# Fractal dimensions from a three-dimensional intermittency analysis in $e^+e^-$ annihilation

CELLO Collaboration

H.-J. Behrend, L. Criegee, J.H. Field [1], G. Franke, H. Jung [2], J. Meyer, O. Podobrin, V. Schröder, G.G. Winter
*Deutsches Elektronen-Synchrotron, DESY, W-2000 Hamburg, FRG*

P.J. Bussey, A.J. Campbell, D. Hendry, S.J. Lumsdon, I.O. Skillicorn
*University of Glasgow, Glasgow G12 8QQ, UK*

J. Ahme, V. Blobel, M. Feindt, H. Fenner, J. Harjes, J.H. Köhne [3], J.H. Peters, H. Spitzer, T. Weihrich
*II. Institut für Experimentalphysik, Universität Hamburg, W-2000 Hamburg, FRG*

W.-D. Apel, J. Engler, G. Flügge [2], D.C. Fries, J. Fuster [4], K. Gamerdinger [5], P. Grosse-Wiesmann [6], H. Küster [7], H. Müller, K.H. Ranitzsch, H. Schneider
*Kernforschungszentrum Karlsruhe und Universität Karlsruhe, W-7500 Karlsruhe, FRG*

W. de Boer [3], G. Buschhorn, G. Grindhammer, B. Gunderson, C. Kiesling, R. Kotthaus, H. Kroha [8], D. Lüers, H. Oberlack, P. Schacht, S. Scholz, W. Wiedenmann [6]
*Max Planck-Institut für Physik und Astrophysik, W-8000 Munich, FRG*

M. Davier, J.F. Grivaz, J. Haissinski, V. Journé, F. Le Diberder [9], J.-J. Veillet
*Laboratoire de l'Accélérateur Linéaire, F-91405 Orsay, France*

K. Blohm, R. George, M. Goldberg, O. Hamon, F. Kapusta, L. Poggioli, M. Rivoal
*Laboratoire de Physique Nucléaire et des Hautes Energies, Université de Paris, F-75251 Paris, France*

G. d'Agostini, F. Ferrarotto, M. Iacovacci, G. Shooshtari, B. Stella
*University of Rome and INFN, I-00185 Rome, Italy*

G. Cozzika, Y. Ducros
*Centre d'Etudes Nucléaires, Saclay, F-91191 Gif-sur-Yvette, France*

G. Alexander, A. Beck, G. Bella, J. Grunhaus, A. Klatchko [10], A. Levy and C. Milsténe
*Tel Aviv University, 69978 Tel-Aviv, Israel*

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The intermittency structure of multihadronic $e^+e^-$ annihilation is analyzed by evaluating the factorial moments $F_2$–$F_5$ in three-dimensional Lorentz invariant phase space as a function of the resolution scale. We interpret our data in the language of fractal objects. It turns out that the fractal dimension depends on the resolution scale in a way that can be attributed to geometrical resolution effects and dynamical effects, such as the $\pi^0$ Dalitz decay. The LUND 7.2 hadronization model provides an excellent description of the data. There is no indication of unexplained multiplicity fluctuations in small phase space regions.

## 1. Introduction

Some years ago Bialas and Peschanski [1] suggested the one-dimensional analysis of high energy processes via the factorial moments of e.g. rapidity distributions at different resolution scales. Since then much attention has been given to studies of this kind, especially when experiments reported failure of commonly used hadronization models to reproduce the observed effects (for recent reviews see e.g. refs. [2,3]). In this context the analysis of multihadronic $e^+e^-$ annihilations is important for two reasons: The initial state is well defined and the subsequent QED and QCD processes can be calculated, or are described in a phenomenological way, which is known to account for a variety of effects observed in $e^+e^-$ annihilation from 10 to 90 GeV. Recently the TASSO Collaboration [4] reported a quantitative (but not qualitative) discrepancy between their data and Monte Carlo calculations concerning factorial moments derived from a one-dimensional analysis of rapidity distributions. Similar studies from the DELPHI [5] and CELLO [6] Collaborations revealed good agreement between the data and the Lund parton shower model [7]. The results of the HRS Collaboration [8] cannot easily be compared due to the lack of Monte Carlo calculations. However, they do not confirm the strong rise of $F_3$ as seen by the TASSO experiment.

Here we present the first experimental study of factorial moments in three-dimensional phase space. In this approach sensitivity for intermittent effects is retained, even at very small resolution scales, whereas in conventional analyses too much information is lost due to the projection on to a one-dimensional axis. The importance of the phase space dimension has also been pointed out by Ochs [9]. This letter is organized as follows: In the following section the analysis method is explained and the results are interpreted in terms of fractal dimensions. In the third section we give a conventional explanation of our data within the framework of the LUND Monte Carlo. In addition Monte Carlo investigations of single dynamical or kinematical effects in the hadronization process are presented.

## 2. Analysis

The present analysis is based on data taken at the PETRA $e^+e^-$ collider operating at beam energies of 17.5 GeV, where the CELLO detector [10] recorded a luminosity of 86 $pb^{-1}$. Multihadronic events are identified by a standard selection procedure [11] which required a minimum of five charged particles. 18 433 events were accepted and are the basis for the subsequent analysis.

We study the multiplicity fluctuations of charged particles in Lorentz invariant phase space. For this we consider the differential phase space element $dLIPS = d^3p/E$, which is decomposed into $dp_x/\sqrt[3]{E}\, dp_y/\sqrt[3]{E}\, dp_z/\sqrt[3]{E}$. In contrast to the decomposition $dy\, dp_\perp^2\, d\phi$, this parametrization allows a unique definition of a resolution scale, since all variables are of the same dimensionality. A further advantage of this approach is the invariance of the factorial moments under rotational transformations of

[1] Present address: Université de Genève, CH-1211 Geneva 4, Switzerland.
[2] Present address: RWTH, W-5100 Aachen, FRG.
[3] Present address: Universität Karlsruhe, W-7500 Karlsruhe, FRG.
[4] Present address: Institute de Física Corpuscular, Universidad de Valencia, E-46100 Bujassot (Valencia), Spain.
[5] Present address: MPI für Physik und Astrophysik, W-8000 Munich, FRG.
[6] Present address: CERN, CH-1211 Geneva 23, Switzerland.
[7] Present address: DESY, W-2000 Hamburg, FRG.
[8] Present address: University of Rochester, Rochester, NY 15627, USA.
[9] Present address: Stanford Linear Accelerator Center, Stanford, CA 94305, USA.
[10] Present address: University of California, Riverside, CA 92521, USA.

the analyzed variables, thus making the definition of a preferred axis (such as a jet axis) unnecessary. The results presented below have been checked to be identical if the variables are defined in the laboratory frame or by the eigenvectors of the sphericity tensor. The factorial moments $F_2$–$F_5$ of the charged multiplicity distribution are evaluated at different resolution scales. For this purpose the original phase space volume ΔLIPS containing an average of $\langle N\rangle$ particles is successively divided into $M^3$ cubes of size $\delta\text{LIPS}=\Delta\text{LIPS}/M^3$, each containing $n_m$ particles. The normalized factorial moment of rank $i$ is then computed according to the following formula:

$$\langle F_i(M)\rangle = \frac{M^{3i}}{\langle N\rangle^i}\left\langle \frac{1}{M^3}\sum_{m=1}^{M^3} n_m(n_m-1)...(n_m-i+1)\right\rangle. \quad (1)$$

This formula implies two averages: The *horizontal* average running over $M^3$ cubes (indicated by the sum) and the *vertical* average running over all events (indicated by the brackets).

For reasons that will become clear later, ΔLIPS has been chosen to be much larger than the volume actually occupied by our events, namely $10^9$ GeV$^2$. This arbitrary choice influences the absolute value of factorial moments at a given scale, but has no effect on the physically relevant slopes.

The analysis is performed with M ranging from 1 to $1.6\times10^5$, corresponding to phase space volumes in the range $10^9$–$2.5\times10^{-7}$ GeV$^2$. Note that it is not necessary for the computation of $F_i$ to perform $M^3\approx4\times10^{15}$ additions. The sum in (1) is completely determined by bins which contain at least $i$ particles, thus the computing time depends only on $\langle N\rangle$.

Since the three momentum variables used in this analysis are defined in the centre of mass system and are therefore centered at zero, strong fluctuations in the moments are expected for very coarse resolutions: The central region may either be contained in one cube or may be split into $2^3=8$ cubes, depending on whether the number of subdivisions is odd or even. This effect is circumvented by employing translational invariance: The complete event is moved at random inside the large phase space volume, which for this purpose is continued periodically. A further reduction of random fluctuations is achieved by analyzing each event at e.g. ten different random positions and taking the average. We have checked that the results are not influenced by this procedure, except that artificial fluctuations are damped. In fig. 1 the factorial moments are displayed on a log-log diagram. In principle, the moments $F_i$ can be measured up to a resolution scale where one event still has a phase space box containing at least $i$ particles. As shown in our conventional intermittency analysis [6], a reliable error estimate is however only possible, if at least ≈ 5 events still contribute. Therefore, in fig. 1 we show those data points to which at least 5 events gave a non-zero contribution. The detector resolution is better than the smallest bin sizes shown. The errors shown in fig. 1 are taken from the covariance matrix, which has been computed from the fluctuations of the horizontally averaged factorial moments of all events:

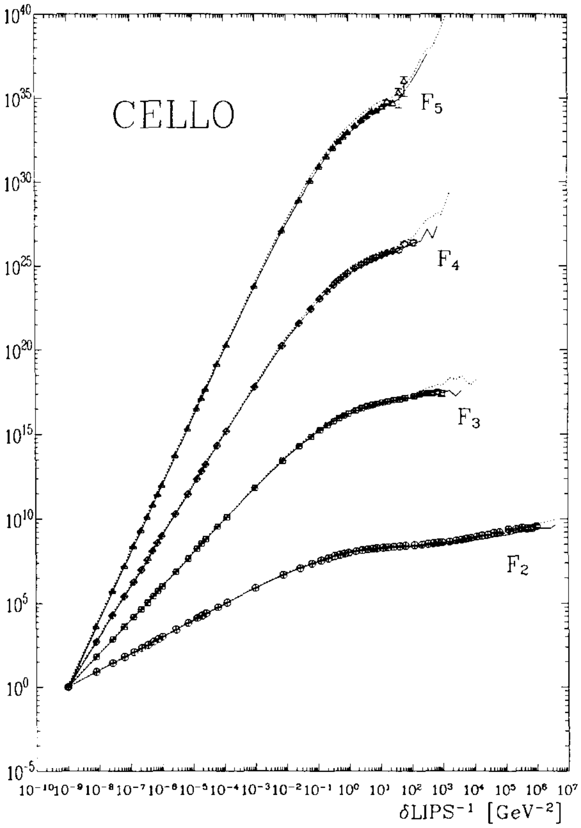


Fig. 1. Factorial moments $F_2$–$F_5$ as function of $\delta\text{LIPS}^{-1}$. The open symbols show the data and the dotted and solid curves correspond to the LUND 7.2 PS simulation prior to and after detector simulation.

$$C_i(L, M) = \frac{\langle F_i(L)F_i(M)\rangle - \langle F_i(L)\rangle\langle F_i(M)\rangle}{N_{tot}} . \tag{2}$$

The brackets indicate averaging over all events, and $N_{tot}$ is the total number of events analyzed. The diagonal elements of this matrix give the (squared) errors of the factorial moments consistent with ref. [1]. In ref. [6] we have shown in Monte Carlo investigations that these errors are a good estimate for the true standard deviation. With the statistics shown here, the errors are gaussian to a good approximation. Note that the correlation between neighbouring points is large and positive.

In fig. 1 different slopes are seen at different resolution scales with high statistical significance. A very interesting behaviour is observed for $F_2$, which after a strong initial rise flattens out, but then starts to rise again. The results from a Monte Carlo simulation using the LUND 7.2 parton shower model with default parameters after inclusion of initial state radiation are seen to be in perfect agreement with the data. The same holds for a second order matrix element ansatz. It is also visible in fig. 1 that detector effects are of minor importance in this analysis.

The local intermittency exponents (=slopes in the logarithmic plot) $\alpha_i$ are related to the factorial moments by the following derivative:

$$\alpha_i = -\frac{\partial(\log F_i)}{\partial(\log \delta\text{LIPS})} . \tag{3}$$

The theory of multifractal objects [12] allows these local intermittency exponents to be interpreted as a (multi-)fractal dimension of the object under study via the linear relation

$$D_i^F = D_0\left(1 - \frac{\alpha_i}{i-1}\right) . \tag{4}$$

A simple example may explain this relation: Assume that $N$ particles are randomly distributed in a $D_0$ dimensional phase space. Then the particle content of every phase space box follows a poissonian distribution with $\langle n_m\rangle = N/M^{D_0}$, and the $F_i$ calculated using (1) are observed to be independent of $M$. Consequently all slopes vanish and $D_i^F = D_0$ for all ranks $i$. Now consider the other extreme case, that all $N$ particles of an event are placed in one singular point, then the sum in eq. (1) is constant for any given $M$ and $F_i$ is given by the factor $M^{D_0(i-1)}$, consequently $\alpha_i = i-1$. Inspecting eq. (4) gives $D_i^F = 0$ for all ranks $i$, which coincides with the intuitive expectation for a point-like object. This example can be generalized to distributions which are phase space like in $D_i^F$ dimensions and singular in $D_0 - D_i^F$ dimensions. The factorial moments $F_i$ of such distributions can be shown to exhibit a power law behaviour with slope

$$\alpha_i = \left(1 - \frac{D_i^F}{D_0}\right)(i-1) . \tag{5}$$

Solving this equation for $D_i^F$ leads to the definition of fractal dimensions, eq. (4).

Fig. 2 shows the fractal dimension inferred from $F_2$ (i.e. the *correlation dimension*) as a function of the

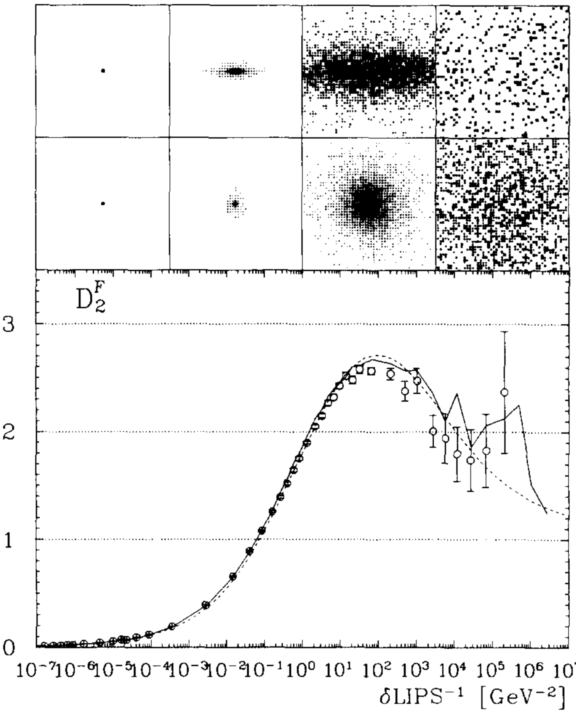


Fig. 2. Fractal dimension $D_2^F$ as a function of $\delta\text{LIPS}^{-1}$. The open symbols show the data with errors propagated from $F_2$. Superimposed, as the dashed and solid lines are the corresponding results from the LUND 7.2 parton shower calculation prior to and after detector simulation. Displayed above is a pictorial view of the fractal dimension at four typical scales. Upper row: plane along the sphericity axis; lower row: plane perpendicular to the sphericity axis. This figure has been prepared using 400 Monte Carlo events.

phase space volume. The errors shown are propagated from the measurement of $F_2$ using the complete covariance matrix. On top of fig. 2a geometrical interpretation of $D_2^F$ is displayed. For very coarse resolutions, the events appear point-like ($D_2^F=0$), with increasing resolution the structure begins to emerge; first along the jet-axis and then also transverse to the jet-axis. At very small scales all dimensions are resolved, and in the geometrical interpretation one expects $D_2^F=3$. The data, however, show a maximum dimension $D_2^F\approx 2.5$ at $\delta\text{LIPS}\approx 1\times 10^2$ GeV$^2$ followed by a decrease to about $D_2^F\approx 2.0$. The reduction of $D_2^F$ is the manifestation of the strong rise of $F_2$ at small δLIPS. This complex behaviour is well reproduced by the LUND 7.2 parton shower Monte Carlo simulation, both with and without detector simulation.

In the geometrical interpretation given above one would expect the fractal dimension derived from factorial moments to be independent of the rank, i.e. one expects $\alpha/(i-1)=\text{const}$. This scaling law can be violated; a system doing so is called multifractal. To search for such multifractal behaviour we have fitted the slopes of different rank at a given resolution scale to the expression

$$\alpha_i=c(i-1)(\tfrac{1}{2}i)^x, \qquad (6)$$

where $x=0$ indicates geometrical scaling and $x=1$ corresponds to the scaling law predicted by a random cascading model [1]. The fit results are shown in fig. 3: A clear deviation from the simple geometrical scaling law is observed as the resolution increases. This behaviour is well described by the Monte Carlo expectation, both with and without detector simulation, again demonstrating the small influence of detector effects on this type of analysis.

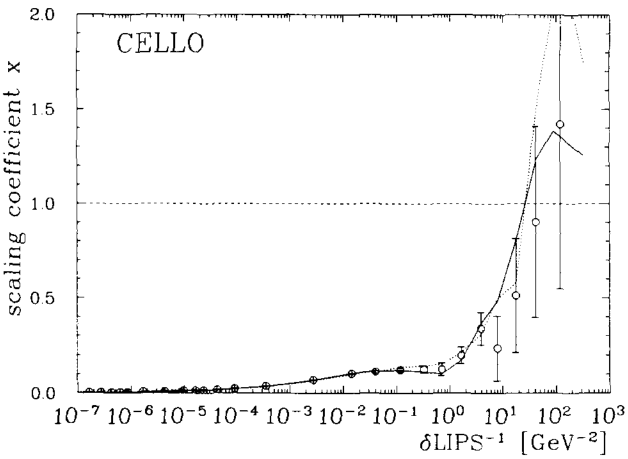


Fig. 3. Anomalous rank scaling law. The open symbols show the data with errors propagated from $F_2$–$F_5$. The dotted and solid curves correspond to the LUND 7.2 PS simulation prior to and after detector simulation. $x=0$ indicates geometrical scaling.

## 3. Monte Carlo studies and discussion

Given the success of the LUND model in describing the data, we have undertaken some generator studies in order to isolate the effects contributing to the observed complex behaviour of the fractal dimension and the rank scaling law. The resulting $D_2^F$ curves are displayed in fig. 4. In these investigations we also use neutral particles, since here no uncertainties due to acceptance and resolution of photons appear. Furthermore, for the investigation of the influence of resonance decays, the charge of stable particles and their decay products are not the same, such that a charge distinction is unnatural. The qualitative features have been checked to be independent of this choice.

The most simple case we considered are primary particles produced in $q\bar{q}$ events with no transverse degrees of freedom, i.e. $\sigma_{p_\perp}=0$. These events exhibit a one-dimensional structure once the jet structure is resolved [solid line (a)]. It is interesting to note that although the LUND fragmentation function is recursive, no deviation from a one-dimensional structure is seen in the high resolution limit, thus excluding it as a source of intermittency. The self-similarity of the particle production process is increased by the fragmentation function $f(z)=\delta(z-a)$ with $a=0.13$. This treatment leads indeed to an increase of $D_2^F$ [dotted line (a)]. The inclusion of initial state QED radiation was found to give a very slight increase of the fractal dimension. The effect of gluon emission is seen after inclusion of QCD radiation; [dashed line (a)] according to $O(\alpha_s^2)$ and [dash-dotted line (a)] according to the leading log approximation (parton shower). The first approach leads to a maximum fractal dimension of $D_2^F\approx 1.4$ which may be identified with the anomalous dimension of QCD as proposed by Gustafson [13]. The parton shower leads to a higher fractal dimension which can be attributed to the self-similarity of the parton branchings. Unfor-

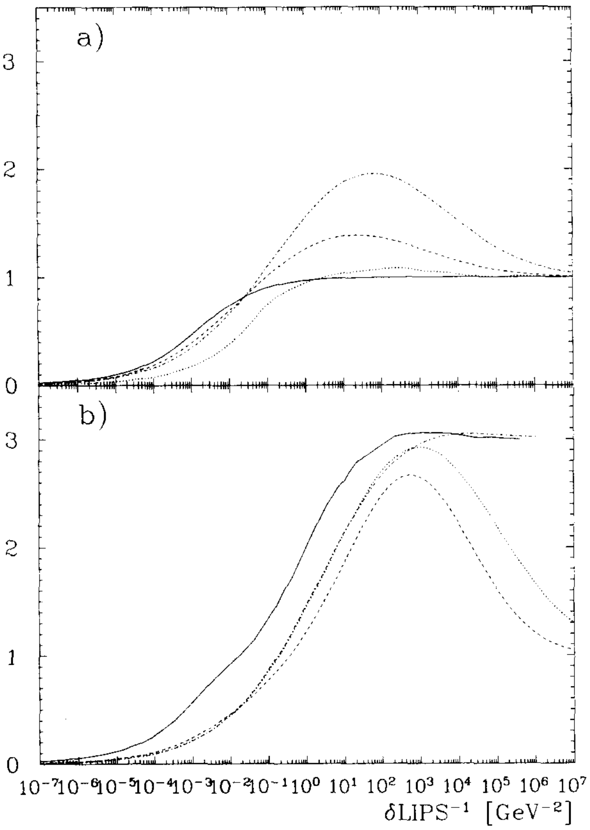


Fig. 4. LUND Monte Carlo generator studies of $D_2^F$. If not stated otherwise, the Lund symmetric fragmentation function has been used. (a) Solid line: primary particles from q$\bar{q}$ events with $\sigma_{p_\perp}=0$; Dotted line: ditto, but using a self-similar fragmentation function with $a=0.13$; Dashed line: primary particles from an $O(\alpha_s^2)$ matrix element simulation with $\sigma_{p_\perp}=0$; Dash-dotted line: ditto, but using the parton shower (PS) algorithm. (b) Solid line: primary particles from q$\bar{q}$ events with $\sigma_{p_\perp}=0.35$ GeV: Dashed line: final stable particles from q$\bar{q}$ events with $\sigma_{p_\perp}=0$; Dotted line: final stable particles in a default PS simulation including initial state radiation; Dash-dotted line: ditto, but neglecting $e^+e^-$ pairs from the $\pi^0$ Dalitz decay.

tunately, a straightforward distinction between the two descriptions is hindered by transverse degrees of freedom, as is made clear after giving the quarks the standard $\sigma_{p_\perp}$ of 0.35 GeV, which leads to a fast rise of $D_2^F$ towards 3 [solid line (b)]. The influence of resonance decays is demonstrated by the dashed line (b), which corresponds to q$\bar{q}$ events with $\sigma_{p_\perp}=0$, where resonances are allowed to decay. A very interesting structure is observed: At intermediate scales the fractal dimension appears much larger than 1, almost reaching 3, followed by a decrease towards 1. This behaviour of $D_2^F$ implies the existence of correlated two-particle production up to the highest resolution. This sounds rather interesting; however the explanation is simple. The Dalitz decay of the $\pi^0$ [14] has been identified as the source of this effect. This is demonstrated by a comparison of the default PS simulation [dotted line (b)] and the same simulation just neglecting the $\pi^0$ Dalitz decay [dash-dotted line (b)]. It is obvious that the rise of $F_2$ observed in the high resolution limit and thus the decrease of $D_2^F$ is caused by the Dalitz decay of neutral pions, which is the only source for correlated two-particle production in this limit. This is understandable since the $e^+e^-$ invariant mass distribution (and thus its phase space distance correlation function) is almost (up to the small electron mass scale) singular at threshold, leading to an intermittent behaviour (for the connection between two-particle correlation function and factorial moments see e.g. ref. [15]). A similar effect is produced by photons converted in the beam pipe. A study of Bose–Einstein correlations as implemented into the LUND 7.2 program [#1] revealed a significant rise of the moments $F_2$–$F_5$ at intermediate and high resolution scales. The size of this effect is of the right order of magnitude to further improve the consistency of data and Monte Carlo concerning the factorial moments $F_2$–$F_5$. Also the fractal dimension $D_2^F$ is reduced by Bose–Einstein correlations, since the pion phase space is effectively decreased. It is apparent in fig. 2 that this effect is present in the data.

Finally, it is clear that these different effects do not simply add in the dimension, such that we cannot argue in favour of one or the other QCD approach. The parton shower and the matrix element ansatz with default parameters both describe our data.

Monte Carlo studies of the rank scaling coefficient $x$ [cf. eq. (6) and fig. 3] indicate that the deviation from geometrical scaling for $\delta\text{LIPS}^{-1} \leqslant 1\ \text{GeV}^{-2}$ is due to the jet structure of the events. The strong violation of the geometrical scaling law above $\delta\text{LIPS}^{-1}=1\ \text{GeV}^{-2}$ can be attributed to transverse degrees of freedom. These are caused by gluon radia-

[#1] Note that there is a programming error in the JETSET 7.2 code such that only $\pi^+$ mesons are affected by the algorithm. In addition the strength parameter had to be considerably increased to reproduce the experimentally measured $Q^2$ distribution.

tion, fragmentation $p_\perp$ and particle decays.

It may be criticized that our particular variable choice leads to strong variations of the inclusive distribution and the large initial slopes are thus trivial. However, this does not invalidate the results derived in the high resolution limit, since for small bin sizes the inclusive distribution is constant to a good approximation. The sensitivity of our approach for genuine density fluctuations, as well as the quality of our data is demonstrated by the observation of a strong effect due to $\pi^0$ Dalitz decays and $\gamma$ conversions in detector material. Moreover, a variation of the inclusive distribution at intermediate scales is inherent in $e^+e^-$ annihilation, independent of the choice of variables. This is due to the mixture of events with different kinematics and topology, such as light and heavy quarks or hard and soft gluon radiation. In addition, the presentation of local slopes (or equivalently dimensions) as a function of the resolution scale clearly shows that there is a smooth variation, rather than a simple power law, as it is implied by the usual straight line fits, whose results of course depend on the fit range chosen. Finally we want to point out the usefulness of *factorial* (in contrast to normal) moments [1]. As demonstrated above, these allow the measurement of an intuitively reasonable *dimension* also for objects consisting of a finite number of points, whose dimension in the strictly mathematical definition in the infinite resolution limit is of course zero.

## 4. Summary and conclusions

We have performed the first intermittency analysis in three-dimensional phase space. Using multihadronic $e^+e^-$ annihilation events taken with the CELLO detector at $\sqrt{s}=35$ GeV, we have measured the factorial moments $F_2$–$F_5$, the fractal dimension and the rank scaling law up to $\delta\mathrm{LIPS}\approx 10^{-6}$ GeV$^2$. The data show several distinct features at different resolution scales which can be related to conventional phenomena in the fragmentation of quarks and gluons. The LUND 7.2 Monte Carlo program (with either parton shower or second order matrix element) using only default parameters provides an excellent description of the data. The moment $F_2$ shows the first unambiguous experimental evidence for a strong rise up to smallest resolution scales. The effect however can be traced back to the Dalitz decay of the $\pi^0$ and $\gamma$ conversions in detector material. No "new" physics beyond our present understanding of multiparticle production in $e^+e^-$ annihilation is needed to describe the data.

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