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ADVANCES IN LATTICE QED

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In this talk I shall review recent lattice work on the ultra-violet behavior of QED. Of particular interest is the question whether QED has a non-trivial continuum limit or not.

1. INTRODUCTION

There has recently been considerable interest in the continuum limit of QED. This began after Miransky [1] investigated a truncated Schwinger - Dyson equation for the fermion propagator and found a second order chiral phase transition, with chiral symmetry being broken spontaneously at strong coupling. Though his equation did not include any vacuum polarization effects, he argued that the critical coupling should be regarded as an ultra-violet stable fixed point, at which the theory admits a non-trivial continuum limit. The existence of a second order chiral phase transition was confirmed by numerical studies of non-compact lattice QED in the quenched approximation and with a small number of dynamical fermions. [2,3,4] These early lattice investigations did also find support for non-trivial critical behavior. The picture turned when Göckeler et al. [5] carried out a more careful analysis and found critical exponents, which are consistent with mean field theory. A similar result was reported by Booth et al. [6] and Horowitz. [7] Recent studies of a coupled set of Schwinger - Dyson equations which include certain effects of fermion loops [8,9] now also find mean field critical exponents. This suggests, contrary to earlier claims, that QED is trivial in the continuum limit.

This is, of course, very indirect information. In fact, there are two issues, which are often confused in the literature. The question whether QED is trivial or not is a question of the evolution of the renormalized charge as one approaches the critical point. This evolution is described by the Callan - Symanzik β-function, and the real task is to find out whether this function has an ultra-violet stable zero or not. The other issue concerns the scaling behavior and the effective action of the continuum theory. In the last year we have made substantial progress on both fronts. It is the purpose of this talk to review the latest results.

The talk is organized as follows. Section 2 deals with some technical aspects of the lattice calculation. At the same time I will introduce the notation. In sec. 3 I will discuss the phase diagram and determine the critical point. Section 4 is devoted to the renormalization group flow. I will present recent results for the renormalized charge and fermion mass. These results are used to derive the Callan - Symanzik β-function. It is found that the β-function does not have an ultra-violet stable zero, but that the renormalized charge vanishes at the critical point. This raises the question to what extent QED can be regarded as a consistent low-energy theory. In sec. 5 I will discuss the scaling behavior at the critical point in some more detail. Though the photon decouples at the critical point, it is possible that the continuum theory is interacting. This would manifest itself in critical exponents, which are different from the predictions of mean field theory. Section 6 contains a few remarks about the four-fermi interaction. This becomes a relevant operator at high momenta. That this is the case has already been argued for some time. [10] But the scaling dimension turns out to be different from what it was thought to be. In sec. 7 I will summarize the status of the quenched calculations. Finally, in sec. 8 I will present my conclusions.

2. PRELIMINARIES

The non-compact formulation of lattice QED shares all the essential features of the continuum theory. [11] The compact formulation, on the other hand, belongs to a different universality class. It has a first order chiral phase transition, [12,13] and therefore admits no continuum limit. The non-compact gauge field action reads

$$\begin{aligned} S_G &= \frac{\beta}{2} \sum_{x,\mu<\nu} (A_\mu(x) + A_\nu(x+\mu) \\ &- A_\mu(x+\nu) - A_\nu(x))^2, \end{aligned} \tag{2.1}$$

where $\beta = 1/e^2$, and e is the bare charge. In eq. (2.1) and in the following the lattice constant has been set equal to one for convenience.

More problematic is the fermionic action. The numerical work I will review is based on staggered fermions. This action reads

$$\begin{aligned} S_F &= \sum_x \{ \frac{1}{2} \sum_\mu (-1)^{x_1+\ldots+x_{\mu-1}} [\bar{\chi}_x e^{iA_\mu(x)} \chi_{x+\hat{\mu}} \\ &- \bar{\chi}_{x+\hat{\mu}} e^{-iA_\mu(x)} \chi_x] + m \bar{\chi}_x \chi_x \}, \end{aligned} \tag{2.2}$$

where m is the bare mass. It describes four Dirac fermions (flavors). The essential feature of S_F is that it has a chiral $U(1) \times U(1)$ symmetry at $m = 0$. Because photons with high momenta can change a staggered fermion from one flavor to another, the chiral $SU(4) \times SU(4)$ symmetry is only approximate. Such interactions will get less as one approaches the continuum limit, leading eventually to an exact flavor symmetry. This might, however, not happen to the pion-type bound states associated with the chiral phase transition if they are pointlike. Wilson fermions, on the other hand, have no continuous chiral symmetry. It may therefore be that the Wilson action falls into a different universality class than the continuum action.

In the following I shall refer to ordinary staggered fermions as $N_f = 4$. Several authors mimic two flavors by taking the root of the fermion determinant. This I will refer to as $N_f = 2$.

Both fermionic actions couple the Grassmann fields to the gauge fields in compact form via the links $e^{iA_\mu(x)}$, rather than being linear in $A_\mu(x)$. This might also drive the theory into a different universality class, as perhaps the study of QED with a large number of flavors shows. [14] For a small number of flavors there seems, however, no problem.

In order to be as close as possible to the continuum theory, Hands et al. [15] have constructed an action within the framework of staggered fermions, which is linear in the transverse components of the gauge fields. The drawback of this construction is, however, that the action does not satisfy reflection positivity. [16] Whether this leads to an ill-defined transfer matrix or not has still to be seen.

I have nothing to add to the calculation of the renormalized charge e_R and the renormalized fermion mass m_R. [17] The reader, who is interested in details, is refered to the talks of Göckeler [18] and Horsley [19] on these subjects.

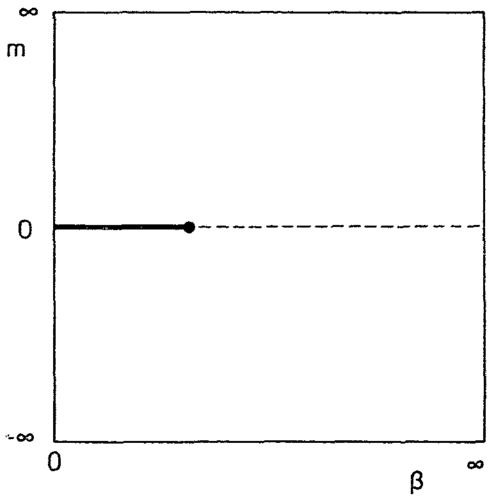

Figure 1: The phase diagram. The dashed line indicates the second order critical line, while the solid line indicates the first order line. The solid circle marks the tricritical point.

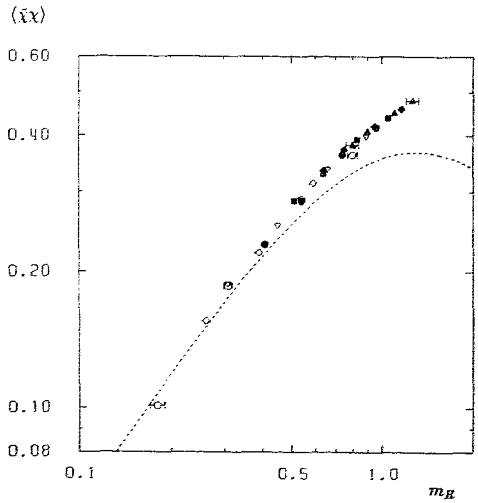

Figure 2: The chiral condensate against the renormalized fermion mass. The symbols refer to the different values of β: β = 0.16 (▲), 0.17 (◆), 0.18 (■), 0.19 (●), 0.20 (▽), 0.21 (◇), 0.22 (○). The open symbols are for β values above β_c, while the solid symbols are for β values below β_c.

3. PHASE DIAGRAM

The phase diagram of non-compact QED has been investigated many times. Based on the results for the order parameter $\langle\bar{\chi}\chi\rangle$ alone, the conclusion was that the theory undergoes a second order chiral phase transition at strong coupling. Recently, it could be shown rigorously [20] that chiral symmetry is broken spontaneously in the strong coupling limit.

If one considers the correlation length $\xi = 1/m_R$ as well, one arrives at the phase diagram [21] shown in fig. 1. It consists of a second order critical line at $m = 0$ extending from $\beta = \infty$ to $\beta = \beta_c$ and a first order line extending from $\beta = \beta_c$ to $\beta = 0$, where $\beta_c = 1/e_c^2$, and e_c is the critical charge. The second order line corresponds to the line $\langle\bar{\chi}\chi\rangle = 0$, whereas on the first order line $\lim_{m\to 0_\pm}\langle\bar{\chi}\chi\rangle > 0$. The critical point is actually a tricritical point.

Let me explain this now. The renormalized fermion mass was computed by Göckeler et al. [17] This is compared with the chiral condensate $\langle\bar{\chi}\chi\rangle$ in fig. 2. The open symbols refer to $\beta > \beta_c$, whereas the solid symbols refer to $\beta < \beta_c$. We will determine β_c in the next paragraph. The dashed curve describes the one-loop relationship [22]

$$\langle\bar{\chi}\chi\rangle = 0.62 m_R - O(m_R^3 \ln m_R^2). \qquad (3.1)$$

Later we shall see that this result becomes exact at the critical point. We find good agreement between this curve and the data for $m_R \lesssim 0.5$. Thus ξ diverges on the line $\langle\bar{\chi}\chi\rangle = 0$. Since

$$\lim_{m\to 0_+}\langle\bar{\chi}\chi\rangle = -\lim_{m\to 0_-}\langle\bar{\chi}\chi\rangle. \qquad (3.2)$$

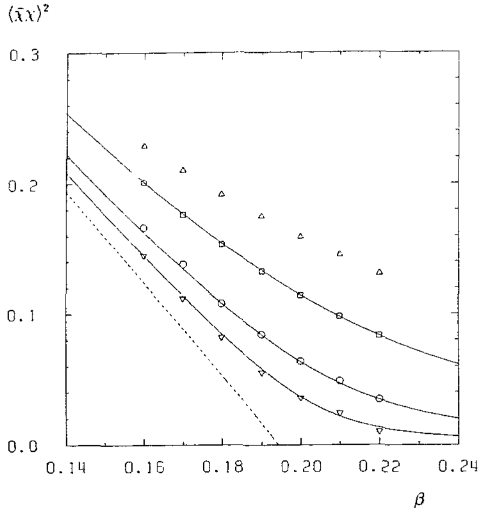

Figure 3: The chiral condensate against β according to Göckeler et al. for $N_f = 4$. The data are compared with a fit of the equation of state. The symbols refer to the different masses: $m = 0.02$ ($\bigtriangledown$), $m = 0.04$ ($\bigcirc$), $m = 0.09$ ($\square$) and $m = 0.16$ ($\triangle$). The fit did not include the data values at $m = 0.16$. The error bars are smaller than the symbols. The dashed curve is the extrapolation to $m = 0$.

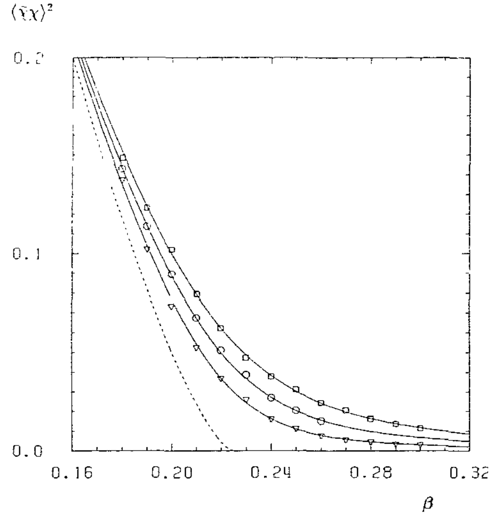

Figure 4: The chiral condensate against β according to Dagotto et al. for $N_f = 2$. The data are compared with a fit of the equation of state. The symbols refer to the different masses: $m = 0.02$ ($\bigtriangledown$), $m = 0.03$ ($\bigcirc$) and $m = 0.04$ ($\square$). The error bars are smaller than the symbols. The dashed curve is the extrapolation to $m = 0$.

the line $\lim_{m\to 0_+}\langle\bar{\chi}\chi\rangle > 0$ is a first order critical line.

A tricritical point is connected with power-like scaling laws. Thus, the critical behavior can be described by the equation of state [23]

$$\begin{aligned} \frac{\partial V_{eff}(\sigma)}{\partial\sigma} &= 0, \\ V_{eff}(\sigma) &= -m\sigma + \kappa\sigma^{\delta-\frac{1}{\dot\beta}+1} + \zeta\sigma^{\delta+1}, \end{aligned} \tag{3.3}$$

where $\sigma = \langle\bar{\chi}\chi\rangle$, and κ, ζ are analytic functions of β: $\kappa = \kappa_1(\beta-\beta_c) + \kappa_2(\beta-\beta_c)^2 + \cdots$, $\zeta = \zeta_0 + \zeta_1(\beta-\beta_c) + \zeta_2(\beta-\beta_c)^2 + \cdots$. In order to avoid confusion of β and the critical exponent named by the same letter I have called the latter $\dot\beta$. For $\delta = 3$ and $\dot\beta = 0.5$, V_{eff} is the effective potential of the σ-model. Equation (3.3) gives

$$(\delta - \frac{1}{\dot\beta} + 1)\kappa\sigma^{\delta-\frac{1}{\dot\beta}} + (\delta - 1)\zeta\sigma^{\delta} - m = 0. \tag{3.4}$$

At $m = 0$

$$\langle\bar{\chi}\chi\rangle \propto (\beta_c - \beta)^{\dot\beta},\ \beta_c > \beta, \tag{3.5}$$

and at $\beta = \beta_c$

$$\langle\bar{\chi}\chi\rangle \propto m^{\frac{1}{\delta}}. \tag{3.6}$$

In order to determine β_c and the critical exponents I have fitted eq. (3.4) to the data. The result of the fit is shown for the $N_f = 4$ data of Göckeler et al. [17] and for the $N_f = 2$ data of Dagotto et al. [24] in figs. 3 and 4, respectively. The dashed curves describe the extrapolation of $\langle\bar{\chi}\chi\rangle$ to $m = 0$. The quality of the fit is certainly very good. The values

Ref.	$N_f = 4$	$N_f = 2$	$N_f = 1$
17	0.194(2)		
24		0.227(3)	
25	0.180(6)	0.143(7)	0.121(5)

Table 1: The values of $\beta_c = 1/\epsilon_c^2$ as obtained from a fit of the equation of state to the data of Göckeler et al. and Dagotto et al., respectively, and analytically by Cornelius.

of β_c are listed in tab. 1. Horowitz [7] finds β_c = 0.187 for the $N_f = 4$ data of Booth et al. [6] and β_c = 0.210 for the $N_f = 2$ data of Kogut et al., [4,24] which is in rough agreement with my result. I will postpone the discussion of the critical exponents to sec. 5.

Very recently, Cornelius [25] studied the phase diagram analytically for Wilson fermions. She computed the renormalized charge and fermion mass and the renormalization constants by an expansion in the hopping parameter $\kappa = 1/(2m + 8)$ up to tenth order. The renormalization scheme she uses is the BPHZ scheme. The line $m_R = 0$ is given by the line $\kappa = \kappa_c$, where κ_c is the radius of convergence of the expansion. In order to determine κ_c, she proceeds in the same way as Lüscher and Weisz. [26] The line is found to extend from $\epsilon = 0$ to $\epsilon = \epsilon_c$, where ϵ_c is the value of the bare charge, beyond which the method is not applicable anymore. A further investigation suggests that m_R is no longer zero for $\epsilon > \epsilon_c$, so that ϵ_c may be taken as the critical charge. The second order critical line is shown in fig. 5. The critical couplings are listed in tab. 1 for $N_f = 1$, 2 and 4. It is striking how close they are to the numbers for staggered fermions.

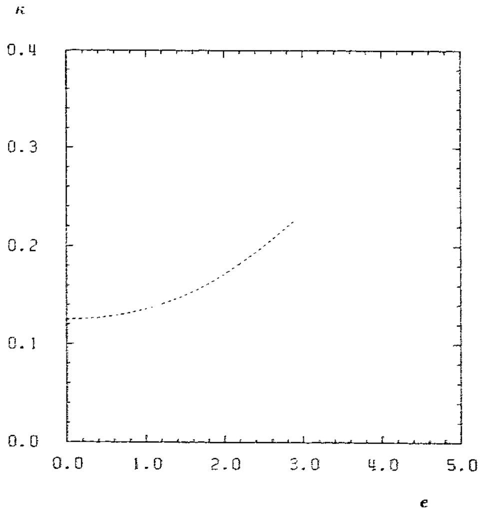

Figure 5: The second order critical line $\kappa = \kappa_c$ found by Cornelius for $N_f = 1$ Wilson fermions.

4. RENORMALIZATION GROUP FLOW AND THE QUESTION OF TRIVIALITY

We can take the continuum limit all along the second order critical line from $\beta = \infty$ to $\beta = \beta_c$. But the theory can only have a non-trivial continuum limit at the tricritical point $\beta = \beta_c$, $m = 0$.

The cut-off dependence of the renormalized charge is described by the renormalization group equation

$$m_R \frac{\partial \epsilon_R^2}{\partial m_R}\bigg|_{\epsilon \text{ fixed}} = \beta(\epsilon_R^2, m_R), \qquad (4.1)$$

where $1/m_R$ acts as the cut-off, and $\beta(\epsilon_R^2, m_R)$ is the Callan - Symanzik β-function. In order that the critical point is a non-trivial fixed point, the β-function must have a zero at $\epsilon_R^2 = \epsilon_R^{*2}$, $m_R = 0$, where $\epsilon_R^* \leq \epsilon_c$. The latter follows from the fact that [27] $Z_3 \leq 1$. One can also define a bare β-

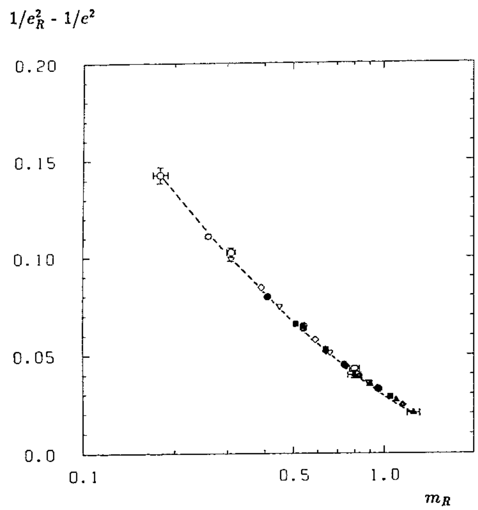

Figure 6: The relationship between e_R, e and m_R. The symbols are the same as in fig. 2. The dashed line is the prediction of the one-loop lattice β-function shifted to fit the data point at the smallest value of m_R, which corresponds to $\beta = 0.22$, $m = 0.2$.

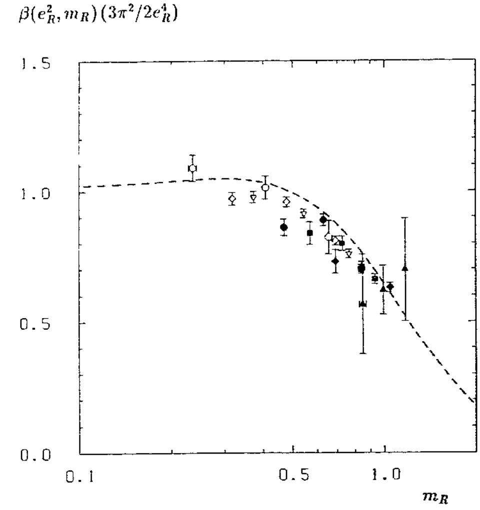

Figure 7: The Callan - Symanzik β-function times $3\pi^2/2e_R^4$. The symbols are the same as in fig. 2. This is compared with the one-loop lattice β-function indicated by the dashed line.

function by

$$m_R \frac{\partial e_R^2}{\partial m_R}\bigg|_{e_R \text{ fixed}} = \beta_0(e^2, m_R), \qquad (4.2)$$

which indicates how the bare charge must run in order to keep the low-energy physics constant. This β-function would have a zero at $e^2 = e_c^2$, $m_R = 0$. In view of the result that the critical charges lie inside the "apparent" radius of convergence of renormalized perturbation theory, it has been argued [27] that it appears rather doubtful that the Callan - Symanzik β-function has an ultra-violet stable zero. If, indeed, QED is trivial, we expect the Callan - Symanzik β-function to be described by renormalized perturbation theory. For staggered fermions ($N_f = 4$) this leads to

$$\beta(e_R^2, 0) = \frac{2e_R^4}{3\pi^2} + O(e_R^6). \qquad (4.3)$$

Since the Callan - Symanzik β-function and the bare β-function are equal in perturbation theory up to two loops, we expect the corresponding result to hold for $\beta_0(e^2, m_R)$.

The data of Göckeler et al. [17] for the renormalized charge and the renormalized fermion mass as well as their analysis is presented in figs. 6 - 8. In fig. 6 I have plotted $1/e_R^2 - 1/e^2$ against m_R. For each symbol the data point with the smallest value of m_R corresponds to $m = 0.02$. The striking result is that the data lie on an approximately universal curve. The slope of the curve is $-\beta(e_R^2, m_R)/e_R^4$ and $-\beta_0(e^2, m_R)/e^4$, respectively. This means that

$$\beta(e_R^2, m_R)/e_R^4 \approx \beta_0(e^2, m_R)/e^4, \qquad (4.4)$$

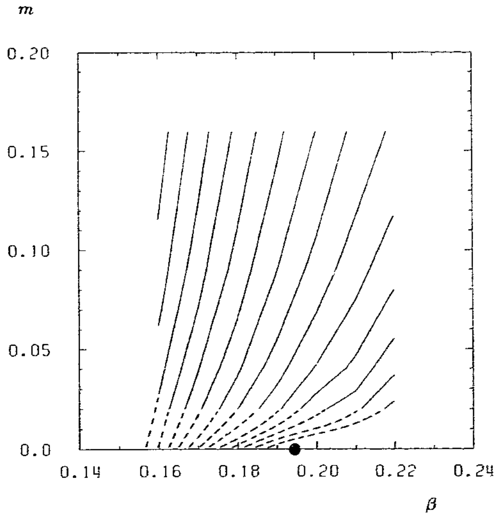

Figure 8: The renormalization group flow in the critical region. The solid lines are lines of constant renormalized charge, where ϵ_R^2 ranges from $\epsilon_R^2 =$ 2.8 (lower right-hand corner) to $\epsilon_R^2 = 5.4$ (upper left-hand corner) in steps of 0.2. The dashed lines arise from integrating the renormalization group equation down to $m = 0$. The solid circle indicates the position of the critical point.

in agreement with lowest order perturbation theory. The Callan - Symanzik β-function is found by differentiating the data. It is shown in fig. 7. The dashed curve in this figure represents the perturbative one-loop lattice result. There is good agreement between the data and this curve. [28] Let us go back to fig. 6 now. The dashed curve in that figure represents the integrated one-loop β-function,

$$-\int^{\ln m_R} d\ln \bar{m}_R\, \beta(\epsilon_R^2, \bar{m}_R)/\epsilon_R^4, \qquad (4.5)$$

normalized such as to match the data point at the lowest value of m_R. The agreement between this data and renormalized perturbation theory is even more impressive. Finally, in fig. 8 are shown the renormalization group trajectories defined by keeping the renormalized charge ϵ_R constant. They are obtained by a suitable interpolation of the data. [17] In the symmetric phase, $\beta \geq \beta_c$, m_R vanishes as $m \to 0$, and so one obtains for a positive β-function $e_R = 0$ in this limit. In particular, $e_R = 0$ at the critical point. In the broken phase, $\beta < \beta_c$, m_R stays finite as $m \to 0$, and so e_R is finite in this limit. Therefore all trajectories will end at $m = 0$ on the first order line. We may use perturbation theory to integrate the renormalization group equations down to $m = 0$. The result is indicated by the dashed lines. The trajectory $e_R = 0$ coincides with the second order line $\beta \geq \beta_c$, $m = 0$.

Horowitz [29] has extracted the renormalized charge and the renormalized fermion mass from the data of Booth et al. [6] for $\langle\bar{\chi}\chi\rangle$ and the average action density $\langle F_{\mu\nu}^2\rangle$. He made the ansatz

$$m_R = 8G\langle\bar{\chi}\chi\rangle + m, \qquad (4.6)$$

based on the gap equation, [7] and fitted $\langle F_{\mu\nu}^2\rangle$ by the one-loop renormalized photon propagator. It turns out that $(8G)^{-1} = 0.62 + O(\beta_c - \beta)$. This is in broad agreement with the one-loop result (3.1), regarding the fact that the bare mass m is relatively small. Qualitatively, the renormalization group trajectories one obtains from his fits look the same as those in fig. 8.

Another group [30] has computed the renormalization group flow analytically for large m and in weak coupling perturbation theory. Unfortunately, the results do not (yet) extend down to the interesting region. But they are consistent with the picture that has emerged.

Finally, I like to mention that Rakow [31] has investigated a set of truncated Schwinger - Dyson equations, which include effects of vacuum polarization. The range of correlation lengths accessible in this approach is far larger than what is possible on the lattice. He finds also that the photon decouples at the critical point.

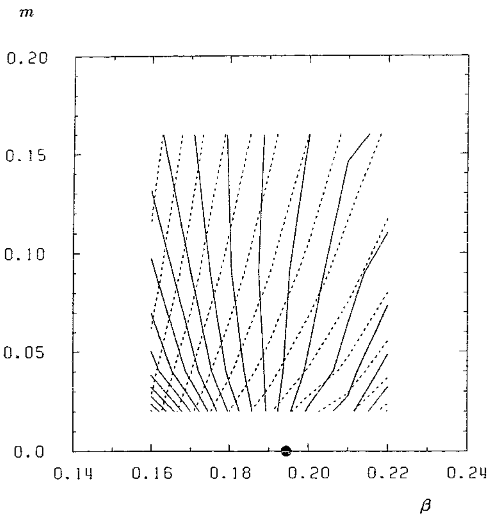

Figure 9: The renormalization group flow in the critical region. The solid lines are lines of constant m_R/m_{PS} ranging from 0.4 (lower right-hand corner) to 2.1 (lower left-hand corner) in steps of 0.1. The dashed lines are the lines of constant charge from fig. 8. The solid circle indicates the position of the critical point.

Though the photon decouples in the continuum limit, QED can be a valid description of charged particles and their interactions up to some finite momentum scale. It is interesting to know what that scale is, because it indicates the onset of new physics. An upper bound on that scale is given by the maximal value of the cut-off. This turns out to be [17]

$$1/m_R \leq (0.038 \pm 0.004)e^{3\pi^2/2e_R^2}, \qquad (4.7)$$

for $N_f = 4$. Note that this value is more than a magnitude smaller than the position of the Landau pole. The true scale is, however, where the low-energy physics starts to depend on the cut-off. Göckeler et al. [17] have compared the renormalization group flow of different dimensionless quantities. Besides e_R they have considered the mass ratio m_R/m_{PS}, where m_{PS} is the pseudoscalar Goldstone boson mass. The trajectories are compared in fig. 9. The two flows are obviously completely different in the parameter range studied. The mass ratio trajectories flow into the critical point in contrast to the e_R trajectories. The inconsistency is most striking for $\beta < \beta_c$, where the mass ratio trajectories move in the direction of larger β, while the e_R trajectories move in the direction of lower β. The correlation length does not have to be very large before the difference between the flows becomes apparent. For example, for $e_R^2 = 3.6$ ($\alpha_R = 0.29$) the difference becomes marked when $m_R \lesssim 0.5$. This corresponds to a cut-off, which is only two times as large as the fermion mass. Thus, there are no lines of constant physics, except possibly for very small values of e_R^2, which we did not explore. This contradicts renormalizability: a change in the cut-off cannot be compensated for by a change in the bare parameters.

5. CRITICAL SCALING BEHAVIOR

I mentioned that the continuum theory may be interacting though the photon decouples. The direct way to find out whether this is the case would be to compute, e.g., the renormalized four-fermi coupling. This has not been done, and so we depend on other information. The critical exponents are one source. Preliminary studies have revealed [5,6,7] mean field critical behavior. In this section I shall discuss some new developments.

Consider the anomalous dimension of the fermion mass, which is given by

$$\gamma_m = \frac{m_R}{m} \frac{\partial m}{\partial m_R}\bigg|_{e\ \text{fixed}} - 1, \qquad (5.1)$$

and the anomalous dimension of $\bar{\chi}\chi$, which is given by

$$\gamma_{\bar{\chi}\chi} = \frac{m_R}{\langle\bar{\chi}\chi\rangle} \frac{\partial\langle\bar{\chi}\chi\rangle}{\partial m_R}\bigg|_{e \text{ fixed}} - 3. \qquad (5.2)$$

At the critical point γ_m and $\gamma_{\bar{\chi}\chi}$ are related by $\gamma_m + \gamma_{\bar{\chi}\chi} = 0$. Figure 2 shows rather convincingly that $\langle\bar{\chi}\chi\rangle \propto m_R$ as $m_R \to 0$, which leads to $\gamma_m = -\gamma_{\bar{\chi}\chi} = 2$. This result is supported by a direct evaluation of γ_m in ref. 23. Combining eqs.(5.1) and (5.2), we obtain

$$\frac{m}{\langle\bar{\chi}\chi\rangle} \frac{\partial\langle\bar{\chi}\chi\rangle}{\partial m}\bigg|_{e=e_c} = \frac{3-\gamma_m}{1+\gamma_m} = \frac{1}{3}. \qquad (5.3)$$

The left-hand-side of eq.(5.3) is equal to $1/\delta$ (cf. eq.(3.6)). It then follows that

$$\delta = 3, \qquad (5.4)$$

which is the mean field result. This result is not surprising. The validity of renormalized perturbation theory indicates already that there are no further relevant interactions. Vice versa, $\delta = 3$ leads to $\gamma_m = 2$. The scaling relations among the critical exponents [32] connect δ to $\dot{\beta}$ and the critical exponent γ by

$$\gamma = \dot{\beta}(\delta - 1). \qquad (5.5)$$

This relation follows also directly from the equation of state (3.3), (3.4). Kocic et al. [33] have argued that $\gamma = 1$ in the ladder approximation. This would give the mean field value $\dot{\beta} = 0.5$.

N_f	β_c	δ	$\dot{\beta}$	γ
4	0.194(2)	3.14(18)	0.49(2)	1.05(9)
2	0.227(3)	2.23(18)	0.74(6)	0.91(13)

Table 2: The critical exponents as obtained from a fit of the equation of state to the data of Göckeler et al. ($N_f = 4$) and Dagotto et al. ($N_f = 2$), respectively.

Let me now turn to the data. In tab. 2 I have compiled the critical exponents that I obtain from the fits described in sec. 3. (The data and the fitted curves are shown in figs. 3 and 4.) In case of the $N_f = 4$ data of Göckeler et al. [17] we find critical exponents, which are in good agreement with mean field theory. The $N_f = 2$ data of Dagotto et al., [24] on the other hand, give critical exponents which deviate significantly from the mean field values. One should remember though that the $N_f = 2$ theory with staggered fermions is non-local: it contains spinless fermions, which may have influenced the result. Quite possibly, locality will not even be restored at the critical point. The situation here is different from QCD, where the running coupling constant goes to zero. The message from investigations of Schwinger - Dyson equations [9] is that one has to go to increasingly large correlation lengths as N_f is decreased, before one sees mean field critical exponents. This might also be the case here.

Based on the report [34] that the critical points of QED and the Nambu - Jona-Lasinio model are connected by a second order critical line, Horowitz [7] has argued that one should be able to describe the critical behavior of QED in terms of a gap equation of a four-fermi interaction. Indeed, the gap equation is identical with the one-loop equation (3.1) for $\langle\bar{\chi}\chi\rangle$, where m_R is expressed by eq.(4.6). Like the σ-model it yields mean field critical exponents. Horowitz has fitted the $N_f = 4$ data of Booth et al. [6] to the solutions of the gap equation. He obtained a chi-squared per degree of freedom of 1.3, which speaks for mean field critical exponents. It indicates furthermore that the chiral transition of non-compact QED is indeed in the same universality class as the Nambu - Jona-Lasinio model. The $N_f = 2$ data of Kogut et al. [4,24] gave a chi-squared per degree of freedom of 7.8.

6. FOUR-FERMI INTERACTION

In the previous section two independent methods of determining γ_m have been discussed, which both gave $\gamma_m = 2$. Hence, the scaling dimension of $\bar{\chi}\chi$ is one. This and the fact that the theory is non-interacting suggests that the scaling dimension of the four-fermi interaction is two. It means it is renormalizable and becomes a relevant operator. Since the four-fermi interaction is generated anyway, it should be included in the action from the beginning. Indeed, it has been argued on the grounds of Schwinger - Dyson equations [9] that the apparent non-renormalizability of QED discussed in sec. 4 is mitigated if a four-fermi interaction is included. [35]

Booth et al. [6] have simulated non-compact QED with the chiral $U(1) \times U(1)$ invariant four-fermi interaction

$$G \sum_{x,\mu} \bar{\chi}_x \chi_x \bar{\chi}_{x+\hat{\mu}} \chi_{x+\hat{\mu}}. \tag{6.1}$$

They confirm the existence of a second order line [34] connecting the critical point of QED with that of the Nambu - Jona-Lasinio model. It has been pointed out [36,37] that the expression (6.1) does not produce a $U(4) \times U(4)$ invariant four-fermi interaction in the classical continuum limit. One could argue that the remnant $U(1) \times U(1)$ symmetry is sufficient to study the breakdown of continuous chiral symmetry. But this has to be investigated further.

It is known that $\gamma_m = 2$ at the critical point of the Nambu - Jona-Lasinio model. Our results suggest that $\gamma_m = 2$, and hence $\delta = 3$, on the whole second order line. This fits in with the claim of Horowitz [7] that the chiral transitions of QED and of the Nambu - Jona-Lasinio model are in the same universality class. It would mean that the theory is trivial everywhere on that line.

7. QUENCHED APPROXIMATION

The quenched approximation is sometimes a useful tool for exploratory studies. In QCD it may even capture the essential features of the full theory. In non-compact QED this is, however not the case, so that one should not take the results literally. For example, an investigation of the gauged Nambu - Jona-Lasinio model [34,38] in the ladder approximation suggests that on the second order line, connecting the critical point of the Nambu - Jona-Lasinio model with that of QED, the anomalous dimension γ_m varies continuously from $\gamma_m = 2$ at the critical point of the Nambu - Jona-Lasinio model to $\gamma_m = 1$ at $\epsilon = \epsilon_c$. This is in conflict with the predictions of the full theory, discussed in the last section, and also with the solutions of truncated Schwinger - Dyson equations including certain effects of fermion loops, [8,9] which both indicate that $\gamma_m = 2$ on the whole critical line.

Nevertheless, let me mention briefly what the status of the quenched calculations is. We have stated some time ago [5] that the critical exponents are consistent with the predictions of mean field theory, when everybody else argued that $\dot{\beta} = \infty$. In my opinion this statement has not been disproved yet, in spite of different claims. [39] I would not be surprised though if the critical exponents came out differently. Motivated by the recent work of Dagotto et al., [39] I have re-fitted all published quenched data [3,5,24] by the equation of state (3.3), (3.4). The result of the fit is shown in fig. 10 in form of the scaling plot introduced in ref. 5. I find $\beta_c = 0.253(3)$ and the critical exponents $\delta = 2.55(24)$ and $\dot{\beta} = 0.64(9)$. Regarding the errors, one cannot claim that the result is inconsistent with mean field theory. Dagotto et al. [39] reported $\beta_c = 0.257(1)$ and $\delta = 2.2(1)$, $\dot{\beta} = 0.78(8)$. But I cannot see that these authors have done a proper fit of the equation of state to their data. I intended

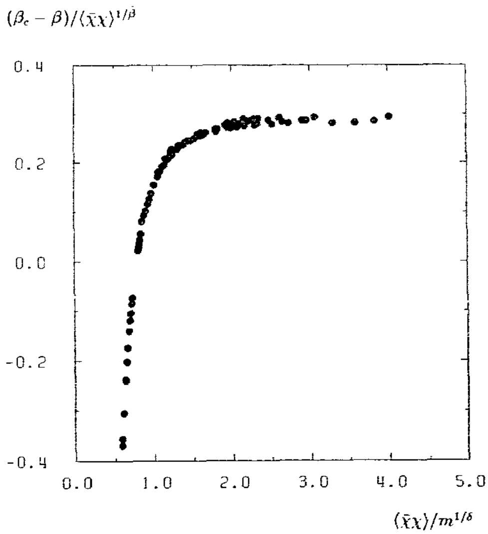

Figure 10: Scaling plot of $(\beta_c - \beta)/\langle\bar{\chi}\chi\rangle^{1/\hat{\beta}}$ against $\langle\bar{\chi}\chi\rangle/m^{1/\delta}$ for β_c and the critical exponents given in the text.

to do that. But I was not able to get the new data from Dagotto.

8. CONCLUSIONS

There is no doubt, that the photon decouples at the critical point which means, in the customary sense, that QED is trivial. It may take a long time though to prove this rigorously. An essential element in the analysis was renormalized perturbation theory, which allowed us to extrapolate the lattice results down to $\epsilon_R = 0$.

There is strong evidence also for mean field critical exponents. The only fact that speaks against it is the $N_f = 2$ result of Dagotto et al. [24] But given the inherent problems of this theory, further studies will be needed in order to dispel all doubts. It should also be mentioned that these authors use the hybrid algorithm, [40] which is not exact in contrast to the hybrid Monte Carlo algorithm [41] everybody else uses.

The most far-reaching result is presumably that the four-fermi interaction is renormalizable. We expect a whole series of other interactions to be renormalizable as well. In this light we have only explored a single point of a multi-dimensional critical surface. So there is still hope to find a non-trivial continuum limit.

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