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SCALAR-FERMION MODELS WITH MIRROR PAIRS OF FERMION FIELDS*

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Scalar-fermion models with mirror pairs of fermion fields on the lattice are discussed. Numerical simulation data in a $U(1)_L \otimes U(1)_R$ symmetric model on the phase structure and on lower and upper limits for the Higgs-boson mass and perturbative estimates of the finite size effects are presented.

1. INTRODUCTION

In the standard model of electroweak interactions the Higgs self-coupling as well as the Yukawa couplings are related to the Higgs mass and fermion masses, respectively. In particular, if the Higgs or the top quark were heavy the corresponding couplings could be large. In this case nonperturbative methods would be needed to study the physics of these particles. Neglecting gauge couplings, which are weak, leads to Yukawa models describing interacting scalar and fermion fields.

In past years the Higgs sector has been studied extensively in the framework of the pure ϕ^4 theory by various groups. It turned out that the triviality of ϕ^4 theory implies cutoff dependent upper bounds on the scalar self-coupling such that it never becomes strong for physically reasonable cutoffs. The inclusion of fermions which couple to the scalar field via Yukawa couplings might, however, change the situation. Therefore nonperturbative investigations of Yukawa models on a lattice are required.

If fermions are put on a lattice the notorious doubling problem occurs. A Dirac field which is transcribed naively to the lattice turns out to describe 16 fermions. The methods to remove the unwanted doublers include the Wilson term, which gives a mass of the order of the cutoff to the doublers. This term breaks chiral symmetry. It can be used for QCD, but for chiral gauge theories like the electroweak standard model it is important to have chiral symmetry preserved on the lattice. Presently two approaches are popular to remove doublers in a chirally invariant way. In the Smit-Swift model (see J. Smits contribution to this conference) the Wilson term is replaced by an analogous chiral invariant Yukawa coupling. The second approach, which we follow, uses models with mirror fermions [1,2]. In addition to the original fermion field ψ a mirror fermion field χ is introduced, which transforms oppositely with respect to the chiral symmetry group. Each fermion-mirror-fermion pair describes 32 fermions on the lattice. But now it is possible to write down a generalized Wilson term $r\bar{\psi}_{R,x+\hat{\mu}}\chi_{Lx} + \ldots$, which mixes ψ and χ and is chirally invariant. This term removes 30 doublers by giving them masses of the order of the cutoff and we are left with one mirror pair. The mirror fermion method has the following advantages: i) the remaining mirror pair constitutes the minimal possible doubling, ii) the mirror field χ is easier to control than the usual doublers since it is explicitly contained in the action and has its own couplings, iii) we have perturbation theory at our disposal to study the vicinity of the Gaussian fixed point, iv) reflection positivity can be proven in large parts of the bare parameter space.

Before we write down explicit actions for lattice models with mirror fermion fields we would like to discuss some general aspects of these mod-

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els. There exists a symmetric phase in which chiral symmetry is unbroken. In the symmetric phase the masses m_ψ and m_χ of fermions and corresponding mirror fermions are equal. On the other hand, in a phase with spontaneously broken chiral symmetry these masses are unequal. What happens to the mirror fermions in the continuum limit? Approaching the continuum limit in a phase with broken chiral symmetry different scenarios are possible:

A. The mirror fermions remain in the physical spectrum at some higher scale. Indeed, mirror fermions above 100 GeV are not excluded phenomenologically [3].

B. Mirror fermions are decoupled, i.e. completely removed from the physical spectrum

B1. either if their masses go to infinity in the continuum limit

B2. or if their masses as well as their couplings to the other particles vanish in the continuum limit [4].

Which of these possibilities can be realized in a given model has to be determined by nonperturbative methods.

The $SU(2)_L \otimes U(1)_Y$ symmetry of electroweak interactions is a subgroup of the chiral group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, where $B-L$ means baryon minus lepton number. In a model with a chiral $SU(2)_L \otimes SU(2)_R$ symmetry we have a fermion doublet ψ_A, $A = 1, 2$, and a mirror fermion doublet χ_A, which transform as

$$\psi'_L = U_L \psi_L \,, \qquad \psi'_R = U_R \psi_R \,,$$
$$\chi'_L = U_R \chi_L \,, \qquad \chi'_R = U_L \chi_R \,. \tag{1.1}$$

The pure fermionic part of the action is

$$S_{\psi\chi} = \sum_x \Big\{ -K \sum_\mu \Big[(\bar\psi_{x+\hat\mu}\gamma_\mu\psi_x) + (\bar\chi_{x+\hat\mu}\gamma_\mu\chi_x)$$
$$- r \Big((\bar\chi_x\psi_x) - (\bar\chi_{x+\hat\mu}\psi_x) + (\bar\psi_x\chi_x) - (\bar\psi_{x+\hat\mu}\chi_x) \Big) \Big]$$
$$+ \mu_{\psi\chi} \Big[(\bar\chi_x\psi_x) + (\bar\psi_x\chi_x) \Big] \Big\} \,. \tag{1.2}$$

Here x is a lattice point and the sum $\sum_\mu$ runs over eight directions of the neighbours. The term proportional to r is the generalized Wilson term. We always set $r = 1$ and normalize the mass mixing term by $\mu_{\psi\chi} + 8rK = 1$. The scalar field is a complex doublet which we write as

$$\varphi_x = \phi_{0x} + i \sum_{s=1}^{3} \tau_s \phi_{sx} \,, \tag{1.3}$$

where ϕ is a four-component real vector and τ_s are the Pauli matrices. It transforms as

$$\phi' = U_L \phi U_R^{-1} \,. \tag{1.4}$$

The action for the scalar field is

$$S_\varphi = \sum_x \Big\{ \frac{1}{2}\mathrm{Tr}(\varphi_x^+ \varphi_x) + \lambda \Big[\frac{1}{2}\mathrm{Tr}(\varphi_x^+\varphi_x) - 1 \Big]^2$$
$$- \frac{\kappa}{2} \sum_\mu \mathrm{Tr}(\varphi_{x+\hat\mu}^+ \varphi_x) \Big\} \,. \tag{1.5}$$

Written in terms of the components ϕ_s this is the action of an O(4)-symmetric ϕ^4 model. Finally the Yukawa interaction is given by

$$S_Y = \sum_x \Big\{ (\bar\psi_{Rx} G_\psi \varphi_x^+ \psi_{Lx}) + (\bar\psi_{Lx} \varphi_x G_\psi \psi_{Rx})$$
$$+ (\bar\chi_{Rx} \varphi_x G_\chi \chi_{Lx}) + (\bar\chi_{Lx} G_\chi \varphi_x^+ \chi_{Rx}) \Big\} \,. \tag{1.6}$$

Here, for later purpose, the Yukawa couplings G_ψ, G_χ are considered to be $2 \otimes 2$ matrices proportional to the unit matrix. The global symmetry of this model is $SU(2)_L \otimes SU(2)_R \otimes U(1)_F$, where F is fermion number. The gauging of $SU(2)_L \otimes SU(2)_R \otimes U(1)_F$ or some of its subgroup can be implemented in a standard manner.

The fermion determinant in this $SU(2)_L \otimes SU(2)_R$ model is real, but in numerical simulations the Hybrid Monte Carlo algorithm requires a doubling of flavours which leads to a positive fermion determinant. This means that we have two flavours $\psi^{(1)}, \psi^{(2)}$ of fermions and two flavours $\chi^{(1)}, \chi^{(2)}$ of mirror fermions. The second flavour transforms with opposite chirality compared to the first flavour. The global symmetry now is $SU(2)_L \otimes SU(2)_R \otimes U(1)_{F1} \otimes U(1)_{F2}$ since both flavours are conserved separately.

In the framework of the electroweak interactions the global $SU(2)_R$ (neglecting gauge couplings) is broken through different masses within the fermion doublets. In the above model a breaking of $SU(2)_R$ to its subgroup $U(1)_{R3}$ can be achieved by replacing the Yukawa couplings G_ψ and G_χ by diagonal matrices

$$G_\psi = \begin{pmatrix} G_{\psi u} & 0 \\ 0 & G_{\psi d} \end{pmatrix}, G_\chi = \begin{pmatrix} G_{\chi u} & 0 \\ 0 & G_{\chi d} \end{pmatrix}. \tag{1.7}$$

This model with split doublets can be gauged with the physical gauge group $SU(2)_L \otimes U(1)_Y$ by replacing the $SU(2)_L \otimes SU(2)_R$ lattice gauge fields

$$U_L(x,\mu) \Longrightarrow U_L(x,\mu)U_Y(x,\mu) ,$$

$$U_R(x,\mu) \Longrightarrow U_Y(x,\mu) . \tag{1.8}$$

The hypercharge of the fields is given by $Y = 2T_{R3} + B - L$.

Even nearer to reality is the *top-bottom model* where, in addition to the electroweak quantum numbers, 3 species of colour for the fermions are introduced. It has three degenerate fermion doublets with a heavy top quark and a light bottom quark each. The mirror doublets are all heavy. (For a fermion-mirror-fermion mass matrix consistent with phenomenology see [2].) Below the mirror-fermion mass scale (or if the mirror fermions are decoupled) this scalar-fermion model describes the electroweak standard model with all small gauge and Yukawa couplings being neglected. Since the number of doublet pairs is odd the fermion determinant is complex. Therefore the top-bottom model unfortunately can not be simulated numerically by presently available Monte Carlo methods.

In our numerical work we have so far mainly considered a model with a chiral $U(1)_L \otimes U(1)_R$ symmetry. The action looks like the one of the $SU(2)_L \otimes SU(2)_R$ model above with the difference that now the fermions are singlets and the scalar field has one complex component. The flavour doubling of the fermion fields leads to an extension of the chiral symmetry from $U(1)_L \otimes U(1)_R$ to $U(1)_L \otimes U(1)_R \otimes U(1)_{1-2}$. If the model is described in terms of the left-handed fields (the right-handed fields are represented by the left-handed component of the charge-conjugate field as $\psi_{cL} \equiv C\bar{\psi}_R^T$), then the quantum numbers of the 8 fermion fields and of the Higgs-field are

$$\begin{array}{lccc} & U(1)_L & U(1)_R & U(1)_{1-2} \\ \psi_L^{(1)}: & 1 & 0 & 1 \\ \psi_{cL}^{(1)}: & 0 & -1 & -1 \\ \chi_L^{(1)}: & 0 & 1 & 1 \\ \chi_{cL}^{(1)}: & -1 & 0 & -1 \\ \psi_L^{(2)}: & 0 & 1 & -1 \\ \psi_{cL}^{(2)}: & -1 & 0 & 1 \\ \chi_L^{(2)}: & 1 & 0 & -1 \\ \chi_{cL}^{(2)}: & 0 & -1 & 1 \\ \phi: & 1 & -1 & 0 \end{array} \tag{1.9}$$

The mass terms allowed by the chiral symmetry $U(1)_L \otimes U(1)_R \otimes U(1)_{1-2}$ are those connecting $\psi^{(1)}$ with $\chi^{(1)}$ or $\psi^{(2)}$ with $\chi^{(2)}$ but not $\psi^{(1)}$ with $\psi^{(2)}$ and $\chi^{(1)}$ with $\chi^{(2)}$ (these latter are forbidden by $U(1)_{1-2}$). The vacuum expectation value of the scalar field breaks $U(1)_L \otimes U(1)_R$ to its diagonal subgroup, but $U(1)_{1-2}$ is not spontaneously broken. Therefore, after removing the mirror fermions $\chi_{1,2}$ even this doubled model is chiral in the sense that the mass terms are absent. This model has been discussed at length in [5,6,7].

2. PHASE STRUCTURE

We explore the phase structure of our model both analytically and numerically (on $4^3 \cdot 8$ lattices). In this section, we report what we know about it in the Ising limit $\lambda = \infty$. In our simulations, we always set $r = 1$ in order to make sure that in some parts of the parameter space we have site-reflection positivity. If $r \neq 1$, the procedure to prove site-reflection positivity simply does not go through [7].

There are four limiting cases in which the model reduces to the pure 2-component ϕ^4 model at $\lambda = \infty$, i.e.: $K = 0, \infty$, and $|G_\psi| = |G_\chi| = 0, \infty$. In these four limits, the model has a second order phase transition from the ferromagnetic phase (denoted by FM phase) to the symmetric phase (denoted by PM phase) at $\kappa_0 \simeq 0.15$, and from the PM phase to the anti-ferromagnetic phase (denoted by AFM phase) at $\kappa_0 \simeq -0.15$. In the FM and AFM phases, the chiral symmetry

$U(1)_L \otimes U(1)_R$ is spontaneously broken down to the vector-like symmetry by the vacuum expectation value of the scalar field.

In the very weak and strong Yukawa coupling limits, we carry out small- and large-G expansions to next to leading order and find

$$\kappa_c = \kappa_0 - 2K^2(G_\psi^2 - 2G_\psi G_\chi + G_\chi^2) \qquad (2.1)$$

for weak Yukawa couplings, and

$$\kappa_c = \kappa_0 - 4K^2(\frac{1}{G_\psi^2} - \frac{2}{G_\psi G_\chi} + \frac{1}{G_\chi^2}) \qquad (2.2)$$

at strong couplings. Note that the above results are at $N_f = 2$ (flavour doubled model which can be simulated by Hybrid Monte Carlo). The coefficients of the leading corrections are proportional to N_f.

At intermediate values of Yukawa couplings, we need to rely on Monte Carlo simulations to explore the phase structure.

Since there are two mass scales (the scalar and fermion masses) in our model, the continuum limit should be defined such that both masses go to zero while their ratio is kept constant. Therefore we need to study also the critical plane on which the fermion mass vanishes.

At $G_\psi = G_\chi = 0$, we have $K_c = 1/8$. At small $|G_\psi|$ and $|G_\chi|$, one can estimate K_c by using 1-loop bare perturbation theory. (See Ref. [6].) We find, qualitatively,

$$K_c \searrow \quad \text{as } |G_\psi G_\chi| \nearrow \quad \text{if } G_\psi \cdot G_\chi \geq 0 \ ,$$

$$K_c \nearrow \quad \text{as } |G_\psi G_\chi| \nearrow \quad \text{if } G_\psi \cdot G_\chi < 0 \ .$$

We have seen the above qualitative behaviours in our simulations.

From the above analysis, we know the phase structure near the Gaussian fixed point of the pure scalar model. The phase structure at $G_\psi \cdot G_\chi < 0$ is schematically shown in fig. 1.

Recently a new phase with both ferromagnetic and antiferromagnetic long-range orders has been found with Monte Carlo simulations in the $U(1)_L \otimes U(1)_R$ model. We call it the ferrimagnetic (FI) phase. A similar phase structure with FM, PM, AFM and FI phases was observed earlier in other scalar-fermion models on the lattice [8,9,10,11,12]. At $G_\psi = -G_\chi = 2.0$, we tune κ and K on the $4^3 \cdot 8$ lattice to make $m_R \simeq 1.0$ in the symmetric phase. As K gradually grows from 0.01 to 0.39, the fermion mass decreases to about 2.0 when we are very close to the boundary of the PM and FI phases (see fig.2). This shows that at $G_\psi = -G_\chi = 2.0$, the K_c plane on which the fermion mass vanishes will pass through the FM and FI phases. Since we know that K_c increases as $|G_\psi G_\chi|$ increases when $G_\psi \cdot G_\chi < 0$, it is very plausible that the K_c plane will intersect the critical line along which PM, FM and FI phases coexist at some smaller G_ψ, G_χ values (e.g.: $G_\psi = -G_\chi \simeq 1.7$). If this does happen, then the intersecting point will be the candidate for a possible nontrivial fixed point [9]. We will pursue this interesting matter in the future.

3. RENORMALIZED QUANTITIES AND VACUUM STABILITY

We use the unbiased Hybrid Monte Carlo method to study the model at weak and intermediate Yukawa couplings, and at very small and infinite bare scalar self-coupling. The renormalized quantities in the PM and FM phases are defined in Refs. [6,7]. In the PM phase, we tune κ and K such that the mass ratio is close to one and the correlation length is not larger than 1/4 of the spatial lattice size to avoid large finite size effects. In the FM phase, we tune κ and K to achieve no fermion mixing and some desired value of the magnetization. Other quantities like the fermion and mirror masses, scalar mass etc. will be determined. This means we cannot always avoid large mass ratios and therefore we sometimes have large finite size effects. Hence, data on small lattices in the FM phase can only tell us the qualitative behaviour of the model.

In the PM phase we find that the fermion doublers can be made very heavy. However, here the mirror fermion is degenerate with the fermion. The low enery spectrum is still vector-like. Also, we see that the renormalized Yukawa coupling $G_{R\psi}$ is linearly proportional to the bare coupling G_ψ and can become at least 2 to 3 times as large as the tree

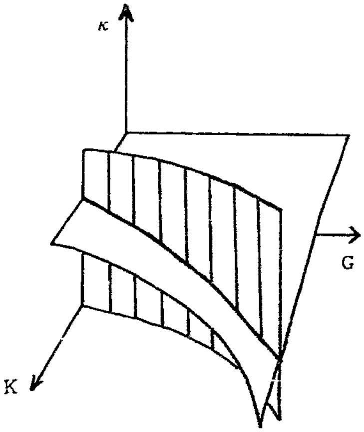

Figure 1: The phase structure near the Gaussian fixed point at $\lambda = \infty$ and $G_\psi = -G_\chi \equiv G$. The fermion mass μ_R vanishes on the shaded surface while the scalar mass m_R becomes zero at another surface.

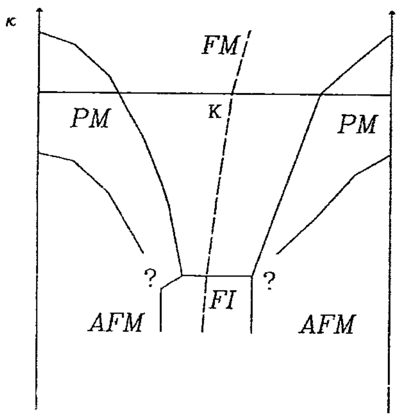

Figure 2: The phase structure at $\lambda = \infty$ and $G_\psi = -G_\chi = 2.0$ is shown. A new phase (FI) is found. The detailed topology in the middle is still not clear. The dashed line is the critical line on which the fermion mass is zero. It seems that this line does no go through the PM phase.

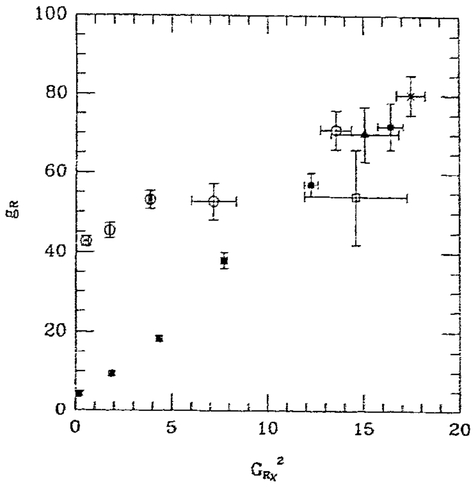

Figure 3: Data of g_R vs. $G^2_{R\chi}$ at $G_\psi = 0.1$ and $-G_\chi = 0.1$ to 0.6 are plotted. Open circles are at $\lambda = \infty$ on $4^3 \cdot 8$ lattice. The open square is at the same λ on $6^3 \cdot 16$ lattice. Full squares are at $\lambda = 10^{-4}$ on $4^3 \cdot 8$ lattice. The cross is at $\lambda = 10^{-6}$ on $4^3 \cdot 8$ lattice. The full triangle is at the same λ on $6^3 \cdot 16$ lattice.

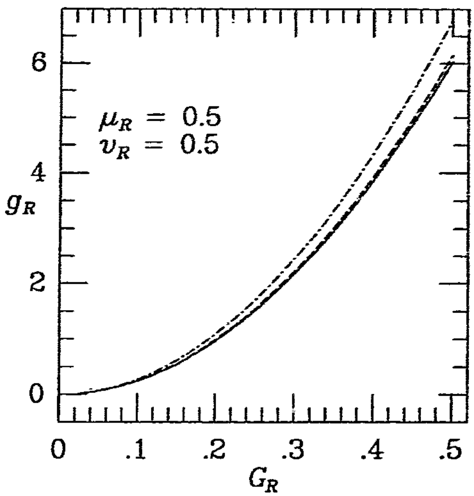

Figure 4: The vacuum stability bound in lattice perturbation theory on different lattices (see text).

unitarity upper limit. We observe very weak or even no λ-dependence of $G_{R\psi}$. We find that the renormalization of $G_{R\psi}$ is much smaller than the 1-loop prediction. Whether this is related to the existence of a nontrivial fixed point is yet to be studied and found out.

In the FM phase not only the fermion doublers can be made heavy, but also the mirror fermion can acquire a large mass by tuning the bare mirror Yukawa coupling G_χ. If the theory is nontrivial, they can all be decoupled by giving them infinite masses. If the theory is trivial, then they can be pushed to the cutoff scale and become part of the "new physics".

In the FM phase we also study the vacuum stability issue by numerically simulating the model at very small λ (e.g.: $\lambda = 10^{-4}, 10^{-6}$). This gives a lower limit for the Higgs-boson mass for fixed Yukawa couplings. At the same time, we run at $\lambda = \infty$ in order to get the upper boundary of the "allowed region" in the space of renormalized couplings (see fig. 3).

The meaning of the vacuum stability bound in the framework of the lattice regularized theory can be explained as follows. In order that the theory be well defined the bare coupling λ has to be positive. Using the Callan-Symanzik β-function, the renormalization group flow may be followed downwards from the cutoff scale to the physical scale, where the renormalized couplings are defined. Those values of the renormalized couplings that can be reached starting from any positive value of the bare coupling form the physically admissible region. Those outside would not correspond to any positive bare λ and cannot be realized for the given cutoff. In particular the boundary corresponding to $\lambda = 0$ yields the *vacuum stability bound.*

The exact effective potential and exact β-functions are, of course, not known. Therefore one has to rely on some approximations like perturbation theory or numerical simulations. Without knowing the qualitative behaviour of the β-functions it is impossible to derive the vacuum stability bound. In particular, the qualitative discussion is different in case of a trivial continuum limit, which is qualitatively represented by the 1-loop β-functions, or if a non-trivial fixed point at non-zero couplings exists, as suggested by the qualitative features of the 2-loop approximation.

Later on one can, of course, obtain information about the β-functions from numerical simulations by studying the cutoff dependence of the allowed values of renormalized Yukawa- and quartic couplings. The allowed region A in the space of renormalized couplings can be mapped out by studying the λ-dependence of $G_{R\psi}, G_{R\chi}, g_R$ for every bare Yukawa couplings within the broken phase near the Gaussian fixed point. The region A will, in general, depend on the cutoff, which can be defined, for instance, by the value of the Higgs-mass in lattice units. The first possibility corresponding to a trivial continuum limit is that for increasing cutoff A is shrinking to the origin $G_{R\psi} = G_{R\chi} = g_R = 0$. Another possibility is that, maybe after some shrinking for low cutoffs, the region A starts to expand and fills a 3-dimensional part of the $(G_{R\psi}, G_{R\chi}, g_R)$-space (or even the whole space) for infinite cutoff. In this case the continuum limit is non-trivial. A third possibility is that at infinite cutoff the region A becomes a lower dimensional subset, say, a surface. In this case the continuum theory is again non-trivial but the quartic coupling is a function of the Yukawa couplings. In other words, for given Yukawa couplings the lower and upper limit on the renormalized quartic coupling coincide.

In the numerical simulation we find that when the mirror fermion is very heavy, the renormalized scalar coupling g_R is within errors independent of λ (see fig. 3). This means that g_R is basically a function of (or strongly correlated with) $G_{R\chi}$ when $G_{R\chi}$ is strong. Therefore, at the given finite cutoff, when the mirror fermion is heavy, we can predict the scalar mass once we know the mirror fermion mass. This reminds us of the famous plot of the joint bounds on the Higgs and heavy fermion masses derived from 1-loop perturbation theory (see, for instance, [13]). The difference is that here we obtain the bounds nonperturbatively. We also notice that in our model, the value of g_R can be pushed to some very high value, about twice that of the triviality upper limit on g_R in the pure ϕ^4-models. (The triviality upper limits on g_R in 2- and 4-component ϕ^4 models are around 40 and 20, respectively.)

4. FINITE SIZE EFFECTS

The question to what extent the results of the numerical simulations are affected by changes of the lattice size $L^3 \cdot T$ is particularly interesting in the broken phase of our model. There, finite size effects are expected to be rather large due to the existence of a massless Goldstone-boson which accounts for the singular infrared behaviour of some Green's functions at zero external momentum. The finite lattice acts as a regulator of these infrared singularities, and one expects big finite size effects even on large lattices. In addition, the mass ratio of bosons to fermions cannot separately be tuned. This leads sometimes inevitably to large mass ratios and hence to large finite size effects. Our numerical simulations were performed on $4^3 \cdot 8$ and $6^3 \cdot 16$ lattices, and the question naturally arises if finite size effects are under control.

Finite volume effets can be studied by means of lattice perturbation theory. In a previous publication [7], the 1-loop expressions for the renormalized parameters are listed. Those relations can be inverted in order to express the bare parameters in terms of renormalized ones. Hence one can use nonperturbative input for the renormalized parameters used in the perturbative expressions for finite size effects. Imposing the renormalization conditions at infinite volume (i.e. $L = T = \infty$, $m_R = m_R^{(\infty)}$, $g_R = g_R^{(\infty)}, \ldots$), we define the finite size difference δX_R for any renormalized quantity X_R as

$$\delta X_R \equiv X_R(L,T) - X_R(\infty,\infty)\,. \tag{4.1}$$

Calculating δX_R essentially amounts to calculating the difference of the 1-loop integrals evaluated for different lattice sizes, viz.

$$\delta \int_p f(p) \equiv \frac{1}{L^3 \cdot T} \sum_p f(p) - \frac{1}{(2\pi)^4} \int d^4p\, f(p), \tag{4.2}$$

where $f(p)$ is some function of the lattice momenta. The 1-loop expressions in the broken phase are rather complicated and can be enormously simplified by considering the special case where $G_\chi = -G_\psi$. As an example we display the expression for the finite size effect on the renormalized fermion mass μ_R:

$$\begin{aligned}\delta\mu_R = -\mu_R \frac{G_R^2}{2}\, \delta \int_q & \frac{\bar{q}^2}{(\bar{q}^2 + \mu_q^2 + G_R^2 v_R^2)} \\ & \cdot \left[(\hat{q}^2 + \widetilde{m}_R^2)^{-2} + (\hat{q}^2)^{-2}\right] \\ + G_R^2\, \delta \int_q & \frac{\mu_q}{(\bar{q}^2 + \mu_q^2 + G_R^2 v_R^2)} \\ & \cdot \left[(\hat{q}^2 + \widetilde{m}_R^2)^{-1} + (\hat{q}^2)^{-1}\right],\end{aligned} \tag{4.3}$$

where $G_R = G_{R\psi} = -G_{R\chi}$, $\mu_q = \mu_R + \hat{q}^2/2$, $\widetilde{m}_R = 2\sinh(m_R/2)$, and the lattice momenta are defined as $\bar{q}_\mu = \sin(q_\mu)$, $\hat{q}_\mu = 2\sin(q_\mu/2)$. The perturbative expressions for δX_R are evaluated numerically with suitable computer programs. The results of a sample computation are shown in table 1. The values of the parameters at infinite volume are chosen such as to give an estimate for finite size effects for the point at which $G_\psi = -G_\chi = 0.1$ in the plot of the vacuum stability bound ($\lambda = 0$). It turns out that the effects for μ_R are rather small, such that $\mu_R = 0.22$ throughout. The 1-loop predictions for lattice size $4^3 \cdot 8$ are not reliable since $G_R^{(\infty)}$ is already quite large (note that $Z_\pi^{(\infty)} = 0.9727$) which precludes the extrapolation to very small lattices. Nevertheless one gets a hint on the possible size of the effects which amount to approx. 16% for m_R and about 30% for g_R with respect to the 6^4-lattice.

Table 1

$L^3 \cdot T$	m_R	g_R	G_R	v_R	Z_π
∞	0.532	6.20	0.45	0.370	0.973
$16^3 \cdot 32$	0.534	6.20	0.45	0.372	0.969
$8^3 \cdot 16$	0.555	6.07	0.43	0.379	0.898
$6^3 \cdot 16$	0.580	5.66	0.41	0.386	0.783
6^4	0.620	4.17	0.34	0.424	0.502

Another interesting question one can study in perturbation theory is whether the vacuum stability bound is affected largely by finite volume effects. The 1-loop formula for the relation between the renormalized scalar and Yukawa couplings for $\lambda = 0$ is [7]

$$g_R = \frac{96\,G_R^2}{v_R^2} \int_q \left(\bar{q}^2 + (\mu_R + \hat{q}^2/2)\right)^{-1}, \quad (4.4)$$

where again $G_R = G_{R\psi} = -G_{R\chi}$. Evaluating the integral for different lattice sizes shows the finite volume effects on the vacuum stability bound. Figure 4 shows curves of g_R versus G_R for infinite volume (solid line) and lattice sizes $6^3 \cdot 16$ (dashed line) and $4^3 \cdot 8$ (dashed-dotted line), respectively. It is seen that the finite size effect on the vacuum stability bound is rather small in the regime where 1-loop perturbation theory is applicable.

5. SUMMARY

To summarize:

(1) Fermion and mirror doublers can be made very heavy in both phases. In the FM phase the mirror fermion can aquire a very large mass by tuning G_χ.

(2) Renormalized Yukawa couplings have very weak λ-dependence. The renormalization of $G_{R\psi}$ investigated at $G_\chi = 0$ in the PM phase is much smaller than predicted by the continuous 1-loop β-function.

(3) In the PM phase, at $G_\chi = 0$ and $G_\psi = -G_\chi$, $G_{R\psi}$ is seen to rise linearly as a function of G_ψ and can become very strong at $G_\psi = 0.6, 1.0$. (Here by strong, we mean that $G_{R\psi}$ is about 2 to 3 times the value of its tree unitarity upper limit.)

(4) We study the vacuum stability issue nonperturbatively by running at $\lambda = 10^{-6}, 10^{-4}$ in the FM phase. We find that when the mirror fermion is heavy, g_R is basically a function of $G_{R\chi}$. This means when the mirror fermion is heavy, we can predict g_R once we know $G_{R\chi}$ and vice versa.

(5) One-loop perturbation theory predicts large finite size effects (of order $20-30\%$) for the renormalized parameters on the lattices under study, except for the fermion mass where the effect is substantially lower. On the other hand, the vacuum stability bound is far less sensitive to finite volume effects, a fact that is also seen in the Monte Carlo simulations.

In the near future, we plan to run on larger lattices (e.g.: $6^3 \cdot 16$, $8^3 \cdot 16$) in the FM phase to have a better knowledge of the finite size effect and to get a quantitative statement of the allowed region for the renormalized couplings. We would also like to study the detailed topology of the phase structure at intermediate G_ψ and G_χ and find out whether there is a nontrivial fixed point there.

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