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Heavy to light transitions in the heavy quark limit and the determination of $|V_{ub}|$

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Abstract. We discuss semileptonic decays of heavy mesons into light pseudoscalar and vector mesons. Exploiting the symmetries arising in the heavy quark limit we use the known data on semileptonic D decays to predict the corresponding rates for semileptonic B decays. These consideration may serve as a model independent way to extract the ub matrix element of the CKM matrix.

1 Introduction

Recently theoretical progress [1–6] has been made in the description of systems involving heavy quarks. The main idea is to treat the heavy quark in a static limit which corresponds to the limit of infinitely large masses for the heavy quarks such that the velocity $v = p/m$ is kept constant. This limit may be formulated in terms of an effective field theory [5, 6] which nicely exhibits the usefulness of this limit.

In this limit two additional symmetries arise. Since all the heavy quarks are treated on the same footing the effective theory exhibits an $SU(N_f)$ symmetry – N_f being the number of heavy flavors – for each velocity of the heavy quark.

The second symmetry is due to the fact that the spin degrees of freedom of the heavy quark decouple in this limit. Consequently there is an additional $SU(2)_{\text{spin}}$ symmetry corresponding to spin rotations of the heavy quark moving with a fixed velocity. This so called spin symmetry predicts that 0^- mesons made of a heavy quark and a light antiquark and the corresponding vector mesons should be degenerate. Comparing this prediction with experiment we conclude that the b and the c quark may certainly be treated as heavy and the heavy flavor symmetry is thus an $SU(2)$ symmetry denoted in the following $SU(2)_{HF}$.

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Especially the spin symmetry has been applied in various ways, e.g. to semileptonic $B \to D$ transitions [2, 4, 14], e^+e^- annihilation into heavy mesons and baryons [7–9], and also to nonleptonic decays [10]. The main point of all these applications is that the symmetry reduces the number of independent form factors describing the matrix elements. In fact, all the matrix elements of heavy quark bilinear operators, i.e. operators of the type $\bar{h}'_{v'} \Gamma h_v$ with Γ some arbitrary combination of gamma matrices, are described in terms of only one universal function, the so called Isgur–Wise function [4].

As far as the heavy to light transitions are concerned spin symmetry does not reduce the number of independent form factors for mesonic transitions. The only additional information which may be extracted from the heavy quark limit is that form factors of the heavy to light transitions for different types of heavy quarks are related. If we use in addition the usual $SU(3)_F$ flavor symmetry among the light quarks we may formulate the current matrix elements of semileptonic decays in terms of universal form factors, where universal means that they are the same for any decay of this type in the heavy quark limit.

This is of phenomenological importance, since this approach allows to relate the rates for semileptonic B decays into light mesons to the rates of the corresponding D decays. For example we can relate the matrix elements of the following charged left handed currents

$$\langle B|L_\mu|\pi\rangle, \quad \langle D|L_\mu|\pi\rangle \quad \text{and} \quad \langle D|L_\mu|K\rangle$$

and also for the corresponding decays into a light vector meson

$$\langle B|L_\mu|\rho\rangle, \quad \langle D|L_\mu|\rho\rangle \quad \text{and} \quad \langle D|L_\mu|K^*\rangle.$$

In particular this allows a determination of $|V_{ub}|$ by using the data on the semileptonic D decays.

Heavy to light transitions in connection with rare B decays have been considered by Isgur and Wise [11]. They relate the matrix elements for rare B decays to the semileptonic decay matrix elements of D mesons, which have a completely different spin structure. They employ relations between form factors which hold at the Voloshin Shifman point, i.e. at maximum momentum transfer. But it is not clear how these relations change if one moves away from this point. Our approach is different: We consider only a given spin structure namely a left handed current and show that the form factors are universal for any heavy to light transition in the heavy quark limit. Moreover, the Voloshin–Shifman point becomes singular in our approach due to the resonances which become degenerate with the heavy mesons in the heavy quark limit. This does not contradict the considerations by Isgur and Wise [11], since their expressions for the form factors at the point of maximum momentum transfer explicitly depend on the small scale Λ_{QCD}.

Uncertainties of our predictions for semileptonic heavy to light transitions arise from the breaking of the two symmetries $SU(2)_{\mathrm{HF}}$ and $SU(3)_{\mathrm{F}}$. First we shall use the full symmetries and then try to estimate the breaking effects. These are discussed in two steps: First we include trivial phase space effects still assuming the universality of the form factors. In a second step we use specific parametrizations of the form factors keeping only the normalization universal.

There have been many calculations of heavy meson decays based on various model assumptions [12–17]. Depending on the kind of model the predictions for the rates vary by a large margin, e.g. the variation of the predictions for the rate for $B \to \pi e \nu$ is about a factor of 10. In contrast to these model assumptions our approach starts from model independent symmetry considerations. Models of form factors are used only to estimate uncertainties due to symmetry breaking effects.

The paper is divided into two pieces. The first part deals with the semileptonic decay of a heavy pseudoscalar meson into a light one. After writing the general heavy to light matrix element in terms of universal form factors we discuss the rates and give the predictions for $B \to \pi e \nu$ based on the D decay data. Then we discuss symmetry breaking effects first by including phase space effects and finally by using specific parametrizations for the form factors. The second part deals in the same way with the heavy 0^- decays into light vector mesons with the special emphasis on the longitudinal rate.

2 The decays of a heavy 0^- into a light 0^- meson

In this section we shall discuss the decays $D \to \pi e \nu$, $D \to K e \nu$ and $B \to \pi e \nu$ which are all decays of a heavy 0^- into a light 0^- and are described all in the same way in the heavy quark limit.

2.1 *The current matrix element and universal form factors*

In this subsection we shall exploit the heavy flavor symmetry to discuss the hadronic matrix element of the left handed current which is needed to describe decays of the type

$$H(P = m_H v) \to l(p) + e(k) + \nu(k'), \quad q = k + k' \tag{1}$$

where H denotes a B or a D meson and l any light pseudoscalar meson. The main result is (6) where the hadronic current is expressed in terms of two form factors which are universal for all transitions of the type (1).

In order to do this we employ the trace formalism for the heavy mesons [6, 14]. In this formalism a heavy pseudoscalar meson moving with a velocity $v = P/m_H$ is represented by a matrix:

$$H(v) = \frac{1}{2}\sqrt{m_H}\,\gamma_5(\not{v} - 1) \tag{2}$$

A generic transition matrix element between a heavy and a light meson (both pseudoscalar) of a left handed current is then given by

$$\langle H, v|\bar{h}_v \gamma_\mu(1+\gamma_5) l|l, p\rangle = \sqrt{v\cdot p}\,\mathrm{Tr}\left\{\bar{H}(v)\gamma_\mu(1+\gamma_5) \cdot\left[\xi_1 + \xi_2 \frac{\not{p}}{v\cdot p}\right]\right\}. \tag{3}$$

The two form factors ξ_1 and ξ_2 in (3) are dimensionless functions of the variable $v\cdot p$. In the rest frame of the heavy meson $v\cdot p$ is the pion energy which is in most of the phase space of the order of the heavy meson mass. Thus it is convenient to introduce the variable

$$x = \frac{2v\cdot p}{m_H}. \tag{4}$$

Then, in general, ξ_i depends on x and the ratios $r = m_l/m_H$ and $r_\Lambda = \Lambda_{\mathrm{QCD}}/m_H$, where Λ_{QCD} is the scale parameter of QCD.

In the limit $m_H \to \infty$ the form factor ξ_i is a function of x only, where x ranges between zero and one. If in addition $SU(3)_F$ symmetry holds the form factors are universal functions of x in the sence that the two functions $\xi_1(x)$ and $\xi_2(x)$ in (3) are the same for all heavy to light transitions via a left handed current. We shall call this limit in which the form factors become universal the heavy quark limit.

Evaluating the traces in (3) one finds

$$\langle H, v|\bar{h}_v \gamma_\mu(1+\gamma_5) l|l, p\rangle = \sqrt{2x}\,\xi_1 P_\mu - \sqrt{\frac{8}{x}}\,\xi_2 p_\mu \tag{5}$$

where $P_\mu = m_H v_\mu$ denotes the momentum of the heavy meson and $\xi_i = \xi_i(x, r, r_\Lambda)$. It is convenient to rewrite (5):

$$\begin{aligned}\langle H, v|\bar{h}_v \gamma_\mu(1+\gamma_5) l|l, p\rangle &= \left[\sqrt{\frac{x}{2}}\,\xi_1 - \sqrt{\frac{2}{x}}\,\xi_2\right] p_\mu^+ \\ &\quad + \left[\sqrt{\frac{x}{2}}\,\xi_1 + \sqrt{\frac{2}{x}}\,\xi_2\right] p_\mu^- \\ &\equiv F_+ p_\mu^+ + F_- p_\mu^- \end{aligned}\tag{6}$$

where

$$p^+ = P + p$$
$$p^- = P - p. \tag{7}$$

We shall neglect the lepton masses and thus the rates will depend only on F_+. The form factors $F_\pm$ are universal functions of the single variable x in the heavy quark limit, i.e. they are the same for any heavy to light decay via a left handed current.

2.2 The differential rate $d\Gamma/dx$ and relations between total rates

From the current matrix element one may easily calculate the differential rate in x for a heavy to light semileptonic decay in terms of the form factor F_+. We exploit the universality of the form factors to derive relations between total rates.

We start from the amplitude for a heavy meson decaying into a light pseudoscalar (cf (1))

$$\mathscr{A} = \frac{G_F}{\sqrt{2}} V_{Hl} \langle H, v | \bar{h}_v \gamma_\mu (1+\gamma_5) l | l, p \rangle \cdot \bar{e}(k,\alpha)\gamma_\mu(1+\gamma_5)v(k',\beta) \tag{8}$$

where G_F is the Fermi coupling and V_{Hl} the appropriate matrix element of the CKM matrix. The doubly differential rate is then given by

$$\frac{d^2\Gamma}{dx\,dy} = \frac{1}{2m_H}|V_{Hl}|^2 \frac{G_f^2}{2}|F_+|^2 p_\mu^+ p_\nu^+ L^{\mu\nu}\Phi(x,y) \tag{9}$$

where $L^{\mu\nu}$ is the leptonic tensor

$$L_{\mu\nu} = 8(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k\cdot k') + i\varepsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta) \tag{10}$$

and the variable y is defined by

$$y = \frac{2v\cdot k}{m_H}. \tag{11}$$

In general F_+ depends on x, r and r_Λ where r denotes the ratio of the masses of the light meson to the heavy one, $r = m_l/m_H$ and $r_\Lambda = \Lambda_{\mathrm{QCD}}/m_H$. Furthermore, the phase space factor is given by

$$\Phi(x,y) = \int \tilde{d}p\tilde{d}k\tilde{d}k' (2\pi)^4 \delta^4(m_H v - p - k - k') \cdot \delta\left(x - \frac{2v\cdot p}{m_H}\right)\delta\left(y - \frac{2v\cdot k}{m_H}\right) \tag{12}$$

with

$$\tilde{d}p = \frac{d^3\mathbf{p}}{(2\pi)^3 2E(p)}, \quad E(p) = \sqrt{\mathbf{p}^2 + m^2}.$$

Expressed in these variables the doubly differential rate takes the form

$$\frac{d^2\Gamma}{dx\,dy} = |V_{Hl}|^2 \frac{G_f^2}{2}\frac{m_H^5}{16\pi^3}|F_+|^2 \Theta(y)\Theta(1-x-r^2) \cdot \Theta(y[2-x-y]-[1-x+r^2])\{y[2-x-y] - [1-x+r^2]\}. \tag{13}$$

The integration over y may be carried out without specifying a special form of F_+ and yields

$$\frac{d\Gamma}{dx} = \frac{1}{192\pi^3}|V_{Hl}|^2 G_f^2 m_H^5 |F_+|^2 \Theta(x-2r) \cdot \Theta(1-x+r^2)(x^2-4r^2)^{3/2}. \tag{14}$$

This may be also expressed in terms of a spectrum for the invariant mass $q^2 = (1-x)m_H^2 + m_l^2$ of the lepton pair:

$$\frac{d\Gamma}{dq^2} = \frac{1}{192\pi^3}|V_{Hl}|^2 G_f^2 \frac{1}{m_H^3}|F_+|^2 \Theta(q^2)\Theta((m_H-m_l)^2 - q^2) \cdot [((m_H-m_l)^2 - q^2)((m_H+m_l)^2 - q^2)]^{3/2} \tag{15}$$

which is the rate in the form given e.g. by Körner and Schuler [14]. We defer a discussion of parametrizations for the form factor F_+ to the next section and rather go to the heavy quark limit, in which the factor F_+ becomes a function of x only and $r = r_\Lambda = 0$. Integrating (14) over x we find

$$\Gamma = \frac{1}{192\pi^3}|V_{Hl}|^2 G_f^2 m_H^5 \int_0^1 dx |F_+(x)|^2 x^3. \tag{16}$$

Since in the heavy quark limit the form factors are universal, we may predict the rates for heavy to light transitions by using the data for one of these transitions:

$$\Gamma(H' \to lev) = \frac{|V_{H'l}|^2 m_{H'}^5}{|V_{Hl}|^2 m_H^5}\Gamma(H \to lev). \tag{17}$$

For the transitions $B \to \pi e v$ we may predict the rate in terms of the CKM matrix element V_{bu} by using the data on $D \to \pi e v$. With the input value [19]

$$\Gamma(D^0 \to \pi^- e^+ v) = (6.10^{+3.59}_{-1.87})\cdot 10^{-15}\,\mathrm{GeV} \tag{18}$$

we get as the prediction for $B \to \pi e v$

$$\Gamma(B^0 \to \pi^- e^+ v) = |V_{ub}|^2 (2.27^{+1.32}_{-0.69}) \cdot 10^{-11}\,\mathrm{GeV} \quad \text{from} \quad D^0 \to \pi^- e^+ v. \tag{19}$$

The errors quoted here and in the following is only the error originating from the input data.

Furthermore, we may also use the data on the decay $D \to Kev$ [19] as input. However, in this case we expect large corrections from symmetry breaking effects, since in this case the ratio of the heavy to the light meson mass for the input decay $D \to Kev$ is already $r \approx 0.26$. In order to take some of these effects into account we multiply the rate for $D \to Kev$ by a correction factor κ which takes into account phase space effects and trivial kinematical factors from the matrix element. It is obtained by taking the ratio of the integral of (14) and (16) calculated with constant form factor:

$$\kappa = (1-r^4)(1-8r^2+r^4) - 24r^4 \ln r \tag{20}$$

This correction factor is about $\kappa \approx 0.59$ for $D \to Kev$ while it is practically one for $D \to \pi e v$ and $B \to \pi e v$ and we shall neglect it for the latter two decays. Using the input value from [19]

$$\Gamma(D^0 \to K^- e^+ v) = (5.32 \pm 0.62)\cdot 10^{-14}\,\mathrm{GeV} \tag{21}$$

we obtain including the correction factor for the rate of $B^0 \to \pi^- e^+ v$

$$\Gamma_{\mathrm{corr}}(B^0 \to \pi^- e^+ v) = |V_{ub}|^2 (1.71 \pm 0.12) \cdot 10^{-11}\,\mathrm{GeV} \quad \text{from} \quad D^0 \to K^- e^+ v. \tag{22}$$

We find consistent predictions for $B \to \pi e v$ from both

$D \to \pi e\nu$ and $D \to K e\nu$ once the phase space corrections for $D \to K e\nu$ is taken into account. In the next section we shall discuss uncertainties of the above predictions by including additional $SU(2)_{HF}$ breaking terms in the form factors.

2.3 Parametrization of the form factor

Finally we discuss model assumptions of the form factors. First we discuss parametrizations of the form factors in the heavy quark limit and fix their parameters using experimental data. In the second part of this section we shall use a monopole parametrization with realistic resonance masses to estimate breaking effects of the $SU(2)_{HF}$ as well as of the $SU(3)_F$ symmetries.

In general the matrix element of a hadronic left handed current is parametrized in terms of the form factors (cf. (6))

$$\langle p_1 | j^\mu | p_2 \rangle = f_+(q^2)(p_1^\mu + p_2^\mu) + f_-(q^2)(p_1^\mu - p_2^\mu) \tag{23}$$

where only f_+ contributes to the semileptonic amplitude if the lepton masses are neglected.

There are several phenomenologically successful parametrizations for the form factors appearing in (23) and we shall refer to one of them, namely a parametrization in terms of monopoles and dipoles. This kind of parametrization was recently checked in one of the E691 experiments for the decay $D \to K e\nu$ [21] and seems to agree well with the data.

It is assumed that the q^2 dependence of the form factor $f_+(q^2)$ of (23) is dominated by the nearest resonance with the appropriate quantum numbers. In the case of a semileptonic B to π transition the appropriate resonance state is the B^* and f_+ appearing in (23) is chosen to be a monopole:

$$f_+(q^2) = \mathcal{N} \frac{m_H^{*2}}{m_H^{*2} - q^2} \tag{24}$$

where m_H^* is the mass of the B^* and $\mathcal{N}$ is an unknown normalization constant which we determine from data using the heavy quark limit.

The same idea for a parametrization may be applied for the form factor in semi-leptonic $D \to \pi$ or $D \to K$ transitions. In the former case the resonance is the D^* while in the latter one chooses D_s^*.

In order to relate $f_+(q^2)$ in (23) to F_+ in (6) we rewrite (24) using the kinematic relations between q^2 and $v \cdot p$:

$$\mathcal{N} \frac{m_H^{*2}}{m_H^{*2} - q^2} = \mathcal{N} \frac{m_H^{*2}}{m_H^{*2} - m_H^2 - m_l^2 + 2m_H v \cdot p}$$

$$= \mathcal{N} \left[\frac{m_H^{*2} - m_l^2}{m_H^2} - 1 + x \right]^{-1} \frac{m_H^{*2}}{m_H^2}. \tag{25}$$

Before we discuss symmetry breaking effects in the form factors we consider them in the heavy quark limit. In this limit $r = m_l/m_H$ is zero; furthermore the heavy 0^- states become degenerate with the 1^- resonance due to spin symmetry. Thus m_H^*/m_H equals one which corresponds to $r_\Lambda = 0$. Therefore we find a universal form factor $F_+(x)$ which is

$$F_+(x) = \mathcal{N} \frac{1}{x}. \tag{26}$$

The normalization factor $\mathcal{N}$ may be determined from experimental data on e.g. $D \to \pi e\nu$.

The parametrization (26) of the form factor has a singularity at the edge of the phase space $x = 0$ corresponding to $q^2 = m_H^2$. However, the rate (14) contains an additional factor x^3 in the heavy quark limit from trivial kinematic dependences of the matrix element and the phase space which compensates this singularity completely.

The situation becomes different if a dipole parametrization is chosen. By the same argument used above the universal form factor then reads

$$F_+(x) = \mathcal{N} \frac{1}{x^2}. \tag{27}$$

In this case, however, the phase space factors do not compensate the singularity any more and a cut-off is needed to get a finite total rate. This cut-off naturally emerges if we would keep the small quantity

$$s = \frac{m_H^{*2} - m_H^2 - m_l^2}{m_H^2} \neq 0. \tag{28}$$

Thus the endpoint region depends strongly on terms of the order of $r_\Lambda = \Lambda_{\mathrm{QCD}}/m_H$.

One may now inject the monopole parametrization of the form factor into the expressions for the rates. Using (26) in (17) from the last section one finds in the heavy quark limit

$$\frac{d\Gamma}{dx} = \frac{\mathcal{N}^2}{192\pi^3} |V_{Hl}|^2 G_f^2 m_H^5 |\mathcal{N}|^2 \Theta(x)\Theta(1-x)x \tag{29}$$

and thus the spectrum is a linear function. Note that (29) is the limit $r \to 0$ and $s \to 0$ of the model of Körner and Schuler [14]. Their numerical studies agree well with the linear spectrum (29) showing that the heavy quark limit is a good approximation.

The normalization may be determined by integrating over x and using experimental input for one heavy to light decay. The total rate is given in terms of the normalization constant $\mathcal{N}$ by

$$\Gamma = \frac{\mathcal{N}^2}{384\pi^3} |V_{Hl}|^2 G_f^2 m_H^5. \tag{30}$$

Using the experimental input for the $D \to \pi e\nu$ and the $D \to K e\nu$ decays one may thus determine the normalization. We obtain for the normalization constants in the heavy quark limit

$$\mathcal{N} = 0.69^{+0.18}_{-0.11} \quad \text{from} \quad D \to \pi e\nu \tag{31}$$

$$\mathcal{N}_{\mathrm{corr}} = 0.59 \pm 0.02 \quad \text{from} \quad D \to K e\nu \tag{32}$$

where we again have taken into account the correction factor (20) for the decay $D \to K e\nu$.

Up to here we have worked in the limit of $SU(2)_{HF}$ and $SU(3)_F$, which in particular implies $r = m_l/m_H = 0$ and also $s = 0$. In the case $D^0 \to \pi^- e^+ \nu$ we have $r \approx 0.07$,

$s \approx 0.15$ and the heavy quark limit should be a good approximation, while for $D^0 \to K^- e^+ \nu$ one has $r \approx 0.26$, $s \approx 0.20$ which indicates a breaking of both symmetries. In order to study the effects of the breaking we shall keep r nonzero in (14) and use again a monopole parametrization for the form factor but with realistic masses for the resonances. Thus we have a nonzero value for the quantity s defined in (28), which is different for the various decays due to the differences in the resonance masses. However, we still keep the normalization constants $\mathcal{N}$ universal and determine it from experimental input. In this way we may still compare B decays with the corresponding D decays.

We use for the decays of a heavy meson into a light pseudoscalar meson

$$F_+(q^2) = \mathcal{N} \frac{m_H^{*2}}{m_H^{*2} - q^2} \tag{33}$$

with $m_H^* = 2.01\,\text{GeV}$ for $D \to \pi e \nu$, $m_H^* = 2.11\,\text{GeV}$ for $D \to K e \nu$ and $m_H^* = 5.33\,\text{GeV}$ for $B \to \pi e \nu$. We determine $\mathcal{N}$ from the data on D decays numerically and obtain

$$\mathcal{N} = 0.78^{+0.20}_{-0.13} \quad \text{from} \quad D^0 \to \pi^- e^+ \nu \tag{34}$$

$$\mathcal{N} = 0.74 \pm 0.04 \quad \text{from} \quad D^0 \to K^- e^+ \nu. \tag{35}$$

These two values agree quite well showing that symmetry breaking effects are properly taken into account by the model. Thus we obtain similar predictions for $B^0 \to \pi^- e^+ \nu$ from the two input values:

$$\Gamma(B^0 \to \pi^- e^+ \nu) = |V_{ub}|^2 (2.67^{+1.54}_{-0.82}) \cdot 10^{-11}\,\text{GeV} \quad \text{from} \quad D^0 \to \pi^- e^+ \nu \tag{36}$$

$$\Gamma(B^0 \to \pi^- e^+ \nu) = |V_{ub}|^2 (2.40 \pm 0.27) \cdot 10^{-11}\,\text{GeV} \quad \text{from} \quad D^0 \to K^- e^+ \nu. \tag{37}$$

Comparing (36) and (37) shows that after taking into account the symmetry breaking effects in the decay $D \to K e \nu$ the value of the prediction for $B \to \pi e \nu$ from this input is now close to the value obtained from $D \to \pi e \nu$ as input. We find that the values (36) and (37) did not change much compared to the ones obtained from the heavy quark limit given in the last section in (19) and (22). We conclude that the heavy quark limit allows a rather model independent prediction for $B \to \pi e \nu$ and the value is in the range given by (19), (22), (36) and (37).

These values are higher than all the model predictions [14–18]. Compared to the model of Körner and Schuler [14] the present value is about a factor of four larger than their prediction, which yields the largest value of the commonly used models. This difference is mainly due to the smaller normalization $\mathcal{N}$; most models use the estimate by Wirbel, Stech and Bauer [13] for the normalization at zero momentum transfer, which is $\mathcal{N} \approx 0.33$ for $B \to \pi e \nu$. Another prediction based on PCAC is contained in [18] which gives a similar value for the normalization. The smallest value is predicted by a model of Kramer and Palmer [17] which is about a factor of 100 smaller, since the normalization $\mathcal{N}$ was fixed at maximum momentum transfer using the value from Wirbel, Stech and Bauer [13].

There are also estimates for $B \to \pi e \nu$ based on QCD sum rules [20] which lead to an upper and a lower bound for the rate of $B \to \pi e \nu$

$$|V_{ub}|^2 0.57 \cdot 10^{-11}\,\text{GeV} \leqq \Gamma(B^0 \to \pi^- e^+ \nu) \leqq |V_{ub}|^2 4.4 \cdot 10^{-11}\,\text{GeV} \tag{38}$$

which are consistent with our predictions.

3 The decays of a heavy 0^- into a light 1^- meson

The presentation of the decays into vector mesons proceeds along the lines of the decays into light pseudoscalars. The additional complication due to the longitudinal contribution is discussed in detail.

3.1 The hadronic current for the decays of a heavy 0^- into a light 1^- meson

In this subsection we shall discuss the decay

$$H(P = m_H v) \to l^*(p, \varepsilon) + e(k) + \nu(k'), \quad q = k + k' \tag{39}$$

where H denotes a B or a D meson and l^* a light vector meson with the polarization ε. Using the same arguments as in the case of the decay into a light pseudoscalar we show that in the heavy quark limit the decay (39) is described by only two universal form factors.

The starting point is the trace formalism [6], in which the hadronic matrix element is written as

$$\langle H(v) | \bar{h}_v \gamma_\mu (1 + \gamma_5) l | l^*, p, \varepsilon \rangle = \sqrt{v \cdot p} \cdot \text{Tr}[\bar{H}(v) \gamma_\mu (1 + \gamma_5) \mathcal{M}] \tag{40}$$

The Dirac matrix $\mathcal{M}$ is the most general Lorentz covariant combination of gamma matrices which is linear in ε; it is given in terms of four form factors:

$$\mathcal{M} = (v \cdot \varepsilon) A + \not{\varepsilon} B + \frac{1}{(v \cdot p)} \not{\varepsilon} \not{p} C + i \frac{1}{(v \cdot p)} \gamma_5 \gamma^\mu \varepsilon_{\mu\nu\rho\sigma} \varepsilon^\nu v^\rho p^\sigma D. \tag{41}$$

Inserting for $H(v)$ the expression (2) from Sect. 1 one may evaluate the traces and get the matrix element of the left handed current in terms of four form factors. By the same arguments as in the case of decays into light pseudoscalars the form factors depend only on the variable x which was defined in (4). In order to compare (40) to the form factors defined in [14] we chose appropriate linear combinations of A, B, C and D and omit the form factor proportional to q which does not contribute to the rate for vanishing lepton masses. We find

$$\langle H(v) | \bar{h}_v \gamma_\mu (1 + \gamma_5) l | l^*, p, \varepsilon \rangle = m_H \left\{ \varepsilon_\mu F_1^A + \frac{1}{m_H^2} P_\mu (P \cdot \varepsilon) F_2^A + \frac{i}{m_H^2} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^\nu P^\rho p^\sigma F^V \right\}. \tag{42}$$

As in the case for the pseudoscalar to pseudoscalar transitions the form factors F are dimensionless functions of the variables x defined in (4), r and r_A.

In the heavy quark limit the form factors become universal functions of x and thus are the same for any heavy to light transition involving the left handed current. In general there are three form factors, namely two transverse and one longitudinal. However, in the heavy quark limit the longitudinal rate should vanish. This reduces the number of independent form factors to two. This is discussed in more detail in Sect. 3.3.

3.2 Relations between transverse rates

In this section we discuss the ratios between transverse rates in the heavy quark limit. Corrections due to trivial kinematic effects turn out to be large and are taken into account by a correction factor.

The differential rate for a decay of a heavy meson into a transversely polarized light 1^- meson is given in terms of the two form factors F_1^A and F^V by

$$\frac{d\Gamma_T}{dx} = \frac{G_F^2}{96\pi^3}|V_{Hl}|^2 m_H^5 (1+r^2-x)\sqrt{x^2-4r^2} \cdot [(F_1^A)^2 + \tfrac{1}{4}(x^2-4r^2)(F^V)^2]. \tag{43}$$

In the heavy quark limit this reduces to

$$\frac{d\Gamma_T}{dx} = \frac{G_F^2}{96\pi^3}|V_{Hl}|^2 m_H^5 (1-x)x[(F_1^A(x))^2 + \tfrac{1}{4}x^2(F^V(x))^2]. \tag{44}$$

Since the form factors become universal in the limit, we may relate different decays as we did in the previous section and predictions may be made by putting in data on one heavy to light transition.

The only detailed data published for a heavy 0^- decaying into a light vector meson are the E691 data [21] on the decay mode

$D^+ \to \bar{K}^{*0} e^+ \nu$.

From this measurement the normalization as well as the dependence on the momentum transfer have been extracted; both agree nicely with recent lattice calculations [22]. From these data we get the total rate of [21]

$$\Gamma(D^+ \to \bar{K}^{*0} e^+ \nu) = (2.37 \pm 0.43)\cdot 10^{-14}\,\mathrm{GeV}. \tag{45}$$

Using the ratio of longitudinal to transverse polarization rates

$$\frac{\Gamma_L(D^+ \to \bar{K}^{*0} e^+ \nu)}{\Gamma_T(D^+ \to \bar{K}^{*0} e^+ \nu)} = 1.8^{+0.9}_{-0.7} \tag{46}$$

we obtain the transverse rate

$$\Gamma_T(D^+ \to \bar{K}^{*0} e^+ \nu) = (0.85 \pm 0.48)\cdot 10^{-14}\,\mathrm{GeV}. \tag{47}$$

Employing the universality of the form factors in the heavy quark limit allows us to predict the transverse decay rate for

$B^0 \to \rho^- e^+ \nu$

by integrating (44):

$$\Gamma_T(B^0 \to \rho^- e^+ \nu) = \frac{|V_{ub}|^2}{|V_{cs}|^2}\left(\frac{m_B}{m_D}\right)^5 \Gamma_T(D^+ \to \bar{K}^{*0} e^+ \nu). \tag{48}$$

Note, however, that the heavy quark limit is not well satisfied in the decay $D \to K^* e\nu$, since here the ratio of the masses is $r \approx 0.48$ and thus is not small. As before we first take into account the effects induced by phase space and by trivial kinematical factors by multiplying the results obtained in the heavy quark limit by a correction factor. This factor is obtained by calculating the ratio of the integrals of (43) and (44) with constant form factors

$$F_1^A = \mathcal{N}(1+r) \tag{49}$$

$$F^V = -2\mathcal{N} R^V \frac{1}{1+r}. \tag{50}$$

In the ratio needed for the correction factor the overall normalization $\mathcal{N}$ drops out and the correction factor depends on the parameter R^V.

For the correction factor κ_T of the transverse rates we use $R^V = 2$ as measured by E691 [21] and obtain

$$\kappa_T(D \to K^* e\nu) = 0.106 \tag{51}$$

$$\kappa_T(B \to \rho e\nu) = 0.721. \tag{52}$$

Since κ_T is different from one we also calculate the correction factor for the quark model value $R^V = 1$ [14]

$$\kappa_T(D \to K^* e\nu) = 0.157 \tag{53}$$

$$\kappa_T(B \to \rho e\nu) = 0.824. \tag{54}$$

We conclude that trivial phase space effects do not depend strongly on the value of R^V and thus may be savely taken into account by the above correction factor.

Including the correction factor with the measured value of R^V we find for the transverse rate from (48)

$$\Gamma_T(B^0 \to \rho^- e^+ \nu) = |V_{ub}|^2 (1.08 \pm 0.60)\cdot 10^{-11}\,\mathrm{GeV}. \tag{55}$$

This result still relies on the universality of the form factors; symmetry breaking effects in the form factors are considered in Subsect. 3.4.

3.3 The longitudinal rate

In the heavy quark limit the longitudinal rate vanishes with the light quark mass going to zero, if we assume that the light vector particle is coupled in a gauge invariant way. Therefore it is difficult to make predictions off the limit. Note first that the limit for vanishing light meson mass does not exist for the longitudinal polarization vector. This can be seen by inspecting the longitudinal polarization vector, which is given by

$$\varepsilon_\mu^L = \frac{1}{m_l\sqrt{(p\cdot v)^2 - m_l^2}}((p\cdot v)p_\mu - m_l^2 v_\mu) \to \frac{p_\mu}{m_l} \quad \text{for} \quad m_l \to 0 \tag{56}$$

with v being the velocity of the heavy meson. Thus ε_μ^L diverges in the limit $m_l \to 0$.

We obtain for the differential rate for a decay of a heavy meson into a longitudinally polarized light 1^- meson

$$\frac{d\Gamma_L}{dx}=\frac{G_F^2}{768\pi^3}|V_{Hl}|^2 m_H^5 \frac{1}{r^2}\sqrt{x^2-4r^2}$$
$$\cdot((x-2r^2)F_1^A+\tfrac{1}{2}(x^2-4r^2)F_2^A)^2. \quad (57)$$

The $1/r^2$ dependence arises from the polarization vector. This dependence has to be compensated by cancellations between the form factors F_1^A and F_2^A. Since in the limit $m_l \to 0$ the longitudinal rate must vanish the longitudinal structure function

$$F_L=(x-2r^2)F_1^A(x)+\tfrac{1}{2}(x^2-4r^2)F_2^A(x) \quad (58)$$

must vanish faster than $r=m_l/m_H$:

$$F_L(x,r)\to r^\alpha f_L(x) \quad \text{with} \quad \alpha>1.$$

The large value measured by E691 [21] for the ratio of longitudinal to transverse polarizations for the decay $D^+ \to \bar{K}^{*0}e^+\nu$ (cf. (46)) shows that we are far away from the above limit where we expect Γ_L/Γ_T to be zero.* This is no surprise, since r is about one half. For $B\to\rho e\nu$ we are closer to the limit ($r\approx 0.15$) and we expect a smaller value for Γ_L/Γ_T. We can only conclude that this ratio for $B\to\rho e\nu$ must lie between zero and one. Therefore the total rate $\Gamma=\Gamma_T+\Gamma_L$ will be

$$\Gamma(B^0\to\rho^- e^+\nu)=(1\cdots 2)|V_{ub}|^2\cdot 10^{-11}\,\text{GeV}. \quad (59)$$

3.4 *Parametrization of the form factors*

In this section we shall try to estimate the breaking effects of both the $SU(2)_{HF}$ as well as the $SU(3)_F$ symmetries by using specific parametrizations for the form factors. In particular we shall use realistic resonance masses. In this way heavy flavor symmetry breaking effects in the form factors are taken into account. The universality of the form factors in the heavy quark limit is now reduced to the assumption that the normalizations of the form factors are the same for all decays of heavy mesons into light vector mesons.

In general the hadronic matrix element of the left handed current L_μ between a pseudoscalar and a vector meson is given in terms of four form factors defined by

$$\langle 0^-,P|L_\mu|1^-,p,\varepsilon\rangle$$
$$=f_1^A\varepsilon_\mu+f_2^A P_\mu(P\cdot\varepsilon)+if^V\varepsilon_{\mu\nu\rho\sigma}\varepsilon^\nu P^\rho p^\sigma+f^q(P\cdot\varepsilon)q_\mu \quad (60)$$

where $q=P-p$ and f^q does not contribute to the rate since we neglect lepton masses. We first discuss f_1^A and f^V. We shall parametrize these form factors with monopoles

$$f_1^A(q^2)=(m_H+m_l)\mathcal{N}'\left(\frac{m_H^{*2}}{m_H^{*2}-q^2}\right) \quad (61)$$

$$f^V(q^2)=-\frac{2R_V}{m_H+m_l}\mathcal{N}'\left(\frac{m_H^{*2}}{m_H^{*2}-q^2}\right). \quad (62)$$

The values for the masses are for the decay $D^+\to\bar{K}^{*0}e^+\nu$, $m_H=m_D$, $m_l=m_{K^*}$ and $m_H^*=2.11$ GeV, while for $B^0\to\rho^-e^+\nu$ $m_H=m_B$ we have $m_l=m_\rho$ and $m_H^*=5.33$ GeV. The two form factors are thus given in terms of one common normalization factor $\mathcal{N}'$ and the ratio R_V. We shall discuss two cases for these input parameters. The first case is to use the data of the E691 experiment, which measured these parameters in $D^+\to\bar{K}^{*0}e^+\nu$, and the second is to use the quark model value $R^V=1$ as given in [14] and to fit the normalization to the data on $D\to K^*e\nu$.

In the first case we use as input [21]

$$\mathcal{N}'=0.46\pm0.07 \quad R_V=2.0\pm0.7. \quad (63)$$

With these input values we get for the transverse rate of $B\to\rho e\nu$

$$\Gamma_T(B^0\to\rho^-e^+\nu)=|V_{ub}|^2(3.1\pm1.3)\cdot10^{-11}\,\text{GeV}. \quad (64)$$

In the second case we obtain $\mathcal{N}'=0.50\pm0.07$ and find for the transverse rate of $B\to\rho e\nu$

$$\Gamma_T(B^0\to\rho^-e^+\nu)=|V_{ub}|^2(2.7\pm0.7)\cdot10^{-11}\,\text{GeV}. \quad (65)$$

Note that the first estimate has a larger error simply because the error of the measurement of R^V was included. We find that for the transverse rate the numerical studies are consistent with the value (55) obtained in Sect. 3.2 from the heavy quark limit.

We now turn to the longitudinal rate. Choosing for f_1^A a monopole as in (61) the value $R_2=0$ measured by E691 is not compatible with the heavy quark limit. In fact, the longitudinal rate diverges as r goes to zero. Even with $R_2\neq0$ and a monopole parametrization the heavy quark limit of the rate does not exist. A finite rate in the heavy quark limit is arrived at by choosing f_2^A to be a dipole

$$f_2^A(q^2)=-\frac{2R_2}{m_H+m_l}\mathcal{N}'\left(\frac{m_H^{*2}}{m_H^{*2}-q^2}\right)^2 \quad (66)$$

where R_2 has to be one. This was already suggested by Körner and Schuler [14] using quark model reasoning and the Brodsky–Lepage counting rules [23]. With these choices we obtain the longitudinal rate

$$\Gamma_L(B^0\to\rho^-e^+\nu)=|V_{ub}|^2(1.4\pm0.4)\cdot10^{-11}\,\text{GeV}. \quad (67)$$

This yields a ratio

$$\frac{\Gamma_L(B^0\to\rho^-e^+\nu)}{\Gamma_T(B^0\to\rho^-e^+\nu)}=0.5 \quad (68)$$

consistent with the estimate of the previous section where we argued that it will lie between zero and one. Finally we estimate the total rate

$$\Gamma(B^0\to\rho^-e^+\nu)=(4.1\pm1.1)|V_{ub}|^2\cdot10^{-11}\,\text{GeV} \quad (69)$$

where the error quoted is only the experimental error of the input data.

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* There are, however, recent data from the MARK III collaboration [24] indicating a much smaller value of about one half for the ratio Γ_L/Γ_T

4 Summary and conclusions

We discussed the heavy quark limit for heavy to light meson transitions. In contrast to heavy to heavy transitions the number of independent form factors is not reduced in this limit. We can, however, derive relations between form factors for different heavy to light transitions. We showed that in the limit of infinitely heavy quarks ($m_H \to \infty$) all form factors become only functions of a scaling variable x which ranges between zero and one. Moreover, the form factors are even universal, i.e. they are the same for all heavy to light transitions via a left handed current, if the usual flavor symmetry among the light quarks holds in addition. In the heavy quark limit we can thus relate, for example, processes involving $b \to u$ semileptonic transitions to experimentally measured processes of semileptonic D decays.

Yet the heavy quark limit is not exact. Corrections arise due to breaking terms of both the $m_H \to \infty$ limit and the light flavor symmetry. Quantitatively, we have nonzero values for the parameters $r = m_l/m_H$ and $s = (m_H^{*2} - m_H^2 - m_\pi^2)/m_H^2$ which is of the order of $r_\Lambda = \Lambda_{QCD}/m_H$. We investigated symmetry breaking effects in two steps. First we maintained the universality of the form factors valid in the heavy quark limit but included terms arising from relativistic kinematics. We conclude that the uncertainties of predictions obtained in the heavy quark limit are small for decays in which the result is rather stable. One such predictions is the rate for the decay $B \to \pi e \nu$ based on data on the decay $D \to \pi e \nu$ for which we estimate $\Gamma(B^0 \to \pi^- e^+ \nu) = (1.5 \cdots 4)|V_{ub}|^2 \cdot 10^{-11}$ GeV.

Second we also relaxed the universality of the form factors and investigated their symmetry breaking terms by using specific parametrizations based on nearest resonance approximation. The remainder of the heavy quark limit is the normalization of the form factors at a fixed value of x. We have chosen a value which corresponds to zero momentum transfer. This is also the normalization point of the models [13, 14]. There, however, the normalization for the B decays differs from the one for D decays. In [11, 15, 16] the normalization is taken at maximum momentum transfer. In [11] it was argued that this choice is motivated by the heavy quark limit. If the q^2 dependence is, however, of the nearest resonance type the normalization becomes sensitive to effects of order Λ_{QCD}: such parametrizations diverge at maximum momentum transfer in the heavy quark limit.

Using monopole parametrizations of the form factors we obtained estimates for rates of decays into light pseudoscalars and for the transverse rates of decays into light vector mesons. We find that the resulting estimates for $B \to \pi e \nu$ based on data on $D \to \pi e \nu$ and for $B^0 \to \rho^- e^+ \nu$ from $D^+ \to \bar{K}^{*0} e^+ \nu$ are quite consistent with the estimates we obtain in the heavy limit augmented by the trivial kinematic factors. The prediction for $B \to \pi e \nu$ moreover agrees with the one obtained from $D \to \pi e \nu$. For these decays symmetry breaking effects mostly arise from phase space. The rates can therefore be estimated quite reliably.

Such model independent predictions can, however, not be obtained for the longitudinal rate of the decay into vector mesons. In the heavy quark limit the ratio Γ_L/Γ_T is zero and extrapolations off this limit are necessarily model dependent. The decay $D \to K^* e \nu$ is so far the only measured decay to which we can relate $B \to \rho e \nu$. Therefore we can only conclude that the ratio Γ_L/Γ_T has to lie somewhere between zero and a value somewhat smaller than the value 1.8 measured by E691. An estimation based on nearest resonance approximation of the form factors yields a value of one half. We note that such a parametrization requires a dipole form factor f_2^A with relative normalization $R_2 = 1$ once f_1^A is chosen to be a monopole. Otherwise the longitudinal rate becomes singular in the heavy quark limit. The total rate for $B \to \rho e \nu$ we estimate by including the measured error on $D \to K^* e \nu$ in our prediction for the transverse rate and by allowing Γ_L/Γ_T to vary between zero and one. This gives $\Gamma(B^0 \to \rho^- e^+ \nu) = (2 \cdots 5)|V_{ub}|^2 \cdot 10^{-11}$ GeV.

Up to now only limits are published on exclusive semileptonic $b \to u$ decays [20–22]. Using the latest limits published by ARGUS [25]

$$\Gamma(B^- \to \rho^0 e \bar{\nu}) < 6.14 \cdot 10^{-16}\,\text{GeV} \tag{70}$$

$$\Gamma(\bar{B}^0 \to \pi^+ e \bar{\nu}) < 5.02 \cdot 10^{-16}\,\text{GeV} \tag{71}$$

we get as limits on $|V_{ub}|$

$$|V_{ub}| < 0.0034 - 0.0057 \quad \text{from} \quad B \to \pi e \nu \tag{72}$$

$$|V_{ub}| < 0.005 - 0.008 \quad \text{from} \quad B \to \rho e \nu \tag{73}$$

using (19) and (36), (37) for $B \to \pi e \nu$ and (69) for $B \to \rho e \nu$ where we have taken an additional factor of 2 in the rate into account, since the ARGUS limits are on a decay with a different combination of charges.

The range of the upper bounds is mainly determined by the experimental errors of the input data: further data on both the semileptonic D as well as the noncharmed semileptonic B decays are needed to determine $|V_{ub}|$ using the rather model independent framework of the heavy quark limit.

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