition effect is not very thickness dependent since e/mip drops only from 0.7 to 0.6 (see fig. 4a). This smooth behaviour has been pointed out in previous Monte Carlo calculations [3,4] and confirmed experimentally [9].

The dependence of the transition effect on the readout thickness s is shown in fig. 4b where we plot e/mip calculated for a fixed lead absorber thickness ($t = 5$ mm) and a scintillator thickness ranging from 0.2 to 100 mm. We observe that the transition effect vanishes for both very thin and very thick readout layers. This result, surprising at first sight, can be explained in the following way (see also fig. 5). If the readout layers are very thin the shower develops in lead essentially undisturbed. As the scintillator layers increase in thickness, the low-energy electrons concentrate in the absorber giving rise to the transition effect as discussed in section 2. If the scintillator layers are very thick they also produce radiation and become sensitive to the low-energy component of the shower. Therefore a vanishing of the transition effect is expected for thicknesses such that $s/X_s > t/X_t$, X_t and X_s being the radiation length of absorber and readout respectively. This corresponds in our case to scintillator layers of 380 mm. For most practical calorimeters, s varies between 2 and 10 mm and the thickness dependence of the transition effect in this range is very small (see fig. 4b).

6. *Z* dependence

In order to investigate the Z dependence of the transition effect we have generated 1 GeV electron showers in calorimeters made of various absorbers and read out by 5 mm thick scintillator layers. We have considered the following thicknesses for the different absorbers: 0.2, 0.4 and $1X_0$. The e/mip values are listed in table 3a. The quantity $\text{mip}/e - 1$ is plotted versus the charge Z_t of the absorber in fig. 6a. We observe that the transition effect increases with Z_t as pointed out in previous publications [3,4]. If the absorber layers are thick enough, the quantity $\text{mip}/e - 1$ is proportional to Z_t, as suggested in section 2 for $Z_t \gg Z_s$.

We have also investigated the dependence of the transition effect on the charge number Z_s of the readout

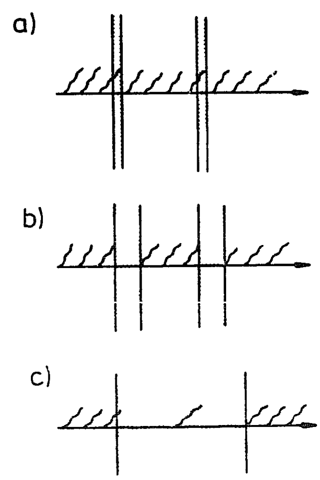

Fig. 5. Graphic explanation of the transition-effect dependence on the readout thickness. For very thin layers (a) the low-energy radiation produced in the absorber is not perturbed. For normal layers (b) the transition effect appears. For very thick layers (c) radiation is also produced in the readout material.

Table 3
e/mip ratios for calorimeters made of scintillator and various absorbers. Three values are considered for the thickness of the absorber

Absorber	Z_t	e/mip ($t = 0.2X_0$)	e/mip ($t = 0.4X_0$)	e/mip ($t = 1X_0$)
C	6	1.01±0.01	1.00±0.01	1.01±0.01
Al	13	0.93±0.01	0.92±0.01	0.89±0.01
Fe	26	0.90±0.01	0.88±0.01	0.85±0.01
Sn	50	0.81±0.01	0.77±0.01	0.74±0.01
W	74	0.81±0.01	0.75±0.01	0.66±0.01
Pb	82	0.77±0.01	0.70±0.01	0.64±0.01
U	92	0.78±0.01	0.70±0.01	0.62±0.01

material. To this purpose we have generated 1 GeV electron showers in lead calorimeters ($t = 0.5$ mm) read out by 0.5 mm thick layers of various materials, mainly noble liquids. The e/mip values resulting from these calculations are listed in table 4 and plotted in fig. 6b. These results suggest the following parametrization:

$$\text{mip}/e - 1 = a(Z_t - Z_s).$$

The parameter a is thickness dependent as seen in fig. 6c. For absorber layers of $1X_0$ the value $a \approx 0.007$ describes well the Monte Carlo data (see fig. 6a and 6b). If the absorber thickness is much smaller than $1X_0$ the parameter a becomes also dependent on Z_t as seen in fig. 6a. If it is much larger than $1X_0$, a becomes energy dependent as discussed in section 4.

7. Density dependence

For completeness we have studied the density dependence of the transition effect. We have first generated electron showers of 1 GeV in lead-scintillator calorimeters with a lead density varying from 1 to 20 g/cm^3. The absorber thickness t was kept at a fixed value of $1X_0$ for all densities and the scintillator thickness at 5 mm. The e/mip values calculated in this way are plotted versus lead density in fig. 7a. We do not observe any significant density effect as expected.

We have also generated 1 GeV electron showers in lead–argon calorimeters (t and s fixed at 5 mm) and the argon density varying from 0.1 to 10 g/cm^3. The result is shown in fig. 7b. We observe a decrease of the transition effect towards low densities. In fact, low-energy electrons in low-density media are far from being perfect mips: they can travel long distances in the readout gap and produce large energy depositions (Landau fluctuations), as discussed in the context of gaseous calorimeters [10]. It is, however, well known that by an adequate choice of the readout technique these effects can be minimized (e.g. by clipping of large pulses etc.). In fact, the e/mip values measured with

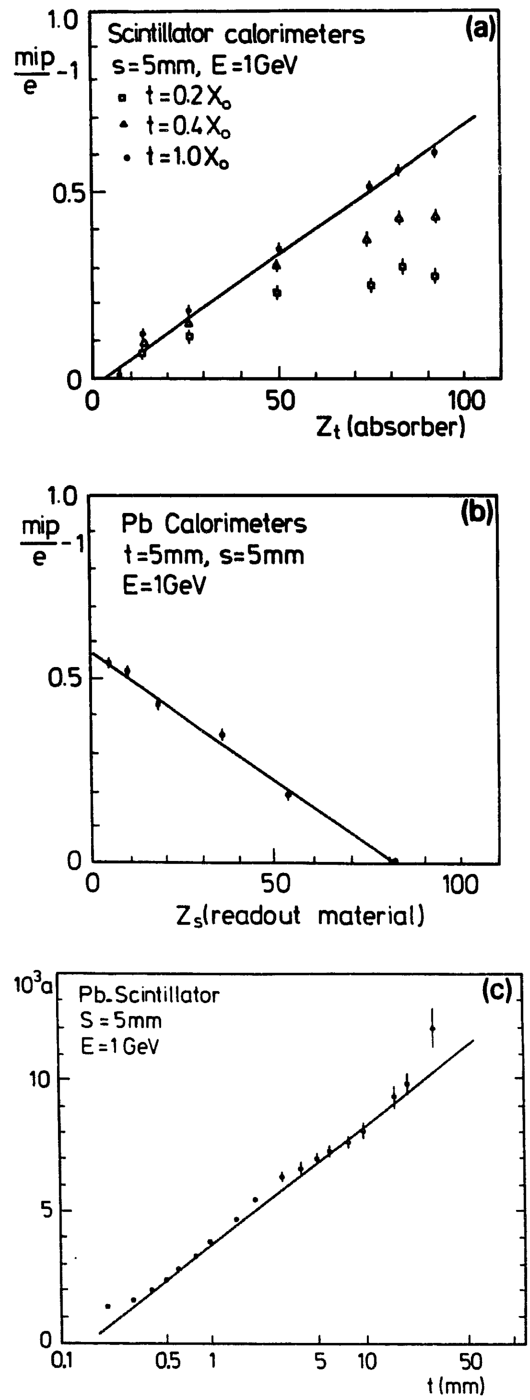

Fig. 6. (a) Dependence of e/mip on the charge number Z_t of the absorber. The electron energy is 1 GeV and the readout material scintillator of 5 mm thickness. Three absorber thicknesses are considered (0.2, 0.4 and $1X_0$). The formula mip/e − 1 = $a(Z_t - Z_s)$ with $a = 0.007$ is indicated in the figure. (b) Dependence of e/mip on the charge number Z_s of the readout. The electron energy is 1 GeV and the absorber material lead of 5 mm thickness. The readout thickness is also fixed at 5 mm. The formula mip/$e - 1 = a(Z_t - Z_s)$ with $a = 0.007$ is indicated in the figure. (c) Thickness dependence of the parameter a used to compute e/mip values. The e/mip values used are those presented in fig. 4a.

Table 4
e/mip ratios for calorimeters made of lead (5 mm layer thickness) and various readout materials

Readout	Z_s	e/mip (t = 5 mm)
Scintillator	3.4	0.65±0.01
Liquid Ne	10	0.66±0.01
Liquid Ar	18	0.70±0.01
Liquid Kr	36	0.74±0.01
Liquid Xe	54	0.85±0.01
Liquid Rn	86	1.03±0.01

gaseous readout do not reveal any suppression of the transition effect (see next section).

8. Experimental situation

In order to obtain experimentally e/mip values, it is necessary to measure the electron response and in addition some other quantity which provides the energy scale. This quantity is usually the average muon response or the most probable energy deposition of muons. Both quantities can be calculated theoretically and compared to the measurements. The analytical calculations can be done very accurately for homogeneous media [11], but are difficult to apply to sampling calorimeters

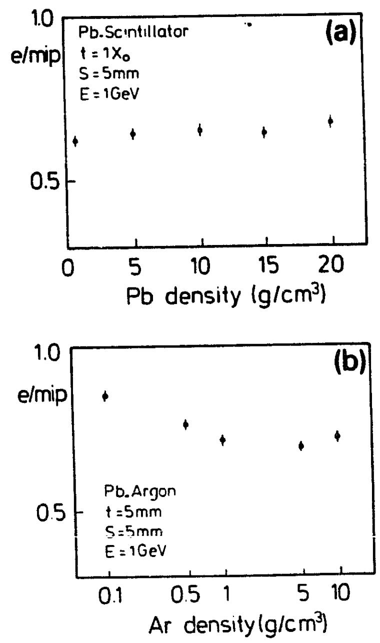

Fig. 7. (a) Dependence of e/mip on the absorber density. The electron energy is 1 GeV. The absorber is lead of $1X_0$ thickness and varying density. The readout is scintillator of 5 mm thickness. (b) Dependence of e/mip on the readout density. The electron energy is 1 GeV. The absorber is lead of 5 mm thickness. The readout is argon of 5 mm thickness and varying density. Note that the density scale is logarithmic.

Table 5
Selection of experimental e/mip values obtained by any of the various methods employing muons. The error is quoted whenever available in the original reference.

Absorber	Readout	t [mm]	s [mm]	e/mip	Ref.
Al	liquid argon	1.0	3.0	1.00	[13]
Fe	liquid argon	1.5	2.0	0.90	[14]
Cu	scintillator	5.0	2.5	0.84±0.05	[15]
Pb	scintillator	2.1	6.3	0.51	[16]
Pb	liquid argon	3.0	2.4	0.54	[17]
Pb	scintillator	5.0	5.0	0.65±0.10	[18]
Pb	scintillator	6.4	6.0	0.77	[19]
Pb	scintillator	10.0	2.5	0.67±0.03	[20]
U	liquid argon	1.6	4.4	0.51	[17]
U	liquid argon	1.7	2.0	0.61	[14]
U	scintillator	3.0	2.5	0.70±0.05	[21]
U	scintillator	3.0	2.5	0.65±0.03	[12]
U	scintillator	3.2	3.0	0.74±0.03	[18]
U	scintillator	3.2	5.0	0.68±0.04	[18]
U	liquid argon	4.0	3.2	0.50±0.05	[22]
U	gas	4.5	6.0	0.65±0.04	[23]
U	scintillator	10.0	5.0	0.61	[24]

due to the following effects: on one hand energetic δ-rays emitted by the muon in the readout can be stopped in the absorber and on the other hand the bremsstrahlung and δ-rays emitted by the muon in the absorber may leak into the readout medium. To account for these effects a detailed Monte Carlo simulation of the calorimeter response to muons is necessary. To our knowledge such a detailed study has only been performed in ref. [12] using the muon tracking Monte Carlo MUDEX coupled to EGS4. All other experimental values of e/mip have been extracted using theoretical approximations and suffer therefore from systematic errors in the determination of the energy scale. These errors are typically 5 to 10%. We present in table 5 a selection of experimental e/mip values obtained by various methods based on the response to muons. A comparison between all these values should not be done without care due to the variety of employed methods. We can however conclude that a clear Z dependence is observed. However, no clear thickness dependence can be deduced except in the dedicated measurements presented in ref. [9].

9. Summary

According to Monte Carlo calculations using the EGS4 shower generator, the sampling fraction of electrons in sandwich calorimeters can be described for energies above 1 GeV by

$$\frac{e}{\text{mip}} = \frac{1}{1 + a(Z_t - Z_s)},$$

where mip is the sampling fraction of minimum ionizing particles and Z_t and Z_s are the charge numbers of the absorber and readout media respectively. The parameter a depends logarithmically on the absorber thickness t and in particular $a \approx 0.007$ for $t = 1X_0$. If $t \ll 1X_0$, a becomes also dependent on Z_t and if $t \gg 1X_0$, a becomes energy dependent. The thickness of the readout has very little influence on the transiton effect for practical calorimeters. Its density might have some influence: in the case of low-density active media (like gases) a detailed simulation of the readout technique is necessary. Finally, the experimental values provide a strong indication in favour of the Z_t dependence of the transition effect but are still not precise enough to confirm the dependence on the absorber thickness.

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