Cross section measurement and spin–parity analysis of the reaction $\gamma\gamma \rightarrow \omega\rho$

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We present results of an analysis of the reaction $\gamma\gamma \rightarrow \omega\rho^0$ in the two-photon process $e^+e^- \rightarrow e^+e^- 2\pi^+2\pi^-\pi^0$ in the untagged mode. The cross section is largely compatible with previous determinations; however, we do not confirm the enhancement claimed at 1.9 GeV. All observed $\omega$'s are accompanied by $\rho$'s in the recoiling $\pi\pi$ spectrum. An angular correlation analysis shows that the data is not dominated by a single spin–parity state, thus establishing severe constraints on $qqar{q}q$ models which predict $J^P=2^+$. 

The production of vector meson pairs in $\gamma\gamma$ collisions became a subject of controversy when the first measurements of the reaction $\gamma\gamma \rightarrow \rho^0\rho^0$ [1] revealed a large cross section, considerably exceeding the vector meson dominance model (VDM) expectation. Subsequent upper limits and measurements of the $\rho^+\rho^-$ production cross section [2] show that the latter is considerably lower, thus ruling out a single resonance interpretation. Achasov et al. [3] and Li and Liu [4] have interpreted these unusual experimental features as the manifestation of several interfering isospin 0 and 2 $qq\bar{q}q$ states which have been predicted in the MIT bag model [5]. Using vector meson dominance to relate the $\gamma\gamma$ coupling to the Zweig-superallowed ("fall apart") vector meson pair ($VV'$) decays of these states, they are able quantitatively to predict $\gamma\gamma \rightarrow VV'$ cross sections with three parameters: the mass, one general coupling constant $g_0$ common to all reactions involving these four-quark states, and the branching fractions into non-superallowed decay modes. In general, these predictions work quite well (see e.g. the reviews in ref. [6]), in particular if also conventional (one-particle exchange) processes are taken into account. An exception might be the process $\gamma\gamma \rightarrow \rho\omega$ [7] whose cross section is largely overestimated in the four-quark scheme.

Several authors [8] argue that an exotic signal should be searched for above a conventional background. They explain the difference between neutral and charged $\rho$ pair production by the fact that the first is diffractive in the VDM approach, whereas the latter has to proceed via $\rho$ or $\pi$ exchange. Using measured photoproduction and proton–proton scattering data and extending $t$-channel factorisation to the kinematical threshold, they are able to describe many of the experimental features. However, this threshold extrapolation procedure is not unique and has been criticized [9,10]. 

The reaction $\gamma\gamma \rightarrow \omega\rho$ is sensitive to excitation of $s$-channel resonances with isospin 1. Previous experimental determinations from ARGUS [11] and TPC/27 [12] resulted in a cross section which suggests an enhancement around 1.9 GeV. This structure is not confirmed by a new JADE measurement [13]. Apart from the enhancement, all models were able to qualitatively describe the data. ARGUS has also investigated some angular distributions but did not come to a definite conclusion about spin and parity.

In view of these controversies and the still limited statistical significance of the experimental data, an investigation of the reaction $\gamma\gamma \rightarrow 2\pi^+2\pi^-\pi^0$ with special emphasis on $\omega\rho$ production is of obvious interest. In particular, an analysis of decay distributions may help to settle one or the other approach. A definite prediction of the $qq\bar{q}q$ models is the spin–parity of the expected signals: the only four-quark resonance expected to contribute to the $\omega\rho$ final state is the isovector C=(36) ($\approx1650$) with the quantum numbers $J^{PC}=2^{++}$. Experimental data on spin–parity of the $\rho^0\rho^0$ final state are available, but not completely compatible between different experiments [1]. It seems that both $0^+$ and $2^+$ contribute to the threshold enhancement. In this letter we present the production cross section and also a spin–parity analysis of the reaction $\gamma\gamma \rightarrow \omega\rho$ employing the extended maximum likelihood method.

The experiment was performed with the CELLO detector at the $e^+e^-$ storage ring PETRA at a centre of mass energy of 35 GeV. The data correspond to an
integrated luminosity of 86 pb$^{-1}$. A detailed description of the detector is given elsewhere [14]. Charged tracks are reconstructed in the central detector consisting of nine cylindrical drift chambers and five proportional chambers in a 1.3 T magnetic field provided by a superconducting solenoid, yielding a momentum resolution of $\sigma(p)/p = 0.02p$ (p in GeV/c) which can be improved by a vertex constraint. The central detector is surrounded by a fine grained lead-liquid-argon calorimeter consisting of 16 barrel and 4 end cap modules. The identification and removal of background due to noise and charged pion reactions in the coil is made possible by means of its good spatial resolution in both the longitudinal and the lateral direction.

Low multiplicity untagged $\gamma\gamma$ events were triggered by a fast track-finding processor [15], which basically required two tracks with transverse momentum $p_T$ above 650 MeV/c or two tracks above 250 MeV/c with an opening angle $\Delta\phi$ in the plane perpendicular to the beam larger than 45° (135° for part of the experiment). By applying the same algorithm to the hit pattern of Monte Carlo events, the trigger decision was reliably simulated.

We selected events with two positively and two negatively charged particles and two photons in the lead-liquid-argon calorimeter. The charged and the neutral particle reconstruction procedures employed here are extended versions of the standard reconstruction, which are optimized for low momenta and energies typical for exclusive two-photon events. Some details of these modifications can be found in ref. [16]. No additional energy deposited in the forward tagger or the hole tagger is allowed.

To improve resolution and reject some background, mainly from photons faked by charged pion interactions in detector material and from non-exclusive events, we apply a series of kinematical fits and cuts: first we apply a constrained least squares fit demanding transverse momentum conservation. The invariant $\gamma\gamma$ mass shows a clear $\pi^0$ signal with some background. $\gamma\gamma$ pairs with masses between 0.09 and 0.18 GeV are taken as $\pi^0$ candidates. To select exclusive events we then perform a second fit (on the original data), this time demanding the $\gamma\gamma$ pair to have the nominal $\pi^0$ mass. The transverse momentum squared of these events is required to be smaller than 0.003 GeV$^2$. Finally, we perform a full fit simultaneously requiring $p_\perp$ conservation and the nominal $\pi^0$ mass. We require the $\gamma$ decay angle in the $\pi^0$ CMS relative to its direction of motion in the laboratory to fulfill $|\cos \theta_\gamma| < 0.70$, thus suppressing asymmetric $\pi^0$ decays, which had a small detection efficiency and are heavily contaminated by background. There are four possible $\pi^+\pi^-\pi^0$ mass combinations per event.

To enhance the $\omega$ signal against background we take advantage of its $I^G(J^P) = 1^-(1^-)$ Dalitz plot population. For the following analysis we accept only $3\pi$ combinations whose Dalitz plot radial parameter (normalized to the actual kinematical boundary, depending on the observed $m_{3\pi}$ and $\varphi_{\text{Dalitz}}$ values) is lower than 0.7. Fig. 1 shows the invariant mass combinations of all neutral $3\pi$ combinations fulfilling all of the above cuts. A clear signal of $50 \pm 9$ events due to $\omega$ production is observed.

For the quantitative analysis we have generated $5\pi$ and $\omega\pi\pi$ Monte Carlo events for invariant $\gamma\gamma$ masses from threshold up to 4 GeV with a cross section constant in $W_{\gamma\gamma}$. For the transverse photon flux the exact QED formulae are used as given by Budnev et al. [17], the $Q^2$ dependence of the cross section being modelled according to a $\rho$ form factor as predicted by VDM and measured in many exclusive processes. The contribution of longitudinal photons to the cross section vanishes at $Q^2 \rightarrow 0$ due to gauge invariance [18]. It can be neglected in the kinematical region of this experiment, in which the $Q^2$ are limited to small values through anti-tag and $p_\perp$ cut requirements. The decay of the $\gamma\gamma$ system into $\omega\pi^+\pi^-$ is simulated according to isotropic phase space. The subsequent $\omega$
Decay is modelled using the matrix element appropriate for a $1^-$ isoscalar decay into $\pi^+\pi^-\pi^0$ [19].

The events are passed through a detailed detector simulation program. Event reconstruction and selection then proceed as for real data, starting from simulated wire hits and deposited energies in the calorimeter and continuing with trigger and filter simulation.

The topological cross section for the reaction $\gamma\gamma\to2\pi^+2\pi^-\pi^0$ is shown in fig. 2. It has been derived from the $\gamma\gamma$ mass spectrum in bins of $W_{\gamma\gamma}$, after a successful fit requiring transverse momentum conservation. We assume a mixture of 5\pi, $\omega\pi$ and $\omega\rho$ phase space distributions for the acceptance correction, in the $W_{\gamma\gamma}$ dependent ratio as determined below. A pure 5\pi phase space model would have lead to some 30% lower cross sections. Feeddown from the processes $\gamma\gamma\to\omega\omega$ [20], $\omega3\pi$ [20], $6\pi$ [21] and $\pi\pi$ (QED) has been estimated using Monte Carlo simulations and found to be small. The observed cross section is in good agreement with previous experiments [22,11]. The errors shown are statistical only. A systematic normalisation error of 15\% must also be taken into account, and has been estimated through quadratic addition of the following contributions: uncertainty in luminosity (3\%), effect of $Q^2$ evolution of form factors (3\%), uncertainties in trigger simulation (5\%), photon (8\%) and track (4\%) reconstruction efficiencies, and background parameterisation (8\%).

We now investigate the intermediate resonance contents. First we fit the 3\pi mass spectrum (fig. 1) to the sum of the Monte Carlo expectations for $\omega\pi\pi$ (including combinatorial background) and 5\pi phase space. In the second step, we investigate the recoil $\pi\pi$ mass spectrum for events in the $\omega$ band ($0.735 < m_{3\pi} < 0.835$ GeV) as well as in the $\omega$ sidebands ($0.66 < m_{3\pi} < 0.71$ GeV and $0.86 < m_{3\pi} < 0.91$ GeV). Here we employ the relevant Monte Carlo expectations from 5\pi phase space (normalised to the result of the fit to the 3\pi mass spectrum) as well as $\omega\pi\pi$ and $\omega\rho$ decay models. All these fits are carried out in $W_{\gamma\gamma}$ bins of 0.25 GeV width. The result is that no $\omega\pi\pi$ contribution is needed for $W_{\gamma\gamma} < 2.5$ GeV, all $\omega$ events being associated with a $\rho$. Performing the same fit in the $\omega$ sidebands results in a $\rho$ yield compatible with zero, thus demonstrating a complete correlation between the $\omega$ and $\rho$ signals. Only between 2.5 and 3.0 GeV is the non-$\rho$ $\gamma\gamma\to\omega\pi^+\pi^-\pi^0$ cross section non-vanishing: $5.6 \pm 2.6$ nb (statistical error). The resulting $\omega\rho$ cross section is shown in fig. 3. It is largely compatible with previous measurements of ARGUS [11] and TPC/2\gamma [12]; however, it does not show an enhancement around 1.9 GeV as possibly observed by these experiments. It is also compatible with new results obtained by JADE [13] which in this energy region are even lower than ours. Detailed numerical values of the fit results and comparisons with other experiments and model predictions can be found elsewhere [23]. The mass spectrum can be described
by four-quark and $t$-channel factorisation ansätze with reasonable agreement; however, not in detail. A more detailed comparison with the four-quark model prediction will be performed below.

Having established clear evidence for the exclusive reaction $\gamma\gamma \rightarrow \omega \pi^0$, we now turn to a study of the dynamics of this process. We first point out that the appearance of $\rho$'s recoiling against the $\omega$'s is no surprise [9]: Due to the positive $C$-parity and the even spin of a (quasi-)real two photon system and the negative $G$-parity of the five-pion system the quantum numbers of a $\pi\pi$ pair recoiling against an $\omega$ are essentially restricted to be $J^G(J^{PC}) = 1^+ (1^-)$, and thus according to Watson's theorem [24] - the $\pi\pi$ pairs in the accessible energy range are $\rho$'s. In an isobar model description one can also first pair the $\omega$ a pion and then combine it with the other pion. Since it decays via symmetric SU(3)-(i.e. $D$-) coupling, the $C$-parity of such an $\omega\pi$ system must be negative and the isospin 1. The $1^- - b_1(1270)$ is the only known $\omega\pi$ resonance [25]; other possibilities ($1^-, 0^-, 2^+$ and $2^-$) are either exotic or expected not to be important below $\approx 1.5$ GeV. In a spin–parity analysis one has to consider the following low-lying states [26]: $J^P = 0^+(0), 0^-(0), 2^+(2), 2^+ (0), 2^-(0, S=1)$ and $2^-(0, S=2) \omega \rho$ and $0^+(0), 2^+ (2), 2^+ (0), 2^-(0, S=2) b_1 \pi$. These amplitudes have been calculated in the $LS$ scheme using spherical harmonics (see the appendix) and have been cross checked using tensor amplitudes [19,26]. Cross sections for different assumptions about spin–parity and intermediate resonances are calculated by weighting $\omega\pi\pi$ Monte Carlo events with the squared amplitudes. A more detailed description of amplitude construction and the connection to $\gamma\gamma$ cross sections can be found in ref. [27].

Each event $i$ is characterized by six final state phase space variables $\xi_i$ which contain information about spin and parity. These can be taken to be two $\omega\pi$ invariant masses and the two Euler angles $\cos \beta$ and $\gamma$ describing the orientation of the $\omega\pi\pi$ plane (the third angle $\alpha$ is not measurable in an untagged experiment) as well as two angles describing the orientation of the $\omega$ decay plane normal. To obtain information about the quantum numbers of the observed events, we construct for each event and each of the model amplitudes $j$ a probability measure [26]

$$P_j(\xi_i) = \frac{|M_j(\xi_i)|^2}{\epsilon_j(W_{\gamma\gamma}) \langle |M_j(W_{\gamma\gamma})|^2 \rangle}.$$ 

Here $\epsilon_j$ denotes the acceptance for model $j$ at the given $W_{\gamma\gamma}$. This quantity compares the matrix element squared of the observed event with the Monte Carlo expectation of the model at the observed $W_{\gamma\gamma}$, integrated over the phase space variables $\xi$. We then define the negative log likelihood function $\mathcal{L}_j = -\sum_i \log P_j(\xi_i)$ by combining the probabilities of all events. It is also possible to calculate probabilities for an incoherent sum of various models using event fractions $a_i$, which then can be fitted to minimize the likelihood function:

$$\mathcal{L}(a_i) = -\sum_j \left( \log \sum_j a_j P_j(\xi_i) - \sum_j a_j \right).$$

In such an extended maximum likelihood fit the maximum available information including all correlations is taken into account. Implicitly one assumes that a priori all model amplitudes are equally probable.

There is no interference between positive and negative parity states nor between different helicity states [26]. Coherence between different isobar amplitudes of a given $J^P$ state as well as interference between $0^-$ and $2^-$ states and $0^+$ and $2^+$ (0) is in principle possible, but will be neglected in the fit. To handle the background events under the $\omega$ signal, we also introduce a “garbage collection” phase space amplitude [28]. In all fits the resulting phase space percentage was compatible with the number obtained in the fit to the $m_{\omega\pi}$ mass spectrum. We also performed the fits with data from the $\omega$ sidebands. Reassuringly, these fits always resulted in 100% phase space, demonstrating that the phase space model for background events is a reasonable assumption.

With the limited statistics, fits in small $W_{\gamma\gamma}$ bins with all parameters allowed to vary are not feasible. We report on qualitative features of the data in the mass bin $1.5 < W_{\gamma\gamma} < 2.0$ GeV, where the $C_{\pi}(36)$ contribution is expected.

In agreement with the fits to the recoil $\pi\pi$ mass spectra described above, $b_1\pi$ contributions turn out to be compatible with zero, thus confirming the complete $\omega\rho$ correlation. Fits to all allowed amplitudes lead to a mixture of different contributions, indicating that the data is not dominated by a single spin-
parity state. Restricting negative and insignificant (lower than 0.1σ) contributions to zero during the fit procedure, the result is 25% 0⁺⁺, 3% 2⁻ (S=1), 34% 2⁻ (S=2) and 38% phase space background. All other contributions are zero. The fit result is stable against changing initial values. In particular, the quantum numbers 2⁺ predicted by the qqqq models are not favoured by the fit at all: it results in negative contributions for both helicity states of 2⁺. This result cannot be an artefact of neglected interference. Given the expected dominance of the helicity 2 state (6:1 by simple polarization state counting, higher for qq mesons [27]) and the smallness of the bπ amplitudes, there is no appreciable interference term.

We now test the hypothesis that the signal is due to one single spin–parity state plus a phase space background contribution fixed to the expectation from the ω sidebands. The result is summarized in table 1. For Jπ=2⁺ we also allow the fit to choose the most likely helicity mixture. Even this mixture yields a likelihood difference ΔL=6 worse than the best single Jπ model, 2⁻ (S=2), corresponding to n=√2ΔL=3.5 standard deviations. In a number of Monte Carlo experiments we have checked that single resonance models cannot be erroneously excluded by the likelihood procedure employed [23]. These tests give us confidence that our result excludes the usual assumption that the ω signal is predominantly 2⁺ which is implicit in qqqq model fits.

Following ref. [25] we establish upper limits for the Jπ=2⁺ partial wave cross sections in the Wγγ range between 1.5 and 2.0 GeV using the following procedure: we calculate the likelihood values in the σ(2⁺(0)) versus σ(2⁺(2)) plane, at each point maximizing with respect to all other parameters. The probability densities (i.e. the likelihood function without taking the logarithm) are integrated in the physical region (both cross sections zero or positive), this integral is then normalized to 1.95% CL upper limits are then obtained by finding the contours of equal likelihood which separate 95% of the integral from the rest. It turns out that the contour is approximately linear [23] and can be parametrized in the following form:

\[
\frac{\sigma_{\gamma\gamma\to q\bar{q}}^{2^{-}},J=2}{11 \text{ nb}} + \frac{\sigma_{\gamma\gamma\to q\bar{q}}^{2^{-}},J=0}{7.5 \text{ nb}} < 1
\]

at 95% CL for 1.5 < Wγγ < 2.0 GeV.

We now investigate the consequences of these findings for the model of Achasov et al. [3]. In fig. 4 we plot the 95% CL lower limits of the parameter a₀ inferred from our data. This parameter describes the relative branching fraction of the four-quark state C⁺(36) into all non-superallowed channels: a₀ = (Γtot - Γsup) / Γtot. These limits are given as a function of the mass of the state and for three common assumptions about the general decay constant g²/4π = 9.3, 16.4 and 21.4 GeV² [3]. Phenomenologically

### Table 1

<table>
<thead>
<tr>
<th>Jπ contribution</th>
<th>ΔL (all)</th>
<th>ΔL (best single)</th>
</tr>
</thead>
<tbody>
<tr>
<td>all free</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2⁻ (S=2)</td>
<td>3.99</td>
<td>0</td>
</tr>
<tr>
<td>0⁺</td>
<td>4.34</td>
<td>0.35</td>
</tr>
<tr>
<td>2⁺ (λ=2)</td>
<td>9.98</td>
<td>5.99</td>
</tr>
<tr>
<td>0⁻</td>
<td>14.32</td>
<td>10.33</td>
</tr>
<tr>
<td>2⁻ (S=1)</td>
<td>14.39</td>
<td>10.40</td>
</tr>
<tr>
<td>2⁺ (λ=0)</td>
<td>16.22</td>
<td>12.32</td>
</tr>
<tr>
<td>2⁺ (best λ mixture)</td>
<td>9.86</td>
<td>5.87</td>
</tr>
</tbody>
</table>

Fig. 4. Lower limits for the parameter a₀ in the four-quark model of Achasov et al. [3] as function of the resonance mass and three sets of coupling constants g²/4π.
one expects the “fall-apart” decay modes to dominate and thus $a_0$ to be small and a mass around or lower than 1.65 GeV, just in the center of the excluded area. It should be noted that a non-zero parameter $a_0$ implies that the resonance should also show up in other final states. A natural candidate final state is $\rho^{\pm}\pi^{\mp}$ [cf. the $a_2$, which has the same spin-parity as the $C_{\pi}(36)$]. However, data on $\gamma\gamma \rightarrow \pi^{+}\pi^{-}\pi^{0}$ [27] do not show any evidence for such a signal. The same is true for the $\eta\pi^0$ final state [25,30]. Another possible decay mode would be $b_\pi$. As the above fits have shown, there is no visible $b_\pi$ contribution in our $\omega\pi\pi$ data. We thus conclude that the data is incompatible with the four-quark model as formulated by Achasov et al. in the present form [3], i.e. using the superallowed MIT bag model couplings for the $qq\bar{q}q$ decays into vector meson pairs in conjunction with vector meson dominance to calculate the $\gamma\gamma$ coupling. This does not rule out, however, the existence of four-quark states in $\gamma\gamma$ reactions in general.

The good fit for a $2^-$ model might indicate a large contribution due to the $\pi_2(1670)$. However, the measured $\gamma\gamma$ width of the $\pi_2(1670)$ [27,31] combined with upper limits for the branching ratio into $\pi\pi$ final states leads to an expectation which is much smaller than the observed. Below the nominal $\omega\pi^0$ threshold the $a_2$ is expected to contribute to the reaction under study. The few events observed below 1.5 GeV are consistent with the $a_2$ expectation but do not allow a positive identification of spin and parity.

Unfortunately, there is no prediction from $t$-channel factorization [8] on the partial wave structure. Since diffraction and pion exchange can contribute to the reaction $\gamma\gamma \rightarrow \omega\pi$ it might be that a large variety of $s$-channel spin and parity states are excited, consistent with our results. However, a quantitative prediction that could be tested experimentally is desirable before accepting this description.

In summary, we have presented an analysis of the reaction $\gamma\gamma \rightarrow \omega\pi^+\pi^-$ measured with the CELLO detector at PETRA. The $\pi\pi$ system recoiling against the $\omega$ is determined to be fully compatible with being $\rho$ mesons. The cross section $\sigma_{\gamma\gamma \rightarrow \omega\pi}$ agrees largely with previous measurements. Both $qq\bar{q}q$ models and $t$-channel factorization can qualitatively describe the mass spectrum. An extended maximum likelihood fit of the decay distributions leads to a mixture of many states, such that no single resonance can dominate the data. In particular, $J^P=2^+$ as predicted by $qq\bar{q}q$ models gives a bad description of the decay distributions. We have established upper limits for the tensor partial wave sections of both helicities (0 and 2) and for some parameters of four-quark models which are severe enough as to cast doubt on the validity of the models in their present formulation.

Appendix

$\omega\pi$ and $b_\pi\pi$ amplitudes

In this appendix we describe the construction of the $\omega\pi$ and $b_\pi\pi$ amplitudes in the spin–orbit ($LS$) coupling scheme. The square of the amplitudes calculated in the following gives the weight which $\omega\pi\pi$ phase space events need in order to describe a process with specific $J^P$, helicity and subresonance (isobar). We begin with the $\omega\pi$ amplitudes. Both spin-1 particles are first coupled to a total spin denoted by the spin wavefunction:

$$f_{\omega}(S, s_\omega) = \sum_{m_\rho, m_\omega} \left( \frac{1}{m_\rho} \frac{1}{m_\omega} S \right) Y_{\omega}(\cos \theta_{\omega}, \phi_{\omega}) Y_{\rho}(\cos \theta_{\rho}, \phi_{\rho}) .$$

(1)

The angles occurring here describe the $\omega$ in the $\gamma\gamma$ helicity frame. A relativistic Breit-Wigner shape is used for the description of the propagation of the $\rho$. The expression in brackets denotes a standard Clebsch–Gordan coefficient which is non-zero only for $m_\rho + m_\omega = s_\omega$. In the second step the spin functions are coupled with the orbital angular momentum functions to the total spin state:

$$A^{(LS)}(J, s_J) = \sum_{m_J, s_J} \left( \frac{L}{m_J} \frac{S}{s_J} \right) f_{\rho}(s_J)$$

(2)

The angles appearing here describe the $\rho$ in the $\gamma\gamma$ helicity frame. A relativistic Breit–Wigner shape is used for the description of the propagation of the $\rho$. The expression in brackets denotes a standard Clebsch–Gordan coefficient which is non-zero only for $m_\rho + m_\omega = s_\omega$. In the second step the spin functions are coupled with the orbital angular momentum functions to the total spin state:

$$A^{(LS)}(J, s_J) = \sum_{m_J, s_J} \left( \frac{L}{m_J} \frac{S}{s_J} \right) f_{\omega}(s_J)$$

In summary, we have presented an analysis of the reaction $\gamma\gamma \rightarrow \omega\pi^+\pi^-$ measured with the CELLO detector at PETRA. The $\pi\pi$ system recoiling against the $\omega$ is determined to be fully compatible with being $\rho$ mesons. The cross section $\sigma_{\gamma\gamma \rightarrow \omega\pi}$ agrees largely with previous measurements. Both $qq\bar{q}q$ models and $t$-channel factorization can qualitatively describe the mass spectrum.
The deviation from pointlike couplings is modelled according to Blatt–Weisskopf form factors [32]:

\[ g(k^*, L=0) = 1, \]
\[ g(k^*, L=1) = \frac{1}{k^n_0} \sqrt{\frac{1 + (k^n_0 r)^2}{1 + (k^* r)^2}}, \]
\[ g(k^*, L=2) = \frac{1}{k^n_0} \sqrt{\frac{1 + \frac{1}{2} (k^n_0 r)^2 + \frac{1}{2} (k^* r)^4}{1 + \frac{1}{2} (k^* r)^2 + \frac{1}{2} (k^* r)^4}}, \]

with \( k^n_0 \) being the CMS momentum at nominal resonance mass and \( r = 1 \text{ fm} \) the spatial dimension of the interaction region. The angle \( \omega_0 \) is not measurable in an untagged experiment and has to be integrated over. Since it only contributes a phase, this has no consequences for the squared amplitudes, but it follows that no interference between different helicity states can be observed, and there is in general no interference between positive and negative naturality amplitudes [26].

The parity of a \( 1^- 1^- \) system is \((-1)^L\), such that parity conservation restricts \( P=+ \) states to decay via even \( L \) and vice versa. In the interesting low energy regime it is a reasonable assumption to neglect all but the lowest orbital angular momenta. This leads to unique \( L \) and \( S \) values except for \( J^P = 2^- \), where two spin states \( S = 1 \) and 2 are possible at \( L = 1 \).

We now construct \( LS \) amplitudes for the \( b_1 \pi \) final state. Considering the reaction \( \gamma \gamma \to \omega \pi \), parity conservation restricts the orbital angular momentum \( L \) to be odd for \( P = + \) and vice versa. The pion being spinless, the total spin of the \( b_1 \pi \) system trivially is just the \( b_1 \) spin \((S = 1)\), described by the function \( f_b(s_z) \). The \( b_1 \) decay into \( \omega \pi \) has been measured to occur in \( S \) and \( D \) wave with a \( D/S \) amplitude ratio of 0.26 [25]. Thus,

\[
 f_b(s_z) = Y_0^0(\cos \theta_\omega, \varphi_\omega) \ Y_{1z}^\sigma(\cos \theta_{b_1}, \varphi_{b_1}) \\
 + d \sum_{m_d, m_w} \left( \frac{2}{m_d} \right) \left( \frac{1}{m_w} \right) |s_z \rangle \\
 \times Y_{2z}^{m_w}(\cos \theta_\omega, \varphi_\omega) \cdot Y_{1z}^{m_z}(\cos \theta_{b_1}, \varphi_{b_1}),
\]

with \( s = 1/\sqrt{1 + (D/S)^2} \) and \( d = \sqrt{1 - s^2} \). \( \theta_\omega \) is measured in the \( b_1 \) rest frame, and \( \theta_{b_1} \) denotes the \( \omega \) decay plane normal in the \( \omega \) CMS, reached from the \( \gamma \gamma \) CMS by two successive Lorentz transformations via the \( b_1 \) to avoid Wigner rotations. The \( LS \) amplitude into \( b_1 \pi \) reads

\[
 B^{(L)}(J, J_z) = \sum_{m_L, m_z} \left( \frac{L}{m_L} \right) \left( \frac{1}{m_z} \right) \left( \frac{J}{J_z} \right) \\
 \times Y_{Lz}^{m_L}(\cos \theta_\omega, \varphi_\omega) \ f_b(s_z) \left( \frac{k^n_\omega L}{m^2_{b_1} - m^2_{\omega} - i m_{b_1} \Gamma_{b_1}} \right).
\]

Finally, \( b_1 \pi \) may appear in two possible charge states. Since the isospin of any intermediate state in the reaction \( \gamma \gamma \to \omega \pi \) is 1, isospin Clebsch–Gordan leads to the decomposition

\[
b_1 \pi = \frac{1}{\sqrt{2}} b_1^+ \pi^- - \frac{1}{\sqrt{2}} b_1^- \pi^+.
\]

Thus, the two possible \( b_1 \pi \) amplitudes have to be coherently subtracted. The interference term between these two amplitudes leads to one more subtlety which has to be taken into account in a spin–parity analysis: usually \( \lambda = +2 \) and \( \lambda = -2 \) amplitudes lead to the same weight, and any of them may be used in the spin–parity analysis. However, interference between two terms with different phases (e.g. two different Breit–Wigner amplitudes) spoils this symmetry. Since both helicity states are produced at equal rates, we do not introduce two different amplitudes into the maximum likelihood fit (whose contributions must be equal), but take instead the (incoherent) mean of both squared helicity amplitudes as the helicity 2 weight.

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