Nuclear Physics B355 (1991) 38–53
North-Holland

BARYONS IN THE HEAVY QUARK EFFECTIVE THEORY*

Thomas MANNEL[1,**], Winston ROBERTS[2,***] and Zbigniew RYZAK[2,+]

[1]*Deutsches Elektronen Synchroton DESY, Notkestr. 85, 2000 Hamburg 52, Germany*
[2]*Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA*

Received 29 August 1990
(Revised 29 November 1990)

We show how to incorporate baryons in the heavy quark effective theory. A convenient formalism is exhibited and applied to semileptonic weak decays of heavy baryons. In general, the heavy quark effective theory reduces the number of form factors necessary to describe the matrix elements for transitions among heavy baryons. We also note that in semileptonic weak decays of Λ_c one arrives at an important and experimentally testable prediction that two form factors suffice to describe this process. As a further application of the formalism we discuss exclusive production of heavy baryons in e^+e^- annihilation.

1. Introduction

The physics of heavy hadrons has recently attracted a great amount of attention. One of the reasons is that the experimental data have improved considerably over the last couple of years. Another reason is that new theoretical ideas which lead to a formulation of an effective heavy quark theory [1–7] have been put forward. A number of interesting predictions based on this theory have been found, and this paper shows how the effective theory can be applied to heavy baryons.

One of the main points of the heavy quark effective theory is the fact that it possesses two additional symmetries compared to the full theory of QCD. The first one is a heavy flavor symmetry, namely an SU(N_h) (N_h denotes the number of heavy flavors), under which the heavy quarks may be rotated into one another.

The second symmetry is the so-called spin symmetry, which is due to the decoupling of the spin degrees of freedom in the heavy quark limit.

These symmetries have been applied very successfully to weak decays of heavy mesons [4,8,9] and to exclusive heavy meson production in e^+e^- annihilation

* Work supported by the National Science Foundation under grant PHY-8714654.
** Supported by a Grant from Deutsche Forschungsgemeinschaft.
*** Supported in part by the Natural Sciences and Engineering Research Council of Canada.
+ Supported by the Department of Energy under grant DE-AC02-76ER03064.

0550-3213/91/$03.50

[10, 11]. In addition, since the heavy quark effective theory is a well-defined limit of QCD, gluonic radiative corrections may be calculated systematically [8, 10].

The purpose of the present paper is to extend the formalism of spin symmetry to baryons containing a heavy quark. This has also been discussed by Isgur and Wise [12] by explicit application of the commutation relations of spin and heavy flavor symmetry. In the present paper, we exhibit a convenient formalism which strongly simplifies the counting of form factors compared to ref. [12]. A brief outline of this formalism is found in ref. [13]. As applications we discuss semileptonic decays of heavy baryons and exclusive heavy baryon production in e^+e^- annihilation. Note that the issue of heavy baryonic form factors has also been addressed by Hussain et al. [14] using a phenomenological model.

For the sake of clarity later on, let us briefly discuss the nomenclature of the baryons containing heavy quarks. We adopt the scheme of the Particle Data Group [15]. In this scheme, the ground-state baryons are

$$\Lambda_{\mathrm{h}} = [(qq')_0 h]_{1/2}, \qquad \Xi'_{\mathrm{h}} = [(qs)_0 h]_{1/2},$$

$$\Sigma_{\mathrm{h}} = [(qq')_1 h]_{1/2}, \qquad \Xi_{\mathrm{h}} = [(qs)_1 h]_{1/2},$$

$$\Omega_{\mathrm{h}} = [(ss)_1 h]_{1/2}, \qquad \Sigma^*_{\mathrm{h}} = [(qq')_1 h]_{3/2},$$

$$\Xi^*_{\mathrm{h}} = [(qs)_1 h]_{3/2}, \qquad \Omega^*_{\mathrm{h}} = [(ss)_1 h]_{3/2}.$$

Here q, q' refer to u- and d-quarks, $q \neq q'$ for the Λ_{h}, but q may be the same as q' for the Σ_{h} and Σ^*_{h}. The first subscript (0, 1) is the total spin of the light pair, while the second subscript $(1/2, 3/2)$ is the total spin of the baryon. For later convenience, ξ_{h} may refer to any of the two baryons Λ_{h} or Ξ'_{h}. Similarly, we denote any of the three baryons Σ_{h}, Ξ_{h} and Ω_{h} as ω_{h}, and Σ^*_{h}, Ξ^*_{h} and Ω^*_{h} as ω^*_{h}.

Section 2 of this paper mainly contains the group theoretical framework of heavy flavor and spin symmetry, which is applied in sect. 3 to semileptonic decays of heavy baryons and to exclusive heavy baryon production in e^+e^- annihilation.

2. The formalism of spin symmetry for baryons

In this section we exhibit the group theoretical framework of spin symmetry ($\mathrm{SU}(2)_{\mathrm{spin}}$) of heavy hadrons which arises in the limit $m_{\mathrm{q}} \gg \Lambda_{\mathrm{QCD}}$, where m_{q} denotes the mass of the heavy quark. As discussed in ref. [6], heavy quark systems may be described by an effective theory in which the heavy quark part of the lagrangian is given by

$$\mathscr{L}_{\mathrm{heavy}} = \int \frac{d^3 v}{v_0} \left(i\bar{h}_v^+ \not{\mathscr{D}} h_v^+ + i\bar{h}_v^- \not{\mathscr{D}} h_v^- \right). \tag{1}$$

$\mathscr{D}$ is the covariant derivative, and h_v^+ (h_v^-) annihilates (creates) heavy quark (antiquark) states with 4-momentum

$$P^\mu = m_q v^\mu + k^\mu , \tag{2}$$

and k_μ is of the order Λ_{QCD}.

The dynamics described by $\mathscr{L}_{heavy}$ differs from QCD by terms that go to zero as $m_q \to \infty$, at least logarithmically: the differences are due to terms suppressed by powers of $\alpha_{QCD}(m_q)$ or Λ_{QCD}/m_q. If we neglect these effects, different velocity sectors do not communicate leading to the so-called velocity superselection rule. Note also that the lagrangian $\mathscr{L}_{heavy}$ contains no reference to the mass of the heavy quark, which has been formally sent to infinity. Consequently, if we have different species of heavy quarks which obey $m_q \gg \Lambda_{QCD}$, the effective theory has an additional heavy flavor symmetry corresponding to exchanges of heavy quarks.

In each velocity sector of the theory described by (1) there is a second new symmetry, called spin symmetry, which arises because QCD interactions of a heavy quark become independent of its spin degrees of freedom in the heavy quark limit. This means that for each v there exists a separate SU(2) spin symmetry. The generators of this symmetry can be associated with three unit vectors ϵ_i^μ ($i = 1,2,3$) which are orthogonal to $v^\mu : \epsilon_i \cdot v = 0, \epsilon_i^2 = -1$. Let us introduce a triplet of quark spin operators $S^+(v, \epsilon)$ ($\epsilon = \epsilon_1, \epsilon_2, \epsilon_3$). The commutation relations for the operators S are

$$[S^+(v,\epsilon), h_v^+] = \gamma_5 \not{v} \not{\epsilon} h_v^+ . \tag{3}$$

Note that if we consider a system with a heavy antiquark, the QCD interactions again do not depend on the polarization state of the heavy antiquark. Analogously then, one can introduce three operators $S^-(v,\epsilon)$ which obey

$$[S^-(v,\epsilon), h_v^-] = -\gamma_5 \not{v} \not{\epsilon}^* h_v^- . \tag{4}$$

Also

$$[S^{(\pm)}(v,\epsilon), h_v^{(\mp)}] = 0, \tag{5}$$

and $S^+(v,\epsilon)$ and $S^-(v,\epsilon)$ commute with each other.

Now consider a heavy baryon built out of a heavy quark and two light quarks. The mass M of the baryon in the spin symmetry limit tends to the quark mass $M = m_q(1 + O(\Lambda_{QCD}/m_q))$. Such a baryon is moving with an "infinite" momentum $P^\mu = Mv^\mu$ and obviously the only heavy quark states present in the baryon's wave function come from the v^μ sector of the theory. In this sector we have the symmetry generated by the operators $S^+(v,\epsilon)$ and all hadron states in the sector must occur in multiplets of $SU(2)_{spin}$. On the other hand, the operators $S^+(v,\epsilon)$ do not commute with total angular momentum, which means that they may connect states of different spins. In other words, each irreducible representation of $G = \text{Lorentz} \oplus SU(2)_{spin}$ in general may describe states whose spins differ by unity.

Here we would like to identify the representations of G that correspond to the light baryons. The simplest baryons are the ξ_h. The nontrivial Lorentz structure of ξ_h is due solely to the two polarization states of the heavy quark. This means that the $SU(2)_{spin}$ rotations constitute a subset of the Lorentz transformations, i.e. that $SU(2)_{spin}$ connects polarization states of the same particle. It follows that the representation of G which contains ξ_h can be described by an ordinary Dirac spinor $u_{\xi_h}(v)$. The Lorentz transformations are

$$u_{\xi_h}(v) \to D(\Lambda) u_{\xi_h}(\Lambda v)\,, \tag{6}$$

where Λ denotes the fundamental representation matrix of the given Lorentz transformation and $D(\Lambda)$ is its spinorial representation. Note that the spinor indices have been suppressed. The spin transformations are

$$u_{\xi_h}(v) \to -\gamma_5 \not{v} \not{a} u_{\xi_h}(v)\,, \tag{7}$$

which transform one polarization state of ξ_h into another.

On the other hand, consider the baryon states in which the light quarks are paired into spin one. These are the ω_h and ω_h^* states. These states have nontrivial Lorentz structure coming from both the light system polarizations and the heavy quark polarizations. In this representation of G there are spin-1/2 states (ω_h), and spin-3/2 states (ω_h^*). In fact there is a slight ambiguity as to which representation of G describes the ω_h and ω_h^* states. This is because under the Lorentz group the light quarks' indices can transform either as a pseudovector or as an antisymmetric tensor. In the first case one has to introduce a pseudovector–spinor object R^μ, where $v_\mu R^\mu = 0$ and the spinor indices have been suppressed. Note that in general $\gamma_\mu R^\mu \neq 0$ because R^μ describes a spin-1/2 particle together with a spin-3/2 particle. In other words, R^μ contains a Rarita–Schwinger field as well as a Dirac field. Under the Lorentz transformation,

$$R^\mu(v) \to \Lambda^\mu{}_\nu D(\Lambda) R^\nu(\Lambda v)\,, \tag{8}$$

and under the spin transformation,

$$R^\mu(v) \to -\gamma_5 \not{v} \not{a} R^\mu(v)\,. \tag{9}$$

The parity transformation of R^μ is

$$R^\mu(v^\alpha) \to -\gamma^0 R_\mu(v_\alpha)\,. \tag{10}$$

An alternative representation is to introduce a spinorial object with two vector indices, $P^{\mu\nu}$, which is antisymmetric under the exchange $\mu \leftrightarrow \nu$ and obeys $v_\mu P^{\mu\nu} = 0$ (spinorial indices once again are suppressed). The Lorentz transformation of P is

$$P^{\mu\nu}(v) \to \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta D(\Lambda) P^{\alpha\beta}(\Lambda v), \tag{11}$$

and the spin symmetry transformation is

$$P^{\mu\nu}(v) \to -\gamma_5 \not{v} \not{\epsilon} P^{\mu\nu}(v). \tag{12}$$

Under parity,

$$P^{\mu\nu}(v^\alpha) \to \gamma_0 P_{\mu\nu}(v_\alpha). \tag{13}$$

In what follows we discuss the formalism in terms of both P's and R's and we will check whether those two descriptions are equivalent. Before we continue it is important to identify the parts of R (P) that correspond to spin-1/2 and those that correspond to spin-3/2 states. Spin-3/2 states can be easily projected by contraction with γ_μ,

$$\gamma_\mu R^\mu_{\omega_h^*} = 0, \tag{14}$$

and equivalently

$$\gamma_\mu \gamma_\nu P^{\mu\nu}_{\omega_h^*} = 0. \tag{15}$$

The rest of the independent components of R (P) correspond to ω_h,

$$R^\mu_{\omega_h} = \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma_5 u_{\omega_h}, \tag{16}$$

where u_{ω_h} is the Dirac spinor of the ω_h state. Analogously

$$P^{\mu\nu}_{\omega_h} = \frac{1}{\sqrt{3}}(\gamma^\mu v^\nu - \gamma^\nu v^\mu - \tfrac{1}{4}[\gamma^\mu, \gamma^\nu]) u_{\omega_h}. \tag{17}$$

The power of the spin symmetry formalism lies in the observation that a generic operator $\bar{h}_{v_1} \Gamma h_{v_2}$ transforms as a spinor under $SU(2)_{v_1}$ (and of course independently as a spinor under $SU(2)_{v_2}$). Suppose we look at the matrix element of $\bar{h}_{v_1} \Gamma h_{v_2}$ sandwiched between heavy hadron states. One of the hadron states must correspond to the v_1 velocity sector (and the other one to the v_2-sector). The irreducible representation of G which describes the heavy hadron state of velocity v_1 decomposes into a direct sum of the spinors of $SU(2)_{v_1}$. In such a representation the different $SU(2)_{v_1}$ spinors correspond to all the independent polarization states of the light quark system (and for this reason the indices which label the spinors are termed light quark indices). The Wigner–Eckart theorem tells us that for each independent $SU(2)_{v_1}$ spinor in the given representation of G there is only one $SU(2)_{v_1}$-invariant amplitude that describes the matrix element of $\bar{h}_{v_1} \Gamma h_{v_2}$ *.

On the other hand, Lorentz transformations mix the light quark indices, which means that the set of all the $SU(2)_{spin}$-invariant amplitudes transforms as some tensor representation (not necessarily irreducible) of the Lorentz group. Such a Lorentz tensor can only be built from the velocity vectors v_1^μ, v_2^μ and invariant tensors $g^{\mu\nu}, \epsilon^{\mu\nu\alpha\beta}$. In other words, the requirement that the hadron matrix element transforms properly under the Lorentz group means that the light quark indices must be contracted with a Lorentz tensor built out of v_1^μ, v_2^μ, $g^{\mu\nu}$ and $\epsilon^{\mu\nu\alpha\beta}$. The number of irreducible components in such a tensor corresponds to the number of independent form factors describing the heavy hadron matrix element of $\bar{h}_{v_1} \Gamma h_{v_2}$.

As we have discussed above, ξ_h baryons are described by the simplest possible representation of G, a single spinor. This means that the matrix elements which describe processes involving ξ_h particles will be most constrained by the requirement of the spin symmetry. For example let us look at the matrix element

$$\langle \xi_h(v) | \bar{h} \Gamma h' | \xi_{h'}(v') \rangle = A(v \cdot v') \bar{u}_{\xi_h}(v) \Gamma u_{\xi_{h'}}(v'), \tag{18}$$

where we have allowed for the possibility of two heavy quark species h and h'. G-symmetry tells us that there is only one independent amplitude $A(v \cdot v')$. Moreover, if we put $\Gamma = \gamma_\mu$ then the matrix element (18) corresponds to the matrix element of the effective flavor symmetry current and in the forward limit it is uniquely specified,

$$A(1) = \sqrt{m_{\xi_h} m_{\xi_h}} \cdot 1, \tag{19}$$

where the factor $\sqrt{m_{\xi_h} m_{\xi_{h'}}}$ means that the hadron states in eq. (18) are normalized relativistically. The case of ξ_h is special because it is the only hadron for which spin symmetry connects different polarization states of the same particle. This means that the number of independent Lorentz amplitudes will be reduced not only for heavy $\to$ heavy transitions. As we show later in this section, in the case of the semileptonic decays $\xi_h \to$ light baryon the a priori six independent amplitudes describing such a transition get reduced to two.

As a way of verifying our group theory arguments which lead to a result like (18) one can present a derivation of (18) where the spin symmetry constraints are imposed explicitly via commutation relations (3). Let us first introduce matrix

* Likewise, for each spinor in the representation that describes the hadron moving with velocity v_2 there is only one $SU(2)_{v_2}$-invariant amplitude.

elements of the vector and axial currents,

$$\left\langle \xi_h(v) \middle| \bar{h}\gamma_\mu h' \middle| \xi_{h'}(v') \right\rangle \equiv \bar{u}_{\xi_h}(v)\{a(v\cdot v')\gamma_\mu - ib(v\cdot v')\sigma_{\mu\nu}(v-v')^\nu\}u_{\xi_{h'}}(v'), \quad (20)$$

$$\left\langle \xi_h(v) \middle| \bar{h}\gamma_\mu\gamma_5 h' \middle| \xi_{h'}(v') \right\rangle \equiv \bar{u}_{\xi_h}(v)\{c(v\cdot v')\gamma_\mu\gamma_5 + d(v\cdot v')(v-v')_\mu\gamma_5\}u_{\xi_{h'}}(v'), \quad (21)$$

where we have used the flavor symmetry of the effective theory to get rid of the second-class currents. Without any loss of generality we can assume that $v' = (1,0,0,0)$ and that the two polarizations of $\xi_{h'}$ correspond to $s_z = 1/2$ and $s_z = -1/2$ states. We may adopt the phase convention

$$|\xi_{h'}, s_z = 1/2\rangle = S(\epsilon_x, v') \ |\xi_{h'}, s_z = -1/2\rangle, \quad (22)$$

where $\epsilon_x = (0,1,0,0)$. Spin symmetry tells us that

$$\left\langle \xi_h(v) \middle| \bar{h}\gamma_\mu h' \middle| \xi_{h'}(v'), s_z = 1/2 \right\rangle = \left\langle \xi_h(v) \middle| \bar{h}\gamma_5 \not{v}' \not{\epsilon}_x h' \middle| \xi_{h'}(v'), s_z = -1/2 \right\rangle. \quad (23)$$

In particular, if we put $\mu = 3$ and use $\gamma^3\gamma_5\not{v}'\not{\epsilon}_x = i\gamma^2$ together with the form of the vector current matrix element given in eq. (20), we get the consistency condition $b(v\cdot v') = 0$. Next, we may put $\mu = 0$, use $\gamma^0\gamma_5\not{v}'\not{\epsilon}_x = -\gamma^1\gamma_5$ and the form of the axial and vector currents (20),(21) to obtain $a(v\cdot v') = c(v\cdot v')$. Lastly, putting $\mu = 1$ and using $\gamma^1\gamma_5\not{v}'\not{\epsilon}_x = -\gamma^0\gamma_5$ we get $d(v\cdot v') = 0$. In conclusion, we see that the matrix elements of the axial and vector currents are given by one form factor $a(v\cdot v') = c(v\cdot v')$. Similarly, we can use the projection equation $\not{v}'h' = h'$ to show that other matrix elements $\langle \xi_h(v)|\bar{h}\Gamma h'|\xi_{h'}(v')\rangle$ are indeed given in terms of a single form factor.

The little exercise we have just performed shows that the results obtained on the basis of general group theory arguments are indeed correct and from now on we forego the use of explicit symmetry commutators in the discussion of the other heavy baryon matrix elements*.

Now let us discuss a matrix element related to a $\xi_{h'} \to \omega_h(\omega_h^*)$ transition. Again, the constraint of G-invariance tells us that only one amplitude consistent with Lorentz invariance *and* spin symmetry can exist,

$$\left\langle \omega_h^{(*)}(v) \middle| \bar{h}_v \Gamma h_{v'} \middle| \xi_h \right\rangle = G(v\cdot v')\bar{R}^\mu_{\omega_h^{(*)}}(v)\Gamma u_{\xi_{h'}}(v')v'_\mu, \quad (24)$$

where we have used the R-representation to describe ω_h and ω_h^*. The use of the P-representation again leads to a single form factor,

$$\left\langle \omega_h^{(*)}(v) \middle| \bar{h}_v \Gamma h_{v'} \middle| \xi_h \right\rangle = G'(v\cdot v')\bar{P}^{\mu\nu}_{\omega_h^{(*)}}(v)\Gamma u_{\xi_{h'}}(v')\epsilon_{\mu\nu\alpha\beta}v^\alpha v'^\beta. \quad (25)$$

* Let us note that in ref. [12] all of the relations due to spin symmetry have been derived explicitly.

But in fact eqs. (24) and (25) are consistent only with the subset of the proper Lorentz transformations. If we use the parity properties of R or P we see that parity forces $G(v \cdot v') = G'(v \cdot v') = 0$. In ref. [16] spin symmetry arguments in combination with parity and angular momentum conservation explain that (24) and (25) in fact had to be equal to zero.

Now consider the matrix element between $\omega_{\mathrm{h}}^{(*)}$ and $\omega_{\mathrm{h}}^{(*)}$,

$$\left\langle \omega_{\mathrm{h}}^{(*)}(v) \middle| \bar{h}_v \Gamma h_{v'} \middle| \omega_{\mathrm{h}}^{(*)} \right\rangle = \bar{R}^{\mu}_{\omega_{\mathrm{h}}^{(*)}}(v) \Gamma R^{\nu}_{\omega_{\mathrm{h}}^{(*)}}(v') \left\{ B(v \cdot v') g_{\mu\nu} + C(v \cdot v') v_{\mu} v_{\nu} \right\}. \quad (26)$$

G-invariance allows two independent form factors (without parity there could be a third one). The same result is obtained with the P-representation,

$$\begin{aligned} &\left\langle \omega_{\mathrm{h}}^{(*)}(v) \middle| \bar{h}_v \Gamma h_{v'} \middle| \omega_{\mathrm{h}}^{(*)}(v') \right\rangle \\ &\quad = \bar{P}^{\mu\alpha}_{\omega_{\mathrm{h}}^{(*)}}(v) \Gamma P^{\nu\beta}_{\omega_{\mathrm{h}}^{(*)}}(v') g_{\alpha\beta} \left\{ B'(v \cdot v') g_{\mu\nu} + C'(v \cdot v') v'_{\mu} v_{\nu} \right\}, \end{aligned} \quad (27)$$

where again parity was used to eliminate one additional invariant.

As in the case of the $\Lambda_{\mathrm{h}} \to \Lambda_{\mathrm{h'}}$ transitions, we can make a statement about the normalization. Taking a matrix element of the (conserved) vector current between two identical ω_{h}'s, we find that B has to be normalized at zero momentum transfer in the same way as A is normalized in eq. (18).

We conclude our discussion of the formalism of heavy → heavy transitions by noting that all the above arguments hold for pair creation matrix elements of heavy baryons as well. This will be used in sect. 3 to discuss the exclusive production of heavy baryon pairs in e^+e^- annihilation.

Finally we comment on the heavy → light transitions. Note that, in general, spin symmetry multiplets contain particles which decay electromagnetically or strongly. This means that even if spin symmetry relates some weak heavy → light amplitudes for the particles in the given representation, many of these relations have little chance of being tested experimentally. One important exception corresponds to the ξ_{h} particles. Here spin symmetry relates different polarizations of the same particle and imposes interesting constraints. Consider for example the matrix element of an operator $\bar{l} \Gamma h_v$ between a heavy ξ_{h} and a light spin-1/2 baryon B_l. It is described by only two form factors,

$$\left\langle B_l \middle| \bar{l} \Gamma h_v \middle| \xi_{\mathrm{h}}(v) \right\rangle = \bar{u}_l(p) \left\{ F_1(p \cdot v) + \not{v} F_2(p \cdot v) \right\} \Gamma u_{\xi_{\mathrm{h}}}(v). \quad (28)$$

Thus in this particular case spin symmetry greatly reduced the number of the independent Lorentz-invariant amplitudes which may describe the heavy → light transitions.

3. Applications

In this section we discuss some applications of the formalism outlined above. In subsect. 3.1 we discuss weak decays of heavy baryons. In subsect. 3.2 we calculate the exclusive cross sections for the production of heavy baryons in electron–positron annihilation.

3.1. WEAK DECAYS OF HEAVY BARYONS

We focus here on the discussion of semileptonic decays, where we need to give the matrix elements of the left-handed hadronic current only. The semileptonic decays fall into two classes: the ones involving a transition of a heavy quark into a heavy quark, such as a $b \to c$ transition, and the ones with a heavy to light transition, such as $b \to u$ or $c \to s, d$ transitions. In general, due to the higher symmetry the heavy to heavy transitions are described by fewer form factors than the heavy to light decays.

Before we discuss the weak form factors, we point out that all of the form factors discussed in sect. 2 are real. This is easily seen by noting that the general form for the vector current of a specific quark species q between spin-1/2 baryons B and B' is

$$\langle B', p', s'|\bar{q}\gamma_\mu q|B, p, s\rangle = \bar{u}(p', s')\left[f_1(q^2)\gamma_\mu - if_2(q^2)\sigma_{\mu\nu}q^\nu + f_3(q^2)q_\mu\right]u(p, s), \tag{29}$$

where f_1, f_2 and f_3 are real. The form factors of sect. 2 may be expressed in terms of these.

For the heavy Λ's, we find (cf. eq. (18))

$$f_1 = A, \qquad f_2 = f_3 = 0. \tag{30}$$

For the case of a heavy lambda (Λ_h) decaying into a light one (Λ(uds)), we have (cf. eq. (28))

$$f_1 = F_1 + \frac{m}{M}F_2, \qquad f_2 = -f_3 = -\frac{1}{M}F_2, \tag{31}$$

where M is the mass of the heavy Λ and m is the mass of the light one. Finally, for the form factors for the transition between the ω_h we find (cf. eq. (26))

$$f_1 = -\tfrac{1}{3}[B(v\cdot v' - 4) + C(v\cdot v' - 1)(v\cdot v' - 3)],$$

$$f_2 = -\frac{4}{3(M+M')}[B + C(v\cdot v' - 1)],$$

$$f_3 = 0. \tag{32}$$

Let us now discuss the semileptonic decays of a heavy Λ, say Λ_b. Its semileptonic weak decay into Λ_c involves the matrix element of the charged current which, due to spin symmetry, may be parametrized in terms of a single form factor,

$$\left\langle \Lambda_c(v')\middle|\bar{c}\gamma_\mu(1+\gamma_5)b\middle|\Lambda_b(v)\right\rangle = A(v\cdot v')\bar{u}_{\Lambda_c}(v')\gamma_\mu(1+\gamma_5)u_{\Lambda_b}(v). \tag{33}$$

As has been pointed out in sect. 2, the form factor A is normalized at $v\cdot v' = 1$. Consequently, as in the case of the semileptonic decays of heavy mesons, this fact may be used to extract V_{cb} of the Cabibbo–Kobayashi–Maskawa (CKM) matrix using these semileptonic decays.

An even more interesting statement may be made about the weak decays of a heavy Λ_h in which there is a transition of a heavy quark into a light one. Since the Λ_h baryons are the only case where a spin symmetry multiplet consists of the same particle in different polarization states, spin symmetry reduces the possible six amplitudes to two independent ones. Using eq. (28) from sect. 2, the relevant current is given by

$$\begin{aligned}&\left\langle \Lambda(p')\middle|\bar{s}\gamma_\mu(1+\gamma_5)h\middle|\Lambda_h(v)\right\rangle\\ &\qquad = \bar{u}_\Lambda(p')\{F_1(v\cdot p') + \not{v}F_2(v\cdot p')\}\gamma_\mu(1+\gamma_5)u_{\Lambda_h}(v).\end{aligned} \tag{34}$$

The phenomenological and experimentally testable applications of (34) have been worked out in more detail in ref. [17].

In addition, heavy flavor symmetry coupled with these relations may allow access to the value of V_{ub} of the CKM matrix. The idea is similar to the corresponding decays of heavy mesons*. Applying (34) to both Λ_c and Λ_b one may use heavy flavor symmetry to find that the form factors F_1 and F_2 are the same in both cases. In the first process, the c → s transition yields a final state which is a Λ, while in the latter the b → u decay produces a nucleon. Although it is experimentally much more difficult than the corresponding decays of heavy mesons, these relations will relate the Cabbibo angle to V_{ub}.

* We thank M. Wise and N. Isgur for a discussion on how to extract a relation between the Cabibbo angle and V_{ub} from the decays of heavy mesons into light ones.

The above relations may also be used to analyze different polarizations in the decay of the Λ_h into $\Lambda_{h'}$ or Λ. This has been discussed in ref. [18], where Lorentz invariance alone was imposed and six real functions were introduced to parametrize the amplitudes. Using the ideas of the heavy quark effective theory, relations between these functions may be found, and this reduces the number of independent real functions to one in the case of an $h \to h'$ transition and to two in the case of a heavy to light decay.

We turn now to decays of heavy ξ_h baryons into ω_h baryons. As has been pointed out, spin symmetry and parity considerations tell us that the semileptonic decay of a Λ_h into a $\Sigma_{h'}$ baryon is suppressed, since the corresponding matrix element of the left-handed current vanishes in the heavy quark limit. This means that the amplitude for such a decay is of higher order in $\alpha_{QCD}(m_{1_h})$ and Λ_{QCD}/m_{1_h}, so that this decay may serve as a test of higher-order corrections to the heavy quark effective theory.

Matrix elements like (26) for the weak decays of heavy ω_h and ω_h^* will in general not be measurable, since these particles are expected to decay dominantly electromagnetically or strongly. One exception is the $\Omega_h[(ss)_1 h]_{1/2}$*, which can neither decay electromagnetically, nor there is enough phase space for its strong decay. In this case the weak decay will be observable and is described by two form factors,

$$\left\langle \Omega_h(v) \middle| \bar{h}_v \gamma_\mu (1+\gamma_5) h'_{v'} \middle| \Omega_{h'}(v') \right\rangle = \bar{R}^{\lambda}_{\Omega_h}(v) \gamma_\mu (1+\gamma_5) R^{\kappa}_{\Omega_{h'}}(v') \times \left\{ B(v\cdot v') g_{\kappa\lambda} + C(v\cdot v') v'_\lambda v_\kappa \right\}. \quad (35)$$

Our final comment concerns the case where at least one of the light quarks is an s-quark and the heavy quark is a b-quark. In this case the heavy baryon may decay weakly in three different ways, since any of the transitions $s \to u$, $b \to c$, or $b \to u$ may occur. The decay of the heavy quark is suppressed by small CKM matrix elements, but the phase space for the heavy quark decay may be large enough to overcome this suppression. Consider the case of a heavy baryon with quark content bsl, where l denotes any of u, d, s. The phase space for its semileptonic decay is

$$\Phi_3 = \frac{1}{64\pi^3}\left(\frac{1}{4M^2}[M^4 - m^4] - m^2 \ln\left(\frac{M}{m}\right)\right), \quad (36)$$

where we have neglected the lepton mass, and M (m) denotes the mass of the initial-(final-) state baryon. For the $s \to u$ transition the final state still contains a heavy quark and the mass difference between the bsl- and the bul-baryon will be small compared to the total baryon mass. In this case the phase space (36) may be

* We thank N. Isgur for a discussion on this point.

TABLE 1
Comparison of phase space ratios to the ratios of the absolute squares of the CKM matrix elements

$\lvert V_{cb}\rvert^2/\lvert V_{us}\rvert^2$	$\Phi_3(s \to u)/\Phi_3(b \to c)$	m_{Λ_b}/GeV
0.053 ± 0.021	0.018	5.3
0.053 ± 0.021	0.011	6.0
$\lvert V_{ub}\rvert^2/V_{us}\rvert^2$	$\Phi_3(s \to u)/\Phi_3(b \to u)$	m_{Λ_b}/GeV
$(7.5 \pm 5.7) \times 10^{-4}$	7.5×10^{-3}	5.3
$(7.5 \pm 5.7) \times 10^{-4}$	5.5×10^{-3}	6.0

expanded for small mass differences and we find

$$\Phi_3 = \frac{1}{64\pi^3}(M - m)^2. \tag{37}$$

Note that (37) depends only on the mass difference which may be estimated by using, for example, the Λ-nucleon mass difference. We may now analyze the $b \to c$ and $b \to u$ decays versus the $s \to u$ decays by comparing the ratios of the absolute squares of the CKM matrix elements to the ratio of the available phase space. In table 1 we compare these ratios. Since there is no firm data as yet on bottom baryons we compare for different values of its masses. The lower value is the mass of the B-meson which should be a lower limit on the bottom baryon mass, while the higher value is a more realistic guess. Furthermore, we have used in table 1 a value of 200 MeV for the mass difference.

From this simple estimate we see that the phase space for $b \to c$ transitions may indeed be sufficient to compensate for the small CKM matrix element. One would expect that the rate for a $b \to c$ decay would be about five times as large as that for an $s \to u$ decay. In the case of the $b \to u$ decays, phase space does not compensate the very small value of $\lvert V_{ub}\rvert$. This simple estimate yields a branching ratio an order of magnitude smaller than the corresponding $s \to u$ transition.

3.2. EXCLUSIVE CROSS SECTIONS FOR e^+e^- INTO HEAVY BARYONS

In this subsection we describe in some detail the calculation of exclusive production of heavy baryons in e^+e^- annihilation. These cross sections have already been considered in connection with the production of charmed baryons, using spin counting arguments [19], and a helicity amplitude formalism [20]. The heavy quark effective theory provides not only the behavior at threshold, but also shows how the cross sections change as the energy is increased above threshold.

The easiest case is the production of ξ_h baryons. As discussed above, the appropriate description of the ξ_h state is to represent it by the spinor of the heavy quark. From these considerations we infer that the current for the pair creation of a $\xi_h\bar{\xi}_h$ pair may be parametrized in terms of a single form factor and is therefore given by

$$\left\langle \xi_h(v)\bar{\xi}_h(v')\middle|\bar{h}\gamma_\mu h\middle|0\right\rangle = \eta(v\cdot v')\bar{u}_h(v)\gamma_\mu h(v')\,. \tag{38}$$

Thus the hadronic tensor is, up to the form factor $|\eta(v\cdot v')|^2$, the same as for a massive pointlike fermion with the mass of the ξ_h, and we may make several predictions. Aside from the cross sections for the various combinations of polarizations we can also predict the angular distribution of the $\xi_h\bar{\xi}_h$ pairs to be

$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{d\Omega} = \frac{3}{8\pi}\frac{2m^2}{2m^2+s}\left[(1-\cos^2\Theta) + \frac{s}{4m^2}(1+\cos^2\Theta)\right], \tag{39}$$

where Θ is the c.m.s. scattering angle.

Things are a little less trivial for the ω_h and ω_h^* baryons. As discussed in sect. 2, spin symmetry relates these two baryons so that relations between the exclusive cross sections may be found. The matrix elements of the relevant current of the heavy quarks may be expressed in terms of the spinors given in sect. 2. Since the ω_h and the ω_h^* lie in the same multiplet of spin symmetry, all the matrix elements may be parametrized in terms of only two form factors A and B*,

$$\begin{aligned}\left\langle \omega_h(v)\bar{\omega}_h(v')\middle|\bar{h}\gamma_\mu h\middle|0\right\rangle &= \bar{R}^\lambda_{\omega_h}(v)\gamma_\mu Q^\kappa_{\omega_h}(v')\big(A(v\cdot v')g_{\lambda\kappa} + B(v\cdot v')v'_\lambda v_\kappa\big)\\ &= -\tfrac{1}{3}\bar{u}(p)\gamma_5(\gamma_\lambda + v_\lambda)\gamma_\mu(\gamma_\kappa - v'_\kappa)\gamma_5 v(p')\\ &\quad\times\big(A(v\cdot v')g^{\lambda\kappa} + B(v\cdot v')v'^\lambda v^\kappa\big),\end{aligned} \tag{40}$$

$$\begin{aligned}\left\langle \omega_h^*(v)\bar{\omega}_h(v')\middle|\bar{h}\gamma_\mu h\middle|0\right\rangle &= \bar{R}^\lambda_{\omega_h^*}(v)\gamma_\mu Q^\kappa_{\omega_h}(v')\big(A(v\cdot v')g_{\lambda\kappa} + B(v\cdot v')v'_\lambda v_\kappa\big)\\ &= \frac{1}{\sqrt{3}}\bar{R}_{\omega_h^*\lambda}(v)\gamma_\mu(\gamma_\kappa - v'_\kappa)\gamma_5 v(p')\\ &\quad\times\big(A(v\cdot v')g^{\lambda\kappa} + B(v\cdot v')v'^\lambda v^\kappa\big),\end{aligned} \tag{41}$$

$$\left\langle \omega_h^*(v)\bar{\omega}_h^*(v')\middle|\bar{h}\gamma_\mu h\middle|0\right\rangle = \bar{R}^\lambda_{\omega_h^*}(v)\gamma_\mu Q^\kappa_{\omega_h^*}(v')\big(A(v\cdot v')g_{\lambda\kappa} + B(v\cdot v')v'_\lambda v_\kappa\big), \tag{42}$$

where we have denoted the corresponding antiparticle spinors by Q_{ω_h} and $Q_{\omega_h^*}$.

* In contrast to the case of the weak decay matrix elements the form factors in the present case are in general complex functions of $v\cdot v'$.

These relations may be employed to give a prediction of the relative exclusive production rates of $\omega_h\bar{\omega}_h$, $\omega_h^*\bar{\omega}_h$ and $\omega_h^*\bar{\omega}_h^*$ near threshold. Note that in the spin symmetry limit the threshold for all these processes is the same, since the mass splitting between ω_h and ω_h^* is of higher order in Λ_{QCD}/m_h.

Experimentally, these cross sections are only interesting near threshold, since the form factors will decrease rapidly for higher c.m.s. energies $\sqrt{s}$. Thus we focus on the region near threshold and expand $s = 4m^2 + q^2$ for small q^2. We find for the cross section for $\omega_h\bar{\omega}_h$ production

$$\sigma_{e^+e^-\to\omega_h\bar{\omega}_h} \propto \frac{2}{27}\left[|D|^2(12m^2+5q^2) - 8q^2 \operatorname{Re} C^*D\right]\left(1+\mathrm{O}\left(\frac{q^4}{m^4}\right)\right), \quad (43)$$

where C and D are given in terms of A and B as

$$C = A + B\frac{s}{2m^2}, \quad (44)$$

$$D = A\frac{s-2m^2}{2m^2} + B\frac{s(s-4m^2)}{4m^4}. \quad (45)$$

Using for the polarization sum of the Rarita–Schwinger objects [21]

$$\sum_{\mathrm{Pol}} u_\mu(p)\bar{u}_\nu(p) = \tfrac{1}{2}(\not{v}+1)\left[g_{\mu\nu} - \tfrac{2}{3}v_\mu v_\nu - \tfrac{1}{3}\gamma_\mu\gamma_\nu - \tfrac{1}{3}(\gamma_\mu v_\nu - \gamma_\nu v_\mu)\right],$$

the remaining two cross sections are

$$\sigma_{e^+e^-\to\omega_h^*\bar{\omega}_h} \propto \frac{16}{27}\left[|D|^2(12m^2+5q^2) - 8q^2 \operatorname{Re} C^*D\right]\left(1+\mathrm{O}\left(\frac{q^4}{m^4}\right)\right), \quad (46)$$

$$\sigma_{e^+e^-\to\omega_h^*\bar{\omega}_h^*} \propto \frac{20}{27}\left[|D|^2(12m^2+5q^2) - 8q^2 \operatorname{Re} C^*D\right]\left(1+\mathrm{O}\left(\frac{q^4}{m^4}\right)\right). \quad (47)$$

Using this we find the interesting result for the ratios near threshold,

$$\sigma_{e^+e^-\to\omega_h\bar{\omega}_h} : \sigma_{e^+e^-\to\omega_h^*\bar{\omega}_h} : \sigma_{e^+e^-\to\omega_h^*\bar{\omega}_h^*} = 1 : 8 + \mathrm{O}(q^4/m^4) : 10 + \mathrm{O}(q^4/m^4), \quad (48)$$

which vary quite slowly as we move away from threshold*.

At threshold, the heavy quark effective theory exactly reproduces the result of ref. [19]. This is because in general the hadronic tensor has two form factors (after

* Note that due to parity and G-parity invariance, the cross section for production of $\omega_h^*\bar{\omega}_h$ is the same as that for $\omega_h\bar{\omega}_h^*$.

summation over polarizations), one of which is proportional to $g_{\mu\nu}$, and the other is proportional to $v_\mu^- v_\nu^-$, where $v_\mu^- = v_\mu - v'_\mu$. Near threshold, v_μ^- vanishes like $\sqrt{s/4m^2 - 1}$ so that, assuming that the second form factor is not singular at threshold, the cross sections may be described by a single form factor, and the ratios may be calculated independently of any unknown parameters. In addition, since the spin degrees of freedom of the heavy quark decouple in the heavy quark limit, our result *must* agree with spin counting arguments. Finally we note that this result does not disagree with ref. [20], since in the heavy quark limit threshold for the production of the heavy quarks coincides with that for baryon production. When the appropriate threshold conditions of ref. [20] are met, they recover the same results.

Of course, the ratios (48) have to be taken with some caveats. As is known, the prediction in the case of heavy mesons [22] is somewhat different from the experimental results [23], due largely to the effects of the heavy charmonium resonances near threshold. We expect that (48) may also be different from the experimental findings, although at the energies for heavy baryon pair creation resonance effects are much smaller.

4. Conclusions

In this paper we have extended the heavy quark effective theory to describe baryons containing a heavy quark. The group theory of the additional symmetries (heavy flavor and spin symmetry) was elaborated for the case of baryons and the corresponding spin symmetry multiplets were identified.

As was the case for the heavy mesons these additional symmetries strongly reduce the number of form factors for the current matrix elements involving heavy baryons. In addition one may obtain absolute normalizations which may be important for the extraction of CKM matrix elements from future measurements of semileptonic decays of heavy baryons.

An additional interesting statement may be made about the heavy ξ_h-baryons. Since these are the only baryons for which a spin symmetry multiplet consists of the same particle in different polarization states, spin symmetry allows us to restrict the number of form factors even for a heavy → light transition. This will facilitate strongly the analysis of the semileptonic decays $\Lambda_c \to \Lambda e\nu$ [17], which should be observable in the near future.

Finally we have discussed the cross section for exclusive production of heavy baryons in e^+e^- annihilation using the heavy quark effective theory.

The extension of the effective theory for heavy quarks to baryons will allow the calculation of corrections of the order Λ_{QCD}/m_h and QCD radiative corrections to the form factors in heavy baryon semileptonic decays. Furthermore, since the data on heavy baryons might continue to improve, all the predictions, including corrections of the order Λ_{QCD}/m_h and $\alpha_{QCD}(m_h)$, will probably be testable in the near future.

We gratefully acknowledge helpful and enlightening discussions with H. Georgi, N. Isgur and M. Wise. We also thank B. Bruen and B. Wyslouch at MIT for computational advice and facilities. One of us (T.M.) acknowledges the warm hospitality of Lyman Laboratory of Physics at Harvard University, where most of this work was done.

References

[1] M. Voloshin and M. Shifman, Sov. J. Nucl. Phys. 45 (1987) 292; 47 (1988) 511
[2] E. Eichten and B. Hill, Phys. Lett. B234 (1990) 511
[3] H. Politzer and M. Wise, Phys. Lett. B206 (1988) 681; B208 (1988) 504
[4] N. Isgur and M. Wise, Phys. Lett. B232 (1989) 113; B237 (1990) 527
[5] B. Grinstein, Nucl. Phys. B339 (1990) 253
[6] H. Georgi, Phys. Lett. B240 (1990) 447
[7] J. Bjorken, SLAC preprint SLAC-PUB-5278 (1990)
[8] A. Falk, H. Georgi, B. Grinstein and M. Wise, Harvard preprint HUTP-90/A011 (1990)
[9] T. Mannel, W. Roberts and Z. Ryzak, Phys. Lett. B248 (1990) 392
[10] A. Falk and B. Grinstein, Harvard preprint HUTP-90/A042 (1990)
[11] T. Mannel and Z. Ryzak, Phys. Lett. B247 (1990) 412
[12] N. Isgur and M. Wise, Nucl. Phys. B348 (1991) 276
[13] H. Georgi, Nucl. Phys. B348 (1991) 293
[14] F. Hussain, J.G. Körner and R. Migneron, Mainz University preprint MZ-TH/90-10 (1990)
[15] Particle Data Group, Phys. Lett. B204 (1988)
[16] D. Politzer, CALTECH preprint CALT-68-1651 (1990)
[17] T. Mannel, W. Roberts and Z. Ryzak, Harvard preprint HUTP-90/A068 (1990)
[18] J. Bjorken, Phys. Rev. D40 (1989) 1513
[19] A. De Rújula, H. Georgi and S. Glashow, Phys. Rev. Lett. 37 (1976) 895
[20] J.G. Körner and M. Kuroda, Phys. Lett. B67 (1977) 455
[21] H. Umezawa, Quantum field theory (North-Holland, Amsterdam, 1956)
[22] A. De Rújula, H. Georgi and S. Glashow, Phys. Rev. Lett. 37 (1976) 398
[23] A. LeYaouanc, L. Olivier, O. Pene and J.C. Raynal, Phys. Lett. B71 (1977) 397