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HOW STRONG ARE WEAK INTERACTIONS IN THE MULTI-TeV RANGE?

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Weak interactions at high energies ($\sqrt{s} \geqslant m_w/\alpha_w$) may lead to strong multiparticle production of weakly interacting particles. This effect is connected to infrared divergences of the underlying gauge theory. Cross sections for the production of many weakly interacting particles of the order 1 nb to 10 μb above a threshold (parton) energy of 2 to 20 TeV could be observable at future colliders.

1. Introduction

Recently there has been a lot of interest in nonperturbative effects in the standard electroweak theory. It has been speculated that electroweak interactions become strong at high energies, showing new phenomena like high-multiplicity events involving "weakly" interacting particles and baryon (B) and lepton (L) number violation [1-5].

Hints of strong flavour interactions have been observed in ref. [2] in the context of $(B+L)$-violating processes induced by instantons [6]. The instanton-induced on-shell vertices for processes like

$$q + q \to (3n_g - 2)\bar{q} + n_g \bar{l} + n_w W(Z) + n_h H, \qquad (1)$$

are, in leading-order, point-like and proportional to $n! \exp(-2\pi/\alpha_w)$ [2]. Here $n = n_w + n_h$ and n_g denotes the number of fermion generations. The absence of form factors leads to cross sections which rise with powers of the energy as determined by the dimension of the local interaction. In consequence, the unitary limit for the cross section of the exclusive process (1) for large $n \sim 1/\alpha_w$ is already reached at some tens of TeV [2]. In particular, it was found [2-4, 7], that the

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(parton) cross sections of the exclusive $(B+L)$-violating processes (1) behave like

$$\sigma_{n_w,n_h} \simeq \left(\frac{\Lambda_W}{v}\right)^{2b} \frac{1}{n_h!} \frac{1}{(n_w!)^3} \left(\frac{s}{8\pi^2 v^2}\right)^{n_h} \left(\frac{3s^2}{8\pi^2 g^2 v^4}\right)^{n_w} v^{-2}, \tag{2}$$

where $\sqrt{s}$ denotes the (parton) center of mass energy and $v = 246$ GeV denotes the Higgs field vacuum expectation value. Here Λ_W is the energy scale where the SU(2) gauge coupling would become strong if there were no spontaneous symmetry breaking,

$$\Lambda_W = \text{const} \cdot v \cdot \exp\left(-\frac{2\pi}{b\alpha_w(v)}\right) \simeq 10^{-23}\ \text{GeV}, \tag{3}$$

and b is the coefficient of the β-function,

$$b = \tfrac{43}{6} - \tfrac{4}{3} n_g = \tfrac{19}{6}. \tag{4}$$

For $\sqrt{s}$ a few times larger than m_w, the inclusive $(B+L)$-violating cross section is dominated by the multiple production of vector bosons [7–9],

$$\sigma_{\Delta B} = \sum_{n_w, n_h} \sigma_{n_w,n_h} \sim \exp\left[-\frac{4\pi}{\alpha_w}\right] \exp\left[\frac{\sqrt{s}}{M_0}\right]^{4/3}, \tag{5}$$

$$M_0 = \frac{2}{3}\left(\frac{2\pi}{\alpha_w}\right)^{1/4} m_w = 195\ \text{GeV}. \tag{6}$$

The average multiplicity increases $\sim s^{2/3}$,

$$\langle n_w \rangle \simeq \frac{1}{3}\left(\frac{\sqrt{s}}{M_0}\right)^{4/3}. \tag{7}$$

The exponential blowing up of the cross section violates unitarity at the scale $\sqrt{s} \simeq M_1$,

$$M_1 = 2^{-1/4} \frac{8\pi}{3} \frac{m_w}{\alpha_w} \simeq 16\ \text{TeV}, \tag{8}$$

where the mean multiplicity is $\langle n_w \rangle \simeq 4\pi/(3\alpha_w)$. It should be noted that the energy (8) is remarkably close to the sphaleron energy [10], $E_{sp} \simeq \pi m_w/\alpha_w \simeq 10$ TeV, the minimum barrier height between topologically inequivalent vacua [11] in the electroweak theory. The instanton describes the tunneling under this barrier

[6, 11, 12]. The calculations of instanton-induced $(B+L)$-violating processes explicitly confirms earlier conjectures that perturbation theory in the instanton sector breaks down for $E > m_w/\alpha_w$ and $n > 1/\alpha_w$ [10, 13]. If the exponential growth of the inclusive cross section persists up to the energy scale M_1, one can speculate about the exciting possibility of *strong* flavour interactions at high energies with a geometric total cross section for particles with SU(2) gauge interactions [4].

$$\sigma_{\Delta B} \sim \frac{4\pi}{E_{sp}^2} \ln^{\delta}\left(\frac{\sqrt{s}}{E_{sp}}\right) \quad (\delta \leqslant 2). \tag{9}$$

Possibly the strong interaction above M_1 is related to the long ago expected breakdown of perturbation theory (in the topologically trivial sector) in high order $N > 1/\alpha_w$ (see e.g. ref. [14] and references therein). Indeed, it was argued in ref. [5] that perturbation theory breaks down at similar energies and multiplicities for $(B+L)$-conserving and $(B+L)$-violating processes, at least in the high-energy, fixed-angle and fixed-multiplicity regime.

Unfortunately, the perturbative expansion around the one-instanton configuration becomes completely unreliable near the critical energy scale M_1. Since the cross section rises so steeply, it is unclear if the exponential suppression factor can be overcome completely. At this stage we do not think that a satisfactory argument in favour of strong flavour interactioons at high energies can be given on the basis of perturbation theory around instantons which rather describes the low-energy behaviour.

In this paper we address the issue of possible strong flavour interactions from a different viewpoint: What happens in the asymptotic regime where the center of mass energy for quark, lepton or gauge boson scattering is much higher than the scale of spontaneous symmetry breaking, $\sqrt{s} \gg v$? It is rather obvious that this issue depends on the structure of infrared divergences in nonabelian gauge theories (for $v \to 0$). The total inelastic cross section can always be written in the form

$$\sigma_W = s^{-1}\tilde{\sigma}(v^2/s). \tag{10}$$

More precisely, we will define σ_W as a "multiparticle" cross section, i.e. as the total cross section for events producing more than $\bar{n}$ non-hadronic flavoured particles – leptons, W, Z, and scalars – say $\bar{n} = 10$. The dimensionless function $\tilde{\sigma}$ can only depend on the ratio v^2/s since all relevant mass scales are $\sim v$. For an infrared finite cross section $\tilde{\sigma}$ approaches a constant as v goes to zero. Then $\sigma_W \sim s^{-1}$ cannot be strong at very high energies. A strong cross section in an intermediate energy range below the asymptotic regime would seem very unlikely in this case. A logarithmic infrared divergence ($\tilde{\sigma} = \sigma_1 + \sigma_2 \ln^{\delta}(s/v^2)$ or similar) does not change this argument qualitatively. In contrast, an infrared divergence of degree two, $\tilde{\sigma}=\tilde{c}s/v^2$, implies a constant cross section, $\sigma_W=\tilde{c}v^{-2}$ (up to the running of couplings in $\tilde{c}$). Unless $\tilde{c}$ is extremely small this leads to strong interactions at high energies. In conclusion "strong" flavour interactions at high energies require an infrared divergence of degree two for total inelastic cross sections. Unfortunately, an explicit evaluation of the degree of divergence of σ_W amounts to a difficult (nonperturbative) calculation. In this paper we present instead a semi-phenomenological argument that the degree of infrared divergence of σ_W in the weak nonabelian gauge theory is indeed near two. We give in sect. 2 our overall picture based on three simple assumptions. They will be motivated in sect. 3.

2. The QCD–QFD analogy

For our discussion we consider a given energy of the scattering process (say $\sqrt{s}=100$ TeV). We now change smoothly the mass parameter in the Higgs potential such that v decreases, keeping all renormalized dimensionless coupling constants $\alpha_w(s)$ etc. fixed. Let us first consider QFD in the limit $v\rightarrow 0$ and neglect for a moment the QCD interactions. The nonabelian SU(2) interactions become strong at the scale Λ_W, eq. (3). The SU(2) doublets (left handed quarks, leptons, and the almost massless Higgs scalar ($m_h\leqslant\Lambda_W$)* are confined. In this regime the quarks or leptons in the initial state are modified and become SU(2) singlet bound states with the Higgs scalar [16, 17]. The theory resembles in many aspects QCD, with Λ_S replaced by Λ_W. We first use the close analogy to QCD for a phenomenological estimate of σ_W in the range $v\simeq\Lambda_W$ and later extrapolate to realistic values of v.

In QCD the total cross sections for the inelastic scattering of hadrons and mesons are dominated by multiparticle production in the forward direction and become approximately constant at $\sqrt{s}\geqslant 2$ GeV, with a slow logarithmic increase for large s,

$$\sigma_S=\frac{4\pi}{\Lambda_S^2}c_S\left(s/\Lambda_S^2\right),$$

$$c_S=A+C\ln^2\left(\frac{s}{\Lambda_S^2}\right)+D\ln\left(\frac{s}{\Lambda_S^2}\right)+\dots\,. \tag{11}$$

* It is important for our argument that the scalar mass does not exceed Λ_W. This is realized if Coleman–Weinberg [15] symmetry breaking does not occur at a scale much above Λ_W. We therefore require the quartic scalar coupling to be sufficiently large. Of course, this is only a technical requirement and the structure of infrared divergences does not depend on λ. No bound on the physical scalar mass arises.

Here Λ_S is a typical infrared cutoff scale due to mass generation from confinement which we take in the range 0.1–1 GeV. The numerical values for the constants $A, C, \ldots$ depend on the choice of Λ_S and are typically [18]

$$A = (3.5\text{–}12.4)\left(\frac{\Lambda_S}{1\ \text{GeV}}\right)^2,$$

$$C = (0.03\text{–}0.11)\left(\frac{\Lambda_S}{1\ \text{GeV}}\right)^2. \tag{12}$$

The breakdown of the asymptotic* behaviour at low energies manifests itself in power corrections to c_S of the form $\Delta c_S \sim B(M_S/\sqrt{s})^\gamma$ with typical exponents $\gamma \simeq 1.5$–9. The scale M_S characterizes the onset of the asymptotic behaviour and is of order

$$M_S \simeq (2.5\text{–}4)\ \text{GeV}. \tag{13}$$

Our first assumption states that *for $v = 0$ the total cross sections for SU(2) singlet bound states of flavoured particles behave similarly as for hadrons in the SU(3) gauge theory,* namely

$$\sigma_W = \frac{4\pi}{\Lambda_W^2} c_W(s/\Lambda_W^2). \tag{14}$$

Of course, the exact form of the function $c_W(s/\Lambda_W^2)$ is expected to reflect the differences between SU(2) and SU(3) (different Casimir operators, different "triality", additional light scalar). For a very rough estimate, however, we may use the same function c as for QCD and obtain for $v = 0$**

$$\sigma_W(\sqrt{s} = 100\ \text{TeV}) \simeq 10^{23}\ \text{cm}^2. \tag{15}$$

This cross section is enormous, reflecting the large geometrical size Λ_W^{-1}. It depends only weakly on s. We observe that (14) diverges $\sim \Lambda_W^{-2}$ for $\Lambda_W \to 0$. This suggests a strong infrared divergence in σ_W due to the interacting massless W-bosons. The scale Λ_W acts as the effective infrared cutoff: At momenta below Λ_W the relevant excitations are massive bound states, $m \sim \Lambda_W$, rather than massless gauge bosons.

* The "asymptotic behaviour" denotes in our context the region where the detailed structure of the infrared cutoff becomes irrelevant. It is well possible that for extremely large s/Λ_S^2 the parametrization (11) becomes insufficient. Thus eq. (11) should be considered as an approximation for $s/\Lambda_S^2 \leq 10^6$.

** We expect large multiplicities since s/Λ_W^2 is large. This allows us to identify the total inelastic cross section with the multiparticle cross section σ_W.

Next we consider $v > 0$. As long as $v \ll \Lambda_W$ a small amount of electroweak symmetry breaking will not affect our estimate (14) for σ_W. For $v \geqslant \Lambda_W$, however, there is a transition from the confining phase to the Higgs phase. The gauge bosons (except the photon) acquire masses $m_w, m_z \propto v$ and these masses provide a new effective infrared cutoff. In general, c_W depends now on two scales, Λ_W and m_w,

$$\sigma_W = \frac{4\pi}{m_w^2} c_W \left(\frac{s}{m_w^2}, \frac{\Lambda_W}{m_w} \right). \tag{16}$$

Our second assumption states that *in the immediate vicinity of the phase transition the structure of infrared divergences is not strongly affected by the transition from the confined to the spontaneously broken regime*. In particular, the degree of divergence remains the same in the Higgs and confinement phase. In the Higgs phase very near the phase transition (m_w of order Λ_W) we thus have

$$c_W \left(\frac{s}{m_w^2}, \frac{\Lambda_W}{m_w} \right) \simeq c_W \left(\frac{s}{\Lambda_W^2}, m_w = 0 \right). \tag{17}$$

Our third (and perhaps most important) assumption is finally that *in the Higgs phase the infrared divergence in σ_W has the simplest possible form, namely an almost constant degree near two*. This means that c_W is a slowly varying function of Λ_W/m_w over a large range $\Lambda_W \leqslant m_w \ll \sqrt{s}$. This is our main conjecture. We conclude that the total cross section for "weak" processes can be much stronger than the perturbative cross sections for a production of only a few particles at large angles which are $\sim s^{-1}$. In this sense weak interactions become strong at high energies. This is a necessary consequence if there is an infrared divergence with degree two (with a substantial coefficient) in σ_W.

We believe that $\sigma_W \sim m_w^{-2}$ is valid for asymptotic values of $s \gg v^2$. For practical purposes, i.e. physics at future colliders, it is crucial to know at what energy such an asymptotic behaviour sets in. This is an even more subtle question than the determination of the degree of the infrared divergence. A necessary ingredient for a nonperturbative behaviour of σ_W seems to be the production of a large number of weakly interacting particles of order $1/\alpha_w$. This requires an energy larger than a critical value $\sqrt{s} \geqslant \kappa M_2$,

$$M_2 = \frac{m_w}{\alpha_w} \simeq 2.3\ \text{TeV}. \tag{18}$$

The QCD analogy $M_S \simeq \Lambda_S/\alpha_s$ suggests $\kappa \simeq 1$. The threshold for the onset of nonperturbative flavour interactions would then be only a few TeV. More conservatively, one may argue that sufficient phase space must be available and that particle multiplicities may need to be somewhat higher than $1/\alpha_w$, resulting in an asymptotic behaviour only above about 10 TeV (sphaleron energy).

One more feature is crucial for a possible experimental detection of the large electroweak cross sections at future colliders, namely the angular distribution of the multi-W events. Although (16) is substantial, it is still at least four orders of magnitude smaller than the total cross section for strong interactions (11), even for $c_W \simeq c_S$. The major part of the strong interaction events occurs at a very small angle from the beam line, with a typical transverse momentum $p_T \leqslant p_S \simeq 400$ MeV and an exponential decrease for larger p_T. We expect an analogous situation for weak interactions, but with larger critical transverse momentum $p_W \simeq m_w$ (see also ref. [4]). Indeed, for $p_T \ll m_w$ the transverse momentum of the particles can be neglected. In contrast, for $p_T \gg m_w$ the transverse momentum introduces a new effective infrared cutoff superseding m_w. Thus we expect inclusive differential cross sections $\partial\sigma/\partial p_T^2$ which are almost flat for $p_T \leqslant p_W$ and decrease rapidly for $p_T > p_W$. This leaves a large window $p_S \ll p_T \leqslant p_W$ where multi-W production may give the dominant contribution to the cross section!

3. Infrared divergent cross sections in nonabelian gauge theories

After describing our overall picture of strong flavour interactions we next want to motivate our assumptions. We define the degree p of the infrared divergence in σ_W by the use of an effective IR cutoff μ (in our case $\mu = m_w$),

$$p = -\mu \frac{\partial}{\partial \mu} \ln \sigma_W . \tag{19}$$

Here the renormalized dimensionless couplings are kept fixed at the scale s. For $\mu \ll \sqrt{s}$ the dimensionless quantity p only depends on the physics at the long distance scale μ^{-1}, i.e. the relevant fluctuations at this scale ("pomerons", "small-p_T gauge bosons", etc...) and their effective interactions. The degree p is therefore only a function of the dimensionless couplings $\gamma_i(\mu)$ characterizing the (relevant and marginal) effective interactions. It only depends on s through the implicit s dependence of the $\gamma_i(\mu)$,

$$\gamma_i(\mu) = \gamma_i\big(\alpha_w(s), s/\mu^2\big) . \tag{20}$$

In case of an infrared fixed point p becomes a constant as $\mu \to 0$. For slowly running γ_i, p is approximately constant. The same is true more generally if p only depends on certain ratios of γ_i and these ratios are in the vicinity of partial (or approximate) IR fixed points.

For a sufficiently strong IR divergence the cross section itself is entirely dominated by the physics at the scale μ. To the extent that the s-dependence of the γ_i can be neglected (slowly running γ_i) the scale μ is then the only scale present. Dimensional analysis implies

$$\sigma_W(s,\mu) = \frac{4\pi}{\mu^2} c(\gamma_i(\mu)). \tag{21}$$

We conclude that $p = 2$ is the canonical degree for strongly IR divergent cross sections, corresponding to the canonical dimension of σ_W. The anomalous dimension

$$a = p - 2 = -\mu \frac{\partial}{\partial \mu} \ln c \tag{22}$$

results in this regime only from the running of the effective couplings γ_i. We therefore expect small values of a for weak (or slowly running) couplings. In QCD, and according to our first two assumptions also in QFD, the case of a strong infrared divergence (21) seems to be realized.

If $\alpha_w(s)$ is the only relevant coupling at the scale s we obtain by dimensional analysis from eq. (16) or, equivalently, from eqs. (20) and (21),

$$a = a_s + a_\Lambda,$$

$$a_s = 2s \frac{\partial}{\partial s} \ln c_{|\mu,\Lambda}, \tag{23}$$

$$a_\Lambda = \Lambda_W \frac{\partial}{\partial \Lambda_W} \ln c_{|\mu,s} = \frac{b\alpha_w^2(s)}{2\pi} \frac{\partial \ln c}{\partial \alpha_w(s)_{|\mu,s}}.$$

A numerical evaluation of (11) suggests a very small value for $|a_s|$ and we note that a_s is limited by the Froissart bound [19]

$$\lim_{s/\mu^2 \to \infty} a_s \leqslant \frac{4}{\ln(s/\mu^2)}. \tag{24}$$

A decrease of c with increasing μ requires a positive anomalous dimension. The quantity $|a_\Lambda|$, however, is expected to be small for $\mu \gg \Lambda_W$ since the couplings run only slowly. (In this region one has $a_\Lambda \sim \alpha_w$ if c is proportional to some power of α_w.) In consequence, a substantial decrease of c over a small interval in $\ln \mu$ seems only possible in the narrow transition region from weak to strong coupling where μ is in the vicinity of Λ_W.

There have been previous attempts to derive the almost constant total cross sections in QCD from the perturbative parton model [20]. They can be directly applied to a perturbative calculation of the infrared divergence in σ_W in

QFD – only the Casimir operators are different as long as all momenta remain in the "perturbative" regime. (If these approaches will prove quantitatively successful in QFD – in contrast to QCD – no arguments on the behaviour at $v \simeq \Lambda_{W}$ and the structure of the phase transition would be needed anymore.) The total cross section for WW or fermion–fermion scattering in the spontaneously broken SU(2) theory (without electromagnetism) obtains in the leading $\ln s$ approximation for large $s/\mu^2 \gg 1$ and $\alpha_w \ln(s/\mu^2) \lesssim 1$ [20, 21]

$$\sigma_W = \frac{4\pi\alpha_w^{2+\delta}}{\mu^2}\hat{c}\ln^{\delta}\left(\frac{s}{\mu^2}\right)\left(\frac{s}{\mu^2}\right)^{\hat{\alpha}/2},$$

$$\hat{\alpha} = \frac{16\ln 2}{\pi}\alpha_w, \quad \delta = -\frac{3}{2}. \tag{25}$$

The constant $\hat{c}$ is of order one and can be determined from refs. [20, 21]. We infer that also even for weak gauge couplings the degree of the infrared divergence is approximately two, with a small anomalous dimension

$$a = \hat{\alpha} + \frac{2\delta}{\ln(s/\mu^2)}. \tag{26}$$

Previously, the result (25) has not been taken too seriously since it violates the Froissart bound [19] for $s \to \infty$. We interpret it here as a reasonable description of the approach to an approximate IR fixed point for a,

$$\mu\frac{\partial}{\partial\mu}a = \frac{1}{\delta}(a - \hat{\alpha})^2, \tag{27}$$

which is valid as long as the running of $\hat{\alpha}$ can be neglected. The running of $\hat{\alpha}$ has to be taken into account as $\mu/\sqrt{s} \to 0$. This is not described by the usual β-function for the SU(2) gauge coupling. It rather arises from the running of the effective couplings γ_i. Although we believe that multiparticle production is most probably nonperturbative and the true behaviour of the anomalous dimension may differ from (26) in the energy range of interest, the fact that the anomalous dimension is small even in perturbation theory (as suggested by our previous general discussion) greatly enhances our confidence that the degree of the infrared divergence remains almost constant ($p \simeq 2$) over the whole range $\Lambda_W \leqslant \mu \ll \sqrt{s}$ (our third assumption). Indeed, eq. (25) should be valid at very high energy s where $\alpha_w(s)$ is sufficiently small, provided the infrared cutoff μ_1 is in an appropriate range not too far below $\sqrt{s}$. Taking this together with our first two assumptions on the behaviour at a very small IR cutoff scale $\mu_0 \simeq \Lambda_W$ excludes a substantial power-law behaviour of c_W. The average anomalous dimension between the two scales μ_0 and μ_1 must be small. (We give bounds on the average value of a in appendix A.) The combination of information at very different scales ($\mu = \mu_0$ or μ_1) gives a much stronger argument in favour of our central result $\sigma_W \sim \mu^{-2}$ than each of the pieces taken separately! Of course, the smooth behaviour of $c_W(s/\mu^2)$ is not sufficient to determine its actual value at the physical scale $\mu = 80$ GeV.

It remains to motivate our assumptions on the behaviour near Λ_W. (These assumptions are implicitly made in all attempts to understand the diffractive behaviour of QCD within the parton model.) Our first assumption about an infrared divergence of degree two in σ_W for the SU(2) gauge theory in the confined phase (with fermions and an almost massless scalar – confined QFD) will perhaps not be contested so much. A more critical point in our argument is the second assumption that the infrared divergence of σ_W is the same in the confined and spontaneously broken phase. At first sight this seems not unreasonable since the physics of fluctuations with momenta $k^2 \ll s$, which determines the issue of infrared divergences, should not depend on whether the infrared behaviour is finally regulated by a small spontaneous symmetry breaking or a small confinement scale. The situation is, however, more subtle since we can only speak about cross sections for the scattering of given incoming particles. The particles themselves depend on the phase.

A necessary ingredient for our assumption is an identification of the relevant states in the confined and spontaneously broken phase as v crosses the transition between the two regimes. For the coupled system of Higgs scalar and SU(2) gauge bosons this possibility is suggested by the structure of the phase diagram. There is a continuous transition between confinement and spontaneous symmetry breaking for not too small quartic scalar coupling λ and sufficiently strong gauge coupling. This implies a continuous interpolation of the states between the two phases. The strong coupling region is relevant for our discussion since the renormalized gauge coupling is strong for $v \simeq \Lambda_W$. One may object that for a high ultraviolet cutoff the bare coupling is small and one is therefore in a region of the phase diagram where the phase transition is first order. For an appropriate λ, however, the typical mass scale of the phase transition is still given by Λ_W*. In the close vicinity of the phase transition line the renormalized gauge coupling is again large and the qualitative features of a transition with strong gauge coupling are expected. More precisely, the lines of constant physics (particle masses, renormalized couplings) approach the region with continuous transition as the ultraviolet cutoff decreases (fig. 1). This is dictated by the validity of the perturbative β-functions at scales large compared to Λ_W and small compared to the scale where the scalar self-interaction becomes strong. (Strictly speaking, one should use a three-dimensional phase diagram where λ decreases for decreasing cutoff according to perturbation theory.)

* If λ is too small, Coleman–Weinberg symmetry breaking [15] occurs at a scale $M_{CW} \gg \Lambda_W$ and the scales of the first order phase transition (e.g. particle masses on the critical line) are of order M_{CW}.

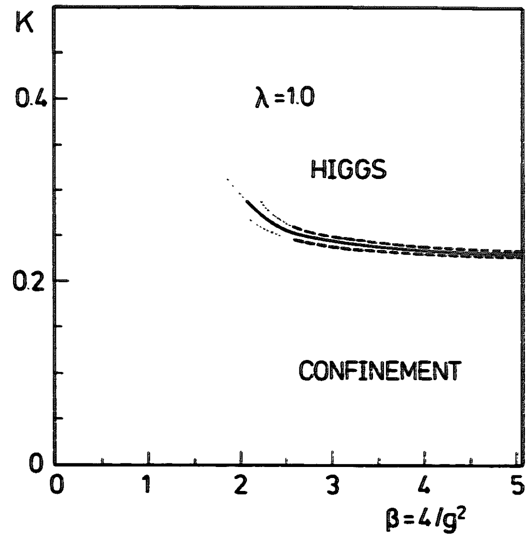

Fig. 1. Phase diagram of the SU(2) Higgs model from lattice simulations [22]. κ determines the bare mass of the scalar and its precise definition is not relevant here. The dashed lines indicate qualitatively the lines of constant physics from perturbation theory in the immediate vicinity of the phase transition (renormalized scalar mass of order Λ_W).

Since two points immediately above and below the transition line for small g^2 stay close together as the UV cutoff decreases we expect a close correspondence of the low mass states in the immediate vicinity of the critical line, e.g. as long as the scalar mass obeys $m_h \leqslant \Lambda_W$. We do not expect a close relation between the low-mass states outside the immediate vicinity of the transition line (i.e. for physical scalar masses $m_h \gg \Lambda_W$). Nevertheless, our restricted version of the "complementarity principle" [16, 17] is sufficient to argue in favour of infrared singularities with the same degree on both sides of the phase diagram.

A similar situation for the light quarks and leptons is suggested by the Abbott–Farhi model [17]. It is well known that the SU(2) gauge symmetry remains unbroken even in the Higgs ("spontaneously broken") phase [16, 17, 23]. The physical quark or lepton states consist of isodoublets surrounded by a cloud of scalars such that the states are gauge singlets. (A typical "size" of the cloud is $\sim m_h^{-1} \sim m_w^{-1}$.) In the Higgs phase massless quarks or leptons are protected by chiral symmetries. If suitable chiral symmetries remain unbroken in the confinement phase 't Hooft's anomaly cancellation conditions [24] imply the existence of massless fermion–scalar bound states with properties very similar to quarks and leptons [17]. For example, an unbroken abelian gauge group (Coulomb phase for electromagnetism) is sufficient to guarantee massless composite fermions. This one to one correspondence of the massless states in the two phases should persist once small Yukawa couplings break the chiral symmetry and induce small fermion masses. We should emphasize that our argument does not require a correspondence of all low mass excitations across the phase transition – similar properties for the light quarks on both sides of the critical surface would be enough.

Of course, a continuous interpolation of states does not guarantee by itself a smooth behaviour of the infrared divergence (nor is it necessary for this). Logically, it is conceivable that the IR divergence in the confinement phase (14) is only a result of the confinement of the initial particles in the scattering process and disappears completely in the Higgs phase. The cross section $\sigma_W(100\ \text{TeV})$ would then jump by 51(!) orders of magnitude as we change v from 0 to a few times Λ_W. In view of the similar relevant states in the confined and spontaneously broken phase near the critical line and the dominant contribution of fluctuations with momenta $k^2 \simeq \Lambda_W^2$ for both p and σ_W we find such an outcome very unlikely.

4. Discussion and conclusions

We end this paper with a few comments:

(i) Even if our picture of asymptotically almost constant multiparticle cross sections turns out to be correct we cannot reliably estimate at present the coefficient c_W. We may try to use the QCD–QFD analogy and take $c_W \simeq c_S(s/m_w^2)$ (11). We choose $\Lambda_S = 1$ GeV, corresponding to the typical particle masses in QCD*. This gives an estimate of σ_W for $m_w = 80$ GeV, $\sqrt{s} = 10$ TeV,

$$\sigma_W \simeq 10\ \mu\text{b}. \tag{28}$$

A suppression of c_W relative to c_S (11), for example $\sim \alpha_w^2(m_w)$, is, however, completely consistent with a smooth behaviour of c_W. Our estimate of σ_W (28) has then to be lowered correspondingly. Unfortunately, perturbative arguments based on the smallness of $\alpha_w(m_w)$ are not reliable in this context. On the other hand, a numerical evaluation of (25) for the same m_w, $\sqrt{s}$ reads

$$\sigma_W = 8.5 \cdot \hat{c}\ \text{nb}. \tag{29}$$

Again it is not clear if (29) can be trusted quantitatively since $\alpha_w \ln(s/m_w^2) \simeq 0.3$ is not much smaller than one. Most probably σ_W is somewhere between a few nb (29) and 10 μb (28) depending on a slow or fast onset of the nonperturbative behaviour. (This depends on the detailed structure of suitable infrared fixed points of the effective (two-dimensional?) theory for the degrees of freedom relevant at the infrared cutoff scale m_w.) As a reasonable lower bound, σ_W may be as low as a few times the tree approximation to the weak elastic qq scattering in the diffractive regime, corresponding to a few times 0.1 nb.

(ii) We have implicitly assumed a large mean multiplicity $\langle n \rangle$ when we identified the total inelastic cross section with the multiparticle cross section σ_W. It seems difficult to conceive that perturbation theory can describe multiplicities of order

* We assume here that the infrared divergence does not depend crucially on the chiral symmetries which are responsible for the light pseudoscalar masses. The effective infrared cutoff should not be too much below the lowest glueball mass in this case.

α_w^{-1}. Nevertheless, a relatively large (and increasing) $\langle n \rangle$ may be suggested within the leading log computations. Whereas cross sections with a fixed number of particles decrease with s, the increase of the total inelastic cross section is related to an increase in multiplicity. (One may expect $\langle n \rangle \sim (s/m_w^2)^\epsilon$, $\epsilon > 0$.) It would be interesting to perform a direct perturbative computation of $\langle n \rangle$ in the framework of ref. [20]. If the true average multiplicity turns out low and perturbation theory is applicable, the almost constant cross sections* for the production of only a few W's etc. may still be of experimental interest, even though our speculations on multiparticle production would not apply in this case.

(iii) We do not expect that the cross sections for the production of only a few weakly interacting particles at large angles become strong. For scattering with large transverse momentum $p_T \gg m_w$ perturbation theory should be reliable. In our picture of the infrared behaviour the decrease of the infrared cutoff μ^2 not only enhances σ_W but also leads to a narrowing of the p_T range. For fixed p_T the cross section decreases strongly once μ^2 becomes much smaller than p_T^2. For $p_T^2 \gg m_w^2$ the naive scaling analysis of exclusive processes [5, 25] should apply. In particular, we expect $\sigma_W(p_T \gg m_w) \sim s^{-1}$.

(iv) Strong electroweak cross sections have first been speculated in the context of $(B+L)$-violating processes. This was based on the point-like form of the effective $(B+L)$-violating interaction which was assumed to grow up to the unitarity limit. The leading-order calculation of the inclusive $(B+L)$-violating cross section certainly breaks down at the energy M_1 (8). The key question is whether the breakdown occurs not already much earlier, say at v? There are many indications that perturbative corrections in the one-instanton sector become important already at $\sqrt{s} \simeq v$ for the *exclusive* cross sections (2) [7, 26]. However, there are also arguments that the corrections to the leading-order *inclusive* cross section exponentiate in such a way that their effects are irrelevant up to energies of order $M_1 \sim E_{sp}$ [9, 26]. If the inclusive cross section (5) really reaches the unitarity limit one would have to take into account multi-instanton configurations which would definitely unitarize the amplitudes [4, 27]. In ref. [4] it was argued that iterations of the instanton amplitudes in the t-channel will give rise to a proliferation of the strong interaction, which occurs first only in the lowest partial wave, to higher partial waves, leading asymptotically to a geometric total cross section for particles with SU(2) gauge interactions like in (9). It should be noted that the iteration of the $(B+L)$-violating amplitude in various channels will lead as well to strong $(B+L)$-*conserving* elastic and inelastic amplitudes. In this sense the $B+L$ violation is not essential at energies above the threshold M_1.

We do not think that the scale of breakdown of the point-like approximation is generally related to the unitary limit. In the instanton computation [2] the domi-

* In this case σ_W may become constant at energies below $M_2 = 2$ TeV (18) and the threshold could be essentially kinematic.

nant contribution comes from instantons of maximal size $\sim \sqrt{n_w + a}/(\pi v)$, where a is of order one. At very high energies, however, the size of the relevant classical configuration is limited by the typical length scale of the scattering process which is proportional to $s^{-1/2}$. The vacuum instanton should be replaced by an instanton in the presence of other particles (sources) [5,28]. For the exclusive $(B+L)$-violating production of only a few particles (e.g. $q + q \to 7\bar{q} + 3\bar{l}$) at high energies we expect that v is replaced in (2) by $\sqrt{v^2 + \eta s}$, with η of order one, resulting in a strong modification of the cross section far below the unitary limit. For the multiparticle production it seems difficult to assess whether the relevant energy scale for the breakdown of the point-like behaviour is determined by the energy of the incoming particles ($\sqrt{s}$) or only reached once the average energy per particle, $\sqrt{s}/n_w$, exceeds $\pi v n_w^{-1/2}$. Only in the second case one could imagine that the point-like structure persists up to the unitarity limit.

We present here a different argument why strong baryon number violation in forward multiparticle production is conceivable. Since perturbation theory breaks down for multiparticle production ($n \geqslant 1/\alpha_w$) the vacuum configuration is not dominant for these processes. (The saddlepoint approximation around the vacuum does not converge.) Other classical configurations (finite-size "instanton" configurations) are equally important. There is then no reason for an exponential suppression factor in $(B+L)$-violating processes, since this factor arises essentially from the difference in the euclidean action between the instanton configuration and the vacuum. The configurations relevant for $(B+L)$-conserving or violating multiparticle production may well have a euclidean action of similar magnitude. We therefore think it is not unreasonable that the strong multiparticle cross sections σ_W at high energies may contain a substantial fraction of baryon number violating events.

(v) So far we have restricted our discussion to only one relevant scale (v) in the model. The real standard model is more complicated. The QCD interactions induce electroweak symmetry breaking through quark condensates even in the symmetric phase ($v = 0$) or in the absence of a Higgs particle. This results in a lower bound on the W-mass of the order $m_{w,\min} \simeq \frac{1}{2} g f_\pi \simeq 31$ MeV. Due to the additional infrared cutoff we have to replace Λ_W and m_w by $m_{w,\min}$ in eqs. (14) and (16) for $v \ll m_{w,\min}$. This does not affect our conclusions on the degree of infrared divergence of σ_W in the nonabelian SU(2) theory at the scale $m_w \simeq$ 80 GeV.

Also, even for $m_w \neq 0$, there remain massless or nearly massless weakly interacting particles, namely the photon and the light quarks and leptons. If these particles would contribute to an infrared divergence of σ_W at momentum scales below m_w, this would further enhance σ_W. Large multiparticle cross sections for weakly interacting particles have been discussed in the context of unexplained cosmic ray events [29]. We emphasize that σ_W exceeding considerably (28) would require an additional strong infrared divergence from particles with mass lower than m_w.

Such a possibility should not be ruled out completely before a proper computation of the structure of infrared divergences for the relevant cross sections has been done.

In conclusion, we have given a very simple argument in favour of weak interactions becoming strong at high energies. It is based on an infrared divergence in σ_{W} of degree two. Intuitively, our result implies that at high energies the weakly interacting particles have a geometrical size $\sim m_{\mathrm{w}}^{-1}$, arising from a cloud of scalars and W's around them. For very high particle multiplicities a description in terms of classical waves seems appropriate. As a result of the infrared structure of nonabelian gauge theories the scattering of such waves does not change their transversal spread (small p_{T}) and the total cross section is proportional to their (transverse) geometrical extension. A change in shape and phase of the classical wave, however, when reexpressed in a basis of particle eigenstates, results in the transmutation of an incoming single particle state into an outgoing multiparticle state. It is not unplausible that these waves have a substantial overlap with sphaleron configurations, producing many baryon and lepton number violating events.

Even though we have neither a precise estimate of the multiparticle cross sections ($1\ \mathrm{nb} \leqslant \sigma_{\mathrm{W}} \leqslant 10\ \mu\mathrm{b}$) nor for the threshold of the onset of strong flavour interactions (2–20 TeV) it should be possible to verify or falsify our picture experimentally at future colliders. Events with many, $\mathcal{O}(1/\alpha_{\mathrm{w}})$, weakly interacting particles with transverse momentum $1\ \mathrm{GeV} \ll p_{\mathrm{T}} \leqslant m_{\mathrm{w}}$ should be seen at LHC or SSC. For a (parton) cross section $\sigma_{\mathrm{W}} \geqslant 1$ nb the colliders should uncover this new phenomenon even if the threshold for the onset of strong flavour interactions is as high as 10 TeV or 30 TeV for LHC or SSC, respectively.

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Appendix A

Assume that eq. (25) is valid at very high s if the scale μ is in an appropriate range,

$$\sqrt{s} = \frac{10}{\alpha_{\mathrm{w}}(s)}\mu_1 , \tag{A.1}$$

provided $\alpha_{\mathrm{w}}(s)$ is already small enough such that

$$\alpha_{\mathrm{w}}(s)\ln\left(\frac{s}{\mu_1^2}\right) = 2\alpha_{\mathrm{w}}(s)\ln\left(\frac{10}{\alpha_{\mathrm{w}}(s)}\right) \ll 1 . \tag{A.2}$$

(For this discussion $\alpha_w(s)$ is chosen at the physical value such that Λ_W is fixed.) Allowing for some uncertainty in $\hat{c}$ we obtain ($\alpha \equiv \alpha_w(s)$)

$$10^{-2}\alpha^{1/2}\ln^{-3/2}\left(\frac{10}{\alpha}\right)\left(\frac{10}{\alpha}\right)^{\alpha(16\ln 2)/\pi} < c_W(\mu_1) < 10^2\alpha^{1/2}\ln^{-3/2}\left(\frac{10}{\alpha}\right)\left(\frac{10}{\alpha}\right)^{\alpha(16\ln 2)/\pi}. \tag{A.3}$$

We use this information to give bounds on the anomalous dimension a. Roughly speaking, a (substantial) power-law behaviour of c is inconsistent with the simultaneous validity of eq. (A.3) at $\mu = \mu_1$ and a cross section $\sim \Lambda_W^{-2}$ at $\mu \simeq \Lambda_W$.

Let us introduce the average anomalous dimension $\bar{a}$ between two scales μ_0 and μ_1,

$$\bar{a} = \ln^{-1}\left(\frac{\mu_1}{\mu_0}\right)\int_0^{\ln(\mu_1/\mu_0)} \mathrm{d}t\, a(t), \quad t = \ln\left(\frac{\mu}{\mu_0}\right), \tag{A.4}$$

such that

$$\frac{c_W(\mu_1)}{c_W(\mu_0)} = \left(\frac{\mu_1}{\mu_0}\right)^{-\bar{a}}. \tag{A.5}$$

We first give an upper bound on $\bar{a}$ using, for a scale μ_0 somewhat above the phase transition,

$$\mu_0^2 = 10\Lambda_W^2, \tag{A.6}$$

a very conservative Froissart type upper bound

$$\sigma_W(\mu_0) < \frac{1}{\Lambda_W^2}\ln^2\left(\frac{s}{\Lambda_W^2}\right), \qquad c(\mu_0) < 10\ln^2\left(\frac{s}{\Lambda_W^2}\right). \tag{A.7}$$

Combining this with the lower bound from (A.3) yields

$$\bar{a} < \frac{b\alpha}{2\pi}\left\{\frac{5}{2}\ln\left(\frac{1}{\alpha}\right) + 3\ln 10 + 2\ln\left(\frac{4\pi}{b}\right) + \frac{3}{2}\ln\ln\left(\frac{10}{\alpha}\right) - \frac{16\ln 2}{\pi}\alpha\ln\left(\frac{10}{\alpha}\right)\right\}$$
$$\times\left\{1 - \frac{b\alpha}{4\pi}\ln\left(\frac{10^3}{\alpha^2}\right)\right\}^{-1}. \tag{A.8}$$

This must hold for all s sufficiently high, including values where $\alpha_w(s)$ is arbitrarily small. Similarly, we use a lower bound based on our first two assumptions,

$$c(\mu_0) > 10^{-4}, \tag{A.9}$$

yielding

$$\bar{a} > \frac{b\alpha}{2\pi}\left\{\frac{1}{2}\ln\left(\frac{1}{\alpha}\right) - 6\ln 10 + \frac{3}{2}\ln\ln\left(\frac{10}{\alpha}\right) - \frac{16\ln 2}{\pi}\alpha\ln\left(\frac{10}{\alpha}\right)\right\}$$
$$\times\left\{1 - \frac{b\alpha}{4\pi}\ln\left(\frac{10^3}{\alpha^2}\right)\right\}^{-1}. \qquad \text{(A.10)}$$

The only reasonable behaviour consistent with the constraints (A.8) and (A.10) is a renormalization group equation for the anomalous dimension with an infrared stable fixed point at $a = 0$ (or $a \sim \alpha_w(s)$). For μ sufficiently small compared to $\sqrt{s}$ the μ-dependence of c_W is then logarithmic (or a very small power). This shows that our third assumption follows from the first two assumptions plus the validity of eq. (25) in the range given by (A.1) and (A.2).

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