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UNQUENCHED INVESTIGATION OF FERMION MASSES IN THE $SU(2)_L \otimes SU(2)_R$ FERMION-HIGGS MODEL

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The fermion masses were studied in the broken phase of the chiral $SU(2)_L \otimes SU(2)_R$ model with dynamical fermions. In our work we follow the proposal by Smit and Swift adding a manifestly chiral gauge invariant Wilson-Yukawa term with strength w to the action of this model in the hope of rendering the unwanted doubler fermions heavy dynamically. In the case of naive fermions ($w = 0$) a strong and weak coupling region were established in the broken phase. The scaling behaviour of the fermion masses and the lines of constant fermion mass were investigated close to the quadruple point A in the broken phase. We show that for large w doublers can be easily decoupled in the strong coupling region of the broken phase while the physical fermion mass and the electroweak scale can be made arbitrarily small.

1. INTRODUCTION

Two complexes of questions will be addressed in this contribution.

a)The nonperturbative investigation of the Yukawa-models has recently stimulated a lot of interest also because of the possible existence of a nontrivial fixed point which might be suitable for the construction of an interacting continuum limit of the standard model [1, 2]. In a recent work we have determined the phase diagram of the $SU(2)_L \otimes SU(2)_R$ fermion-Higgs model and localized the regions of interest [3]. For naive fermions various phase transitions meet in 2 quadruple points A and B which might be possible candidates for a nontrivial fixed point. In this contribution the behaviour of fermion masses is investigated at $w = 0$ in the broken phase close to the various phase transitions [4]. A fine scan on a larger lattice is performed in the vicinity of the point A in the broken phase [5]. This work was done in a collaboration with A. K. De, C. Frick, K. Jansen and T. Trappenberg.

b) In the last years only a few proposals were developed which address the central question of the formulation of a chiral gauge theory on the lattice. In this work we follow the proposal by Smit and Swift and add a manifestly chiral gauge invariant Wilson-Yukawa term to the action of the chiral $SU(2)_L \otimes SU(2)_R$ model in the hope of rendering the doubler fermions heavy dynamically [7, 9, 8]. The removal of the doublers truns out to be intrinsically a nonperturbative phenomenon. In the quenched approximation of this model we could show that the doublers can be decoupled easily in the strong coupling region of the broken phase while approaching the point $\kappa = \kappa_c$, $y = 0$ the physical fermion mass m_F and the electroweak scale v can be made arbitrarily small [10]. In this contribution we confirm these results for dynamical fermions [4]. This work was done in a collaboration with A. K. De.

2. THE MODEL AND ITS PHASE DIAGRAM

The model on the euclidean lattice is defined by the action

$$\begin{aligned} S &= -\kappa \sum_{x\mu} \frac{1}{2} Tr(\Phi_x^\dagger \Phi_{x+\mu} + \Phi_{x+\mu}^\dagger \Phi_x) \\ &+ \frac{1}{2} \sum_{x\mu} \overline{\Psi}_x \gamma_\mu (\Psi_{x+\mu} - \Psi_{x-\mu}) \\ &+ y \sum_x \overline{\Psi}_x (\Phi_x P_R + \Phi_x^\dagger P_L) \Psi_x \\ &+ w \sum_{x\mu} \overline{\Psi}_x (\Phi_x P_R + \Phi_x^\dagger P_L) \Psi_x \\ &- w \sum_{x\mu} \{ \frac{1}{2} \overline{\Psi}_x (\Phi_x P_R + \Phi_{x+\mu}^\dagger P_L) \Psi_{x+\mu} \\ &+ \frac{1}{2} \overline{\Psi}_{x+\mu} (\Phi_{x+\mu} P_R + \Phi_x^\dagger P_L) \Psi_x \}. \qquad (2.1) \end{aligned}$$

Here the radially frozen scalar field Φ_x is a SU(2) matrix, the fermion fields Ψ_x and $\overline{\Psi}_x$ are SU(2)

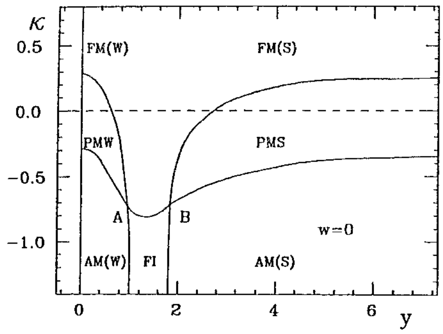

Figure 1: Unquenched phase diagram at $w = 0$.

doublets, κ is the hopping parameter for the scalar field, y is the normal Yukawa coupling, w is the Wilson-Yukawa coupling and $P_{L,R}$ are the left and right handed projectors. The action is invariant under the global $SU(2)_L \otimes SU(2)_R$ transformations $\Psi \to (\Omega_L P_L + \Omega_R P_R)\Psi$, $\overline{\Psi} \to \overline{\Psi}(\Omega_L^\dagger P_R + \Omega_R^\dagger P_L)$, $\Phi \to \Omega_L \Phi \Omega_R^\dagger$ where $\Omega_{L,R} \in SU(2)_{L,R}$. For $y = 0$ the model has an additional symmetry [11] that provides in the context of this model the prediction of the position of the critical value of y at which the fermionic correlation length diverges: the fermion mass m_F vanishes on the whole $y = 0$ sheet (Golterman–Petcher (GP) theorem). The phase diagram at $w = 0$ is shown in fig. 1. The properties of the different phases can be characterized by the order parameters $v\mathbb{1} = \langle \Phi \rangle$ and $v_{st} = \langle \varepsilon_x \Phi_x \rangle$ where $\varepsilon_x = (-1)^{(x_1+x_2+x_3+x_4)}$. We find two symmetric or paramagnetic phases ($v = v_{st} = 0$): PMW and PMS with massless and massive fermions respectively and the following broken symmetry phases: ferromagnetic FM ($v \neq 0$, $v_{st} = 0$), antiferromagnetic AM ($v = 0$, $v_{st} \neq 0$) and ferrimagnetic FI ($v \neq 0$, $v_{st} \neq 0$). For $w = 0$ the various phase transition lines meet, within our precision of their localization, in 2 quadruple points (A and B in fig. 1). At a fixed $w > 0$ the phase diagram can be obtained from the phase diagram at $w = 0$ (fig. 1) by shifting the origin by an amount of $4w$ in the positive y direction. In the 3-dimensional phase diagram with $w > 0$ the quadruple points A and B become lines.

3. FERMION MASSES IN THE REGIONS $y + 4w \ll 1$ AND $y + 4w \gg 1$

$y + 4w \ll 1$: In this region of the 3-dimensional parameter space a perturbative calculation can be applied which in the tree level order leads to

$$m_F \approx vy \quad \text{and} \quad m_D \approx (y + 2nw)v \,, \qquad (3.1)$$

where m_F and m_D denote the mass of the physical and the doubler fermion respectively and n is the number of momentum components equal to π. As the FM(W)–PMW phase transition is approached within the FM phase ($v \searrow 0$) the masses of the doubler fermions stay of the order of the electroweak scale v and cannot be decoupled completely from the physical particle spectrum.

$y + 4w \gg 1$: In this region a strong coupling expansion of the fermion propagator is appropriate [8, 12]. For this purpose it is useful to express the action in eq. (2.1) in terms of the fields $\Psi'_x = (\Phi_x^\dagger P_L + P_R)\Psi_x$ and $\overline{\Psi}'_x = \overline{\Psi}_x(\Phi_x P_R + P_L)$ which are singlets with respect to the SU(2)$_L$ group and doublets with respect to the SU(2)$_R$ group. Using the Ψ'_x fields the Yukawa term becomes a bare mass term and the Wilson–Yukawa term a standard Wilson term. The strong coupling expansion of the propagator $\langle \Psi' \overline{\Psi}' \rangle$ leads to the following expressions for the physical fermion and the doubler fermion masses

$$m_F \approx yz^{-1} \text{ and } m_D \approx (y + 2nw)z^{-1} \,, \qquad (3.2)$$

where $z^2 = \frac{1}{2}\langle \mathrm{Tr}\ \Phi_x^\dagger \Phi_{x+\mu} \rangle$ is the link expectation value [12]. As the FM(S)-PMS transition is approached, $v \searrow 0$ and z also decreases but stays nonzero at the transition. As a result m_F and m_D increase as the FM(S)–PMS phase transition is approached within the FM(S) phase. Both masses stay nonzero at this phase transition and continue to increase in the PMS phase [13]. In agreement with the GP theorem $m_F \to 0$ as $y \searrow 0$. The above formula for m_D already indicates that the doublers might be decoupled in the limit $y \searrow 0$ at $w \gg 1$.

4. SOME DETAILS OF THE SIMULATION

The scalar field configurations were updated using a Hybrid Monte Carlo algorithm. Since the fermion matrix of our model is not hermitian it is necessary to introduce implicitly two replicas of the fermion fields

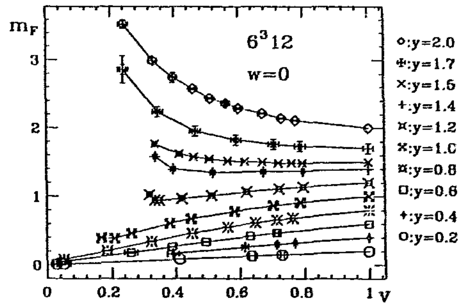

Figure 2: Fermion masses at $w = 0$.

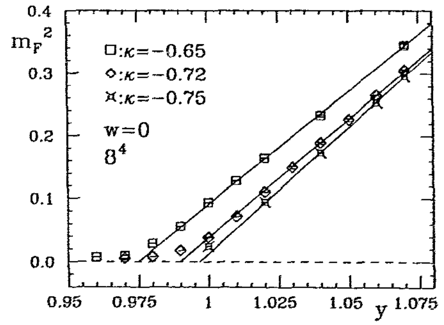

Figure 3: m_F^2 as a function of y for $\kappa = -0.65, -0.72, -0.75$ and $w = 0$.

by squaring the determinant. In all cases we have used lattices with periodic boundary conditions except for antiperiodic ones for the fermion fields in the time direction. We determined the fermion masses from the $\mathrm{Tr}_{SU(2)}\langle \Psi_L \overline{\Psi}_L \rangle$ and $\mathrm{Tr}_{SU(2)}\langle \Psi_R \overline{\Psi}_R \rangle = \mathrm{Tr}_{SU(2)}\langle \Psi'_R \overline{\Psi}'_R \rangle$ propagators. Both propagators should reproduce the same fermion spectrum in the FM phase since the $SU(2)_L \otimes SU(2)_R$ symmetry is broken to its diagonal subgroup $SU(2)_{L=R}$ in this phase. The mass data obtained from these propagator components indeed agree in the FM phase within their error bars. The the numerical determination of the fermion masses is described in more detail in ref. 4.

5. NUMERICAL CALCULATION OF m_F AT $w = 0$

Scan of the fermion mass in the FM phase at $w = 0$: Fig. 2 shows the fermion mass as a function of v for several fixed values of y in the interval $y = 0.2 - 2.0$. The data were obtained on a $6^3 12$ lattice. The runs in the interval $y = 0.2 - 0.8$ hit the FM(W)–PMW phase transition. In agreement with eq. (3.1) the data for m_F fall approximately on straight lines with slope y. Deviations from this behaviour become stronger as y is raised in this interval. At $y = 2.0$ the FM(S)–PMS phase transition is encountered as v decreases and the z^{-1} behaviour is approximately fulfilled for large values of z^{-1}. For $y = 1.0 - 1.7$ the FM–FI phase transition is approached. As pointed out in chap. 2, v does not vanish at this phase transition. The mass data for $y = 1.0, 1.2$ and 1.4 show an interesting behaviour. At moderate values of v the fermion mass first decreases with decreasing v but then very close to the FM–FI phase transition starts increasing again as v is further decreased. This result indicates that the lines of constant fermion mass might flow into the quadruple point A. Providing this scenario is true the point A would have to be regarded as a very probable candidate for a nontrivial fixed point. If this scenario remains valid also for a nonzero value of w it would open up the possibility to decouple doublers within the FM(W) region by tuning toward the point A from the weak coupling side.

Fine scan of the fermion mass in the vicinity of the point A: A nontrivial fixed point could manifest itself also in a deviation of the fermion mass and magnetization data from the Gaussian scaling laws $m_F \propto \{(y - y_c)/y_c\}^{1/2}$ and $v \propto \{(y - y_c)/y_c\}^{1/2}$ in the near vicinity of the point A. In fig. 3 m_F^2 is plotted as a function of y for 3 different values of κ very close to the point A in the FM phase. The data were obtained on an 8^4 lattice. The figure shows that the data fall on straight lines. A similar behaviour was also found for v^2. Thus no significant deviation from the Gaussian scaling law is observed even in the near vicinity of the point A. Even very close to the quadruple point A the FM(W)–PMW phase transition seems to be influenced by the Gaussian fixed point at $y = 0$ and $\kappa = \kappa_c$. By a simple interpola-

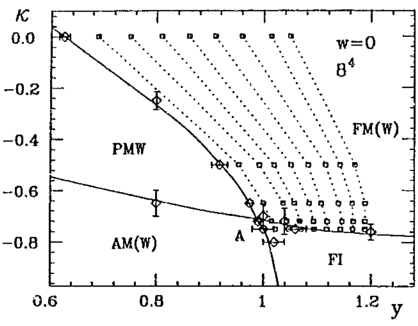

Figure 4: Lines of constant m_F^2 at $w = 0$. m_F^2 changes from 0.1 to 0.8 in steps of 0.1.

tion we have translated all our data for m_F^2 obtained in the FM(W) region into a plot of the lines of constant m_F^2 in the (κ, y) coupling parameter space as shown in fig 4. Fig. 4 shows that the lines of constant m_F^2 really bend a little bit to the point A, but it can be clearly excluded from this figure that the lines of constant m_F^2 flow into the point A.

6. FERMION MASSES AT $w = 0.5$

At $w = 0.5$ the $y = 0$ axis will be shifted to $y = 4w = 2.0$ in fig. 1 intersecting the AM(S), PMS and FM(S) phases. For this value of w and several values of y the masses of the physical fermion m_F and the lowest lying doubler fermion m_D are displayed in dependence on v in fig. 5a. In qualitative agreement with eq. (3.2) both masses increase as the PMS–FM(S) phase transition is approached ($v \searrow 0$). In fig. 5b we have translated the data in fig. 5a by a simple interpolation into a plot of m_F and m_D as a function of y for several fixed values of v. In agreement with the GP theorem the data for m_F nicely extrapolate to zero for all values of v, whereas the data for m_D stay substantially above the cut off. In the continuum limit $y \searrow 0$, $v \searrow 0$ the doublers get decoupled while the mass of the physical termion m_F and the mass of the W–Boson $m_W \propto v$ (if gauge fields are considered perturbatively) can be made arbitrarily small.

7. CONCLUSION

$w = 0$: A continuum limit with vanishing v and m_F can only constructed at the FM(W)–PMW phase transition or at point A. In the continuum limit at the FM(S)–PMS transition fermions are decoupled from the spectrum. The FM–FI phase transition is ruled out for a continuum limit since both v and m_F don't go to zero there. The theory obtained at the FM(W)–PMW phase transition cannot fall into the same universality class as the standard model since doubler fermions are present in particle spectrum. Analyzing fermion masses in the vicinity of point A we could not find an indication for a nontrivial fixed point: a) the lines of $m_F^2 = const.$ don't flow into point A, b) no deviation from the Gaussian scaling behaviour was observed close to the point A.

$w = 0.5$: In the FM(S) region the fermion doublers easily can be given O(1) masses with scaling values for the fermion mass as $y \searrow 0$ and $v \searrow 0$ at the FM(S)–PMS phase transition. This result is very promising for the construction of a chiral gauge theory on the lattice. The physical properties of this continuum limit are as yet not fully understood.

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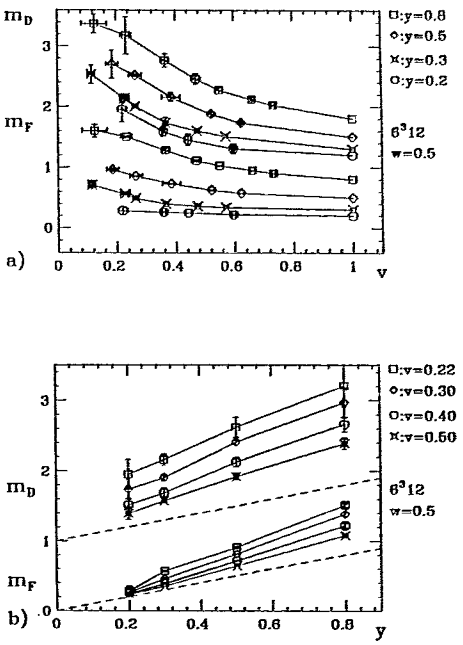

Figure 5: a) The masses m_F and m_D as a function of v for various values of y at $w = 0.5$. b) The masses interpolated from fig. 5a in depencence of y for four fixed values of v. The dashed lines correspond to $\kappa = +\infty$.

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