Unquenched investigation of fermion masses in a chiral fermion theory on the lattice ☆

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Using dynamical fermions we study the decoupling of the fermion doublers of chiral lattice fermions in the broken symmetry phase of a scalar–fermion model. The model uses a chiral invariant Wilson term, called a Wilson–Yukawa term with coupling strength w, in addition to the usual Yukawa coupling y. We find qualitative agreement with our results obtained before in the quenched approximation. First in the $w=0$ case we establish a strong Yukawa coupling region. Then for relatively large but fixed w we confirm that the fermion doublers can be easily decoupled by giving them masses of the order of the cutoff as the symmetry restoring transition is approached.

1. Introduction

Because of the well-known fermion doubling, lattice regularization of chiral gauge theories has proved very difficult. On the lattice the doublers of chiral fermions spoil the chiral couplings in these theories. The central issue, therefore, is how to remove these unwanted doublers so that the theory is left only with light physical fermions in its fermion spectrum.

In two publications [1,2] last year we have persued a nonperturbative investigation of a proposal [3] for such a regularization. The scheme essentially relies on the Wilson method [4], now employed in a manifestly chiral invariant way with a scalar field in the so-called Wilson–Yukawa term such that the doublers are rendered heavy dynamically. This makes the problem of decoupling the doublers intrinsically non-perturbative, in spite of the compelling fact that the physical fermions have to remain light.

Our initial investigations [1,2] did not involve the gauge field dynamics and looked at a chiral $SU(2)_L \otimes SU(2)_R$ invariant scalar–fermion model neglecting fermion loops (quenched approximation). With these approximations we were able to show, as the critical region was approached in the broken symmetry phase, that the physical fermions remained arbitrarily light while the doublers obtained masses of the order of the cut-off [O(1) in lattice units] and thus got decoupled.

In the context of the standard model of electroweak interactions it is presumably sufficient to treat the gauge fields only perturbatively. Radially frozen scalar fields explicitly present in this chiral-invariant formulation of the Wilson mechanism can, nevertheless, be interpreted as gauge degrees of freedom [3].

With the optimism raised by our results in the quenched approximation it is now important to extend the calculations to the case with fermion dynamics. In a recent work [5,6] we have determined the phase diagram of the unquenched model and have located the possible regions of physical interest in the bare coupling parameter space. In the present work we investigate the decoupling of the doublers in the broken phase at relatively strong Wilson–Yukawa coupling.

2. The model

The investigated model on the euclidean lattice is given by the action $S = S_H + S_F$, with

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$$S_{\mathrm{H}}=-\kappa\sum_{x\mu}\tfrac{1}{2}\,\mathrm{Tr}(\Phi_x^\dagger\Phi_{x+\hat{\mu}}+\Phi_{x+\hat{\mu}}^\dagger\Phi_x)\,, \tag{1}$$

$$\begin{aligned}S_{\mathrm{F}}=&\sum_{x\mu}\tfrac{1}{2}[\bar{\Psi}_x\gamma_\mu\Psi_{x+\hat{\mu}}-\bar{\Psi}_{x+\hat{\mu}}\gamma_\mu\Psi_x]\\&+y\sum_x\bar{\Psi}_x(\Phi_xP_{\mathrm{R}}+\Phi_x^\dagger P_{\mathrm{L}})\Psi_x\\&+w\sum_{x\mu}\{\bar{\Psi}_x(\Phi_xP_{\mathrm{R}}+\Phi_x^\dagger P_{\mathrm{L}})\Psi_x\\&-\tfrac{1}{2}[\bar{\Psi}_x(\Phi_xP_{\mathrm{R}}+\Phi_{x+\hat{\mu}}^\dagger P_{\mathrm{L}})\Psi_{x+\hat{\mu}}\\&+\bar{\Psi}_{x+\hat{\mu}}(\Phi_{x+\hat{\mu}}P_{\mathrm{R}}+\Phi_x^\dagger P_{\mathrm{L}})\Psi_x]\}\,.\end{aligned} \tag{2}$$

Here the scalar field Φ_x is radially frozen (the bare quartic self-coupling is infinite) and it is a 2×2 SU(2) matrix, the fermion fields Ψ_x and $\bar{\Psi}_x$ are SU(2) doublets (a summation over two identical doublets is implied), κ is the hopping parameter for the scalar field, y is the usual Yukawa coupling, w denotes the Wilson–Yukawa coupling and $P_{\mathrm{L,R}}$ are left and right handed chiral projectors. From the experience with the pure Φ^4 theories and the related scalar-fermion models [7], we expect that the present model with infinite bare quartic coupling of the scalars is in the same universality class as models with finite quartic coupling.

The action is invariant under the global chiral $\mathrm{SU}(2)_{\mathrm{L}}\otimes\mathrm{SU}(2)_{\mathrm{R}}$ transformations:

$$\Psi\to(\Omega_{\mathrm{L}}P_{\mathrm{L}}+\Omega_{\mathrm{R}}P_{\mathrm{R}})\Psi\,,\quad \bar{\Psi}\to\bar{\Psi}(\Omega_{\mathrm{L}}^\dagger P_{\mathrm{R}}+\Omega_{\mathrm{R}}^\dagger P_{\mathrm{L}})\,,$$
$$\Phi\to\Omega_{\mathrm{L}}\Phi\Omega_{\mathrm{R}}^\dagger\,, \tag{3}$$

where $\Omega_{\mathrm{L,R}}\in\mathrm{SU}(2)_{\mathrm{L,R}}$.

When written in terms of the fermion fields $\Psi'_x=(\Phi_x^\dagger P_{\mathrm{L}}+P_{\mathrm{R}})\Psi_x$, $\bar{\Psi}'_x=\bar{\Psi}_x(\Phi_xP_{\mathrm{R}}+P_{\mathrm{L}})$, which transform as $\mathbb{1}_{\mathrm{L}}\otimes\mathrm{SU}(2)_{\mathrm{R}}$, the Wilson–Yukawa coupling takes the form of the Wilson mass term familiar from QCD. In the broken phase the masses of the Ψ and Ψ' fields are equal as they have the same quantum numbers under the residual symmetry group $\mathrm{SU}(2)_{\mathrm{L=R}}$.

The model (1, 2) is reduced from a lattice formulation of the full standard model [3] by leaving out the $\mathrm{SU}(2)\otimes\mathrm{U}(1)$ gauge fields and specializing to just two doublets [which may have arbitrary U(1) hypercharge]. For $y=0$ the model has an additional symmetry, called Golterman–Petcher (GP) symmetry, which guarantees that the Ψ' fermion mass is zero and that the right-handed fermion decouples in the scaling region [8]. One useful feature of the GP symmetry is that it predicts the critical value of a fermionic hopping parameter. One can of course formulate the standard model on the lattice in a different way such that the symmetry does not prevail any more in its reduction to a fermion–Higgs system, and then tune y by hand to achieve vanishing fermion mass. Getting scaling values of physical fermion masses and decoupling the spurious doublers by making them heavy is independent of the GP symmetry and uses a special behaviour of fermion masses in the strong coupling region ($y+4w\gg\sqrt{2}$) as already stressed before [2,9] and confirmed in this work. Decoupling of the right-handed fermion (neutrino) at zero fermion mass is a different issue and is also expected [3] to hold in models without the GP symmetry.

One can in principle drop the hopping term S_{H} for the scalar field and we shall show some results for $\kappa=0$. Because the scalar hopping term is generated anyway via the fermion determinant, the theory for $\kappa=0$ is not any different from the $\kappa\neq0$ case, provided the intended scaling regions are accessible. But then the only way to tune the theory toward regions of criticality is essentially through w (y has to be kept quite small for light fermions) which takes away the irrelevance of w.

3. The phase diagram

In fig. 1 we show a schematic phase diagram of the model with dynamical fermions at $w=0$ [5]. Briefly we describe the properties of the different phases in terms of the order parameters $v\mathbb{1}=\langle\Phi_x\rangle$ and $v_{\mathrm{st}}\mathbb{1}=\langle\epsilon_x\Phi_x\rangle$ where $\epsilon_x=(-1)^{(x_1+x_2+x_3+x_4)}$. We find two symmetric or paramagnetic phases ($v=v_{\mathrm{st}}=0$): PMW and PMS with massless and massive fermions respectively and the following broken symmetry phases: ferromagnetic FM ($v\neq0$, $v_{\mathrm{st}}=0$), antiferromagnetic AM ($v=0$, $v_{\mathrm{st}}\neq0$) and ferrimagnetic FI ($v\neq0$, $v_{\mathrm{st}}\neq0$). The two regions AM(W) and AM(S) in the AM phase are separated by the FI phase for a wide range of κ and numerically we could not locate where, if at all, the FI phase ends. In the quenched approximation [1] at $w=0$ the FM phase is split by a crossover around $y\approx\sqrt{2}$ into two regions FM(W) and FM(S) where all the fermion masses decrease or increase respectively as $v\searrow0$. Such a crossover was

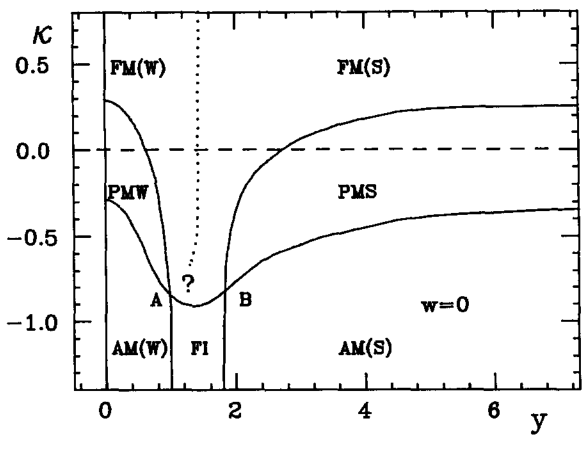

Fig. 1. The phase diagram at $w=0$. The different phases and phase regions are described in the text. We have indicated the crossover by the dotted line. A and B are probable quadruple points where 4 phases meet.

first found in the quenched approximation in a simpler scalar–fermion model [10]. We show in this work that a similar crossover exists also in the unquenched case of our model. The dotted line in fig. 1 indicates its approximate position.

For $w>0$ the phase diagram is approximately given by a part of fig. 1 with the zero of the Yukawa coupling (y) axis shifted to $4w$ in the positive y direction [5]. If one chooses a small value of w, e.g., $\leqslant 0.15$, one still has all the phases and regions as in fig. 1 with the crossover in the FM phase at $y+4w\approx\sqrt{2}$. But for $w\geqslant 0.5$, the crossover and also the funnel-like structure of fig. 1 disappears to the left and there are only the FM(S), PMS and AM(S) phases or phase regions for $y\geqslant 0$.

For $w=0$ the various phase transition lines meet, within our precision of their localization, in 2 quadruple points (A and B in fig. 1). In the 3-dimensional phase diagram with $w>0$ these points become lines which we call lines A, B.

For $w=0$ the coordinates of the points A and B are $\{y\approx 1.0,\ \kappa\approx -0.85\}$ and $\{y\approx 1.9,\ \kappa\approx -0.80\}$ respectively. On the $\kappa=0$ line, again for $w=0$, the FM(W)–PMW transition occurs at $y\approx 0.62$ and the FM(S)–PMS transition at $y\approx 2.75$.

4. Some regions of interest

The scaling regions of the FM phase are the most natural choice for physics of the standard model. One has to approach the phase transitions where $v\searrow 0$ (v/a is a physical scale, a being the lattice constant). The FM–FI transition is immediately ruled out because v does not go to zero there.

As we will see below, the FM(W) scaling region bounded on the right by the possible quadruple line A (fig. 1) is the weak Yukawa coupling region ($y+4w\ll\sqrt{2}$) where perturbative calculations apply leading to $m_F\approx yv$ and $m_D\approx(y+2nw)v$ where m_F and m_D denote the physical fermion and the doubler masses respectively and $n=1, 2, 3, 4$ is the number of momentum components equal to π in lattice units. Since $v\searrow 0$ as the FM(W)–PMW transition is approached from within the broken phase, the doublers cannot be completely decoupled there, unless the crossover in the FM phase near the FM–FI transition bends and approaches the line A or sufficiently influences it so that by tuning toward A from the weak coupling side the low energy phenomenology remains practically unaffected and devoid of doublers. We have tried to locate the crossover with respect to A at $w=0$. Keeping in mind the approximate shift of the phase diagram for $w>0$, we find indications of the possibility that the crossover may approach the line A but a definite conclusion is difficult because of the need to know the position of A very precisely and investigate very close to it.

In this work, however, we concentrate on the strong Yukawa coupling region which is very suitable for the purpose of decoupling the doublers. In the FM(S) region for $y+4w\gg\sqrt{2}$ a hopping parameter expansion for the fermion propagator is appropriate. Introducing a mean field for the product $\Phi_x^\dagger\Phi_{x+\mu}$, this leads to $m_F\approx yz^{-1}$ and $m_D\approx(y+2nw)z^{-1}$ where $z^2=\frac{1}{2}\langle \mathrm{Tr}\,\Phi_x^\dagger\Phi_{x+\mu}\rangle$ is the link expectation value [9]. As the FM(S)–PMS transition is approached, $v\searrow 0$ and z also decreases though staying finite at the transition. As a result m_F and m_D increase at fixed y and w.

With a choice of $w\geqslant 0.5$ one is in the FM(S) region for any $y\geqslant 0$ and then choosing y small one can have arbitrarily light physical fermions while the doublers can have masses of the order of the cut-off since z^{-1} is O(1) at the FM(S)–PMS phase transition.

The PMS phase also has massive fermions. The Ψ' fermion mass follows the approximate z^{-1} behaviour into the PMS phase [6]. This phase could be of great interest for the asymptotically free chiral gauge theories [3]. We postpone its discussion for a future publication.

5. Details of the simulation

Because of the pseudoreality of the SU(2) group the fermion determinant is real. This enables us to use the Hybrid Monte Carlo algorithm with two identical fermion doublets. We have used a $6^3\times12$ lattice with periodic boundary conditions everywhere except for antiperiodic ones for the fermion fields in the euclidean time direction.

We analyzed our results of the $SU(2)_{L,R}$ invariant propagators $\mathrm{Tr}_{SU(2)}\langle \Psi_{L,R}\bar{\Psi}_{L,R}\rangle$ in terms of effective free fermion energy formulas. The time dependence of the L–L or R–R part of the free Wilson fermion propagator at spatial momentum components 0, π with chiral representation of the γ-matrices is given by

$$\frac{Z_{L,R}}{2\sqrt{1+2rm_n+m_n^2}} \times\left(\frac{\exp(-E_+t)-\eta\exp[-E_+(N_t-t)]}{1-\eta\exp(-E_+N_t)} -(-1)^t\frac{\exp(-E_-t)-\eta\exp[-E_-(N_t-t)]}{1-\eta\exp(-E_-N_t)}\right), \tag{4}$$

where $Z_{L,R}=1$, E_+ and E_- are the rest energies of the fermion and its "time doubler", respectively, and $\eta=1(-1)$ for periodic (antiperiodic) boundary conditions in time. The energies are related to the mass parameters by

$$E_{\pm,n}=\ln\left(\frac{\sqrt{1+2rm_n+m_n^2}+r+m_n}{1\pm r}\right), \tag{5}$$

with $m_n=m+2rn$ and n the number of components of the spatial momentum equal to π. From fits of the fermion propagator to the expression (4) we determined $E_{\pm,n}$ for $n=0,1$ and from this m_n and r_n using (5), allowing r to depend on n. In the following we use the notation $m_F\equiv m_0$, $m_D\equiv m_1$. It turned out that within errors $r_0\approx r_1$ and $m_D\approx m_F+2r_{0,1}$, which supports the analysis in terms of a free fermion propagator.

6. Fermion masses at $w=0$

Fig. 2 shows m_F as a function of v with $w=0$ for various fixed $y=0.2$–2.0 on a $6^3\times12$ lattice. For κ deep in the FM phase corresponding to $v\geqslant0.5$ there is a crossover around $y\approx1.4$–1.5 where m_F is almost independent of v. For $y<1.4$, m_F clearly decreases with decreasing v while for $y>1.5$ the behaviour is opposite.

As κ is gradually reduced to approach the FM(W)–PMW, FM–FI and FM(S)–PMS phase transitions, the v-dependence of m_F for fixed y becomes more complex. As can be deduced from fig. 2 the crossover position seems to depend appreciably on κ (or v). For $y=0.2$–0.8 as the FM(W)–PMW transition is approached, v goes to zero and so does m_F. The perturbative tree relation $m_F=yv$ is fulfilled quite well for sufficiently small v. For $y=2.0$ and higher, the FM(S)–PMS phase transition is approached and the z^{-1} behaviour of the mass is approximately valid for sufficiently large z^{-1} (i.e., sufficiently close to this transition). For $y=1.0$–1.7 the FM–FI transition is encountered and as already

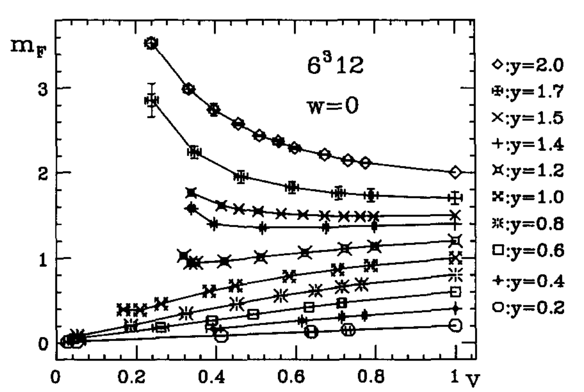

Fig. 2. Fermion mass in dependence of the scalar field vacuum expectation value v at $w=0$ for different values of y in a range from 0.2 up to 2.0. For $y=1.0$, 1.2, 1.4, 1.5 and 1.7 v stays nonzero as the FM–FI phase transition line is approached.

pointed out v does not go to zero there. For $y=1.0$, 1.2, 1.4 at moderate values of v, m_F decreases first with decreasing v, then very close to the transition at $\kappa=-0.7$ and smaller it increases as v further decreases. This indicates that the crossover may shift from around $y\approx1.4$ deep in FM phase to around $y\approx1.0$ at the FM–FI transition. This would be very close to the point A in fig. 1. If the crossover approaches the line A in a similar way also for nonzero w, there is a good chance that by tuning toward A from the weak couplings side (approximately $y\to0$, $w\to0.25$, $\kappa\to-0.85$) one may achieve arbitrarily light physical fermions and very heavy doublers.

7. Fermion masses with $w>0$

Having now established the crossover and the FM(S) region at $w=0$ in the full model with dynamical fermions, we now consider the $w\neq0$ case and try to decouple the doublers using the typical strong coupling mass behaviour, viz., fermion masses increase with decreasing v. We approach the FM(S)–PMS phase transition where $v\searrow0$ and simultaneously have small y to allow for light physical fermions. At this transition we have looked at the distributions of v and z^2 over a modest number of configurations and failed to detect metastable states. Actually our preliminary results of the scalar mass on a 8^4 lattice show that it is close to the corresponding value (for the same v) in the pure O(4) scalar theory which is known to have a second order transition. The above preliminary checks on the order of the transition are so far carried out only at $w=0$. For $w>0$ we assume it to be a continuous transition although more investigation is certainly needed.

A simple possibility to decouple the doublers is to fix y and κ at appropriate values in the FM phase and increase w. If y is small, m_F stays small while m_D increases with w. Fig. 3 shows the physical fermion mass m_F and the first doubler ($n=1$) mass m_D at $\kappa=0$ and $y=0.1$ as functions of w on a $6^3\times12$ lattice. At $w=0.12$–0.13, i.e., at the FM(W)–PMW transition, both masses are consistent with zero. Increasing w we find an almost negligible increase of m_F while m_D increases very rapidly. At $w=0.7$, i.e., very near to FM(S)–PMS transition, m_F is still around 0.1 and $m_D\gg1$, showing a clear indication of decoupling of

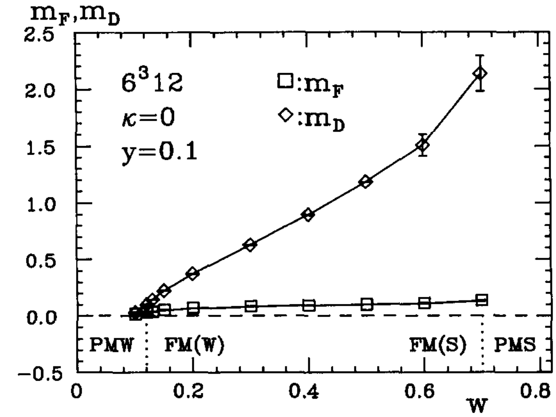

Fig. 3. The masses m_F and m_D as a function of w for $y=0.1$ and $\kappa=0$ in the FM phase.

the doublers in the FM(S) scaling region.

In the following we present a more detailed fermion mass calculation at $w=0.5$. Because of the above mentioned shift of the phase diagram for $w>0$, the $y=0$ sheet will now be approximately at the position $y=4w=2.0$ in fig. 1 so that the FM(S)–PMS phase transition can be approached by decreasing κ in the FM(S) region with arbitrarily small values of y.

Fig. 4a shows m_F and m_D as functions of v at several values of y from 0.8 down to 0.2 with fixed $w=0.5$ on a $6^3\times12$ lattice. Both m_F and m_D increase as v decreases. Lowering y means a smaller m_F but m_D always remains substantially above the cut-off. In fig. 4b we have translated the data of fig. 4a by a simple interpolation into a plot of m_F and m_D as functions of y for several fixed values of v. In this figure it is more apparent how m_F approaches zero as $y\to0$ according to ref. [8] while m_D remains much above the cut-off even very close to the transition and thus the doublers get completely decoupled.

8. Conclusion

In the FM(S) region the fermion doublers can easily be given O(1) masses with scaling values for the fermion masses as $v\searrow0$ at the FM(S)–PMS phase transition. Our results favour strongly the possibility of a lattice regularized theory with chirally coupled fermions. They have to be extended to larger lattices to estimate finite size effects which are certainly pres-

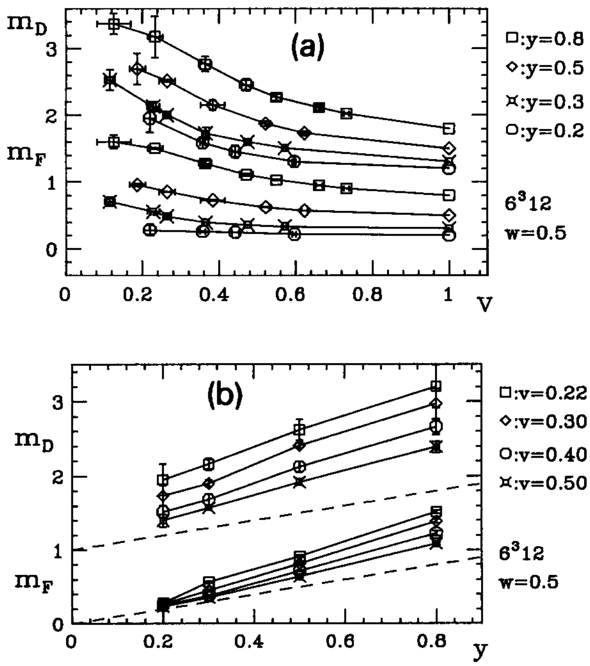

Fig. 4. (a) The masses m_F and m_D displayed as a function of v at $w=0.5$ for various values of y. (b) The masses m_F and m_D interpolated from fig. 4a shown in dependence of y for fixed values of v. The dashed straight lines correspond to $\kappa=\infty$.

ent in our data. Our previous quenched results [1,2] are in qualitative agreement with the unquenched results presented here. A recent quenched [11] and previous exploratory unquenched analyses [7] in related chiral models confirm our results and support the view in ref. [3]. Getting renormalized Yukawa couplings from the lattice and its conformity with perturbation theory in the continuum for the lowest lying spectrum is very important and is presently under investigation.

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