Baryon-Number Violation in High-Energy Collisions

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We study the phenomenology of baryon-number violation induced by electroweak instantons and find that if the naive instanton amplitudes were valid for arbitrarily high energies, the event rate at the Superconducting Super Collider could be a few per hour. A typical event would consist of 3 "primary" antileptons and 7 "primary" antiquark jets, accompanied by ~ 85 electroweak gauge bosons, having a sharp threshold in the total subenergy at about 17 TeV. However, the instanton approximation is not valid at such high energy (*above* the sphaleron energy), so that new theoretical methods are needed.

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't Hooft showed many years ago that the electroweak instanton induces a low-energy effective Lagrangian which simultaneously annihilates or creates one member of each different electroweak doublet.[1] Assuming three generations, each consisting of a lepton doublet and three quark doublets (one of each color), this effective Lagrangian gives rise to processes such as $uu \to \bar{d}e^+\bar{c}\bar{s}\bar{s}\bar{\nu}_\mu\bar{t}\bar{t}\bar{b}\tau^+$, but with an unobservably small amplitude: $\exp(-8\pi^2/g_2^2)\sim 10^{-80}$. Recently, Ringwald[2] and others[3,4] have attempted to calculate the electroweak-instanton-induced amplitudes for scattering to final states in which the fermions are accompanied by large numbers of gauge and Higgs bosons, using the naive instanton approximation. Their results, if used at very high energy and coupled with approximate treatments of the final-state phase space, suggested that the total cross section for $B+L$ violation might become large enough to be observed at Superconducting Super Collider (SSC) energies. This use of the naive instanton approximation for energies much greater than M_W has been challenged by many authors.[5-14] In particular, Refs. 11 and 12 show that a better treatment leads to a *suppression* relative to the naive result, at least in related models. Nonetheless, it is interesting to consider the phenomenology of this system, since the extent of the suppression is not quantitatively known for the interesting case of electroweak gauge theory, and the difference between a factor of 1 or 4π multiplying $-1/g^2$ in the exponential can make the difference between an observable or unobservable effect. Therefore in this Letter we take the naive instanton results for the amplitudes at face value, and investigate their phenomenological consequences. This work lays the foundation for the study of the physics of this system when the correct amplitudes are known.

The most important contribution to these processes in pp scattering arises from the case in which the initial particles are u or d quarks. We begin by considering the case that an arbitrary number n_H of Higgs bosons accompany the fermions. Ringwald[2] was the first to obtain the naive instanton amplitude for this process. Using the extreme relativistic approximation for phase space (which, as we shall see later, is accurate for $n \lesssim E/5m$) he observed that if the naive instanton approximation could be used at arbitrarily large energies, the total cross section for baryon-number violation would become observably large for a parton c.m. energy of "tens of TeV." Since the most important contributions to the cross section in this case came from final states containing $n\sim 3E/5m$, however, the extreme relativistic approximation is not really trustworthy in the regime of interest. This was pointed out by McLerran, Vainshtein, and Voloshin (MVV),[4] and they proposed using the nonrelativistic approximation instead. They concluded that "electroweak interactions become strong at energy above ~ 10 TeV."[4] This is not borne out by a correct quantitative analysis, as we shall see below. (Their formula for nonrelativistic phase space *underestimates* the correct approximate formula,[15] and their approximations in implementing the saddle-point summation of $\sum\sigma_{n_H}$ are valid only at *extremely* large energies. Nonetheless we find, using their amplitudes and formula for phase space, that $\sum\sigma_n$ is more than a factor 10^{140} *below* unitarity at $E\sim 10$ TeV, and grows *very* slowly with energy. Indeed, their formula for the total cross section [Eq. (63)] only becomes large for $E \gtrsim (400\ \text{TeV})[(1/c^2)(250\ \text{GeV})/M_H]$, where c is supposed to be of order 1.)

When is it possible to use simplifying approximations to large-n phase space? We show in Fig. 1 the base-10 logarithm of the extreme relativistic and nonrelativistic approximations to n-body phase space,[15] for $m=81$ GeV and $E=16$ TeV. For comparison, we show the "true" phase space, as computed using either the Monte Carlo phase-space generator RAMBO (Ref. 16) or the analytic method described in Ref. 15 (the results of these two methods are indistinguishable on the logarithmic plot shown). In the analytic approach, the delta function imposing overall energy-momentum conservation is replaced by a Laplace transform and the final expression for the phase space is a one-dimensional integral of the nth power of a function of the Laplace transform variable. In the cases of interest to us, this function can be

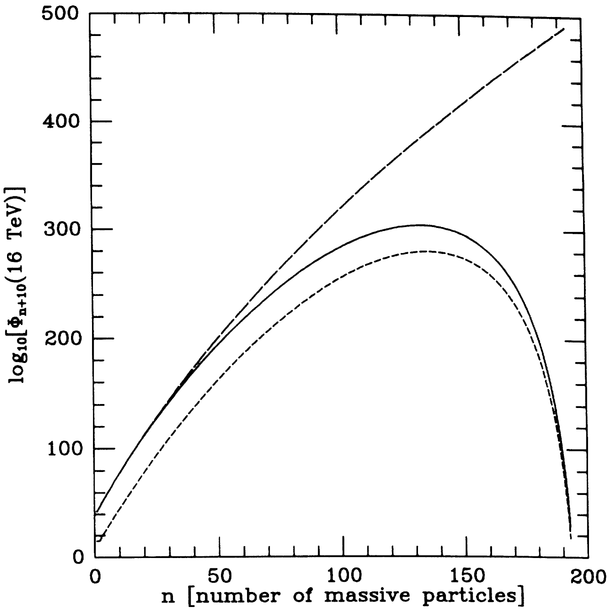

FIG. 1. Phase space vs the number of particles in the final state for $\sqrt{s}=16$ TeV and $M=81$ GeV, as computed using the extreme relativistic (long-dashed line) and nonrelativistic (short-dashed line) approximations, and the analytic method described in the text (solid line).

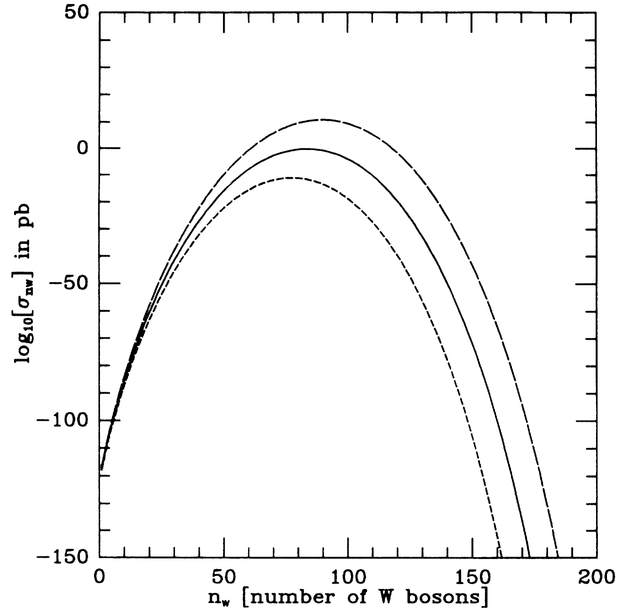

FIG. 2. Quark-level cross section as a function of the number of gauge bosons produced: $\log_{10}\sigma_{n_W}$ (in pb) vs n_W for $\sqrt{s}=17$ TeV (solid line), $\sqrt{s}=18$ TeV (long-dashed line), and $\sqrt{s}=16$ TeV (short-dashed line).

evaluated analytically. The final integration can be done by saddle-point approximation, and yields the usual extreme relativistic and nonrelativistic expressions for small and large n, respectively. For intermediate n, the equation determining the saddle point needs to be solved numerically. Since the analytic method is in excellent agreement with RAMBO, we use it in preference to RAMBO which is rather time consuming for large n. We warn potential users of this method that the numerical work must be done carefully, when n is so large, or large systematic errors can be introduced. For $n\sim 120$, the relevant value according to Ringwald's calculation, Fig. 1 shows that the extreme relativistic approximation overestimates the cross section by a factor of $\sim 10^{60}$, while the nonrelativistic approximation underestimates the cross section by more than a factor of $\sim 10^{25}$, except for $n \gg 150$. This exercise teaches us the importance of doing n-body phase space quantitatively when n is large.

We now turn to the determination of the cross section for $B+L$ violation initiated by the collision of two u and/or d quarks, using Ringwald's amplitudes and the analytic method to evaluate phase space, and adding up the contribution of each σ_n. We find that $B+L$ violation accompanied only by Higgs-boson production is negligible at the SSC, for any value of M_H which is not excluded experimentally. Since the original work of Ringwald, it has been pointed out[3,9,17,18] that the amplitude for production of gauge bosons has a different energy dependence from that for Higgs-boson production. In particular Ringwald's Eq. (66) should be multiplied by $2(3\omega^2+\mathbf{k}^2)/3m_W^2$ for each gauge boson.[19,20] We have therefore computed the cross section for arbitrary n_W and n_H, evaluating the phase-space integrals using the analytic method, now with a matrix element having nontrivial dependence on the momenta of the gauge bosons and fermions. We checked by comparing with RAMBO at 16 TeV that the analytic method also works well in this case. We find that SU(2) gauge-boson production dominates Higgs-boson production, and Higgs-boson production can be ignored altogether to excellent accuracy. Figure 2 shows the cross sections for production of n_W $W^{\pm}$'s for $E_{qq}=16$, 17, and 18 TeV, using the Ref. 17 coefficient for the gauge-boson factor. We have also done the same calculation using the Z^0 mass and the result is essentially the same, indicating that the effects of SU(2) breaking can be neglected for our present purpose. The cross section peaks for $n_W\sim 85$, a smaller number than for pure Higgs-boson production, which is not surprising since that increases the momentum-dependent factor in the amplitude. As can be seen from Fig. 2, there is an extremely abrupt threshold in the total cross section at ~ 17 TeV. The threshold shifts to 21 TeV when the gauge boson factor is multiplied by $\frac{1}{3}$, following Ref. 9, and $\langle n\rangle$ increases slightly.

We next turn to the computation of the cross section for pp collisions, to see if such a parton-level cross section would be observable at currently proposed accelerators. In order to model the parton-level cross section, we take advantage of the sharp threshold and the fact that unitarity must be respected in a physical process. The naive-instanton-approximation amplitudes are pointlike, i.e., purely s wave, so that their unitarity limit is

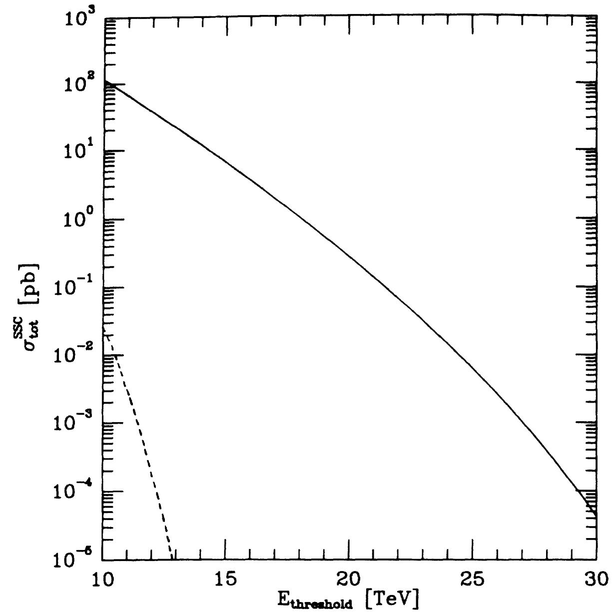

FIG. 3. Total cross section at the SSC (solid line, c.m. energy =40 TeV) and LHC (dashed line, c.m. energy =16 TeV) as a function of the threshold energy.

$\sigma_{tot} \leq 16\pi/s$ (=70 pb at $\sqrt{s}=17$ TeV). We thus model the quark-level cross section as $\theta(\hat{s}^{1/2}-E_{thresh})16\pi/\hat{s}$. Using the structure functions of Ref. 21 and $E_{thresh}=17$ TeV, we find a cross section of ~ 1.5 pb for pp collisions at 40 TeV c.m. energy, corresponding to a few events per hour at the SSC, while at $E_{thresh}=21$ TeV the cross section at the SSC is about an order of magnitude lower. Anticipating that future theoretical developments will result in modifications to the amplitudes, we give in Fig. 3 the cross section at the SSC and at the CERN Large Hadron Collider (LHC), as a function of the energy of the threshold.

We now argue that the events, if they are actually produced, will be easily distinguished from conventional physics, and furthermore will provide a signature of their $(B+L)$-violating nature, even though it is not easy to distinguish between quark and antiquark jets. A typical event would have $E_{qq} \sim 17$ TeV and would contain ~ 85 SU(2) gauge bosons, approximately $\frac{2}{3}$ of them $W^{\pm}$'s and the remainder photons and Z^0's.[22] These events are very distinctive. The gauge particles decay to jets and leptons, such that on average each event contains ~ 120 jets, ~ 10 charged leptons (not including τ's or their decay products), and 30 ν's, each carrying about 95 GeV, with, typically, two $e^{\pm}$ and/or $\mu^{\pm}$ pairs reconstructing to a Z. Even though such a large number of jets will tend to overlap and thus not be completely separable, the large multiplicity of charged particles ($\sim 10^3$) and large total energy (~ 15 TeV) in the central detector, with unbalanced transverse momentum of several hundred GeV (from the ν's), is unlike any ordinary event. Although we cannot reliably estimate the QCD background, it is clearly negligible, since the cross section for producing events with ~ 120 jets of average energy ~ 95 GeV would (barring total breakdown of our understanding of perturbative QCD) be far below the unitarity limit.

Another characteristic feature of this process is the abrupt threshold in the cross section as a function of the total energy of the subprocess. Even if some non-"instanton" mechanism were discovered to produce large numbers of electroweak gauge bosons, it would be unlikely to imitate this structure, since phase space alone would not produce the abrupt threshold, as can be appreciated by studying Figs. 1 and 3 in the relevant region of n. See Ref. 23 for a discussion of "strong" weak interactions.

However, if the cross section for electroweak instanton-induced $B+L$ violation accompanied by large numbers of SU(2) gauge bosons does become large, it is clear that cross sections for a similar process *without* $B+L$ violation would also be large. This is because a sphaleronlike field configuration, if it can be produced by two high-energy particles, will decay with or without $B+L$ violation depending on "which direction" it goes in index space when it decays. However, the gross features of the final-state gauge bosons will be similar in either case, as will be the total cross section. Thus the sharp threshold with energy, and the presence of large numbers of electroweak gauge bosons, is an indicator that $B+L$ violation may have occurred, but not a proof that it has. While such a phenomenon would be tremendously exciting, even without explicit evidence of $B+L$ violation, we would like to be able to experimentally determine the relative rate of $B+L$ violation in order to investigate the dynamics of this phenomenon.

In principle, at least, it should be possible to determine that $B+L$ has been violated, and at what rate. As noted above, besides the $W^{\pm}$'s and Z^0's which decay to lepton pairs or quark jets, with particles and antiparticles evenly represented in their decays, there are ten primary *antifermions* when $B+L$ is violated. Since each member of each fermion weak isodoublet is equally likely to be produced, $\frac{3}{4}$ of the $(B+L)$-violating events will have either a primary e^+ or μ^+, and $\frac{1}{4}$ will have a primary e^+ *and* μ^+. Events with primary $\mu^+\mu^+$ or e^+e^+ will not occur, unless the energy were so high that multi-instanton processes could play a role. Besides the excess in the number of antileptons as compared to leptons, which would require excellent solid-angle coverage and high statistics to establish, there is an asymmetry in the mean energy of the fastest antilepton as compared to that of the fastest lepton. To illustrate its origin, first neglect the difference in the dependence of the matrix element on the energy of the primary antifermions and gauge bosons. In this case the most energetic e^+ or μ^+ would have, on average, twice as much energy as the most energetic e^- or μ^-, which is always the decay product of a gauge boson and never "primary." In the naive instanton approximation, however, the matrix element has a factor $\sim E_f$ for each fermion and a factor $\sim 4E_{\rm gb}^2 - m_{\rm gb}^2$ for each gauge boson, reducing the asymmetry. In this case, for $E=17$ TeV and $n_{\rm gb}=85$, the average energy carried by a primary antifermion is 116 GeV, while $\langle E_{\rm gb}\rangle = 186$ GeV. Thus the most energetic antilepton typically carries about 30% more energy than the most energetic lepton. Just as the magnitude of the $(B+L)$-violating cross section cannot be reliably obtained from the naive instanton approximation, the form of the matrix element is also uncertain and we must await better theoretical tools to predict this asymmetry with confidence. Nonetheless these two simple cases illustrate that a measurable asymmetry can be expected. Furthermore, if events such as these are produced, the gauge-boson spectra can (at least with an ideal detector) be *measured.* One can then compare the observed spectrum of highest-energy $e^\pm$ and $\mu^\pm$'s with that expected from decays of the gauge bosons. If the fraction of events violating $B+L$ is large enough, the excess of high-energy antileptons will be observable. In principle, the spectra of the primary antifermions could be disentangled from that of the gauge bosons, leading to detailed dynamical information about the process, with which to confront theory.

In conclusion, we have seen that use of the naive instanton amplitudes for electroweak-instanton-induced baryon-number violation in high-energy collisions, combined with a correct treatment of the massive, multiparticle phase space, leads to a predicted cross section at the SSC which is large enough for unambiguous identification of the phenomenon. We must caution, however, that it is highly doubtful that the naive instanton approximation is valid at the relevant energies. References 11 and 12 have shown that as the energy is increased from low to intermediate relative to the barrier height, corrections to the naive instanton approximation *reduce* the amplitude. While no quantitative estimate of the suppression has been made for the electroweak theory, it would seem miraculous for it to be no greater than a factor of 10^5, when the natural scale of suppression is e^{-E^2/m_W^2}. Moreover, we have found that even using the naive instanton approximation, an energy *higher* than the sphaleron energy [7–13 TeV (Ref. 24)] is necessary in order to have an experimentally observable effect. Therefore, the two most important results of this work are the following: (i) The question of the experimental observability of baryon-number violation in high-energy collisions cannot be convincingly addressed theoretically until a reliable computational scheme for energies *higher than* the sphaleron energy is developed. (ii) If $(B+L)$-violating events are produced, it would be useful to have a detector which was "hermetic" and had large solid-angle coverage, good energy resolution, the ability to identify both electrons and muons, and the ability to measure the sign of lepton charges, in order to demonstrate their $(B+L)$-violating nature and investigate the dynamics in detail.

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