Anomalous aspects of chiral quantum gravity in two dimensions

Thilo Berger* and Izumi Tsutsui†
II. Institut für Theoretische Physik der Universität Hamburg, D-2000 Hamburg 50, Federal Republic of Germany
(Received 5 November 1990)

We study anomalous aspects of a theory in which a set of chiral fermions is coupled to gravity in two dimensions. We first give a perturbative derivation of the effective action in the conformal gauge. From this the complete fermion propagator is obtained in the path-integral formalism, and also from operator solutions. The fermion is shown to acquire an anomalous dimension purely by quantum effects, i.e., through interactions with scalars which are dynamical only after quantization. The renormalization of the theory is also discussed.

I. INTRODUCTION

Quantization of gauge theories which are afflicted with gauge anomalies is known to be problematical. Several years ago, however, Faddeev and Shatashvili[1] proposed a possible framework to overcome the difficulty by adding the Wess-Zumino action to the original system in order to cancel the anomaly. Independently, Jackiw and Rajaraman[2] showed that the chiral Schwinger model, where a chiral fermion couples to a U(1) field in two dimensions, yields a consistent unitary theory, although it is anomalous. The common rule for the successful quantization in both cases is that one should consider the gauge degrees of freedom as dynamical ones; either they are explicitly dealt with in the Wess-Zumino action[1] (gauge-invariant formulation) or their effect is implicitly accounted for by integrating them out[2] (anomalous formulation). Actually, both formulations have been shown to be equivalent.[3] Since then, many efforts have been made to quantize other anomalous models. For instance, Li[4] considered a theory of gravity coupled to a set of chiral fermions in two dimensions. However, until recently the chiral Schwinger model (and its generalized versions) had been the only soluble, consistent anomalous gauge theory.

The situation changed when Li's theory was reconsidered[5] as a theory of two-dimensional quantum gravity based on the idea of Polyakov.[6] It has been shown in Ref. 5 that although chiral quantum gravity is afflicted with both gravitational and Weyl anomalies, the theory is unitary if the number of fermions of each chirality is less than or equal to 24. The constraint on the number of chiral fermions is also crucial for closing the deformation algebra, which is necessary to set up the physical space consistently.[7] Moreover, it has been found that in the conformal gauge the effective action of chiral quantum gravity has a structure analogous to that of the chiral Schwinger model. This unexpected discovery motivated us to investigate further chiral quantum gravity in order to reveal common features with the chiral Schwinger model, and thereby elucidate some characteristic aspects of anomalous gauge theories.

For this purpose, we discuss in Sec. II the sources of the anomalies in chiral quantum gravity by presenting a perturbative derivation of the effective action in the conformal gauge. In Sec. III we obtain the complete fermion propagator in the path-integral formalism. The propagator exhibits an anomalous dimension that is common not only for the chiral Schwinger model, but also for some other soluble models in two dimensions, e.g., the Thirring model[8] and the Schroer model.[9] Operator solutions for the fermion, which reproduce the complete propagator, are also briefly discussed. To realize the specific feature inherent in chiral quantum gravity which discriminates it from nonanomalous models, a perturbative derivation of the propagator is presented in Sec. IV. It is then shown that induced massless scalars, which become dynamical only at the quantum level, are responsible for the anomalous dimension. Furthermore, the wave-function renormalization constant is shown to be equal to the vertex renormalization constant, which usually follows from the Ward-Takahashi identity for nonanomalous gauge theories. This equality ensures that there is no renormalization for the gravitational coupling with the fermion. Section V is devoted to the conclusion and discussion. In the Appendix we provide a detailed calculation of the effective action discussed in Sec. III.

II. EFFECTIVE ACTION $I_{\rm eff}$

In this section we discuss how the effective action of chiral quantum gravity is constructed from anomalies. In the conformal gauge we can do this in a way parallel to the one discussed for the chiral Schwinger model[2,10] so as to compare the two models easily.

To begin with, we recapitulate what is known about chiral quantum gravity,[5] where a set of chiral fermions ψ (n_R right-handed and n_L left-handed fermions) interacts with gravity (zweibein e^a_μ) through the action

$$I_F=\int dx\sqrt{-g}\,\frac{i}{2}e^\mu_a(\bar\psi\gamma^a\overleftrightarrow{\partial}_\mu\psi)\ . \tag{2.1}$$

[Notation: $\gamma^0=\sigma_1$, $\gamma^1=i\sigma_2$, $\gamma^5=-\gamma^0\gamma^1=\sigma_3$, $\epsilon^{01}=1$, $\gamma^5\psi_{R,L}=\pm\psi_{R,L}$, $x^\pm=(1/\sqrt2)(x^0\pm x^1)$.] The effective action $I_{\rm eff}=-i\ln\int d\psi\, d\bar\psi\, e^{iI_F}$ has been obtained by Leutwyler[11] by means of heat-kernel regularization for the fermion determinant:

$$I_{\text{eff}}=\frac{1}{48\pi}\int dx\left[\tfrac{1}{4}\sqrt{-g}R\frac{1}{\sqrt{-g}\nabla^2}(\alpha\sqrt{-g}R+\beta\sqrt{-g}g^{\mu\nu}\nabla_\mu\omega_\nu)+\mu\sqrt{-g}+\frac{a}{2}\sqrt{-g}g^{\mu\nu}\omega_\mu\omega_\nu\right], \tag{2.2}$$

where $\alpha=n_R+n_L$, $\beta=n_R-n_L$, and two arbitrary parameters μ and a represent regularization ambiguities. Here we define $\omega_\mu=\epsilon^{ab}\omega_{\mu,ab}=\epsilon^{ab}e_a^\nu\nabla_\mu\epsilon_{b\nu}$, with $\omega_{\mu,ab}$ being the spin connection. In order to quantize the gravitational sector in the path-integral formulation, we consider an integration over the zweibein modulo the diffeomorphism volume, de_μ^a/V_{diff}. Then it may be effectively replaced by integrations of the Weyl and the Lorentz degrees of freedom, ϕ and F, which are the variables of the zweibein in the conformal gauge:

$$g_{\mu\nu}=e^\phi\eta_{\mu\nu},\quad e_\mu^a=e^{\phi/2}\begin{bmatrix}\cosh\frac{F}{2} & -\sinh\frac{F}{2}\\ -\sinh\frac{F}{2} & \cosh\frac{F}{2}\end{bmatrix}. \tag{2.3}$$

Taking into account the Weyl anomaly associated with the conformal gauge fixing amounts to adding (24 times) the Liouville action I_L,

$$Z=\int\frac{de_\mu^a}{V_{\text{diff}}}e^{iI_{\text{eff}}}=\int d\phi\,dF\,e^{iI_{\text{eff}}+24I_L}. \tag{2.4}$$

(The Einstein-Hilbert gravity action is a topological invariant in two dimensions and can be disregarded.) The total effective action $I_T=I_{\text{eff}}+24I_L$ can be diagonalized and reads

$$I_T=\frac{1}{48\pi}\int dx\left[\frac{b}{2}(\partial_\mu\phi)^2+\mu e^\phi+\frac{a}{2}(\partial_\mu\tilde{F})^2\right], \tag{2.5}$$

with

$$\tilde{F}=F+\frac{\beta}{4a}\phi,\quad b=\left[24-\frac{\alpha}{2}-a\right]-\frac{1}{a}\left[\frac{\beta}{4}\right]^2. \tag{2.6}$$

In the simplest case where the arbitrary parameter μ is chosen to be zero, one finds that the theory admits two massless scalars ϕ and $\tilde{F}$. They have positive-norm states if the number of each of the chiral fermions satisfies

$$n_R\le 24,\quad n_L\le 24. \tag{2.7}$$

By the unitarity requirement the arbitrary parameter a is limited by $(m_A-m_G)/2\le a\le(m_A+m_G)/2$, where m_A and m_G are the arithmetic and the geometric means of $24-n_R$ and $24-n_L$, respectively: $m_A=[(24-n_R)+(24-n_L)]/2$, $m_G=\sqrt{(24-n_R)(24-n_L)}$.

Now we show how anomalies related to symmetries inherent in the classical theory make up the effective action and eventually render the theory nontrivial. This point has already been discussed by Alvarez-Gaumé and Witten[12] in a gauge where the zweibein is symmetrized. In the conformal gauge, however, we should give an alternative argument in which some modifications are needed, which is also appropriate for comparison with the chiral Schwinger model. For simplicity, we focus on a theory with one right-handed fermion, governed by the action

$$I_R=\int dx\frac{i}{2}e^h(\bar{\psi}_R\gamma^-\overleftrightarrow{\partial}_-\psi_R), \tag{2.8}$$

where we set $h=(\phi-F)/2$. At the classical level, the theory is symmetric under Weyl and Lorentz transformations. As a result the energy-momentum tensor $T_{\mu\nu}=(-1/\sqrt{-g})e_{a\nu}(\delta I/\delta e_a^\mu)$ should be traceless, $T_{+-}+T_{-+}=0$, and symmetric, $T_{+-}=T_{-+}$. (In the conformal gauge, one trivially has $T_{++}=T_{--}=0$.) Since T_{+-} vanishes identically due to the chirality of the interaction, the only meaningful equation is

$$T_{-+}=0. \tag{2.9}$$

Indeed, with the help of the equation of motion for the fermion

$$(\partial_-+\tfrac{1}{2}\partial_-h)\psi_R=0, \tag{2.10}$$

we confirm that the energy-momentum tensor derived from the action (2.8),

$$T_{-+}=\frac{i}{2}e^h(\bar{\psi}_R\gamma^-\overleftrightarrow{\partial}_-\psi_R), \tag{2.11}$$

vanishes at the classical level. We may recall a similar situation for the chiral Schwinger model, where chiral symmetry results in a conserved chiral current at the classical level, which is confirmed by use of the fermion's equations of motion. In chiral quantum gravity the divergence of the chiral current is replaced by the energy-momentum tensor.

Let us derive the fermionic effective action I_{eff} by treating h as a background field in perturbation theory. In order to define a propagator $(i\partial\!\!\!/)^{-1}$ for the fermion, we add a free left-handed fermion to I_R in Eq. (2.8):

$$I_R=\int dx\,i\bar{\psi}\partial\!\!\!/\psi+\int dx\frac{i}{2}h(\bar{\psi}_R\gamma^-\overleftrightarrow{\partial}_-\psi_R)+O(h^2). \tag{2.12}$$

Since the interaction part in Eq. (2.12) is proportional to the energy-momentum tensor T_{-+} in Eq. (2.11), each of the perturbative contributions, which are correlation functions of T_{-+}'s, appear to vanish due to the Weyl and the Lorentz symmetries. Unfortunately, since the interaction term contains a derivative, the contributions are quadratically divergent so that we cannot neglect them naively. This situation differs from the case of the chiral Schwinger model, where all the perturbative contributions of current correlation functions, except the logarithmically divergent one with two external lines, are seen to vanish because of their finiteness.[13] Therefore we generally have to evaluate diagrams with n external h fields. However, assuming that the Lorentz (or the Weyl) anomaly of the theory is proportional to the Riemann curvature, we may fix the effective action to be quadratic in ϕ. Then we are allowed to consider only the diagram with two external lines of h, depicted in Fig. 1. It is

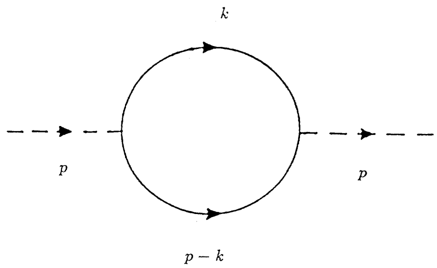

FIG. 1. Diagram which contributes to the effective action in $O(h^2)$.

$$I_{\rm eff}=\tfrac{1}{2}\int \frac{dp}{(2\pi)^2} h(p)\Pi(p)h(-p) , \qquad (2.13)$$

with

$$\Pi(p)=-\frac{i}{4}\int \frac{dk}{(2\pi)^2}\frac{(2k-p)_-^2}{k_-(p-k)_-} , \qquad (2.14)$$

which appears, as stated above, quadratically divergent. Note that in the symmetrized gauge[13,14] we encounter an integral which is similar to Eq. (2.14) but has a different tensorial structure and accordingly is readily evaluated by using tricks introduced in Ref. 15. However, in the conformal gauge the tricks does not work and one has to find out a way to give meaning to the integral (2.14). This is done by employing dimensional regularization, where no divergent part shows up in this case, which gives

$$\Pi(p)=-\frac{p_+p_-}{24\pi} . \qquad (2.15)$$

Using a relation $\omega_-=-2\partial_- h$ in the conformal gauge, we end up with

$$I_{\rm eff}=-\frac{1}{192\pi}\int dx\, \omega_-(x)\frac{\partial_+}{\partial_-}\omega_-(x) . \qquad (2.16)$$

In arbitrary coordinates Eq. (2.16) becomes exactly the effective action $I_{\rm eff}$ in Eq. (2.2) for the case $\alpha=\beta=1$, up to general coordinate-invariant local counterterms $\int dx\sqrt{-g}\,g^{\mu\nu}\omega_\mu\omega_\nu$ and $\int dx\sqrt{-g}$.

For dimensional regularization we encounter the ambiguity in defining γ matrices in d dimensions. Here we only take into account the two-dimensional part of them and disregard the ambiguity of order $\epsilon=d-2$. This ambiguity reflects the arbitrariness of the effective action (2.2). A detailed discussion for it is provided in the Appendix.

III. FERMION PROPAGATOR

Coming back to the general case of α and β, we can study the physical behavior of the fermions by observing the complete propagator, e.g., for the right-handed fermion,

$$\langle 0|T\psi_R(x)\bar\psi_R(y)|0\rangle=\int \frac{de_\mu^a}{V_{\rm diff}} d\psi\, d\bar\psi\, \psi_R(x)\bar\psi_R(y)e^{iI_F} . \qquad (3.1)$$

By employing a familiar trick in the path-integral formalism,[16] we can easily evaluate it in the following way. First we note that the fermion can be made free by the chiral transformation

$$\psi_R \to \psi_R'=e^{h/2}\psi_R , \qquad (3.2)$$

in the action (3.1). However, as is well known, this transformation induces an anomalous Jacobian,[5,17] $d\psi\, d\bar\psi=d\psi' d\bar\psi' e^{iI_{\rm eff}}$. As a consequence Eq. (3.1) becomes

$$\begin{aligned}\langle 0|T\psi_R(x)\bar\psi_R(y)|0\rangle&=\int \frac{de_\mu^a}{V_{\rm diff}} d\psi\, d\bar\psi\, e^{-h(x)/2}\psi_R'(x)e^{-h(y)/2}\bar\psi_R'(y)e^{iI_F}\\ &=\int d\phi\, d\tilde F\, d\psi' d\bar\psi' e^{-[h(x)/2+h(y)/2]}\psi_R'(x)\bar\psi_R'(y)e^{iI_T}\\ &=P_\phi(x,y)P_{\tilde F}(x,y)iS_R^f(x-y) , \end{aligned} \qquad (3.3)$$

where $S_R^f(x-y)$ is a free fermion propagator for a right-handed fermion, and P_ϕ and $P_{\tilde F}$ represent the ϕ and $\tilde F$ path integrals, respectively, after diagonalizing the action with $\tilde F$ in Eq. (2.6).

The integral P_ϕ is involved for $\mu\neq 0$ because we need the propagator for ϕ in the Liouville theory. However, when μ is chosen to be zero, it is easy to calculate P_ϕ by rescaling $\phi\to\sqrt{b/48\pi}\,\phi$:

$$\begin{aligned}P_\phi(x,y)&=\int d\phi \exp\left[c[\phi(x)+\phi(y)]+i\int dz\, \tfrac{1}{2}[\partial_\mu\phi(z)]^2\right]\\ &=\int d\phi \exp\left[i\int dz\{\tfrac{1}{2}[\partial_\mu\phi(z)]^2+J(z;x,y)\phi(z)\}\right]\\ &=\exp\{-ic^2[D_F(x-y)+D_F(0)]\} . \end{aligned} \qquad (3.4)$$

Here we defined a source $J(z;x,y)=-ic[\delta(z-x)+\delta(z-y)]$ with $c=-(1+\beta/4a)\sqrt{3\pi/b}$, and $D_F(x)=-(i/4\pi)\ln(-m^2x^2+i0)$ is the propagator for a massless scalar with an infrared-cutoff mass m. Together with an analogous procedure for $P_{\tilde F}$, we finally arrive at

$$\langle 0|T\psi_R(x)\bar\psi_R(y)|0\rangle =\exp\{-4\pi i\kappa[D_F(x-y)+D_F(0)]\}iS_R^f(x-y) , \qquad (3.5)$$

where $\kappa=(3/4ab)(24-n_L)$, with b given by Eq. (2.6).

From this, one may define the renormalized propagator for the renormalized fermion $\psi_R^r=Z_2^{-1/2}\psi_R$ with

$$Z_2=e^{-4\pi i\kappa D_F(0)} . \qquad (3.6)$$

(Usually, wave-function renormalization is performed to normalize the residue of a pole at the physical mass. Here we use "renormalization" simply to mean the rescaling which eliminates the infinity in the fermion propagator.) Then, for $(x-y)^2<0$, one has

$$\langle 0|T\psi_R^r(x)\bar{\psi}_R^r(y)|0\rangle=\frac{1}{[m^2|(x-y)^2|]^{\kappa}}iS_R^f(x-y)\ . \tag{3.7}$$

It implies that the fermion is not free for $x\rightarrow y$ and acquires an anomalous dimension κ. Owing to the unitarity condition for the theory it has the lower limit

$$\kappa\geq\frac{3}{24-n_R}\ . \tag{3.8}$$

For the case of Dirac fermions, $n_R=n_L=n_D$, one is naturally led to choose $a=0$ to ensure the Lorentz invariance of the theory. As a result, the Weyl anomaly associated with the Lorentz degrees of freedom is absent and accordingly the number 24 is shifted to 25. In this case the anomalous dimension of the fermion is uniquely determined:

$$\kappa=\frac{3}{4(25-n_D)}\ . \tag{3.9}$$

We also see from Eq. (3.7) that the canonical anticommutation relation is anomalous: $\{\psi_R^r(x),\psi_R^{r\dagger}(y)\}_{ET}\neq\delta(x^1-y^1)$. Proceeding analogously, the propagator for the left-handed fermion can also be seen to have a form identical to Eq. (3.7) with obvious replacement of S_R^f by S_L^f, the free propagator for the left-handed fermion.

The renormalized propagator (3.7) can also be obtained directly in the operator formalism. For this, we have only to notice that the equation of motion for the fermion (2.10) can be formally solved by $\psi_R(x)=e^{-h(x)/2}\psi_R^f(x^+)$ in terms of a free right-handed fermion $\psi_R^f(x^+)$. This implies that the operator solution for the renormalized fermion may be given by

$$\psi_R^r(x)=:\exp\left[-\frac{1}{4}\left[1+\frac{\beta}{4a}\right]\phi+\tfrac{1}{4}\tilde{F}\right]:\psi_R^f(x^+)\ . \tag{3.10}$$

For $\mu=0$, one can readily confirm that it reproduces the renormalized propagator (3.7), with the help of the identity $:e^A::e^B:=e^{[A^{(+)},B^{(-)}]}:e^{A+B}:$, valid for free fields A and B.

Remarkably, even for $\mu\neq0$, there is a particular point of regularization,

$$a=-\frac{\beta}{4}\ , \tag{3.11}$$

where the right-handed fermion (3.10) consists of only free fields $\tilde{F}$ and ψ_R^f. Because of the positivity constraint on a, Eq. (3.11) is possible only if the theory contains more left-handed fermions than right-handed ones, i.e., $n_L>n_R$. Of course a similar situation exists for the left-handed fermions: ψ_L can be solved analogously to (3.10), and at a point $a=\beta/4$, the left-handed fermions become free if $n_R>n_L$. Such a point does not exist for Dirac fermions.

IV. RENORMALIZATION

The appearance of an anomalous dimension for the fermion is a common feature of several soluble two-dimensional models, such as the chiral Schwinger model, the Thirring model[8] and the Schroer model.[9] In fact, because of conformal symmetry, the latter two models have the same fermion propagator, Eq. (3.7), up to the value of the anomalous dimension. For the chiral Schwinger model in the anomalous formulation, Eq. (3.7) also holds as long as we look at the short-distance region which recovers conformal invariance. In each of the models the anomalous dimension arises from the contribution of a scalar field which interacts with the fermions in the theories. However, the nontrivial aspect of chiral quantum gravity lies in the fact that the massless free scalar fields (the Weyl and the Lorentz degrees of freedom) appear only at the quantum level. In other words, the interaction is solely due to the quantum effect of the anomaly. We have a similar situation in the chiral Schwinger model, where the longitudinal degrees of freedom in the U(1) field is partly responsible for the anomalous dimension.[16,18] It is a salient feature common to anomalous models that the fermion propagator exhibits an anomalous dimension through the scalar fields being made physical by the anomaly at the quantum level.

To gain a better intuitive understanding of this nontrivial situation, let us evaluate the fermion propagator perturbatively. The vertices needed to calculate diagrams up to two loops are given in Fig. 2. Here we note that the Weyl and the Lorentz degrees of freedom acquire their kinetic terms in the effective action (2.2) owing to the quantum effect of the fermion. Accordingly the propagators for those degrees of freedom are "complete" ones, namely, they require no further corrections. Their combined effect on the fermion shows up in the propagator for h:

$$\langle 0|Th(x)h(y)|0\rangle=-16\pi i\kappa D_F(x-y)\ . \tag{4.1}$$

From this we recognize that the power of the anomalous

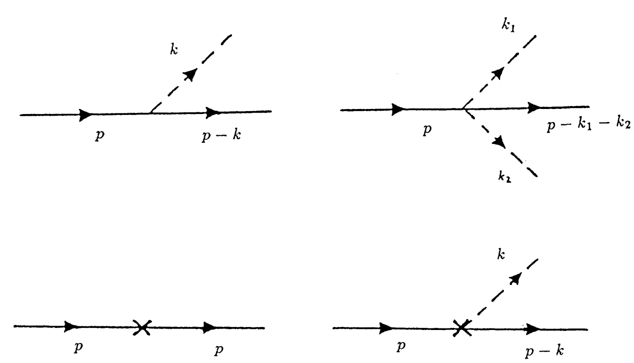

FIG. 2. Vertices appearing up to $O(h^2)$; they read $(i/2)(2p-k)_-\gamma^-$, $(1/2)(2p-k_1-k_2)_-\gamma^-$, $(Z_2^{(2)}-1)$ and $(Z_1^{(1)}-1)(1/2)(2p-k)_-\gamma^-$, respectively. $Z_2^{(2)}$ is the two-loop wave-function renormalization constant and $Z_1^{(1)}$ is the one-loop vertex renormalization constant, which are necessary to renormalize contributions up to $O(\kappa^2)$.

dimension κ corresponds to the number of loops in calculating the fermion propagator perturbatively. Hence it is clear that the anomalous dimension is generated by the massless scalars coupled to the fermion. Actually if we sum up all the relevant diagrams up to two loops collected in Fig. 3, we obtain

$$S_R^{2\text{ loops}}(p)=S_R^f(p)\left\{1+\kappa\ln\left[\frac{-p^2}{m^2}\right]+\frac{\kappa^2}{2!}\left[\ln\left[\frac{-p^2}{m^2}\right]\right]^2\right\}. \quad (4.2)$$

The expression (4.2) is in agreement with the complete renormalized propagator in the momentum space up to $O(\kappa^2)$.

In the above perturbative calculation one notices a quite interesting point: the vertex renormalization constant Z_1 is identical to the wave function renormalization constant Z_2: $Z_1=Z_2$. Actually this aspect is suggested by the fact that only the rescaling of the fermion is needed to get the renormalized fermion propagator (3.7). It becomes clear by looking at the self-energy correction and the vertex correction. The corresponding one-loop diagrams are depicted in Fig. 4. The self-energy correction $\Sigma^{(1)}(p)$ is given by

$$\Sigma^{(1)}(p)=-\pi\kappa\gamma^-\int\frac{dk}{(2\pi)^2}\frac{(2k-p)_-^2}{k^2(p-k)_-^2}$$
$$=-ip_-\gamma^-\left\{1-Z_2^{(1)}+\kappa\ln\left[\frac{-p^2}{m^2}\right]\right\}. \quad (4.3)$$

On the other hand, if we set the momentum insertion

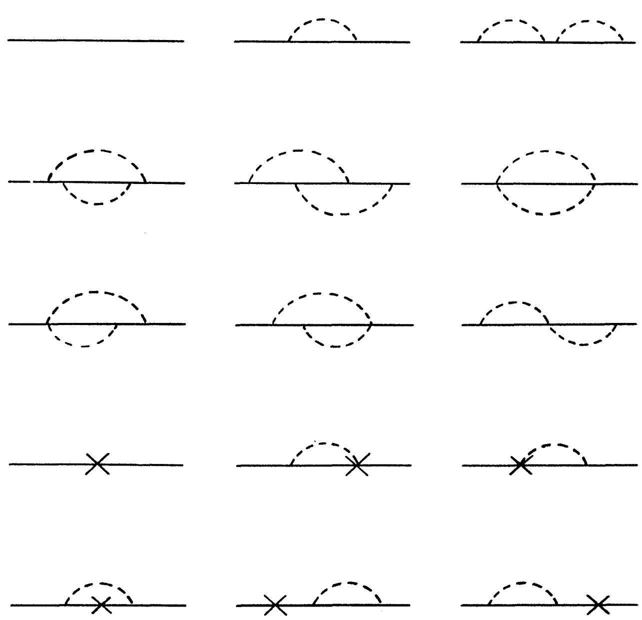

FIG. 3. Diagrams which contribute to the fermion propagator up to $O(\kappa^2)$.

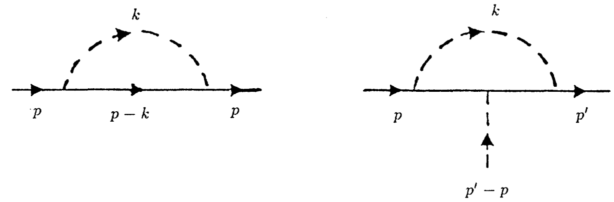

FIG. 4. The self-energy correction $\Sigma^{(1)}(p)$ and the vertex correction $\Gamma^{(1)}(p,p')$ in $O(\kappa)$.

$p'-p$ to be zero for the vertex correction, $\Gamma^{(1)}(p,p')$, we find that it yields exactly the same integral expression as the right-hand side of (4.3). It then follows that both renormalization constants are equal and read

$$Z_1^{(1)}=Z_2^{(1)}=1-i\lim_{p_-\to\infty}\frac{1}{p_-}\Sigma^{(1)}(p)=1-4\pi i\kappa D_F(0)\ . \quad (4.4)$$

This gives correctly the complete wave function renormalization constant Z_2 in Eq. (3.6) up to $O(\kappa)$.

As a result of the equality (4.4), there is no "coupling constant" renormalization in our theory; that is, the bare constant 1 which is the coefficient of the interaction between h and fermions is not renormalized. This remarkable aspect of renormalization is also common to other soluble models in two dimensions, as the Thirring model or the chiral Schwinger model.[18]

V. CONCLUSION AND DISCUSSION

In this paper we first presented a perturbative derivation of the effective action of the chiral quantum gravity in the conformal gauge. It coincides with the one originally derived by Leutwyler with a more involved method,[11] if only the diagram with two external lines is considered. This restriction could arise from the form of the anomalies the effective theory should possess. If this criterion is disregarded, the perturbative effective action in the conformal gauge would contain terms of arbitrary order in the external field. We have seen that the energy-momentum tensor in the chiral quantum gravity corresponds to (the divergence of) the chiral current in the chiral Schwinger model; both vanish classically but nevertheless their correlation function gives the effective action because of the anomalies at the quantum level.

From the effective action, we obtained the complete fermion propagator for the case $\mu=0$ and found that it acquires an anomalous dimension. Further it has been shown that the equality of the wave function and the vertex renormalization constants holds although the symmetries are broken by the anomalies. It is however known that these aspects, the appearance of the anomalous dimension and the equality between the two renormalization constants, also exist in other soluble models in two dimensions. Indeed, at the quantum level in the conformal gauge the chiral quantum gravity resembles the Schroer model in its massless case. What makes the chiral quantum gravity distinct is that the scalars, which give rise to the anomalous dimension to the fermion, become dynamical only at the quantum level due to the Weyl and the Lorentz anomalies. Since a similar quantum effect is also present in the chiral Schwinger model, it seems to be a typical feature of anomalous gauge theories in two dimensions.

ACKNOWLEDGMENTS

We would like to thank F. Brandt and G. Kramer for useful discussions. We are supported through funds provided by the Studienstiftung des deutschen Volkes (T.B.) and by the Alexander von Humboldt Foundation (I.T.).

APPENDIX

In this Appendix we present the perturbation theoretic derivation of the effective action including one of the ambiguous terms in Eq. (2.2). For this purpose we use arbitrary coordinates in flat Minkowski metric, instead of the light-cone coordinate used in obtaining Eq. (2.16).

The interaction part of the action (2.12) linear in $h=\frac{1}{2}(\phi-F)$ reads

$$I_R^{\rm int}\int dx \frac{i}{4} h\,[\bar\psi\gamma^\mu \overleftrightarrow{\partial}_\mu(1+\gamma^5)\psi] \ . \qquad \text{(A1)}$$

In the perturbative expansion we employ dimensional regularization to evaluate the contribution of $O(h^2)$. We then immediately encounter the well known difficulty with the definition of γ^5 in d dimensions. However, it may be circumvented by means of the Breitenlohner-Maison scheme,[19] where the d-dimensional γ^5 is defined in such a way that it no longer anticommutes with d-dimensional γ_μ matrices, but still satisfies $(\gamma^5)^2=1$. γ_μ in the vertex may generally be written as

$$\bar\gamma_\mu = r\gamma_\mu + s\theta_{\mu\nu}\gamma^\nu\gamma^5 \ , \qquad \text{(A2)}$$

where $\theta_{\mu\nu}=\epsilon_{\mu\nu}+O(\epsilon)$ is an antisymmetric tensor ($\epsilon=d-2$), and r and s are arbitrary parameters which satisfy

$$r+s=1+O(\epsilon) \ , \qquad \text{(A3)}$$

so that $\bar\gamma_\mu$ in (A2) satisfies $\bar\gamma_\mu\to\gamma_\mu$ as $d\to 2$. This prescription has been originally proposed to derive the general effective action of the chiral Schwinger model.[20,21] In our case one realizes that the interaction vertex in (A1) can also be generalized:

$$V_\mu=(\phi\bar\gamma_\mu - F\theta_{\mu\alpha}\bar\gamma^\alpha\gamma^5)(1+\gamma^5) \ . \qquad \text{(A4)}$$

V_μ is the most general vertex which gives the original one in (A1) as $d\to 2$. (ϕ and F may be exchanged without affecting the final result.)

Then the effective action is given by

$$\begin{aligned} I_{\rm eff}[\phi,F] &= -\left[\frac{i}{8}\right]^2 \int \frac{dp}{(2\pi)^2}\int \frac{d^dk}{(2\pi)^d}\frac{(2k-p)^\mu(p-k)^\sigma(2k-p)^\nu}{k^2(p-k)^2}\Gamma_{\mu\sigma\nu\rho}(p) \\ &= \frac{1}{64}\int\frac{dp}{(2\pi)^2}\left[\frac{1}{\epsilon}I_{-1}^{\mu\sigma\nu\rho}(p)+I_0^{\mu\sigma\nu\rho}(p)+O(\epsilon)\right]\Gamma_{\mu\sigma\nu\rho}(p) \ , \end{aligned} \qquad \text{(A5)}$$

where $\Gamma_{\mu\sigma\nu\rho}(p)={\rm Tr}[V_\mu(p)\gamma_\sigma V_\nu(-p)\gamma_\rho]$ and

$$\begin{aligned} I_{-1}^{\mu\sigma\nu\rho}(p) &= -\frac{i}{12\pi}[-p^2(g^{\mu\sigma}g^{\nu\rho}+g^{\sigma\nu}g^{\mu\rho}+g^{\mu\nu}g^{\sigma\rho})+4(p^\nu p^\rho g^{\mu\sigma}+p^\sigma p^\rho g^{\mu\nu}) \\ &\qquad +10p^\sigma p^\nu g^{\mu\rho}-8p^\mu p^\rho g^{\nu\sigma}-2p^\mu p^\sigma g^{\nu\rho}-5p^\mu p^\nu g^{\sigma\rho}] \ , \\ I_0^{\mu\sigma\nu\rho}(p) &= -\frac{i}{36\pi}\Bigg[4p^2(g^{\mu\sigma}g^{\nu\rho}+g^{\sigma\nu}g^{\mu\rho}+g^{\mu\nu}g^{\sigma\rho})-13(p^\nu p^\rho g^{\mu\sigma}+p^\sigma p^\rho g^{\mu\nu}) \\ &\qquad -31p^\sigma p^\nu g^{\mu\rho}+23p^\mu p^\rho g^{\nu\sigma}+5p^\mu p^\sigma g^{\nu\rho}+14p^\mu p^\nu g^{\sigma\rho}+\frac{3}{p^2}p^\mu p^\sigma p^\nu p^\rho\Bigg] \ . \end{aligned} \qquad \text{(A6)}$$

As $I_0^{\mu\sigma\nu\rho}(p)$ is not singular in ϵ, the contraction with $\Gamma_{\mu\sigma\nu\rho}(p)$ can be performed with $d=2$. This leads straightforwardly to the unambiguous contribution of the effective action. The contraction of $(1/\epsilon)I_{-1}^{\mu\sigma\nu\rho}(p)$ and $\Gamma_{\mu\sigma\nu\rho}(p)$ has to be done in d dimensions. The result is

$$\begin{aligned} I_{\rm eff}[\phi,F] &= \frac{1}{384\pi}\int dx\,[(r+s)^2(\phi-F)\Box(\phi-F)+(r^2-s^2)(\phi-F)\Box(\phi+F)] \\ &= \frac{1}{48\pi}\int dx\,[\tfrac{1}{4}\phi\Box(\phi-F)+(a/2)(\phi-F)\Box(\phi+F)] \ , \end{aligned} \qquad \text{(A7)}$$

where we used the condition (A3) and set $s=-2a$. This expression (A7) is just the conformal gauge version of (2.2) for $\alpha=\beta=1$ without the μ term, as we have $R=-e^{-\phi}\Box\phi$, $\omega_\pm=\partial_\pm(F\pm\phi)$ in this gauge. It is not obvious how to get the μ term in the conformal gauge, because it would require to calculate perturbative contributions in all orders of h.

We finally comment on a subtlety in defining the theory. We evaluated the effective action by generalizing the vertex to d dimensions [see Eq. (A4)]. Instead, one may define the interaction part (the energy-momentum tensor) by a usual point-splitting procedure, as it has been done for the current of the chiral Schwinger model to get the ambiguous term in the model.[10] This however fails to get the ambiguous a term in the present model, since the "phase" part which restores the Weyl and the Lorentz invariances is real, in contrast with the imaginary phase in the current of the chiral Schwinger model.

*Present address: Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW, England.

†Present address: Dublin Institute for Advanced Studies, School of Theoretical Physics, 10 Burlington Road, Dublin 4, Ireland.

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