Nuclear Instruments and Methods in Physics Research A306 (1991) 485–499
North-Holland

Fast simulation of electromagnetic showers in the ZEUS calorimeter

J. del Peso *
DESY, Hamburg, Germany

E. Ros
II Inst. für Exp. Phys., Hamburg University, Hamburg, Germany

Received 25 February 1991

We present a fast Monte Carlo algorithm for the generation of electromagnetic showers in the uranium–scintillator sampling calorimeter of the ZEUS experiment. This algorithm includes a simulation of longitudinal and transverse profiles, their fluctuations and the correlation between these fluctuations as well. The tuning of this fast Monte Carlo with data generated with EGS is described and its performance together with some applications is discussed.

1. Introduction

Data analysis in experiments at high energy colliders like LEP or HERA requires the simulation of the detector response via the Monte Carlo technique for large numbers of events. In the case of electromagnetic calorimeters, the existing Monte Carlo programs like EGS [1] or GEANT [2], provide a detailed and reliable simulation of the detector response. However these programs are very time consuming at high energies, since the required CPU time is proportional to the energy of the showering particles. On the other hand there are many studies where a detailed simulation of showers with their full complexity is not needed, but just some approximate description of the energy distribution and fluctuation. In this case, the use of a "fast Monte Carlo" seems to be more efficient and adequate. A fast Monte Carlo aims at the reproduction of global rather than detailed shower properties and is usually obtained by parametrization of showers generated with detailed Monte Carlo programs or directly with experimental data. The shower fluctuations are introduced by smearing of the parameters or by fluctuating the number of points which simulate in the fast Monte Carlo the energy hits in the active volumes of the calorimeter. A certain number of these fast Monte Carlo programs have been written (see for example refs. [3] and [4]). In this article we propose a new program developed for the ZEUS electromagnetic calorimeter [5] with some improvements, as compared to previous work, in the description of the shower transverse profile and in the simulation of shower fluctuations.

2. EGS reference data

In order to tune our fast Monte Carlo program we have used 7 reference samples of 1000 electromagnetic showers generated with EGS. These samples correspond to the following energies: 1, 2, 5, 10, 20, 50 and 100 GeV. The showers were initiated by electrons entering under normal incidence a calorimeter structure consisting of 2.6 mm thick scintillator plates sandwiched with 3.3 mm thick uranium plates. This calorimeter structure is a simplified version of the ZEUS electromagnetic calorimeter. For each generated shower we have recorded the longitudinal and transverse energy deposition in the scintillator layers. These samples provide therefore a detailed information on individual shower fluctuations. The problem related with the influence of EGS cut-off parameters in the results is discussed in appendix A. In other appendices (B and C) we discuss also the simulation of showers initiated by photons or positrons and the simulation of showers as a function of the angle of incidence.

3. Simulation of the longitudinal profile

The longitudinal profile of electromagnetic showers is well described by a function of the following type (see ref. [6] and references therein):

$$f_L(z) = \left(\frac{z}{\lambda_z}\right)^{\alpha_z - 1} \frac{e^{-z/\lambda_z}}{\lambda_z \Gamma(\alpha_z)}$$

* Supported by DAAD.

0168-9002/91/$03.50

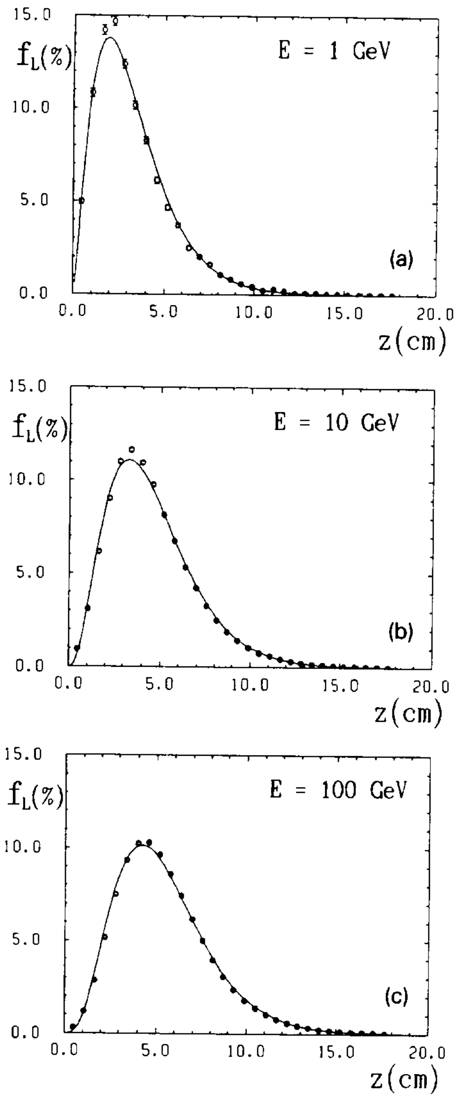

Fig. 1. Average longitudinal profiles for energies of (a) 1 GeV, (b) 10 GeV and (c) 100 GeV. The points are data generated with EGS and the continuous line a fit to the function $f_L(z)$ introduced in section 3.

with

$$\int_0^\infty f_L(z)\, dz = 1,$$

where z is the coordinate along the shower axis and α_z and λ_z are two free parameters which can be obtained by fitting the average shower profile as shown in fig. 1 for 1, 10 and 100 GeV. Such a fitting procedure however, gives no information on the fluctuations of the shower. As pointed out in ref. [4], the average parameters and their fluctuations can be obtained from the moments of the distribution f_L. In general the moment of order n is

$$Z_n = \int_0^\infty z^n f_L(z)\, dz = \lambda_z^n \Gamma(\alpha_z + n)/\Gamma(\alpha_z)$$

and in particular the first moments are

$$Z_1 = \lambda_z \alpha_z \quad \text{and} \quad Z_2 = \lambda_z^2 \alpha_z (\alpha_z + 1),$$

so the parameters are simply given by the equations

$$\alpha_z = \frac{Z_1^2}{Z_2 - Z_1^2} \quad \text{and} \quad \lambda_z = \frac{Z_2 - Z_1^2}{Z_1}.$$

The parameters α_z and λ_z obtained in this way from the 1 GeV reference sample are displayed in figs. 2a and 2b respectively. We observe that they have non-Gaussian distributions, but log α_z and log λ_z, on the contrary, are Gaussian distributed as shown in figs. 3a and 3b. Since Gaussian distributions can be entirely described by just two quantities, the mean and the width, we will use in the following the logarithm of the parameters rather than the parameters themselves. The widths of the distributions inform us of course on the profile fluctuations as explained in the next section.

4. Fluctuations of the longitudinal profile

The longitudinal profiles of three individual showers of 1, 10 and 100 GeV respectively generated with EGS are displayed in fig. 4. We observe that individual

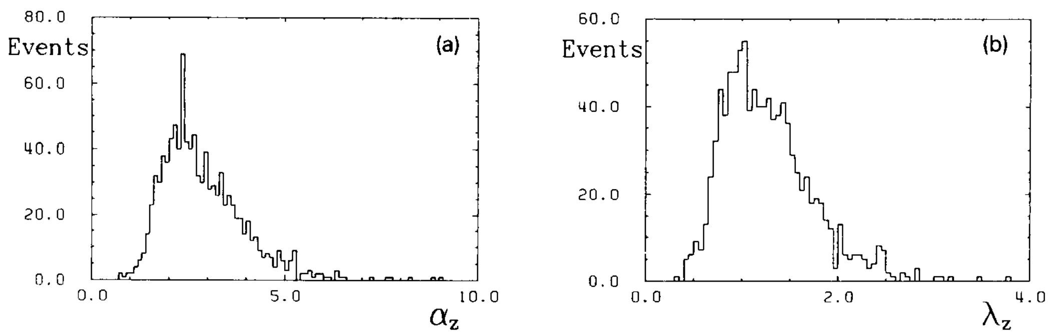

Fig. 2. Distribution of the two shower parameters describing the longitudinal profile (a) α_z and (b) λ_z. These parameters have been calculated from 10 GeV showers generated with EGS.

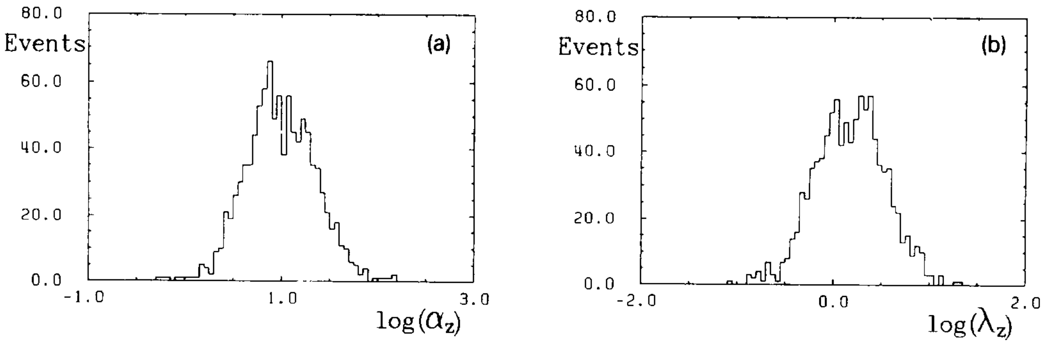

Fig. 3. Distribution of the logarithm of the parameters (a) log α_z and (b) log λ_z at 10 GeV. These distributions are well described by Gaussian functions.

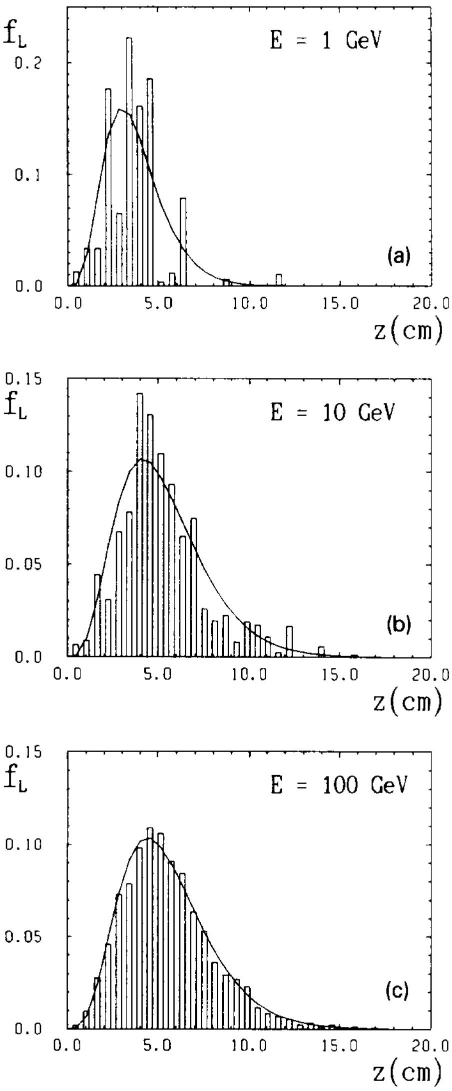

Fig. 4. Longitudinal profiles for individual showers generated with EGS at (a) 1 GeV, (b) 10 GeV and (c) 100 GeV. The function $f_L(z)$ with parameters obtained from the first moments of he EGS profile is overlayed.

shower profiles might deviate considerably from average profiles, especially towards low energies. We observe to kinds of fluctuations:

– global fluctuations, which can be simulated by fluctuating the parameters around their mean values, and

– local fluctuations, which can be simulated by fluctuating the position of the energy hits in the fast Monte Carlo.

In the fast Monte Carlo the longitudinal profile is generated by N random hits following the distribution $f_L(Z)$. Each random hit produces an energy deposition $\epsilon = E/N$, E being the total shower energy. The number of hits N is not an arbitrary parameter, but plays a fundamental role in the description of local fluctuations. In general the fluctuation σ of any Gaussian parameter related to the shower is the sum of two independent fluctuations:

$\sigma = \sigma_i \oplus \sigma_s$ ($\oplus$ means a quadratic sum),

where σ_i (intrinsic fluctuation) is related to global shower fluctuations and is therefore independent of N, whereas σ_s (sampling fluctuation) is related to local shower fluctuations and is proportional to $1/\sqrt{N}$ (see appendix D). These two contributions to the total fluctuation can be extracted from the EGS data by the following technique [#1]:

– for a calorimeter where all scintillator plates are read out we obtain $\sigma_1 = \sigma_i \oplus \sigma_s$, and

– for a calorimeter where only every second plate is read out we obtain $\sigma_2 = \sigma_i \oplus \sqrt{2}\,\sigma_s$, since the number of hits is reduced by a factor 2.

These equations allow the extraction of intrinsic and sampling fluctuations

$$\sigma_s = \sigma_2 \ominus \sigma_1 \quad \text{and} \quad \sigma_i = \sqrt{2}\,\sigma_1 \ominus \sigma_2.$$

[#1] This technique has been employed to determine intrinsic and sampling energy fluctuations of hadronic showers in sampling calorimeters (see ref. [7]).

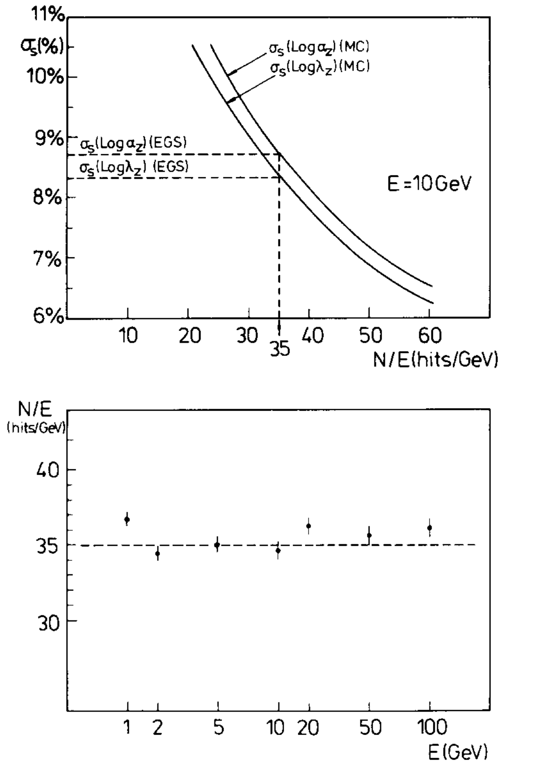

Fig. 5. (a) Statistical fluctuations of the parameters log α_z and log λ_z for 10 GeV showers as a function of the number of hits N used in the fast Monte Carlo. The EGS values can be reproduced with $N = 35$ hits per GeV. (b) The optimum number of hits N in order to reproduce the statistical fluctuations of EGS as a function of the shower energy.

Fig. 5a shows the values of σ_s given by the fast MC [#2] for log α_z and log λ_z at 10 GeV as a function of N. The values of σ_s extracted from the EGS reference data are also indicated in the figure. We conclude that in order to match the reference values, N has to be fixed at a value around 35 hits/GeV. This value is energy independent as shown in fig. 5b. It is also compatible with the number of hits required to reproduce in addition the energy sampling fluctuations which obey the law

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{N}} = \frac{17\%}{\sqrt{E}}$$

in good agreement with the EGS result of $(16.5 \pm 0.5)\%/\sqrt{E}$.

As a conclusion, local fluctuations of the longitudinal profile are reproduced by just fixing the number of Monte Carlo hits N to 35 per GeV. Global fluctuations are reproduced by smearing the logarithmic parameters with Gaussian fluctuations of width σ_i around their mean values. The quantities σ_i can be extracted from the EGS reference data as explained before.

5. Simulation of the transverse profile

The integrated transverse profile of electromagnetic showers can be described by the sum of two exponentials [6] but for our purpose a detailed description of the profile as a function of depth is required. In fig. 6 the average transverse (or radial) profile of 10 GeV showers is displayed at various depths inside the calorimeter, namely for scintillator plates 3, 6, 9, 12, 15 and 20 (we note that the maximum of the shower is around plate 6). These transverse profiles are well described, except for the first plates [#3], by the distribution function

$$f_T(z, r) = \frac{1}{2\lambda_r}\left(\frac{r}{\lambda_r}\right)^{\alpha_r/2-1} \frac{e^{-\sqrt{r/\lambda_r}}}{\lambda_r \Gamma(\alpha_r)},$$

with

$$\int_0^\infty f_T(z, r)\,dr = 1,$$

as can be seen in the fits shown in fig. 6. The parameter λ_r can be kept constant for all plates ($\lambda_r = 0.10$ cm), whereas α_r has the following z dependence, resulting from the fits (see fig. 7):

$$\alpha_r(z) = 1 + \alpha_r' z.$$

The integrated transverse profile is therefore

$$f_T(r) = \int_0^\infty f_T(z, r) f_L(z)\,dz.$$

No analytical expression can be given. For the purpose of our fast Monte Carlo just random numbers generated according to $f_T(z, r)$ are needed and this can easily be achieved (see section 9). The moments of the integrated transverse profile are important because they can be used to determine the corresponding parameters. These moments can be calculated analytically:

$$R_n = \int_0^\infty r^n f_T(r)\,dr = \int_0^\infty f_L(z)\,dz \int_0^\infty r^n f_T(z, r)\,dr$$
$$= \int_0^\infty f_L(z)\,dz\,\lambda_r^n \Gamma(\alpha_r + 2n)/\Gamma(\alpha_r),$$

[#2] This value is the width in the absence of any intrinsic fluctuation in the parameters. It can also be calculated analytically as shown in appendix D.

[#3] Due to back scattering the distributions develop long tails in the first plates. Since these first plates contain anyway a small fraction of the energy, no attempt has been made to reproduce them.

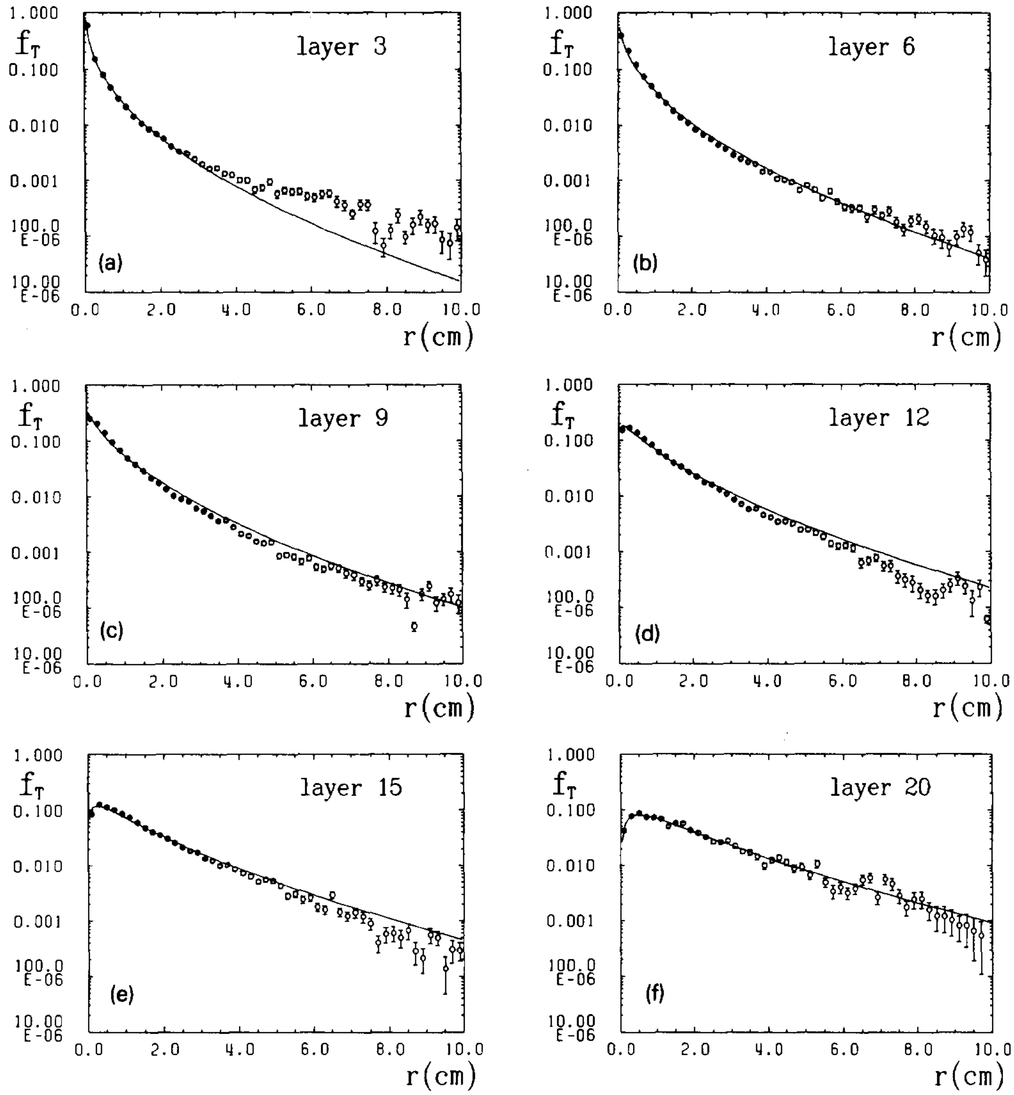

Fig. 6. Average transverse profiles of 10 GeV showers generated with EGS at various depths inside the calorimeter corresponding to the following scintillator layers: (a) layer 3, (b) layer 6 (maximum of the shower), (c) layer 9, (d) layer 12, (e) layer 15 and (f) layer 20. The corresponding fits to the function $f_T(z, r)$ are overlayed.

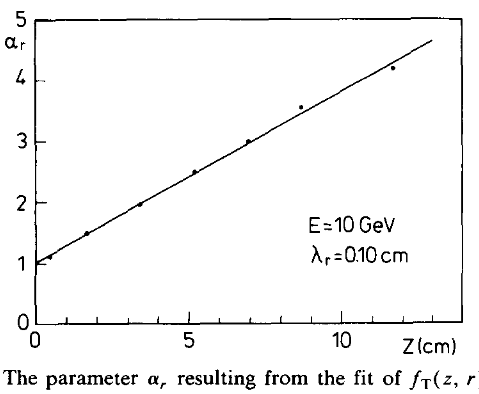

Fig. 7. The parameter α_r resulting from the fit of $f_T(z, r)$ to the 10 GeV radial profiles as a function of depth inside the calorimeter.

and in particular:

$$R_{1/2} = \sqrt{\lambda_r} \int_0^\infty f_L(z) \alpha_r \, dz = \sqrt{\lambda_r} (1 + \alpha_r' Z_1),$$

$$R_1 = \lambda_r \int_0^\infty f_L(z) \alpha_r (\alpha_r + 1) \, dz$$
$$= \lambda_r \left(2 + 3\alpha_r' Z_1 + \alpha_r'^2 Z_2\right),$$

Z_1 and Z_2 being the moments of order 1 and 2 of the longitudinal profile.

6. Fluctuations of the transverse profile

The fluctuations of the transverse profile can be simulated in the same way as those of the longitudinal profile. For each event, the two parameters of the transverse profile, α_r' and λ_r are calculated from the two moments of the integrated transverse profile $R_{1/2}$ and R_1. The distributions of $\log \alpha_r'$ and $\log \lambda_r$ are approximately Gaussian and their widths σ can be decomposed into a statistical contribution σ_s and an intrinsic one σ_i. Since the number of hits N has been fixed by the fluctuations of the longitudinal profile, σ_s is also fixed. The only remaining work is therefore to find the values of σ_i such that the total fluctuations match the EGS reference values.

Concerning the calculation of α_r' and λ_r event by event, we have found more convenient to use the following approximate formulae:

$$\alpha_r' = \frac{1}{Z_1}\frac{2-\epsilon_r}{\epsilon_r - 1} \quad \text{and} \quad \lambda_r = (\epsilon_r - 1)^2 R_{1/2}^2$$

(with $\epsilon_r = R_1/R_{1/2}^2$), which lead to the exact values after some iterations (the exact formulae involve a square root and are more difficult to handle).

This method for the determination of α_r' and λ_r leads to a satisfactory description of the integrated transverse profiles as shown in fig. 8 for energies of 1, 10 and 100 GeV.

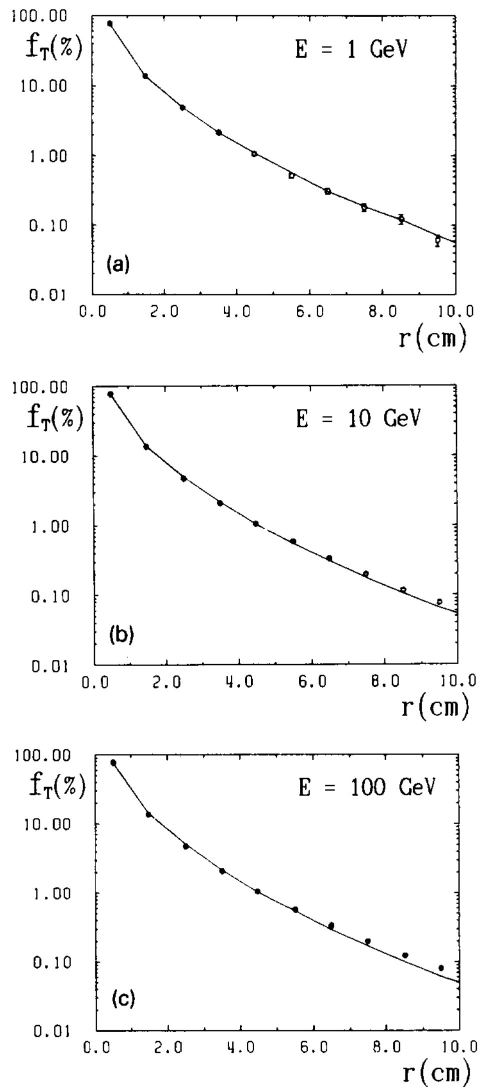

Fig. 8. Integrated transverse profiles of (a) 1 GeV, (b) 10 GeV and (c) 100 GeV showers generated with EGS. The fast Monte Carlo values are overlayed as a continuous line.

7. Correlation between parameters

The four quantities $\log \alpha_z$, $\log \lambda_z$, $\log \alpha_r'$ and $\log \lambda_r$ calculated event by events as described in the previous sections are correlated and this correlation should be taken into account in the fast Monte Carlo. This correlation contains again a statistical contribution and an intrinsic one which has to be extracted (in appendix D we show that the statistical part can be calculated analytically). This extraction can be done via the formula

$$C_{12}\sigma_1\sigma_2 = C_{12}^{i}\sigma_1^{i}\sigma_2^{i} + C_{12}^{s}\sigma_1^{s}\sigma_2^{s}$$

where C_{12} is the correlation coefficient between parameters 1 and 2, and σ_1 and σ_2 the corresponding fluctuations (indices i and s stand for intrinsic and statistical respectively). We have assumed that intrinsic and statistical fluctuations are independent. The values of C_{12}^{s} can be obtained analytically or by use of the fast Monte Carlo in the same way as σ_1^{s} and σ_2^{s}.

In order to reproduce the proper correlation between parameters, the fast Monte Carlo has therefore to generate 4 Gaussian random numbers with a correlation matrix given by C_{lm}^{i} $(l, m = 1,\dots,4)$. This can be achieved via the transformation

$$R_l' = \sum_{m=1}^{4} U_{lm}\sqrt{e_m}\,R_m,$$

where R_m are uncorrelated Gaussian random numbers with rms $= 1$, e_m are the eigenvalues of the matrix C^{i} and U a matrix formed with its eigenvectors as columns. Once the random numbers R' with proper correlations are obtained, the four logarithmic parameters are fluctuated around their mean values by adding the quantities $\sigma_l^{i}R_l'$ $(l = 1,\dots,4)$.

At 10 GeV we obtain the following correlation matrix C^{i}:

$$\begin{pmatrix} 1 & -0.74 & -0.60 & -0.08 \\ & 1 & -0.10 & 0.07 \\ & & 1 & -0.22 \\ & & & 1 \end{pmatrix},$$

where indices $1,\ldots,4$ stand for $\log \alpha_z$, $\log \lambda_z$, $\log \alpha'_r$ and $\log \lambda_r$, respectively. The values obtained at 10 GeV are rather energy independent and can in fact be used at all energies.

8. Simulation of transverse deformations

The average profile is of course symmetric around the shower axis (z axis). This does not exclude however departures from this symmetry for individual showers. In fact, this kind of asymmetries or deformations in the transverse profile are expected simply by statistical fluctuations. In addition, there are also intrinsic deformations which are studied below.

We have considered two types of transverse deformations which are depicted in figs. 9a and 9b. In the first case the shower becomes larger along a given direction and in the second case the shower axis is displaced. The first type of deformation can be simulated by the following transformation applied to all hits (x, y) of the shower:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta_{xy} & \sin\theta_{xy} \\ -\sin\theta_{xy} & \cos\theta_{xy} \end{pmatrix} \begin{pmatrix} 1+\delta_{xy} & 0 \\ 0 & 1-\delta_{xy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

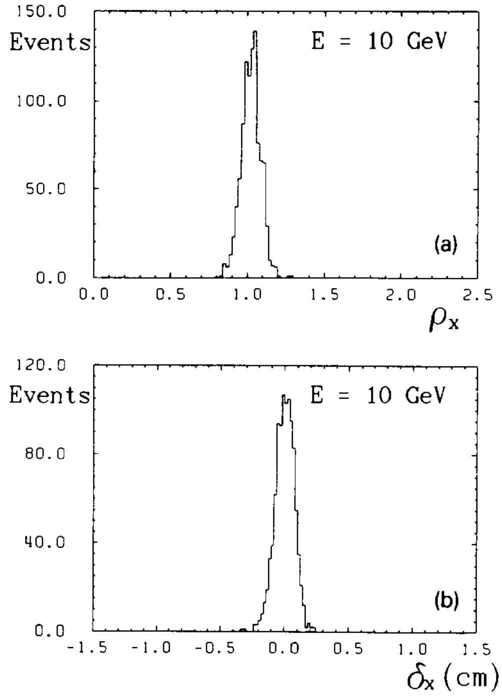

Fig. 10. Distribution for the two parameters (a) ρ_x and (b) δ_x describing the shower transverse deformations. These distributions have been obtained with 10 GeV showers generated with EGS.

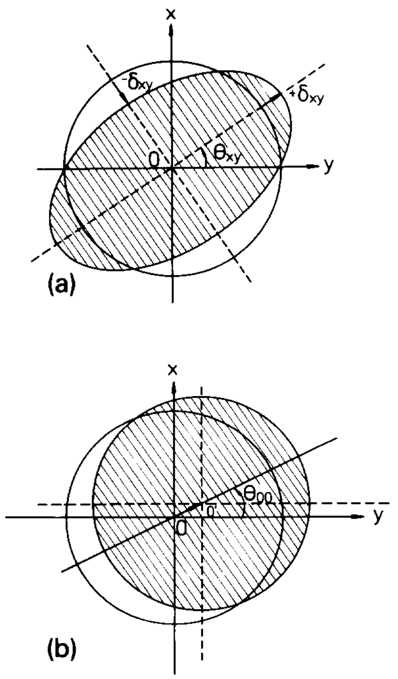

Fig. 9. Drawings illustrating the two kinds of shower transverse deformations considered in section 8: (a) the shower becomes larger along a given direction and (b) the shower axis gets displaced along a given direction.

where θ_{xy} gives the direction of the deformation and δ_{xy} its magnitude. The second type of deformation can be simulated by

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} z\delta_{00}\cos\theta_{00} \\ z\delta_{00}\sin\theta_{00} \end{pmatrix},$$

where θ_{00} gives the direction of the shift and $z\delta_{00}$ its magnitude (z is the coordinate of the hit along the shower axis). We have assumed that this shift is proportional to z, so δ_{00} is in fact an angle of rotation for the shower axis.

For the purpose of the fast Monte Carlo, θ_{xy} and θ_{00} are random numbers uniformly distributed in the interval $[0, 2\pi]$ whereas δ_{xy} and δ_{00} are Gaussian random numbers centered at 0 and with rms tuned to the reference data. The quantities used to do this tuning are

$$\rho_x = \frac{2}{\pi}\sum_i \epsilon_i |x_i| \Big/ \sum_i \epsilon_i r_i \quad \text{and} \quad \delta_x = \frac{1}{E}\sum_i \epsilon_i x_i,$$

where x_i and r_i are hit coordinates and ϵ_i hit energy depositions. The sum run over all shower hits. The quantities ρ_x and δ_x (and the corresponding ones along the y axis) take the values 1 and 0 respectively in the absence of any shower deformation. However, as figs. 10a and 10b show for the 10 GeV reference sample, they are Gaussian distributed with nonvanishing widths $\sigma(\rho_x)$ and $\sigma(\delta_x)$ respectively. These widths can be re-

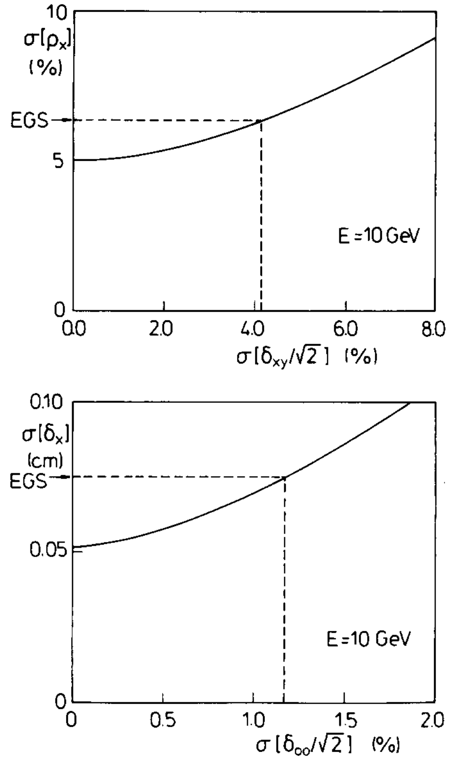

Fig. 11. The width of (a) ρ_x and (b) δ_x as a function of the Monte Carlo parameters $\sigma(\delta_{xy})$ and $\sigma(\delta_{00})$ at 10 GeV. The EGS result can be matched with proper values for the Monte Carlo parameters.

produced in the fast Monte Carlo with proper values for $\sigma(\delta_{xy})$ and $\sigma(\delta_{00})$ as shown in figs. 11a and 11b. We note that even in the absence of any shower intrinsic deformation (that is for $\sigma(\delta_{xy}) = \sigma(\delta_{00}) = 0$) the quantities ρ_x and δ_x fluctuate, this effect being due to fluctuations in the position of the N hits inside the shower (statistical fluctuation).

9. Energy dependence and generation of parameters

In order to generate fast MC showers as described in the previous sections, 16 quantities are needed, in addition to the shower energy E and the number of hits N. These 16 quantities are the four average values of the logarithmic parameters describing the longitudinal and radial profiles ($\log \alpha_z$, $\log \lambda_z$, $\log \alpha'_r$ and $\log \lambda_r$), their intrinsic fluctuations (four parameters) and correlations (6 parameters). The way to extract all these quantities from the EGS reference samples has been described in sections 4 to 7. The last two quantities needed are the widths $\sigma(\delta_{xy})$ and $\sigma(\delta_{00})$ describing shower transverse deformations introduced in section 8.

As shown in figs. 12 to 14, these parameters have a

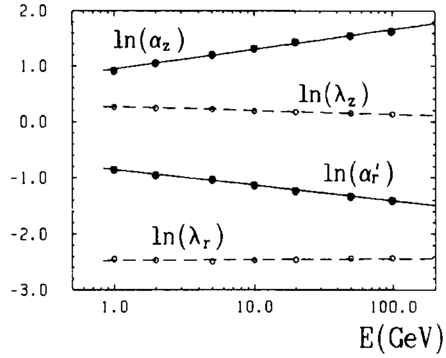

Fig. 12. Average values of the four logarithmic parameters $\log \alpha_z$, $\log \lambda_z$, $\log \alpha'_r$ and $\log \lambda_r$ describing the shower profiles as a function of energy. A fit to a linear function of $\log E$ is overlayed.

simple dependence [#4] on $\log E$ which can be used to interpolate values in order to generate showers at any energy in the interval of 1 to 100 GeV. The six correlation parameters are energy independent as already commented in section 7.

Concerning the generation of random numbers, the fast MC requires for each shower the generation of seven Gaussian random numbers used to smear the number of hits N (with a width of $\sqrt{N}$), the four logarithmic profile parameters and the two deformation parameters, and two uniform random numbers which define the angular directions of the shower deformations. The coordinates of each shower hit are obtained with random numbers which generate the radial distributions $f_L(z)$ and $f_T(z, r)$ introduced in sections 3 and 5 respectively. We have used for this purpose the random generator Rangam of the CERNLIB [8]:

$$z = \lambda_z \text{Rangam}(\alpha_z) \quad \text{and} \quad r = \lambda_r [\text{Rangam}(1 + \alpha'_r z)]^2.$$

Finally the x and y hit coordinates are generated with another uniform random number ϕ in $[0, 2\pi]$:

$$x = r \cos \phi \quad \text{and} \quad y = r \sin \phi.$$

The slower radial deformations are then applied to x and y with help of the transformations introduced in section 8.

10. Performance of the fast Monte Carlo

As shown in figs. 1, 6 and 8, the fast Monte Carlo is able to reproduce well enough all average profile distri-

[#4] In the case of the average values, the $\log E$ dependence is clear. In the case of fluctuations, a dependence of the kind $a/\sqrt{E} + b$ cannot be excluded. We have used the $\log E$ dependence for all quantities except $\sigma(\delta_{00})$ where the $\sqrt{E}$ dependence gives clearly a better description of the data.

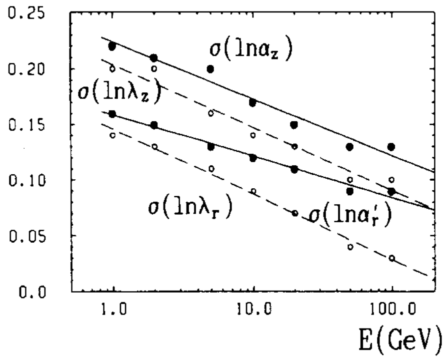

Fig. 13. Intrinsic fluctuations of the four logarithmic parameters $\log \alpha_z$, $\log \lambda_z$, $\log \alpha_r'$ and $\log \lambda_r$ describing the shower profiles as a function of energy. A fit to a linear function of $\log E$ is overlayed.

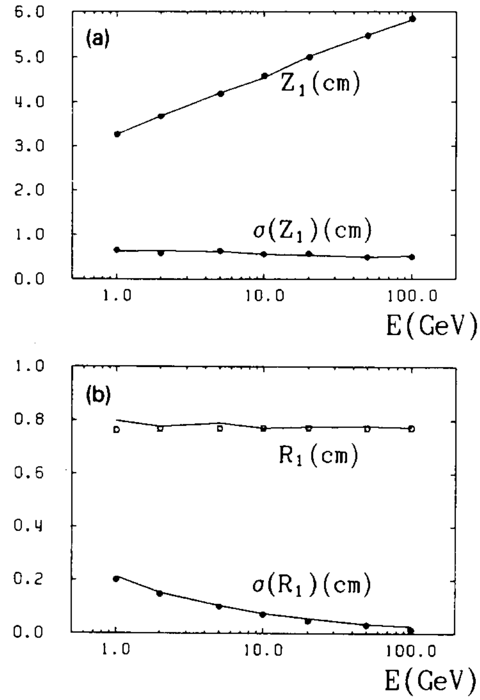

Fig. 15. (a) Average value and fluctuation of the first moment of the longitudinal shower profile Z_1 as a function of energy. The points are EGS data and the continuous line is the fast Monte Carlo result. (b) Average value and fluctuation of the first moment of the transverse shower profile R_1 as a function of energy. The points are EGS data and the continuous line is the fast Monte Carlo result.

butions: longitudinal shower profiles, integrated radial profiles and radial profiles at various depths inside the calorimeter as well. Concerning shower fluctuations, the fast Monte Carlo can also reproduce those where the shower as a whole is concerned. For example, the fluctuations $\sigma(Z_1)$ an $\sigma(R_1)$ of the first longitudinal and radial distribution moments are well reproduced as shown in figs. 15a and 15b respectively and the correlation between these fluctuations as well. It is not expected that the fast Monte Carlo can properly reproduce shower fluctuations in very localized regions. One can see however in fig. 16 that the energy distributions in various scintillator layers generated with EGS are relatively well matched by the fast Monte Carlo distributions.

Obviously the fast Monte Carlo cannot reproduce the real shower development in inhomogeneous regions of a calorimeter like boundaries between modules or cracks between calorimeter components (e.g. barrel and end-cap parts) where significant distorsions are expected. This problem is certainly a weakness of he fast Monte Carlo approach.

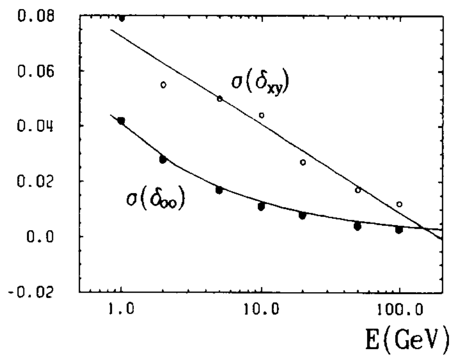

Fig. 14. Intrinsic fluctuations of the two parameters δ_{xy} and δ_{00} describing the shower transverse deformations as a function of energy. Fits to linear functions of $\log E$ and $1/\sqrt{E}$ respectively are overlayed.

Concerning CPU time considerations, the fast Monte Carlo needs about 0.002 s to generate a 1 GeV shower in the DESY IBM-3090. As a comparison EGS4 with the energy cut-offs applied to generate the reference samples needs 2 s, that is 1000 times more (the CPU time needed by EGS strongly depends on these cut-offs). Otherwise, for both EGS and the fast Monte Carlo, the CPU time required is proportional to the shower energy.

Concerning the energy range of validity of the fast Monte Carlo, the parameters can be extrapolated to any energy outside the range of 1 to 100 GeV. There is however no garantee that the results obtained in which way reproduce the reality. Since the elementary energy deposition is about 30 MeV (1/35), showers with energy below 30 MeV cannot be generated. At very high energies, around 1000 GeV, a breakdown of the model is also expected since all intrinsic widths become negative (see fig. 13).

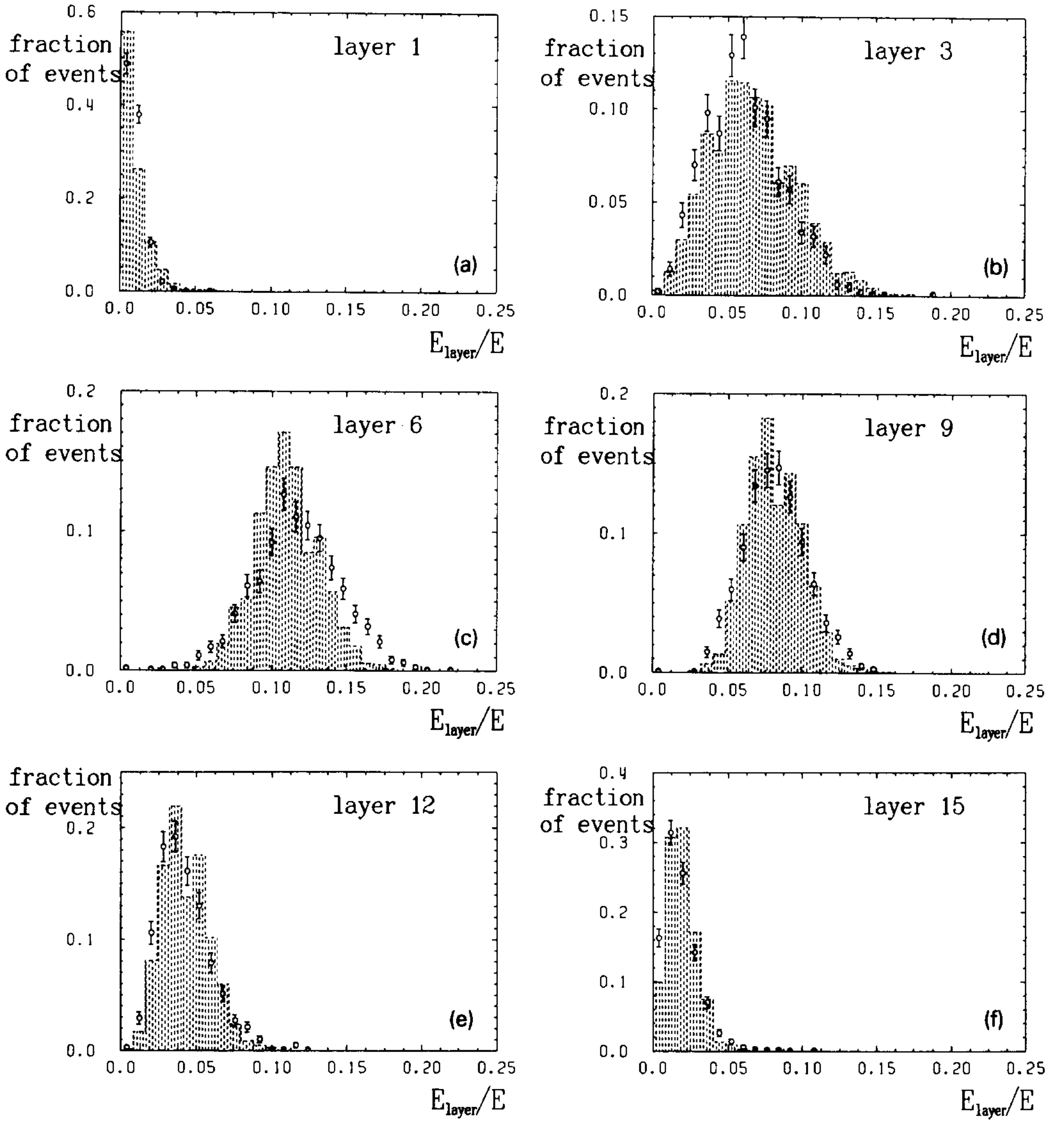

Fig. 16. Energy distribution for the energy deposited at various depths inside the calorimeter by 10 GeV showers, corresponding to the following scintillator layers: (a) layer 1, (b) layer 3, (c) layer 6 (maximum of the shower), (d) layer 9, (e) layer 12, (f) layer 15. The points are EGS data and the distributions filled with dots are the fast Monte Carlo result.

11. Some applications

In this section we consider briefly some applications of the fast Monte Carlo, in order to test its power but also to learn its limitations.

11.1. Shower containment and energy resolution

In fig. 17a we show the fraction of energy contained in two hypothetical calorimeters with total lengths of 15 and $20X_0$ and the corresponding resolution relative to infinitely long calorimeters. We observe that the result strongly depends on the energy and that the resolution degrades faster than the containment at high energies. At 50 GeV, for example, a $20X_0$ long calorimeter would contain 98% of the energy, but its resolution would be 15% worse than for a calorimeter with complete containment. The fast Monte Carlo reproduces fairly well the energy containment and reasonably well the energy resolution. In fig. 17b the same exercise has been worked out for calorimeters with incomplete containment in the transverse direction ($1r_M$ and $2r_M$, r_M being the Molière radius, 2 cm in our case). We observe a rather energy independent EGS result: 90% containment for $1r_M$ (by

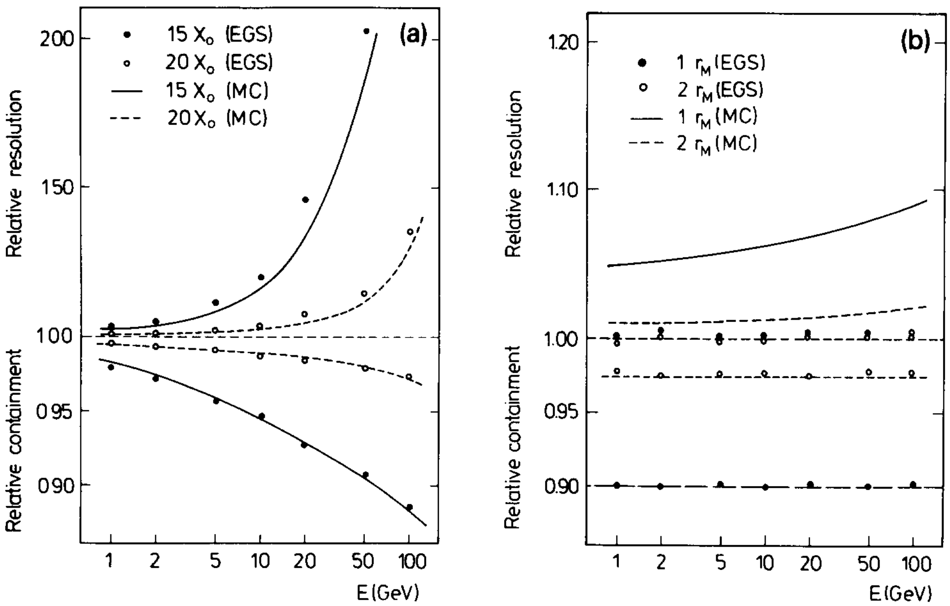

Fig. 17. (a) Energy resolution (upper part of the plot) and containment (lower part) relative to an infinitely long calorimeter, as a function of energy. Filled and open dots correspond to EGS data for 15 X_0 and 20 X_0 long calorimeters respectively. The lines correspond to the fast Monte Carlo result. (b) Energy resolution (upper part of the plot) and containment (lower part) relative to a calorimeter infinitely large in the transverse direction, as a function of energy. Filled and open dots correspond to EGS data for 1 r_M and 2 r_M large calorimeters respectively (r_M = 2 cm is the Molière radius). The lines correspond to the fast Monte Carlo result.

definition of r_M) and about 97.5% for $2r_M$. This result is well reproduced by the fast Monte Carlo. EGS predicts however no degradation of the energy resolution by incomplete transverse containment and the fast Monte Carlo fails to reproduce this result.

11.2. Matter in front of the calorimeter and energy resolution

If dead material is located in front of the calorimeter, both an incomplete energy containment and a degradation of the energy resolution are expected. Fig. 18 shows the result obtained with EGS at 1 and 5 GeV for various thicknesses of aluminium as dead material. We observe that the effect of this dead material is more important at low energies. In the fast Monte Carlo it is not possible to generate showers in two different media, the aluminum first and then the calorimeter. It is possible however to define an "equivalent aluminum length" to propagate the shower before entering the calorimeter. The optimum equivalent length to reproduce the EGS result is in our case 0.8 cm per X_0 of aluminum. This value yields a satisfactory fast Monte Carlo result as shown in fig. 18.

11.3. Position resolution under normal incidence

The electromagnetic part of the ZEUS calorimeter is segmented in the transverse direction into towers with dimensions $D_x = 20$ cm and $D_y = 5$ cm. The right and left side of each tower in the x direction are read out by wavelength shifter plates and photomultipliers. The shower impact position is reconstructed according to the formulae (see ref. [9] for more details)

$$x = \frac{\lambda}{2}\log\frac{E_R}{E_L} \quad \text{and} \quad y = \frac{1}{E}\sum_i E_i y_i,$$

where λ is the attenuation length of scintillator, E_R and E_L the energies measured on the right and left side of the tower, E the total energy, E_i the energy in tower number i and y_i the coordinate of its center.

In the limit of infinite photostatistics ($\sigma_{ph} \to 0$) and infinite segmentation ($D_y \to 0$), the position resolution would be simply

$$\sigma_x = \sigma_y = \sigma(\delta_x), \quad \text{where } \sigma(\delta_x) = \frac{0.24}{\sqrt{E}} \text{ [cm]}.$$

This quantity $\sigma(\delta_x)$ has already been introduced in section 8. In practice the resolution along the x direction is dominated by photostatistics [9]:

$$\sigma_x = \lambda\sigma_{ph} = \frac{5.4}{\sqrt{E}} \text{ [cm]}$$

and along the y direction by the calorimeter segmentation D_y. Fig. 19 shows the position resolution σ_y as a function of D_y. For very granular calorimeters, where the segmentation is comparable to typical shower trans-

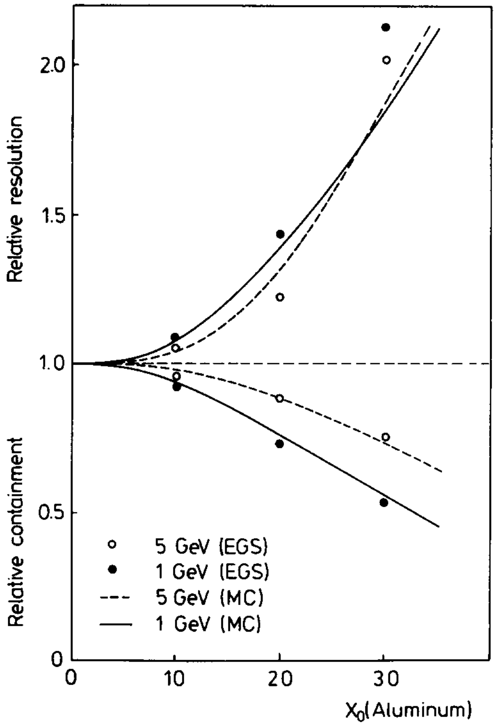

Fig. 18. Energy resolution (upper part of the plot) and containment (lower part) relative to a calorimeter without any dead matter in front, as a function of the equivalent-aluminum distance traversed by showers before entering the calorimeter. Filled and open dots correspond to EGS data for 1 GeV and 5 GeV respectively. The lines correspond to the fast Monte Carlo result.

verse dimensions (about 0.5 cm), the position resolution is again dominated by shower fluctuations and the fast Monte Carlo can account for this result.

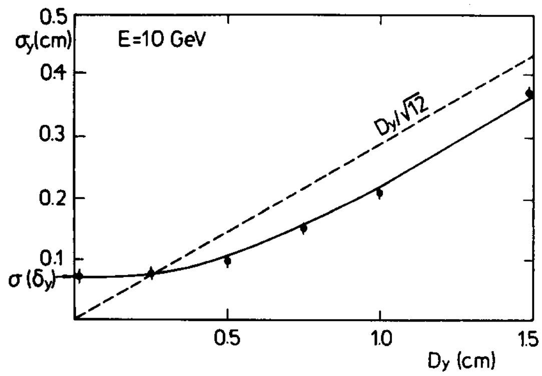

Fig. 19. Position resolution in the y direction, σ_y, as a function of calorimeter segmentation D_y for 10 GeV showers under normal incidence. The points are EGS data and the continuous line the fast Monte Carlo result.

12. Summary

We have constructed a fast Monte Carlo algorithm to simulate electromagnetic showers in the ZEUS uranium–scintillator sandwich calorimeter. The longitudinal shower profile is parametrized by the distribution function

$$f_L(z) = \left(\frac{z}{\lambda_z}\right)^{\alpha_z - 1} \frac{e^{-z/\lambda_z}}{\lambda_z \Gamma(\alpha_z)},$$

z being the coordinate along the shower axis. The transverse profile at a depth z inside the calorimeter is parametrized by the distribution function

$$f_T(z, r) = \frac{1}{2\lambda_r}\left(\frac{r}{\lambda_r}\right)^{\alpha_r/2 - 1} \frac{e^{-\sqrt{r/\lambda_r}}}{\lambda_r \Gamma(\alpha_r)},$$

r being the radial coordinate and $\alpha_r(z) = 1 + \alpha_r' z$. We have used electron showers generated with EGS in the energy range of 1 to 100 GeV to calculate event by event the four shower parameters α_z, λ_z, α_r' and λ_z. The logarithms of the parameters are Gaussian distributions with mean and width linearly depending on the logarithm of the shower energy. Shower fluctuations are reproduced by a combination of smearing in the shower parameters (intrinsic fluctuations) and fluctuations in the position of the energy hits generated by the fast Monte Carlo (statistical fluctuations). The optimum number of hits in order to simulate the fluctuations observed in EGS has been found to be 35 per GeV of incident energy. The fast Monte Carlo includes also the correlation between the various shower parameters, as observed in EGS. Showers initiated by photons or by particles entering the calorimeter under an angle can also be simulated. This fast Monte Carlo provides a satisfactory description of average shower distributions and fluctuations which involve the shower as a whole. Very local effects are not expected to be well reproduced. Finally, the algorithm is about 1000 faster than programs with detailed shower tracking like EGS.

Acknowledgement

We would like to thank Prof. E. Lohrmann for useful comments.

Appendix A

Cut-off dependence of EGS calculations

The generation of showers with EGS requires the use of cut-off energies for low energetic electrons and photons. It is well known that these cut-off parameters have an influence on the result of the calculations (see for example a discussion for the case of the energy resolution in ref. [10]). These cut-off parameters should be chosen low enough to produce accurate results, but still high enough to allow the generation of a significant number of events. In fig. 20 we display as an example the dependence of log α_z on the electron cut-off ϵ_e. We have found that a value $\epsilon_e = 1.4$ MeV is adequate for our analysis. The photon cut-off has been set at $\epsilon_\gamma = 0.01$ MeV and the step parameters at the recommended values (see ref. [10] for more details).

Appendix B

Photon and positron showers

The fast Monte Carlo presented here has been tuned to electron showers. We have also generated with EGS photon and positron showers. In the case of positrons, we have not found any difference with electron showers, at least down to the lower limit of our energy interval (1 GeV). In the case of photons, we observe some significant differences, especially a shift in the longitudinal profile of the showers (see figs. 21a and 21b). These differences appear to be rather energy independent. In first approximation, photon showers can be generated by shifting the starting point of the shower by $7/9X_0$ (corresponding to 0.44 cm) which is the conversion length for photons into electron–positron pairs.

Appendix C

Shower profiles for various angles of incidence

The tuning of the Monte Carlo has been performed for particles under normal incidence. For particles penetrating the calorimeter with some inclination, various effects are expected like deformations in the shower development and changes in the energy resolution as well. We have generated with EGS samples of 10 GeV electron showers entering the calorimeter under angles of 20°, 40° and 60°. We do not observe any significant variation in the energy resolution, at least up to 40°. We also find a satisfactory agreement between EGS and MC profiles by simply rotating the MC shower hits according to the angle of incidence (see figs. 22a to 22d). Therefore, angular effects can be neglected in first approximation.

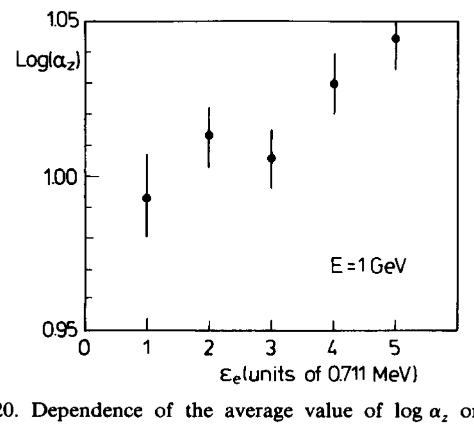

Fig. 20. Dependence of the average value of log α_z on the electron cut-off energy ϵ_e applied to generate showers with EGS. The study has been performed for 1 GeV showers.

Appendix D

Statistical fluctuations in the fast Monte Carlo

The shower generation technique used in the fast Monte Carlo consists in the generation of N hits of equal energy ϵ. For $N \to \infty$, the shower profiles are exactly reproduced, but for any finite value of N, a fluctuation is introduced in the profiles and therefore also in the parameters of the fast Monte Carlo calculated from the first moments as described in sections 3 and 6. This fluctuation, which has been called statistical, can be obtained by analytical calculations.

The first moments of the longitudinal shower profile are

$$Z_1 = \frac{1}{N}\sum_i n_i z_i \quad \text{and} \quad Z_2 = \frac{1}{N}\sum_i n_i z_i^2,$$

where n_i is the number of hits in a bin centered around z_i. The fluctations of n_i, which are Poissonian and uncorrelated (in particular $\langle \delta n_i \delta n_j \rangle = n_i \delta_{ij}$) generate the following fluctuations in the moments:

$$\delta Z_1 = \frac{1}{N}\sum_i \delta n_i (z_i - Z_1)$$

and

$$\delta Z_2 = \frac{1}{N}\sum_i \delta n_i (z_i^2 - Z_2).$$

The widths of these fluctuations are obtained by averaging over many events:

$$\langle (\delta Z_1)^2 \rangle = \frac{1}{N^2} - \sum_i n_i (z_i - Z_1)^2 = \frac{1}{N}(Z_2 - Z_1^2)$$
$$= \frac{\lambda^2}{N}\alpha,$$
$$\langle (\delta Z_2)^2 \rangle = \frac{1}{N^2}\sum_i n_i (z_i^2 - Z_2)^2 = \frac{1}{N}(Z_4 - Z_2^2)$$
$$= \frac{\lambda^4}{N}\alpha(\alpha+1)(4\alpha+6),$$

where α and λ are the average parameters used to

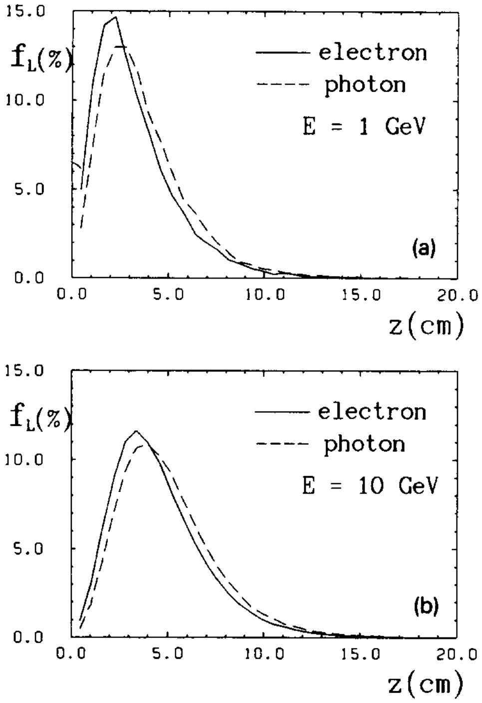

Fig. 21. Longitudinal profiles obtained with EGS for electrons (continuous line) and photons (broken line) for energies of (a) 1 GeV and (b) 10 GeV.

generate the longitudinal profile. We have used the relation

$$Z_n = \lambda^n \alpha(\alpha+1)\cdots(\alpha+n-1).$$

The correlation between the fluctuations of Z_1 and Z_2 is

$$\langle \delta Z_1 \delta Z_2 \rangle = \frac{1}{N}(Z_3 - Z_1 Z_2) = \frac{\lambda^3}{N} 2\alpha(\alpha+1).$$

The fluctuations in the moments induce fluctuations in the Monte Carlo parameters which are calculated as follows:

$$\alpha_z = \frac{Z_1^2}{Z_2 - Z_1^2} \quad \text{and} \quad \lambda_z = \frac{Z_2 - Z_1^2}{Z_1}.$$

These fluctuations are

$$\delta \log \alpha_z = 2(1+\alpha)\frac{\delta Z_1}{Z_1} - \alpha \frac{\delta Z_2}{Z_1^2}$$

and

$$\delta \log \lambda_z = -(1+2\alpha)\frac{\delta Z_1}{Z_1} + \alpha \frac{\delta Z_2}{Z_1^2}.$$

After some algebra we obtain

$$\langle(\delta \log \alpha_z)^2\rangle = \frac{2}{N}\left(\frac{1+\alpha}{\alpha}\right)$$

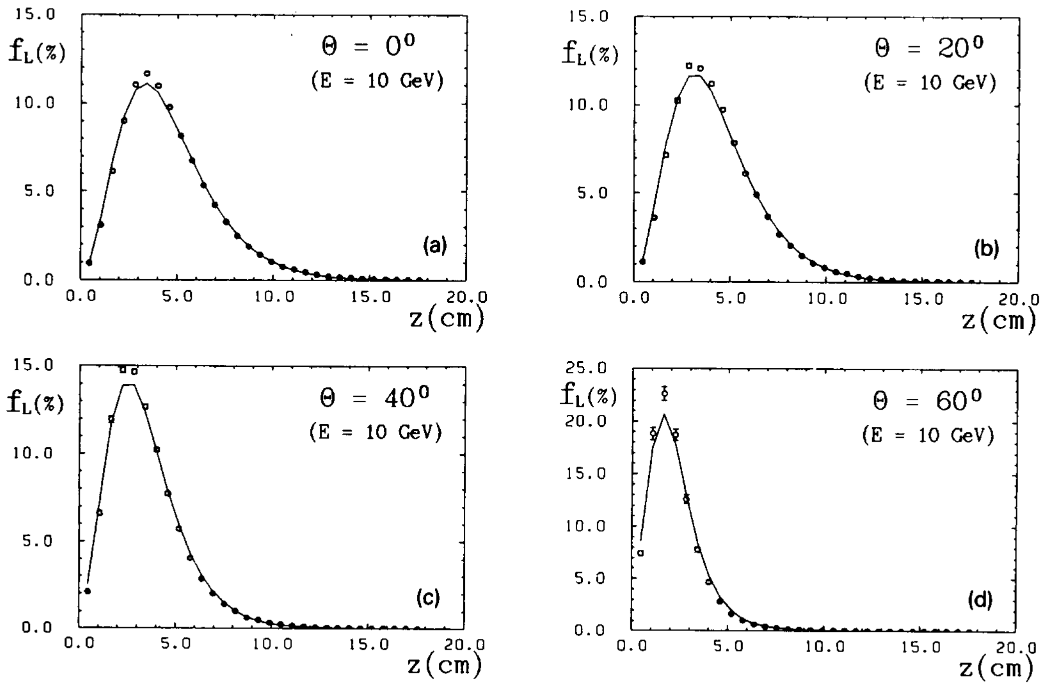

Fig. 22. Longitudinal profiles obtained at 10 GeV for electrons entering the calorimeter with the following angles relative to normal incidence: (a) $\theta = 0°$, (b) $\theta = 20°$, (c) $\theta = 40°$ and (d) $\theta = 60°$. The points correspond to EGS data and the continuous line to the fast Monte Carlo result.

and

$$\langle(\delta \log \lambda_z)^2\rangle = \frac{2}{N}\left(\frac{3/2+\alpha}{\alpha}\right),$$

or equivalently,

$$\sigma(\log \alpha_z) = \sqrt{\frac{2}{N}\left(\frac{1+\alpha}{\alpha}\right)}$$

and

$$\sigma(\log \lambda_z) = \sqrt{\frac{2}{N}\left(\frac{3/2+\alpha}{\alpha}\right)}.$$

These formulae show that the fluctuations of the logarithmic parameters are proportional to $1/\sqrt{N}$ as expected (see also fig. 5a). It is also possible to calculate the correlation between parameters

$$\langle(\delta \log \alpha_z)(\delta \log \lambda_z)\rangle = -\frac{2}{N}\left(\frac{1+\alpha}{\alpha}\right),$$

and therefore the correlation coefficient is

$$C_{12} = \frac{\langle(\delta \log \alpha_z)(\delta \log \lambda_z)\rangle}{\sigma(\log \alpha_z)\sigma(\log \lambda_z)} = -\sqrt{\left(\frac{1+\alpha}{3/2+\alpha}\right)},$$

which is independent of N.

References

[1] W.R. Nelson, H. Hirayama and D.O. Rogers, SLAC-265 (1985).
[2] R. Brun et al., CERN DD/EE/84-1 (1986).
[3] A. de Angelis and P.A. Palazzi, Nucl. Instr. and Meth. A281 (1989) 93.
[4] G. Grindhammer, M. Rudowicz and S. Peters, Nucl. Instr. and Meth. A290 (1990) 469.
[5] The ZEUS Detector, Technical Proposal, DESY (March 1986);
The ZEUS Detector, Status Report 1989, DESY (March 1989).
[6] Review of Particle Properties, Phys. Lett. B204 (1988) 66.
[7] G. Drews et al., Nucl. Instr. and Meth. A290 (1990) 335.
[8] CERN Program Library, ed. T. Lindelof, function code V109 (1986).
[9] U. Behrens et al., Nucl. Instr. and Meth. A289 (1990) 115.
[10] J. del Peso and E. Ros, Nucl. Instr. and Meth. A276 (1989) 456.