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Implementation of 2nd-order QCD 3-jet matrix elements in Monte Carlo generators for e^+e^- annihilation

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Abstract. Matrix elements to be used in second order Monte Carlo generators for e^+e^- annihilation have been derived from the $O(\alpha_s{}^2)$ calculations by Ellis, Ross and Terrrano and by Kramer and Lampe. They were incorporated into the JETSET 6.3 Lund String Monte Carlo program. The recombination scheme dependence of the $O(\alpha_s{}^2)$ jet cross sections are studied in detail on the parton level (ERT) and on the hadron level (ERT and KL).

1 Introduction

The measurement of jet cross sections in the hadronic final states in e^+e^- annihilation offers the possibility of testing detailed features of the theory of strong interactions, quantum chromodynamics (QCD). Experimentally, however, not the QCD quanta, the quarks and gluons are seen, but only the colourless hadrons. Up to now the hadronization process can not be calculated from basic QCD and can be described only phenomenologically. Therefore, due to hadronization no direct comparisons between the predictions of the theory and the measured hadrons can be made. Elaborate computer programs to simulate the fragementation process have been developed in the last decade [1–3]. Two different classes of simulation programs are available. The first type employs fixed order QCD calculations as theory input. The now almost exclusively used program of this type is the JETSET Monte Carlo program of the Lund group [3]. In a first step this program determines the jet multiplicity to be produced. For this purpose the QCD cross sections for jet production have to be provided.

Complete perturbative calculations have only been performed up to order $\alpha_s{}^2$. Therefore only 2, 3 and 4 dressed partons can be calculated. This has the consequence that only up to 4 hadron jets can be simulated realistically. Events with more than 4 hadron jets originate from the fragmentation of the 2, 3 or 4 dressed partons and are not described correctly.

In contrast to this simulation of the number of jets the second class of simulation programs, the parton shower programs [4, 5], can produce any number of jets since they calculate any number of produced partons. The drawback of these programs, though, lies in the application of the leading logarithmic approximation (LLA) to the theory which prohibits the deduction of fixed order parameters, like e.g. the strong coupling strength or equivalently the Λ parameter in a specified renormalization scheme.

In the last four years rather accurate measurements of the 2-, 3-, 4- and 5-jet rates have been reported by the JADE [6] and TASSO [7] collaboration at PETRA, the MARK II [8] collaboration at PEP, the AMY [9], TOPAZ [10] and VENUS [11] collaboration at TRISTAN, and just recently by the MARK II [12] collaboration at SLC and the OPAL [13] collaboration at LEP. These experimental data were compared either to parton shower calculations, or to the complete second order matrix element calculations by Ellis, Ross and Terrano (ERT) [14] or by Kramer and Lampe (KL) [15].

In both matrix elements the complete calculations up to $O(\alpha_s{}^2)$ have been performed using different lines of approach. In the ERT approach the inclusive cross section for e^+e^- annihilation into 3 and 4 partons have been calculated including all virtual and real contributions up to the second order in α_s. No resolution parameter for the separation into 2-, 3- and 4-jet contributions has been introduced. For this reason these calculations are usually referred to as representing the results for vanishing jet resolution. In order to obtain cross sections for a fixed number of jets a resolution criterion must be introduced and partons must be recombined to form jets. This recombination of partons is not unique and causes a recombination scheme dependence of the results [16–18]. The KL approach is complementary to the ERT approach in the sense that the resolution criterion, which allows the recombination of partons into 2, 3, and 4 jets is introduced already in the perturbative calculations to cancel infrared and/or collinear divergences between virtual and real contributions. In this approach the cross

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sections for the production of 2, 3 and 4 jets as a function of the resolution parameter were obtained directly. Nevertheless some arbitraryness exists here also. For example, the 3-jet cross section depends on the way how the variables describing 3 jets were formed out of the momenta of the four partons.

The purpose of this paper is two-fold. First we want to make it known that both second order matrix element calculations, ERT and KL, have been implemented in the Lund string fragmentation program and can be used for analysing e^+e^- annihilation data. Second we want to study the recombination dependence of partons and hadrons, the latter corresponding to different schemes for the clustering of hadrons into jets. The recombination dependence of partons is calculated only for the ERT version, in which recombination of partons is unavoidable if one wishes to calculate jet cross sections. The dependence on the hadron clustering is investigated with both, the ERT and the KL based Monte Carlo generators. Concerning the ERT [14] based studies we follow the earlier work of one of us [17]. Of course, for various jet cross sections we shall compare results obtained with the ERT routine with results obtained with the KL approach [15].

The plan of the paper is the following. In Sect. 2 we describe the preparation of the ERT and KL matrix elements to be used in the Lund string Monte Carlo. In this section we also describe the three schemes which are used for the recombination of partons in the ERT based generator and for the recombination of hadrons in both generators, ERT and KL. For the KL generator we demonstrate that the 3-jet cross section obtained with the Monte Carlo (without fragmentation) agrees with the directly calculated cross section reported in [15]. The results of the recombination studies are presented in Sect. 3. Here we show also how selecting clustering schemes may help to enrich the 4-parton contribution which is useful for investigating details of the 4-parton matrix elements concerning the 3-gluon coupling or other details. Section 4 contains a summary and some concluding remarks.

2 Preparation of the ERT and KL matrix elements

The starting point of the simulation of multi-hadronic events in the Lund Monte Carlo generator is the production of 2-, 3- or 4-QCD jets (resolved partons in $O(\alpha_s^2)$) according to the so-called dressed or resolution dependent matrix elements which are installed in the program.

The matrix element for the production of 4 QCD jets in $O(\alpha_s^2)$ is calculated from tree-level diagrams as given in [14] or [19] and no ambiguities are introduced. The calculation of the $O(\alpha_s^2)$ resolution dependent 3-jet matrix elements involves the introduction of a criterion to define resolved 4-jet events and the definition of a procedure of how to recombine the unresolved 4-parton events into 3-jet events. For this we use the scaled invariant mass y_{ij} of any two partons i and j as resolution criterion with y_{ij} defined as usual

$$y_{ij} = (p_i + p_j)^2/s \tag{2.1}$$

where p_i and p_j are the four-momenta of the partons i and j and $\sqrt{s}$ is the center-of-mass energy. If all possible combinations (i, j) yielded scaled invariant masses larger than a prescribed parameter y the n-parton event was counted as a n-jet event. For any combination $y_{ij} < y$ the partons i and j were recombined into a new "dressed" parton (or "QCD"-jet) and the n-parton event was counted as a $(n-1)$-jet event if all possible combinations in the recombined event satisfied $y_{ij} \geqq y$.

As already pointed out in the introduction the complete $O(\alpha_s^2)$ calculation by ERT [14] is performed for vanishing jet resolution and can therefore not be separated into 2-, 3- and 4-jet contributions and thus cannot be used in a Monte Carlo generator in its original form. To make use of this calculation we employed the results of a numerical integration of the ERT matrix elements done by Ali and Barreiro [20], introduced a resolution criterion y as described above and calculated the $O(\alpha_s^2)$ 3-jet matrix elements. The results of the numerical evaluation were in the form of more than 8 million 3- and 4-parton events with specified kinematics and weights. The dressed $O(\alpha_s) + O(\alpha_s^2)$ 3-jet cross section is given by

$$\sigma_{3\text{-jet}}^{O(\alpha_s+\alpha_s^2)}(y) = \sigma_{3\text{-jet}}^{\text{Born}}(y) + \sigma_{3\text{-jet}}^{\text{virt}}(y) + \left\{ \int_{3\text{-jets}} d\sigma_4 \right\}(y) \tag{2.2}$$

where $\sigma_{3\text{-jet}}^{\text{Born}}(y)$ stands for the resolution dependent (since the 3-jet contribution must be separated from the 2-jet contribution) $O(\alpha_s)$ cross section, $\sigma_{3\text{-jet}}^{\text{virt}}(y)$ for the virtual corrections in $O(\alpha_s^2)$, as defined in [20], and $\left\{ \int_{3\text{-jets}} d\sigma_4 \right\}(y)$ for the 3-jet-like unresolved 4-parton contributions minus the terms already contained in $\sigma_{3\text{-jet}}^{\text{virt}}(y)$. $\sigma_{3\text{-jet}}^{\text{Born}}(y)$ and $\sigma_{3\text{-jet}}^{\text{virt}}(y)$ are calculated from the weighted 3-parton events. To calculate $\left\{ \int_{3\text{-jets}} d\sigma_4 \right\}(y)$ those parton pairs that fall below the specified resolution y have to be recombined into a dressed parton such that a 3-jet event balanced in momentum and energy follows.

The recombination of two partons into one jet can be done in different ways. Three schemes have been used in the past [16, 17, 21]: (i) the energy scheme (ERT E), where the 4-vectors of the partons are added. The resulting parton mass is taken into account when checking whether the recombined event belongs to the 2- or 3-jet class after recombination; (ii) the energy scheme without mass (ERT E_0), where the 4-vectors of the partons are added and the mass of the new parton is neglected in the recombined event; (iii) the momentum scheme (ERT **p**), where the momenta of the partons are added. The mass of the new parton is set to zero and the momenta are rescaled to yield a balanced event. The difference between the E- and E_0 recombination scheme lies in the different treatment of the mass of the recombined events. In the E_0 scheme all partons are regarded as massless in the recombined event, as it is the case for tree graph partons, and no compensation for the neglected energy is performed. The E scheme, however, manifestly conserves energy and momentum. In the **p** scheme energy and momentum are also conserved and the resulting recombined events are indistinguishable from genuine 3-jet events. Although one can give good arguments for either of these three schemes there is no principle that would guide us which scheme should be preferred. Therefore we shall consider all three schemes in the following.

Before we can present our results on the recombination dependence of the 3-jet cross section we must describe some further technical details.

In the JETSET 6.3 Monte Carlo program the $O(\alpha_s^2)$ matrix element for 3-jet production is installed as a correction matrix G to the Born matrix element which depends on the scaled energies x_1 and x_2 of quark and antiquark, respectively, and the resolution cut y, so that the sum of first and second order contribution is:

$$\frac{d^2\sigma_3}{dx_1 dx_2}(x_1, x_2, y)$$

$$= \frac{d^2\sigma_3^{\text{Born}}}{dx_1 dx_2}(x_1, x_2)\left[1 + \frac{\alpha_s}{\pi} G(x_1, x_2, y)\right]. \tag{2.3}$$

Second one needs in the Lund program the integrated 3-jet cross section, which is obtained by integration of (2.3) over the 3-jet region. The 4-jet cross section is obtained by integration of the 4-parton matrix elements with the appropriate cuts as explained above. Then one needs still the 2-jet rate as a function of y. This can be deduced from the sum rule

$$\sigma_{\text{tot}} = \sigma_{2\text{-jet}}(y) + \sigma_{3\text{-jet}}(y) + \sigma_{4\text{-jet}}(y). \tag{2.4}$$

Thus all three jet fractions are known if $G(x_1, x_2, y)$ in (2.3) is supplied. To generate G the following steps are taken. The ERT matrix elements were evaluated by Ali and Barreiro [20] in the form that they supplied 6.5 million 4-parton events ($q\bar{q}gg$ and $q\bar{q}q\bar{q}$) and 1.9 million 3-parton events ($q\bar{q}g$) with their corresponding weights. From these events all three contributions to $\sigma_{3\text{-jet}}(y)$ in (2.2), $\sigma_{4\text{-jet}}(y)$ and $\sigma_{2\text{-jet}}(y)$ from (2.4) could be calculated. Also from the stored events the correction matrix $G(x_1, x_2, y)$ in (2.3) was determined and stored as a matrix in x_1, x_2 in bins of $\Delta x_1 = 0.1$, $\Delta x_2 = 0.1$ in total 100 G-matrix elements for 20 y values 0.015, 0.02, 0.03, ..., 0.20. When the correction matrix $G(x_1, x_2, y)$ is known and 3-jet events are generated with it the integrated 3-jet rate obtained from G need not be identical with the 3-jet rate calculated from (2.2) with the same y. The reason for this difference lies in the fact, that with the correction matrix G the separation of 2- and 3-jet events occurs on the basis of the scaled energies x_1 and x_2, which are considered as the jet energies of massless partons. This difference, which is small, will be largest in the E-scheme. One might think that for this mismatch those events give the largest contribution in which a $q\bar{q}$ pair in $ggq\bar{q}$ or in $q\bar{q}q\bar{q}$ is being combined and considered as one jet. This contribution is non-singular for $y_{\min} \to 0$ and is contained exclusively in the last term of (2.2). This contribution from $gg(q\bar{q})$ with $q\bar{q}$ combined has been incorporated into G by identifying the momentum of the first gluon with x_1, of the second gluon with x_2 and then symmetrized. The difference $\Delta\sigma_3(y)/\sigma_{\text{tot}}$ as a function of y, where $\Delta\sigma_3$ is $\sigma_E^{\text{QCD}}(y) - \sigma_E^{\text{MC}}(y)$ is the difference between the result following from (2.2), denoted $\sigma_E^{\text{QCD}}(y)$, and the result obtained from (2.3) with the correction matrix G, denoted

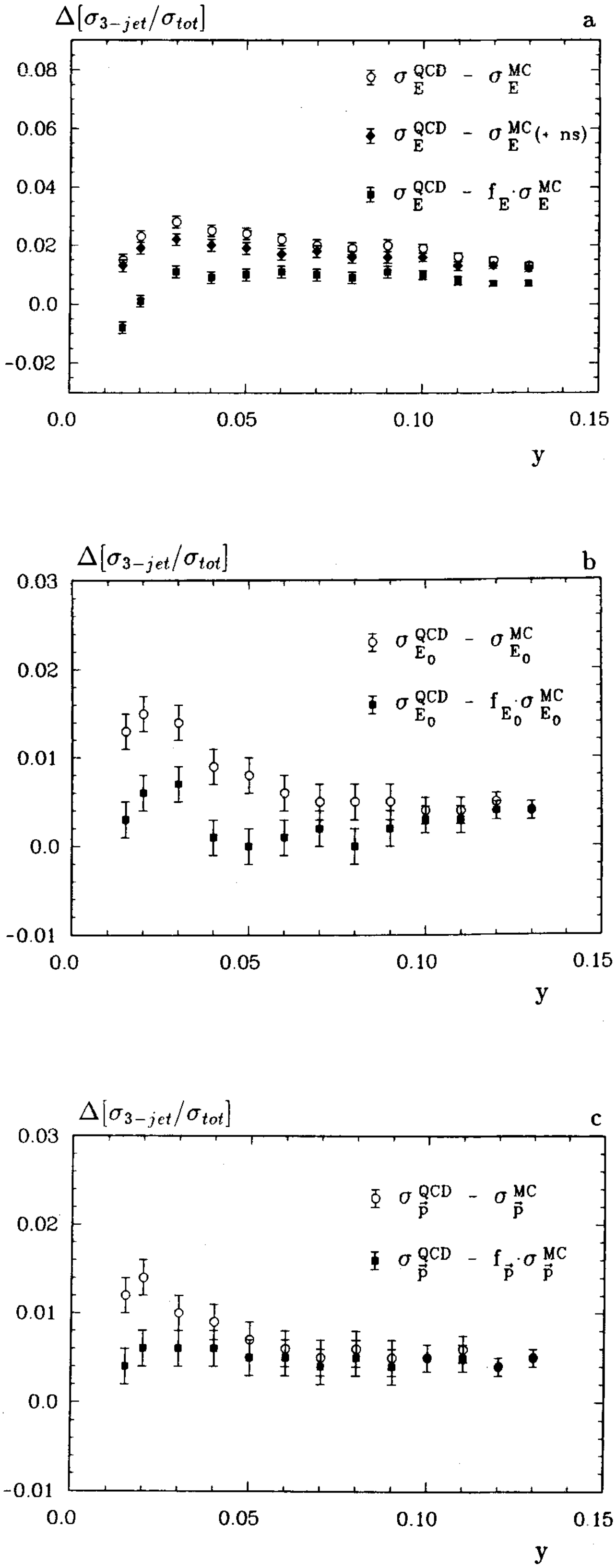

Fig. 1. **a** Deviation of Monte Carlo generated 3-jet rate (σ_E^{MC}) from original 3-jet rate calculated from (2.2) (σ_E^{QCD}) for E recombination scheme as a function of y. Open circles are the full deviations, full diamonds are deviations corrected with non-singular term, full squares are deviations corrected with f_E. **b** Same as **a** for E_0 recombination scheme. **c** Same as **b** for **p** recombination scheme

$\sigma_E^{MC}(y)$ is shown in Fig. 1a. In this figure the open circles give $\Delta\sigma_3$ in the case that the non-singular $gg(q\bar{q})$ or $q\bar{q}(q\bar{q})$ terms are neglected completely. The points labelled $\sigma_E^{QCD} - \sigma_E^{MC}(+ns)$ have the contributions $gg(q\bar{q})$ and $q\bar{q}(q\bar{q})$ included. This changes $\Delta\sigma_3$ by a very small amount which shows that these events contribute very little to $\Delta\sigma_3$. In total $\Delta\sigma_3$ is of the order of 2% and therefore nonnegligible. To restore the correct number of 3-jet events by using the correction matrix G we have renormalized the matrix G by the difference

$$f_E(y) = \sigma_E^{QCD}(y) - \sigma_E^{MC}(y). \tag{2.5}$$

This gives the points denoted "$\sigma_E^{QCD} - f_E\sigma_E^{MC}$" in Fig. 1a. With this renormalization $\Delta\sigma_3/\sigma_{tot}$ is now between 0.5% and 1%. The equivalent results for the E_0 and the **p** scheme are shown in Fig. 1b and c respectively. Since the contribution of $gg(q\bar{q})$ made no difference in Fig. 1a we did not include this contribution for the E_0 and the **p** scheme in Fig. 1b and c. As we expected also in these two schemes the mismatch between the direct calculation of $\sigma_3(y)$ from (2.2) and from G is reduced. For the case with renormalization, using f_{E_0} and f_p defined in analogy to (2.5), the $\Delta\sigma_3/\sigma_{tot}$ are in both cases smaller than 0.5%. The results in Fig. 1a–c are obtained with α_s calculated with the second order formula with $\Lambda_{\overline{MS}} = 100$ MeV at $E_{cm} = 43$ GeV ($\alpha_s = 0.119$). In the following all correction matrices G calculated from the ERT matrix element are renormalized with the functions f_E, f_{E_0} and f_p.

Now we are in the position to study the recombination dependence of the integrated 3-jet cross section as a function of y. For this purpose we have calculated the $O(\alpha_s^2)$ contribution to $\sigma_{3\text{-jet}}(y)$ for the three recombination schemes ERT E, ERT E_0 and ERT **p**. The results are shown in Fig. 2a. We used again $\alpha_s = 0.119$, as in all the following plots, and normalized $\sigma_3 \equiv \sigma_{3\text{-jet}}$ by σ_0, the lowest order cross section with $N_f = 5$. The σ_3 for the three recombination schemes show an appreciable spread. The ERT E scheme produces the largest $O(\alpha_s^2)$ correction to σ_3. The results for the E_0 and **p** scheme lie near together, they differ by approximately 30%. In Fig. 2 we also plotted the $O(\alpha_s^2)$ contribution to σ_3 calculated with the formulas of Gutbrod et al. (GKS) [22], which have been used in the Lund Monte Carlo in the past [3]. In this calculation σ_3 is obtained purely analytically, although in an approximate way. These authors neglected all non-singular contributions to G, which vanish in the limit $y \to 0$. The neglected terms include also the non-singular terms that originate from unresolved 4-parton events of the $ggq\bar{q}$ and $q\bar{q}q\bar{q}$ type with unresolved $q\bar{q}$ pair, already mentioned above. This particular contribution is exhibited in Fig. 2a as $\sigma(ns)$. If we add this to the GKS prediction we come closer to the ERT **p** result, but a small difference, of the order of 0.01–0.02, remains. We notice that all four $O(\alpha_s^2)$ predictions become negative in the vicinity of $y = 0.01$. This is caused by the leading term, which is $(-\ln^4 y)$. The **p** recombination scheme has also been studied by Zhu [21]. His results agree with ours for those y-values where there is overlap. In Fig. 2b we have added the cross sections σ_3 in $O(\alpha_s)$ and $O(\alpha_s^2)$ and normalized with σ_{tot}. This shows clearly the spread in the total σ_3 as a function of y. The GKS prediction is the smallest, the ERT E gives the largest 3-jet cross section.

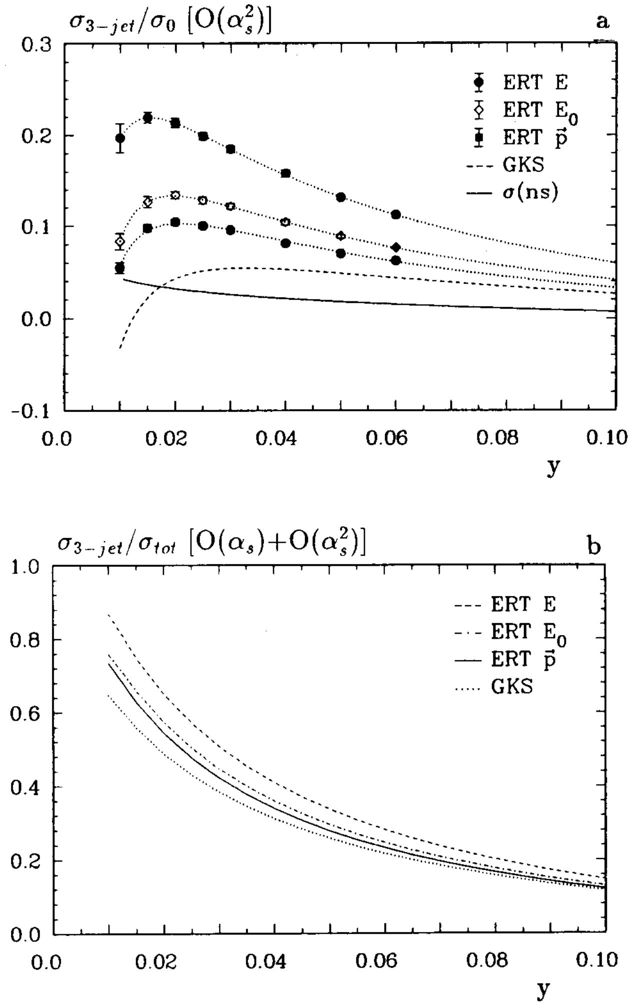

Fig. 2. a $O(\alpha_s^2)$ contributions to the 3-jet rate $\sigma_{3\text{-jet}}/\sigma_0$ as a function of y calculated from ERT matrix element for E, E_0 and **p** recombination and GKS prediction [22] and non-singular contribution (σ(ns)). **b** Comparison of full ($O(\alpha_s) + O(\alpha_s^2)$) 3-jet rates $\sigma_{3\text{-jet}}/\sigma_{tot}$ obtained from ERT E, ERT E_0 and ERT **p** scheme and from GKS

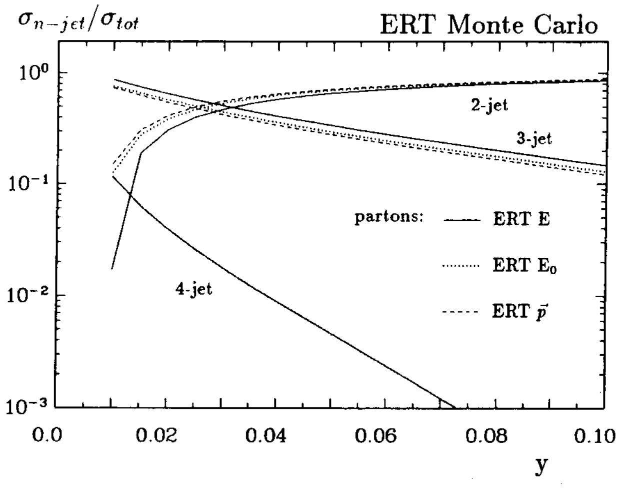

Fig. 3. Recombination dependence of full ($O(\alpha_s) + O(\alpha_s^2)$) n-jet rates $\sigma_{n\text{-jet}}/\sigma_{tot}$ ($n = 2, 3$) for E, E_0 and **p** scheme together with 4-jet rate obtained from ERT Monte Carlo as a function of y

To see the final predictions for all three jet cross sections σ_2, σ_3 and σ_4 as they depend on the recombination scheme we show them in a logarithmic plot in Fig. 3. Of course, the differences are somewhat reduced since the $O(\alpha_s)$ contribution in σ_3 and the $O(1)$ and $O(\alpha_s)$ contributions in σ_2 are independent of the recombination scheme. The 4-jet cross section shows no dependence on the recombination scheme as it was calculated from tree graphs only. Various differential distributions for 3 jets with these ERT E, E_0 and **p** parton dressing schemes [17] have been calculated in [18].

Now we shall describe the KL generator. In [15] the $O(\alpha_s^2)$ matrix elements for 3-jet production have been calculated using a partial fraction decomposition of the 4-parton matrix elements and then applying the invariant mass resolution cut. This allows a separation of the singular and the non-singular terms. The results consist of three parts, (i) the singular contributions, which could be calculated analytically, and are equal to the GKS terms up to small terms of order y, (ii) non-singular contributions which are closely related to the singular terms and which are given as an integral over the y_{ij} invariant in the region $y_{ij} \leqq y$, which produces the singularity in the singular terms and (iii) non-singular terms from the 4-parton phase space region $y_{ij} \geqq y$, which are everywhere finite because of the partial fractioning. The contributions in (ii) are obtained by one numerical integration whereas the contribution (iii) have been evaluated by Monte Carlo methods since an analytical integration of the partial fractions in connection with the complicated phase space boundaries due to the invariant mass resolution cut was not feasable. All three contributions have been incorporated into the correction matrix $G(x_1, x_2, y)$ defined in (2.3). This implementation of the KL results was done for $y = 0.01, 0.02, 0.03, \ldots, 0.14$ and for $y = 0.16, 0.18, 0.20, 0.22, 0.25$. For all these y values $G(x_1, x_2, y)$ is written as a 100×100 matrix in x_1, x_2-bins, 0.01 wide, similar to the implementation of the ERT matrix elements. In [15] the separation of the 3- and 4-jet contribution of the 4-parton matrix elements with the chosen resolution cut y had already been done in the course of the analytical and numerical evaluations. Therefore no subsequent recombination of partons as in the implementation of the ERT matrix elements is needed here. Also the re-evaluation of the $O(\alpha_s^2)$ contribution to σ_3 using the correction matrix $G(x_1, x_2, y)$ should reproduce the results for σ_3 obtained in [15]. This has been checked by Monte Carlo integration of the 100×100 correction matrix G. We write σ_3/σ_0 as usual

$$\sigma_3(y)/\sigma_0 = \alpha_s C_{31}(y) + \alpha_s^2 C_{32}(y). \tag{2.6}$$

Our results for $C_{32}(y)$ for all y values for which G has been converted into matrix form are collected in Table 1. We compare them with the directly calculated values taken from [15] for those y values for which results are reported in this reference. The agreement is very good. For all $y \geqq 0.02$ the deviations are below 0.7%. A small difference is caused by neglecting a small non-singular piece of the $q\bar{q}q\bar{q}$-contribution when calculating G which is below 0.3%. This term was included in C_{32} (analytical). For $y = 0.01$ the deviation is larger which presumably comes from the fact that the higher-order corrections are transformed into the matrix G with finite bins in x_1 and x_2. The achieved accuracy, however, is sufficient for our purposes.

Table 1. Comparison of the 3-jet rate $O(\alpha_s^2)$ coefficient C_{32} for various y values as calculated with correction matrix G (C_{32}(matrix)) with directly calculated C_{32} (analytical) from [15]

y	C_{32} (analytical)	C_{32} (matrix)	Δ(%)
0.01	6.8367	6.7166	−1.8
0.02	9.2478	9.2651	0.2
0.03	8.2678	8.3150	0.6
0.04	7.1111	7.1457	0.5
0.05	6.0494	6.0822	0.5
0.06	5.1574	5.1618	0.1
0.08	3.7260	3.7343	0.2
0.10	2.7045	2.7079	0.1
0.12	1.9686	1.9749	0.3
0.14	1.4343	1.4446	0.7
0.16		1.0504	
0.18		0.7563	
0.20		0.5329	
0.22		0.3628	
0.25		0.1836	

Actually in [15] two different predictions for 2- and 3-jet cross sections have been presented. In both versions all subleading 4-parton contributions were included from the start. It was found, however, that the 3-jet cross section depends on the way the variables describing 3-jets were formed out of the momenta of the 4 partons. In one version, called KL1 in this work (this corresponds to KL in [15]), if the singular term in y_{ij} corresponds to a qg pair, the energy of $\bar{q}$ is unchanged in the recombination procedure, while the energy of the unrecombined gluon is increased by $y_{ij}E_{CM}/2$ and that of the qg pair is decreased by the same amount. If the recombined pair is gg or $q'\bar{q}'$ the treatment is more symmetric. The energy of the combined pair is decreased and the sum of quark and antiquark energies is increased accordingly. We have symmetrized the final result in quark and antiquark energies x_1 and x_2 to eliminate the unsymmetric procedure of quark and antiquark energies in the recombination process.

In the other scheme, the KL′ in [15], denoted KL2 in this work, the definition of 3-jet variables is essentially similar to the E scheme, in that the energy of a recombined parton pair (ij) is $E_i + E_j$ with three-momenta suitably modified. The treatment of gg and $q'\bar{q}'$ pairs is as in the KL1 procedure.

In the study of a singular term in y_{ij} in the region $y_{ij} < y$, it is always the partons (ij) that are recombined, even if another parton pair has a smaller invariant mass, e.g. if $y_{ik} < y_{ij}$. For the numerical integration in other regions, where the 4-parton matrix elements are non-singular because of partial fractioning, the energy recombination is performed for the pair with small invariant mass in that region. The second order corrections in the KL1 scheme are larger than in the KL2 scheme. The different results for the KL1 and KL2 procedure is similar to the recombination dependence for the ERT case described above. In this work we have implemented only the KL2 version into the JETSET 6.3 Lund program.

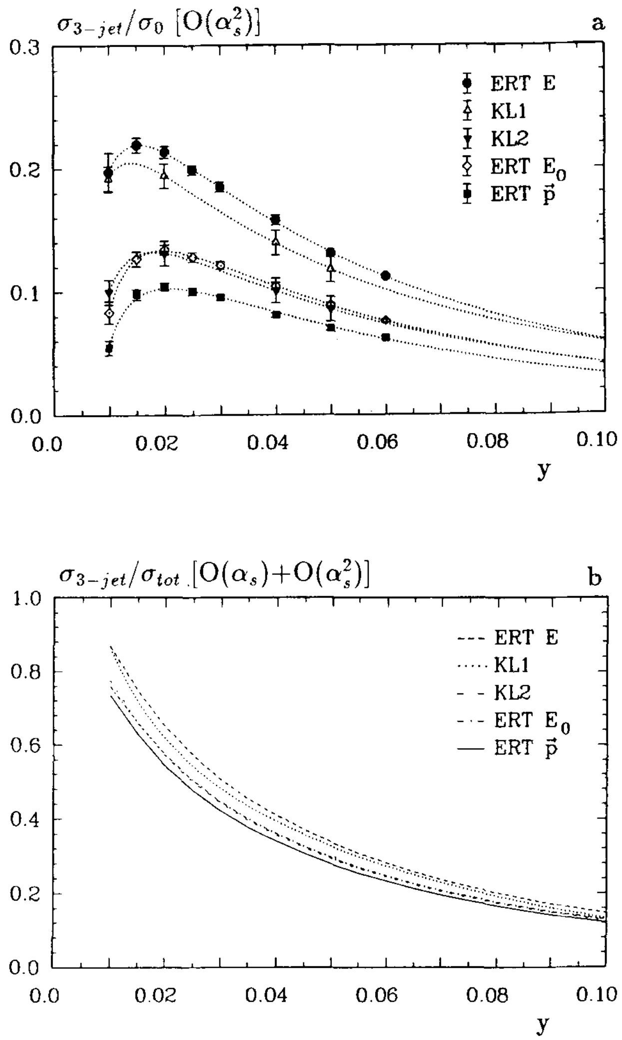

Fig. 4. a Results from Fig. 2a for ERT E, ERT E_0 and ERT **p** compared to KL1 and KL2 results [15]. **b** Results from Fig. 2b for ERT E, ERT E_0 and ERT **p** compared to KL1 and KL2 results [15]

But the integrated 3-jet cross sections as a function of y are available for both versions, however, for a limited number of y values [15] only and can be compared with the ERT E, ERT E_0 and ERT **p** results presented above. This is shown in Fig. 4a and b. In Fig. 4a we compare the $O(\alpha_s{}^2)$ contribution to $\sigma_3(y)/\sigma_0$ from Fig. 2a with the KL1 and KL2 result [15]. We see that the ERT E_0 cross section is almost equal to KL2 whereas KL1 lies only slightly below the result for ERT E. In Fig. 4b we have included the $O(\alpha_s)$ contributions in the plot for $\sigma_3(y)/\sigma_{tot}$. We see that ERT E_0 and KL2 almost coincide and KL1 is smaller than ERT E by 0.01–0.02. This completes our description of the implementation of the ERT and the KL2 matrix elements into the JETSET 6.3 Lund Monte Carlo program. We compared results for the three recombination schemes of the ERT implementation with the two KL versions. In the next section we shall use the two Monte Carlo routines, ERT and KL (we write KL for KL2 in the following), and shall study how the jet cross sections depend on the recombination scheme which is used to define jets out of the hadron distribution with various cluster algorithms.

3 Hadron recombination dependence

If we want to confront the predictions for 2-, 3- and 4-jet cross sections presented in the last section with experimental data we can do this in two ways. One way is to use the implementation of the ERT E, ERT E_0, ERT **p** or KL matrix element in the Lund string Monte Carlo to generate hadron events. To these hadron events one of the popular cluster algorithms for the definition of hadron jets is applied. As cluster algorithms we consider the same E, E_0 and **p** scheme for partons as in Sect. 2. The only difference is that these recombination schemes are applied to hadron four-momenta with a preselected cluster parameter y. For the **p** scheme, however, we have to introduce one important change from the definition in Sect. 2. If we apply the same algorithm to hadronic events, as we did to the partons, the total cross section is almost saturated by 2-jet and 1-jet events for large y values and only very low 3- and 4-jet rates are measured. This occurs because the rescaling procedure tends to drag the jets into the vicinity of the highest momenta tracks and thus favours the q and $\bar{q}$ directions. We therefore have chosen to use the **p** algorithm without rescaling of momenta. It then yields multijet rates comparable to the E and E_0 algorithm. Then one might try the **p**-scheme without rescaling also for the parton recombination. We have estimated the influence of the rescaling procedure on the ERT **p** cross section by comparing our result to the ERT **p** result of [18] who use the **p** scheme without rescaling. Our 3-jet rates for $0.2 \leqq y \leqq 0.10$ are about 1–4% larger than the 3-jet cross sections given in [18]. This causes an even larger spread of the **p** scheme to the two other schemes on the parton level than shown in Fig. 2b.

Now the same cluster algorithm applied to the Monte Carlo generated hadron events is applied to the measured hadron events and then compared to the calculated events. This way we can check whether the measured hadron jet cross sections as a function of y agree with the generated hadron-jet cross sections and use this as a test of perturbative QCD. Such comparisons with the JADE experimental data [6] have been performed in great detail by one of us [17]. The other method is to use the Lund Monte Carlo to calculate correction matrices which allow to deduce the QCD-jet cross sections from the hadron-jet cross sections for a definite Monte Carlo version ERT E, ERT E_0, ERT **p** or KL and for a definite cluster algorithm E, E_0 or **p**. It has also been suggested by Kunzst et al. [18] that there should be a one-to-one correspondence between the recombination scheme to define jets out of partons, i.e. E, E_0 or **p** and the scheme to define jets or clusters from hadrons, i.e. also E, E_0 or **p**, so that these correction matrices should be near unity, if the same scheme is applied for the definition of jets from partons and from hadrons. This is unfortunately not the case as shown in Fig. 5a and b. In Fig. 5a we present curves for the n-hadron jet fractions σ_n/σ_{tot} ($n = 2, 3, 4$) as a function of y for the two cases of E and **p** algorithm. The hadrons for the E(**p**) cluster algorithm are generated

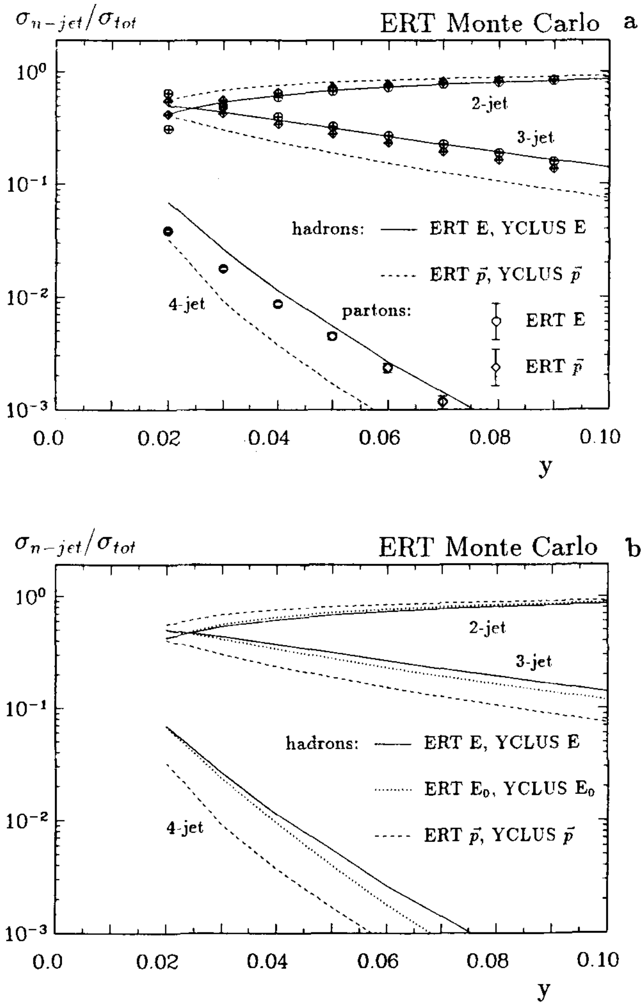

Fig. 5. **a** 2-, 3- and 4-hadron jet fractions $\sigma_{n\text{-jet}}/\sigma_{tot}$ as a function of y for E and $\mathbf{p}$ cluster algorithm (YCLUS) for hadrons based on ERT E and ERT $\mathbf{p}$ parton recombination together with parton jet fractions for ERT E and ERT $\mathbf{p}$. **b** n-hadron jet fractions $\sigma_{n\text{-jet}}/\sigma_{tot}$ ($n = 2, 3, 4$) as a function of y for E, E_0 and $\mathbf{p}$ cluster algorithm based on ERT E, ERT E_0 and ERT $\mathbf{p}$ parton recombination

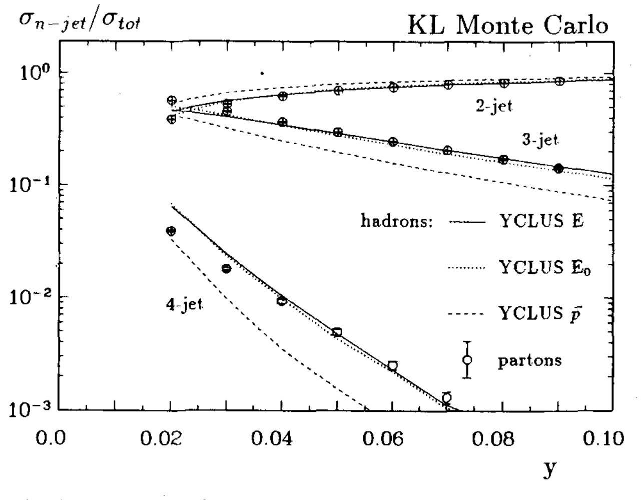

Fig. 6 n-hadron jet fraction $\sigma_{n\text{-jet}}/\sigma_{tot}$ ($n = 2, 3, 4$) as a function of y generated with KL Monte Carlo for E, E_0 and $\mathbf{p}$ cluster algorithm for hadrons

with the ERT $E(\mathbf{p})$ recombination scheme for partons. We see that the results for the E and $\mathbf{p}$ clustering differ by an appreciable amount, in particular σ_3/σ_{tot} and σ_4/σ_{tot}. The curves for the parton predictions lie much nearer together as we had seen already in Fig. 3. Furthermore for the E-scheme the hadron and parton curves almost coincide, so that for the E scheme the correction matrix is approximately the unit matrix. For the $\mathbf{p}$ scheme, on the other hand, the correction from hadron to parton jet-cross sections is certainly large. In Fig. 5b we show the hadron-jet fractions σ_n/σ_{tot} ($n = 2, 3, 4$) with E, E_0 and $\mathbf{p}$ clustering, where the hadrons used for the E, E_0 and $\mathbf{p}$ clustering are generated from the ERT E, E_0 and $\mathbf{p}$ Monte Carlo. The spread in the hadron jet-fractions is much larger than the spread shown in Fig. 3 originating from the recombination of partons. Thus the results depend very much on the clustering scheme. If the scheme dependence on the hadron side would be approximately the same as on the parton side the three curve in Fig. 5b for $n = 2$, 3 and 4, respectively, should coincide. This is not the case. That this does not happen we can perhaps understand from the fact that if jets are formed out of hadrons many more hadrons produce one jet or cluster than partons form one dressed jet. Then on the hadron side the ambiguities inherent in the E, E_0 or $\mathbf{p}$ scheme must matter more than on the parton side.

In Fig. 6 we compare the hadron-jet fractions generated with the KL Monte Carlo and analysed with E, E_0 and $\mathbf{p}$ cluster algorithm. The result is similar to Fig. 5a obtained for the ERT based Monte Carlo. The results for the E, E_0 scheme on one side and for $\mathbf{p}$ on the other side differ very much. Furthermore we observe that for the E_0 scheme the hadron-jet fractions overlap almost completely with the parton-jet fractions. This means that the correction matrix for the E_0 scheme has the effect that there is a "linear correction" factor equal to one. This does not mean, that the correction matrix is the unit matrix. This can even be more clearly seen in the differential jet rates $D_2(y)$ defined as

$$D_2(y) = N \cdot [R_2(y + \Delta y) - R_2(y)] \tag{3.1}$$

with N the total number of events which is shown in Fig. 7a–c for the three jet algorithms. The differential jet rates as determined with the E_0 jet algorithm follow almost exactly the differential parton rates as given by the KL Monte Carlo events. For the E- and $\mathbf{p}$-algorithms the linear correction factor on $D_2(y)$ is larger. The same result is obtained for the ERT E_0 Monte Carlo. We conclude from this that in particular for the E_0 scheme the experimental data for the hadron-jet fractions can directly be compared to the parton predictions, i.e. for example to the results in [15]. In Fig. 5a we observe this also for the ERT based routine. This finding, which had been obtained also in [6, 8, 13] with the parton shower program with subsequent fragmentation into hadrons, is very satisfactory since it offers the possibility to measure the coupling α_s without recourse to complicated hadronization programs. These findings are for $\sqrt{s} = 43$ GeV. In general the linear correction factors are a function of the c.m. energy and the y cut. If the c.m. energy is large

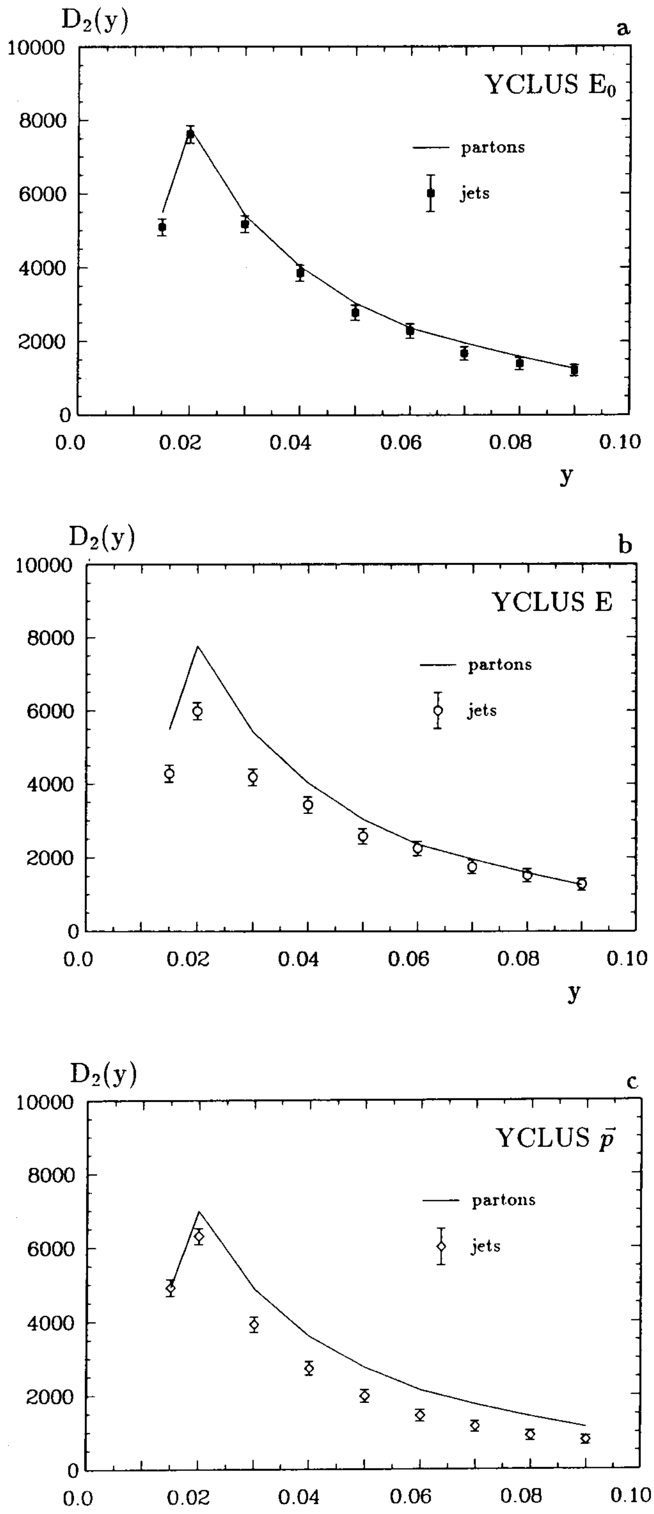

Fig. 7. **a** Differential hadron jet rate $D_2(y)$ as a function of y for E_0 cluster algorithm generated with KL Monte Carlo compared to KL differential parton jet rate. Points are for hadron jets. **b** Same as **a** for E cluster algorithm. **c** Same as **b** for **p** cluster algorithm

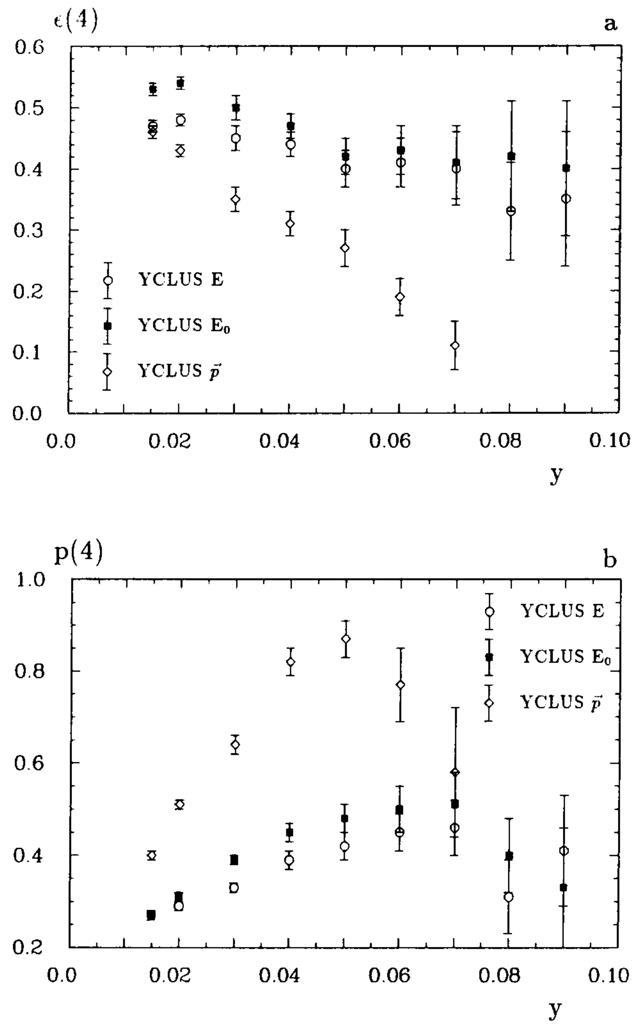

Fig. 8. **a** Efficiency $\varepsilon(4)$ for constructing 4 jets as a function of y for the cluster algorithms YCLUS E, E_0 and **p**. **b** Purity $p(4)$ of reconstructed 4-jet sample as a function of y for the cluster algorithm YCLUS E, E_0 and **p**

enough ($E_{c.m.} \gtrsim 35\,\text{GeV}$) the correction factor for the E_0 scheme is always equal to one for $y \gtrsim 0.08$. For larger energies it is reached for even smaller y values.

As a last subject we want to show a possible application of the different jet algorithms. Using the KL Monte Carlo events we determined for each of the three jet reconstruction algorithms the efficiency $\varepsilon(4)$ for reconstructing 4 jets and the purity $p(4)$ of the reconstructed 4-jet sample for different values of the cut-off y in the algorithm with

$$\varepsilon(4) = \frac{\text{\# of reconstructed 4-jet events}}{\text{\# of 4-parton events}} \tag{3.2}$$

and

$$p(4) = \frac{\text{\# of reconstructed 4-jets stemming from 4-parton events}}{\text{\# of reconstructed 4-jet events}}. \tag{3.3}$$

These results are shown in Fig. 8a for $\varepsilon(4)$ and in Fig. 8b for $p(4)$. From Fig. 8b it is readily seen that using the **p**-algorithm for determining the jets a very clean 4-jet sample can be produced for jet resolution cuts in the range of $0.04 \leqq y \leqq 0.06$. Although the efficiency for reconstructing 4-jet events is only 20–30% the sample has a background of only about 20% compared to the E_0 jet algorithm where $\varepsilon(4)$ is about 40% in this y region but $p(4)$ is as small as 40–50%.

These results show that it is worthwhile to select special hadron recombination schemes for example the **p** scheme if one wants to have large number of genuine 4-parton events in the event sample. Since the 4-parton final state is the simplest state in which the 3-gluon coupling occurs the **p** scheme may be advantageous for studying the non-abelian nature of QCD.

This study also shows that the close relationship between parton-jet fractions and hadron-jet fractions in the E_0 scheme is only correct for the integrated cross sections and is not true for all distributions, i.e. not all hadron n-jet events ($n = 2, 3, 4$) originated from the parton n-jet events.

4 Summary and conclusions

In this paper we described the implementation of second-order 3-jet matrix elements in the JETSET 6.3 Lund String Monte Carlo program. As matrix elements we used the ERT formula supplemented with results of numerical integration of the ERT matrix elements in the non-singular region. With this implementation we studied the dependence of the $O(\alpha_s^2)$ corrections to the 3-jet cross section on the dressing, i.e. the method to combine non-singular 4-parton configurations into 3 jets. For this we employed three recombination schemes, the ERT E, E_0 and **p** scheme. The integrated $O(\alpha_s^2)$ 3-jet rates as function of the mass resolution cut differ appreciably for the three dressing schemes. The ERT E yields the largest, the ERT **p** the smallest $O(\alpha_s^2)$ corrections.

These results are compared with another complete second-order calculation of the integrated 3-jet cross section [15] for which also two versions KL1 and KL2 exist, depending how the 3-jet variables are defined in terms of the 4 parton momenta. It turns out that KL1 agrees approximately with ERT E and KL2 almost exactly with ERT E_0 in the $O(\alpha_s^2)$ 3-jet fraction.

The KL2 version is also implemented into the Lund Monte Carlo program. It has been checked that the implementation produces the same 3-jet fraction as the original calculation. With both Monte Carlo programs ERT and KL we investigated the recombination or cluster algorithm dependence on the hadron side. We used three cluster algorithms YCLUS E, E_0 and **p** which are defined in a similar way as the parton recombination schemes. Concerning ERT we always combined ERT E for partons with YCLUS E for hadrons and similarly for the E_0 and **p** scheme.

For both versions ERT and KL we found an appreciable dependence on the cluster algorithm. The 3-jet rate for YCLUS **p** is always smaller than for YCLUS E and YCLUS E_0. This spread of $\sigma_{3\text{-jet}}$ is much larger than the spread on the parton level in the ERT version. An important result is that the YCLUS E_0 scheme gives almost identical 3-jet rates for hadron jets as the ERT E_0 and KL predicted for the dressed parton jets. This is particularly apparent if we compare the differential jet rate $D_2(y)$ as a function of y. This means that the linear correction factor on $D_2(y)$ is almost equal to one for the E_0 scheme.

As an application of the KL Monte Carlo we determined for each of the three jet reconstruction algorithms the efficiency for constructing 4 jets and the purity of the reconstructed 4-jets concerning the original 4-parton events. It turned out that with the **p** scheme one gets a very clean 4 jet sample (high purity), but with low efficiency. The other two schemes YCLUS E and YCLUS E_0 have large efficiency but less purity than the **p** scheme.

We conclude that the cluster algorithm dependence for defining jets out of hadrons is not negligible and must be taken into account for quantitative tests of perturbative QCD. It leads to a systematic uncertainty, for example, in the determination of the strong coupling constant α_s in agreement with earlier findings [16].

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Note added in proof. The E cluster recombination scheme applied in the definition of the number of jets from hadrons is actually used in such a form that the reconstructed jets are taken to be massless when applying the resolution criterion (2.1) to test for resoluable jets. This means that the E cluster scheme was used only for the recombination of hadrons into jets and the E_0 scheme was applied for the final recombination of jets. Taking the reconstructed jet mass also in the final step of the cluster recombination into account yields a much larger n-jet rate ($n \geq 3$) and thus strengthens the above conclusion (see e.g. [23]).

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