Polarization effects in heavy Λ decays ☆

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Within the framework of the heavy quark effective theory, we study the process $\Lambda_c^+ \to \Lambda \ell^+ \nu$, with emphasis on the polarization structure in the decay. Assuming factorization of the amplitude, we perform a similar analysis for the decay $\Lambda_c^+ \to \Lambda \pi^+$. The asymmetry parameter, α, is predicted to be -1 for this decay. We point out that in a certain limit, the results we obtain are also valid for the corresponding $\Lambda_b \to \Lambda_c$ decays.

1. Introduction

The physics of hadrons containing heavy quarks has recently enjoyed much theoretical interest [1–15]. This has been largely due to the observation that, in the limit of infinitely massive quarks, symmetries above and beyond those usually associated with QCD arise. In the so-called heavy quark effective theory (HQET) that results, these symmetries have been termed heavy flavor symmetry and spin symmetry.

The heavy flavor symmetry is described by the group $SU(N_h)$, where N_h is the number of heavy flavors that exist. This symmetry arises because the physics of any situation is unchanged when one infinitely heavy quark is replaced by any of the other $N_h - 1$ infinitely heavy quarks. Spin symmetry arises because the spin degrees of freedom of the heavy quark decouple from the light QCD degrees of freedom in the effective theory. Interactions, masses and other hadronic properties are independent of the spin of the heavy quark, at least to leading order in Λ_{QCD}/m_Q and $\alpha_{QCD}(m_Q)$, where m_Q is the mass of the heavy quark.

Some of the consequences of these symmetries have been investigated in a number of papers. In particular, the symmetries have been used to derive relations between the form factors that describe the weak decays of hadrons. These relations reduce the number of form factors necessary for the description of any single process. For instance, the six, real, generally independent form factors that must be introduced to describe the semileptonic decays of a baryon are reduced to a single one for the decay $\Lambda_b \to \Lambda_c \ell \nu$, if we assume that b and c quarks satisfy the requirements of HQET. For the transition $\Lambda_c \to \Lambda \ell^+ \nu$ in which a heavy quark c is changed into a light quark s, only two form factors are necessary.

The decrease in the number of independent form factors leads to testable experimental predictions. In addition, when we couple this with the hypothesis that the matrix element for non-leptonic decays factorizes (this is explained in section 3), we find that the same form factors which describe the semileptonic decays also describe the non-leptonic ones. Obviously, it increases the scope of the predictions that may be made.

In this paper, we explore, in some detail, implications of HQET for the semileptonic decays $\Lambda_c^+ \to \Lambda \ell^+ \nu$. In addition, we study a simple two-body non-leptonic decay, $\Lambda_c^+ \to \Lambda \pi^+$, in which we use the additional assumption of factorization. In section 2, we set out the formalism of HQET for Λ_c decays. In section 3, we examine the semileptonic decay of the Λ_c, as well as the process $\Lambda_c^+ \to \Lambda \pi^+$, while we present our conclusions in section 4.

2. Formalism

As discussed in ref. [6], heavy quark systems may be described by an effective theory in which the heavy quark part of the lagrangian is given by

$$\mathscr{L}_{\text{heavy}} = \int \frac{d^3 v}{v_0} (i\bar{h}_v^+ \mathscr{D} h_v^+ + i\bar{h}_v^- \mathscr{D} h_v^-) . \tag{1}$$

$\mathscr{D}$ is the covariant derivative, and h_v^+ (h_v^-) annihilates (creates) heavy quark (antiquark) states with four-momentum

$$P^\mu = m_Q v^\mu + k^\mu , \tag{2}$$

and k_μ is of the order Λ_{QCD}.

The dynamics described by $\mathscr{L}_{\text{heavy}}$ differs from QCD by terms that go to zero as $m_Q \to \infty$, at least logarithmically, as the differences between the effective theory and QCD are due to terms suppressed by powers of $\alpha_{\text{QCD}}(m_Q)$ or Λ_{QCD}/m_Q. If we neglect these effects, different velocity sectors do not communicate, leading to the so-called velocity superselection rule. In each velocity sector of the theory described by (1) there is a new symmetry, called spin symmetry, which arises because QCD interactions of a heavy quark become independent of its spin degrees of freedom in the heavy quark limit. This means that for each v there exists a separate SU(2) spin symmetry. Also, the lagrangian, $\mathscr{L}_{\text{heavy}}$ contains no reference to the mass of the heavy quark, which has been formally sent to infinity. Consequently, if we have different species of heavy quarks which obey $m_Q \gg \Lambda_{\text{QCD}}$ the effective theory has an additional heavy flavor symmetry corresponding to exchanges of heavy quarks.

Let us consider a heavy baryon built out of a heavy quark and two light quarks. The mass M of the baryon in the spin symmetry limit tends to the quark mass $M = m_Q[1 + O(\Lambda_{\text{QCD}}/m_Q)]$. Such a baryon is moving with an "infinite" momentum $P^\mu = M v^\mu$ and obviously the only heavy quark states present in the baryon's wave-function come from the v^μ sector of the theory.

The Λ_c, with structure $[(ud)_0 c]_{1/2}$, is one of the simplest of the heavy baryons. In HQET the nontrivial Lorentz structure of this baryon is due solely to the two polarization states of the heavy quark. This means that the SU(2) of spin symmetry connects different polarization states of the same particle. Consequently, an ordinary Dirac spinor $u_{\Lambda_c}(v)$ may

be used to describe the representation of Λ_c under the large symmetry group $G = \text{Lorentz} \oplus \text{SU}(2)_{\text{spin}}$.

As discussed in ref. [12], spin symmetry implies that the current matrix element for the decay $\Lambda_c \to \Lambda$ may be written as

$$H_\mu \equiv \langle \Lambda(P) | \bar{s} L_\mu c | \Lambda_c(v) \rangle$$
$$= \bar{u}_\Lambda(P) [F_1(P \cdot v) + \slashed{v} F_2(P \cdot v)] L_\mu u_{\Lambda_c}(v) , \tag{3}$$

where $L_\mu = \gamma_\mu(1 - \gamma_5)$ is the left handed current. Note that while we discuss mainly the decays to Λ's, the form above is also valid for the matrix elements of the left handed currents between Λ_c and any other light baryon of spin $\frac{1}{2}$. In particular, one may use the form of eq. (3) to analyze the decay $\Lambda_c \to \Sigma$ as well as the (Cabibbo-suppressed) decays to nucleons in which there is a $c \to d$ transition. Note that the similarity extends only to the form of the matrix element (3). For example the functional structure of the form factors F_1 and F_2 will depend on the light baryon appearing in the final state.

For decays in which the final baryon is also heavy, a decay of the type $\Lambda_h \to \Sigma_{h'}$ is forbidden. This is easily seen by noting that since the spins of the heavy quarks decouple, the light quarks are truly spectators in such a decay. This means that the total angular momenta of the light quarks (and accompanying gluons) must be conserved in the decay. In the case of a light (i.e., normal) Σ, the strange quark's spin is not expected to decouple, so that the spin of the light "spectators" can change during the process. This is discussed more formally in refs. [11-14].

The form of eq. (3) is to be compared with the general form of the current matrix element of the left handed current for weak baryon decays, which is described by six real form factors:

$$\langle B', p' | \bar{q}' L_\mu q | B, p \rangle$$
$$= \bar{u}(p') \{ [f_1(q^2)\gamma_\mu - if_2(q^2)\sigma_{\mu\nu}q^\nu + f_3(q^2)q_\mu]$$
$$+ [g_1(q^2)\gamma_\mu - ig_2(q^2)\sigma_{\mu\nu}q^\nu + g_3(q^2)q_\mu]\gamma_5 \} u(p) . \tag{4}$$

In terms of these form factors, we find

$$f_1 = -g_1 = F_1 + \frac{m}{M} F_2 ,$$

$$f_2 = g_2 = f_3 = g_3 = -\frac{1}{M} F_2 . \tag{5}$$

In the usual notation, this means that HQET *predicts* that $G_A = -G_V$. In the next section, we examine the significance of (3) for the decays $\Lambda_c^+ \to \Lambda \ell^+ v$ and $\Lambda_c^+ \to \Lambda \pi^+$. In the limit when $F_2 = 0$, all of our results are also valid for the corresponding $\Lambda_b \to \Lambda_c$ processes.

3. Spin structure of Λ_c decays

In the analysis that follows, we retain all possible polarization information about the Λ and the Λ_c. We do this since the self-analyzing decay of the Λ ($\Lambda \to p\pi^-$) allows a fairly easy determination of its polarization. In addition, our analysis shows that the polarization of the Λ_c may also be determined from its decay. This of course assumes that the Λ_c's are created with some non-zero polarization. While this is unlikely at e^+e^- machines with unpolarized beams, Λ_c's are expected to be polarized when they are produced at hadron colliders in processes like $pp \to \Lambda_c + X$ or $p\bar{p} \to \Lambda_c \bar{\Lambda}_c$. All of this means that in addition to the spectra of leptons (or Λ's) arising from the decay, the various spin correlation quantities will also be of interest.

As discussed in the previous section, the semileptonic decays of the Λ_c are described by the matrix element of (3). Moreover, if one assumes that other weak decays may be treated in the factorization approximation, then the hadronic current of (3) will be a part of the matrix element of all such decays. This means that the spin structure of the Λ_c and the light spin-$\frac{1}{2}$ baryon in all decays of the form $\Lambda_c \to BX$ is contained in (3) (B is a light spin-$\frac{1}{2}$ baryon). We thus focus some attention on this matrix element.

By the factorization hypothesis we man simply that the matrix element M for the decay $\Lambda_c \to BX$ may be written as

$$M \equiv \langle B(p)X | \bar{q}L_\mu cx^\mu | \Lambda_c(v) \rangle$$
$$= \bar{u}_B(p)[F_1(p \cdot v) + \not{p}F_2(p \cdot v)]L_\mu u_{\Lambda_c}(v)X^\mu , \quad (6)$$

where, symbolically,

$$X^\mu = \langle X | x^\mu | 0 \rangle . \quad (7)$$

We emphasize that the factorization hypothesis is independent of HQET. Decays of the form $\Lambda_c \to BX$ may, in principle, be studied using the factorization hypothesis and the form of the weak current given in

eq. (4). What HQET does for us here is to introduce relationships among the form factors, eq. (5), which lead to a great simplification of the final results.

Note also that, given the assumption of factorization, all weak decays of the form $\Lambda_c \to BX$ are not only described by two form factors as in eq. (6), *but that these form factors are the same for all such processes*, regardless of the form of X. F_1 and F_2 will, in general, depend on the light spin-$\frac{1}{2}$ baryon B involved in the process, but not on X. This means that the decays of the form $\Lambda_c \to BX$ are related to each other, and ultimately to the semileptonic decay $\Lambda_c \to B\ell^+v$. On the other hand, the validity of the factorization hypothesis may depend on X. We expect that the best chance for the factorization to hold corresponds to the case when the invariant mass of X is small. In our opinion states like π or ρ are good candidates and in fact we apply our discussion only to $X = \pi$.

For semileptonic decays, where factorization is exact,

$$X^\mu = \bar{v}\gamma_\mu(1 - \gamma_5)u \quad (8)$$

is the leptonic current. In the case of a decay where $X = \pi$, and factorization is believed to be approximately correct we have

$$X^\mu = f_\pi P_\pi^\mu . \quad (9)$$

The spin structure present in eq. (3) is easily analyzed by reducing the 4×4 Dirac algebra to the 2×2 Pauli algebra. In the notation of ref. [16] the matrix element may then be written as

$$M = |D| \chi_\Lambda^\dagger(s)[1 + \sigma \cdot (R + iI)]\chi_{\Lambda_c}(S) , \quad (10)$$

where s is the spin of the light spin-$\frac{1}{2}$ baryon (Λ, say) measured in its own rest frame and S is the spin of the Λ_c measured in its rest frame. Writing

$$X = X_1 + iX_2 \quad (11)$$

(X_0 is real for the cases in which we are interested), we see that

$$D = D_1 + iD_2 , \quad (12)$$

with

$$D_1 = (F_1 + F_2)(E + m)X_0 - (F_1 - F_2)\mathbf{P} \cdot \mathbf{X}_1 ,$$
$$D_2 = -(F_1 - F_2)\mathbf{P} \cdot \mathbf{X}_2 . \quad (13)$$

Then R and I may be written as

$$R = \frac{a_1 D_1 + a_2 D_2}{D_1^2 + D_2^2} , \tag{14}$$

$$I = \frac{a_2 D_1 - a_1 D_2}{D_1^2 + D_2^2} , \tag{15}$$

with

$$a_1 = (E+m)(F_1+F_2)X_1$$
$$+ (F_1-F_2)(\boldsymbol{P} \times \boldsymbol{X}_2 - X_0 \boldsymbol{P}) , \tag{16}$$

$$a_2 = (E+m)(F_1+F_2)X_2 - (F_1-F_2)\boldsymbol{P} \times \boldsymbol{X}_1 . \tag{17}$$

$\boldsymbol{P}$ is the three-momentum of the Λ, which is assumed to be parallel to the z-axis.

The forms of the various differential decay rates obtainable from eq. (10) will depend on whether or not the polarizations of the Λ and/or Λ_c are measured. If both $\boldsymbol{s}$ and $\boldsymbol{S}$ are measured, the differential decay rates are proportional to

$$\mathrm{Tr} \, |M|^2 = D'(1 + \boldsymbol{s} \cdot \boldsymbol{a} + \boldsymbol{S} \cdot \boldsymbol{A} + \boldsymbol{B} \cdot \boldsymbol{s} \times \boldsymbol{S} + s_i S_j C_{ij}) , \tag{18}$$

where again for ease of comparison, we are using the same notation as Bjorken [16]. Expanding and evaluating the traces yields the Bjorken results

$$D' = |D|^2(1+R^2+I^2) ,$$

$$A = \frac{2(\boldsymbol{R} - \boldsymbol{R} \times \boldsymbol{I})}{1+R^2+I^2} ,$$

$$a = \frac{2(\boldsymbol{R} + \boldsymbol{R} \times \boldsymbol{I})}{1+R^2+I^2} ,$$

$$B = \frac{2I}{1+R^2+I^2} ,$$

$$C_{ij} = \frac{(1-R^2-I^2)\delta_{ij} + 2(R_i R_j + I_i I_j)}{1+R^2+I^2} . \tag{19}$$

While much of this has already been discussed in ref. [16], the new information available here is in the forms of $\boldsymbol{R}$ and $\boldsymbol{I}$ predicted by HQET. The above equations mean that measurement of the spin variables $\boldsymbol{a}$, $\boldsymbol{A}$, $\boldsymbol{B}$, or C_{ij} will serve to test the predictions of HQET.

In eq. (18) above, if the polarization of either the light baryon or that of the Λ_c is not measured, this amounts to putting $\boldsymbol{s}$ (or $\boldsymbol{S}$) = 0. For example, if Λ_c's are produced with no net polarization, $|M|^2$ reduces to

$$|M|^2 = D'(1 + \boldsymbol{s} \cdot \boldsymbol{a}) . \tag{20}$$

Some information is also available from the general forms above. When X is real, and there is no recoil of the final baryon ($\boldsymbol{P}=0$), $\boldsymbol{I}=0$ and $\boldsymbol{R}=X/X_0$. Then $\boldsymbol{B}=0$ and $\boldsymbol{a}=\boldsymbol{A}$. If, in addition, $|X| \ll X_0$, then $C_{ij} \approx \delta_{ij}$, and there is maximal spin transfer from the Λ_c to the light baryon. One possible situation where this may arise is discussed below.

3.1. Λ_c semileptonic decays

We refer to these decays generically as $\Lambda_c^+ \to \Lambda \ell^+ \nu_\ell$. These are probably the most interesting of the Λ_c semileptonic decays, anyway, since the polarization of the Λ may be easily measured. For these decays, the vector X^μ is given by eq. (8). Following Bjorken [16], we evaluate

$$Y = \frac{X}{X_0} = \frac{\hat{p}_\ell + \hat{p}_\nu + i \hat{p}_\ell \times \hat{p}_\nu}{1 + \hat{p}_\ell \cdot \hat{p}_\nu} , \tag{21}$$

where $\hat{p}_\nu$, $\hat{p}_\ell$ are unit vectors in the directions of motion of the neutrino and charged lepton, respectively. Of course $\boldsymbol{p}_\nu$ cannot be measured directly, and one needs to know the momenta of all the other particles involved (Λ_c, Λ, ℓ^+) in order to evaluate Y. This can be done only at collider machines where there is 4π detection. In electron–positron colliders, if the beams are unpolarized, Λ_c's are pair produced with no net polarization, and the decay matrix element is given by eq. (20). Even in this case experiment should be able to provide the values of $|D'|$ and $|a|$ which in principle may allow a calculation of the form factors F_1 and F_2.

Note that in general Y is real when the leptons are back-to-back or collinear. For back-to-back leptons, $Y \approx 0$, $C_{ij} \approx \delta_{ij}$, and there is complete spin transfer from the Λ_c to the Λ. In other words, in this kinematical limit one can directly measure the polarization of the Λ_c by measuring the polarization of the Λ. This might be especially useful in determining whether Λ_c's are produced polarized in hadron colliders.

3.2. $\Lambda_c^+ \to \Lambda \pi^+$

Let us now turn to the two-body non-leptonic decays of the Λ_c, such as $\Lambda_c^+ \to \Lambda \pi^+$. In what follows we employ the factorization hypothesis which so far lacks

any good theoretical justification in general (and for which we offer no justification here). Note, however, that recent work by Dugan and Grinstein [17] argues for the validity of this hypothesis in certain kinematic limits. In addition, factorization seems to work phenomenologically, at least for the decays of heavy mesons. Obviously, we expect that the factorization hypothesis has many limitations. In fact, we agree with the proposition that the only place where the factorization could work corresponds to a two body decay where one of the final particles is a light charged meson that can be produced from the virtual W. Fortunately the decay $\Lambda_c \to \Lambda\pi$ is precisely such a decay.

Mindful of this disclaimer we use eqs. (6) and (9) to write

$$M_{\Lambda_c \to \Lambda\pi} = f_\pi \bar{u}_\Lambda(P)(F_1 + \slashed{P}F_2)\gamma_\mu(1-\gamma_5)u_{\Lambda_c}(v)P_\pi^\mu \,. \tag{22}$$

Substituting $P_\pi = m_{\Lambda_c}v - P$, we find

$$M_{\Lambda_c \to \Lambda\pi} = f_\pi \bar{u}_\Lambda(P)(G_1 + G_2\gamma_5)u_{\Lambda_c}(v) \,, \tag{23}$$

with

$$G_1 = F_1(m_{\Lambda_c} - m_\Lambda) + F_2(m_{\Lambda_c} + m_\Lambda - 2E_\Lambda) \,, \tag{24}$$

$$G_2 = F_1(m_{\Lambda_c} + m_\Lambda) - F_2(m_{\Lambda_c} - m_\Lambda - 2E_\Lambda) \,, \tag{25}$$

and we have again assumed that the Λ_c is at rest. We see immediately that, because of momentum conservation, $X \propto P$, and $I = B = 0$. We remind the reader that F_1 and F_2 are real.

In terms of the new form factors G_1 and G_2, and the usual hyperon decay parameters α, β and γ, the spin variables of eq. (19) are

$$a_x = a_y = A_x = A_y = 0, \quad a_z = A_z = \alpha \,,$$

$$B = 0, \beta = 0 \,;$$

$$C_{xx} = C_{yy} = \gamma, \quad C_{zz} = 1 \,,$$

$$C_{xz} = C_{zx} = C_{xy} = C_{yx} = C_{yz} = C_{zy} = 0 \,. \tag{26}$$

In terms of G_1 and G_2 we find

$$\alpha = \frac{-2G_1G_2P}{(E_\Lambda + m_\Lambda)G_1^2 + (E_\Lambda - m_\Lambda)G_2^2} \,, \tag{27}$$

$$\beta = 0 \,, \tag{28}$$

$$\gamma = \frac{(E_\Lambda + m_\Lambda)G_1^2 - (E_\Lambda - m_\Lambda)G_2^2}{(E_\Lambda + m_\Lambda)G_1^2 + (E_\Lambda - m_\Lambda)G_2^2} \,. \tag{29}$$

In general the quantities α, β and γ depend on both form factors F_1 and F_2. However, in the limit of negligible pion mass, the form factors G_1 and G_2 become

$$G_1 \approx \frac{m_{\Lambda_c} - m_\Lambda}{m_{\Lambda_c}}(m_{\Lambda_c}F_1 + m_\Lambda F_2) \,,$$

$$G_2 \approx \frac{m_{\Lambda_c} + m_\Lambda}{m_{\Lambda_c}}(m_{\Lambda_c}F_1 + m_\Lambda F_2) \,. \tag{30}$$

Then $\alpha \approx -1$ and $\gamma \approx 0$, *independent* of the values F_1 and F_2. This is a very strong *prediction* of HQET. It means that one should expect to observe maximal asymmetry in the angular distribution of the Λ's produced in $\Lambda_c \to \Lambda\pi$ (or of the Λ_c's produced in $\Lambda_b \to \Lambda_c\pi$). Recent experimental results [18] appear to confirm this prediction, since α has been measured to be $-1.0^{+0.4}_{-0.0}$ for the decay $\Lambda_c^+ \to \Lambda\pi^+$. This suggests that both HQET and factorization appear to be working in this decay.

4. Conclusions

In this paper we have discussed some decays of heavy Λ_h particles. Matrix elements corresponding to these decays are severely constrained by HQET. In particular, semileptonic decays $\Lambda_h \to \Lambda_{h'}$ are described by a single real form factor, and semileptonic decays $\Lambda_h \to \Lambda$ are described by two real form factors. Non-leptonic decays for which factorization holds are described by the same form factors.

Spin asymmetry and spin correlation measurements can be used to obtain the values of the form factors describing the weak decays. This should be a good test both of HQET and of the factorization hypothesis. Hopefully the relevant data will soon be available.

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